stants, $\epsilon^{\mu}\epsilon^{\mu'}$ times

$$2C_{\alpha bc}C_{\beta bc}\int k_\mu(k_{\mu'}-p_{\mu'})\Delta^{\nu\nu'}(k)\Delta^{\nu\nu'}(k-p)d^4k$$
$$+C_{b\alpha c}C_{c\beta b}p_\nu p_{\nu'}\int \Delta^{\nu\nu'}(k)\Delta^{\mu\mu'}(k-p)d^4k$$
$$+C_{a\alpha c}C_{c\beta a}p_\nu p_{\nu'}\int \Delta^{\nu\mu'}(k)\Delta^{\mu\nu'}(k-p)d^4k$$
$$+4C_{\alpha bc}C_{b\beta c}p_{\nu'}\int (k_\mu-p_\mu)\Delta^{\nu\nu'}(k)\Delta^{\nu\mu'}(k-p)d^4k, \quad \text{(A1)}$$

where $\epsilon(p)\cdot p=\epsilon_\mu p_\mu=0$ and

$$\Delta_{\mu\nu}(k)=\frac{g_{\mu\nu}-k_\mu k_\nu/m^2}{k^2-m^2+i\epsilon} \quad \text{(A2)}$$

is the vector-meson propagator. We first note that in general $T_{\mu\mu'}(p^2)=g_{\mu\mu'}I(p^2)+p_\mu p_{\mu'}J(p^2)$, and hence $-T_{ii}(p^2)=3I(p^2)-\mathbf{p}^2J(p^2)$, which we then evaluate in the rest frame $\mathbf{p}=0$. The absorptive part of this expression is obtained by putting the intermediate particles on their mass-shells, i.e., by replacing the denominators $(k^2-m^2+i\epsilon)^{-1}$ in the propagator by $\epsilon(k_0)\delta(k^2-m^2)$. This yields finite integrals, which are easily calculable and lead to the results listed in Eq. (2.3). For example, for the coefficient of $C_{b\alpha c}C_{c\beta b}$ in (A1) we obtain

$$T_{ii}(p_0^2)=\int d^4k\,\theta(k_0)\theta(p_0-k_0)\delta(k^2-m^2)\delta[(p-k)^2-m^2]$$
$$\times[p_0^2-(p_0k_0)^2/m^2](-3-\mathbf{k}^2/m^2)$$
$$=-\pi(p_0^2/4-m^2)^{1/2}(p_0-p_0^3/4m^2)(2+p_0^2/4m^2).$$

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Partially Conserved Axial-Vector Current, Charge Commutators, Off-Mass-Shell Correction, and the Broken $SU(3)$ Symmetry

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(Received 15 December 1966; revised manuscript received 6 March 1967)

We point out that the extension of the PCAC (partially conserved axial-vector current) relation $\partial_\mu A_\mu{}^\pi=C_\pi\phi^\pi$ to $\partial_\mu A_\mu{}^K=C_K\phi^K$ and the use of charge commutators typified by $A_K=[V_K,A_\pi]$ are useful in the study of broken $SU(3)$ symmetry. The use of the $\partial_\mu A_\mu{}^K=C_K\phi^K$ condition usually confronts us with a considerable off-mass-shell extrapolation $m_K\to 0$. However, by using the above charge commutators and the approximation we propose, the off-mass-shell extrapolation $m_K\to 0$ may be replaced by a more comfortable one, $m_\pi\to 0$, effectively to first order in the symmetry-breaking interaction. This approach is applied to the study of the $SU(3)$ symmetry breaking. Encouraging results have been obtained in the case of $V\to P+P$ (i.e., $K^*\to K+\pi$ and $\rho\to\pi+\pi$) decays and in the direct determination of the $f-f'$ mixing angle from their decay widths. We also make some estimate of the off-shell extrapolation $m_K\to 0$ compared with the case $m_\pi\to 0$. Another useful application of the above charge commutators is for the weak leptonic decays. We can derive a set of sum rules for the axial-vector coupling constants of the leptonic decays of hyperons which seem to give new insight into the Cabibbo theory of leptonic interactions.

THERE have been many interesting calculations based on the idea of current algebra.[1] In the actual computations the use of the PCAC (partially conserved axial-vector current) hypothesis is essential. One may take a variety of attitudes toward the use of PCAC.

(I) One point of view is to regard the equation

$$\partial_\mu A_\mu{}^\pi=C_\pi\phi^\pi \quad (1)$$

as approximately true.[2] Taking the matrix element of (1) between the proton and the neutron, one obtains a form of Goldberger-Treiman relation

$$C_\pi=\sqrt{2}g_A\frac{m_p}{G_{pp\pi}(m_\pi{}^2=0)}m_\pi{}^2, \quad (2)$$

where g_A is the ratio of the axial-vector to the vector coupling constant of β decay. The calculation[3] from this standpoint involves the extrapolation of the pion off the mass shell ($m_\pi\to 0$).

(II) In a second point of view, $\partial_\mu A_\mu{}^\pi$ is regarded as a highly convergent operator whose matrix element satisfies an unsubtracted dispersion relation in squared momentum transfer q^2. For small q^2, the dominance of

[1] M. Gell-Mann, Physics 1, 63 (1964).

[2] Vector and axial-vector currents are denoted by $V_\mu{}^{\pi+}(x)$, $V_\mu{}^{K+}(x),\cdots$, and $A_\mu{}^{\pi+}(x)$, $A_\mu{}^{K+}(x),\cdots$, respectively, normalized so that in a quark model we would have, e.g., $V_\mu{}^{\pi+}(x)=i\bar{q}\gamma_\mu\times(\lambda_1+i\lambda_2)q/2$, $A_\mu{}^{K+}(x)=i\bar{q}\gamma_\mu(\lambda_4+i\lambda_5)q/2$, etc.; the space integral of, say, $A_0{}^{\pi+}(\mathbf{x},0)\equiv -iA_4{}^{\pi+}(\mathbf{x},0)$ is denoted by A_{π^+}. The PCAC relations are used in the form $\partial_\mu A_\mu{}^{\pi\pm}=C_\pi\phi^{\pi\pm}$ and $\partial_\mu A_\mu{}^{K\pm}=C_K\phi^{K\pm}$, where, e.g., $\phi^{\pi+}$ creates π^+ mesons.

[3] For instance, S. Adler, Phys. Rev. 137, B1022 (1965).

the pion pole term is assumed. In this approximation, one need only deal with on-mass-shell quantities.[4,5]

(III) A third point of view[6] is to regard Eq. (1) as an exact condition which provides a definition of a local pion field.[7] Then C_π can be evaluated directly from the pion decay rate.

For the problem dealing with the charge operators A_π and V_π, the above three approaches do not lead to very different results. This is partly because the value of C_π evaluated by using (2) is not very different from the one obtained by using the pion decay rate and partly because the off-shell effect due to $m_\pi \to 0$ is probably not important. In fact, from standpoint (II) Weisberger[4] obtained a value $|g_A| = 1.15$. However, he also observed that the value

$$|g_A| = 1.21 \quad (3)$$

can be obtained by using the value of C_π determined from the pion decay rate and still using the physical pion-nucleon scattering cross section. Since (3) is also quite close to the experimental number, $|g_A| = 1.18 \pm 0.025$, one may make the following speculation: The off-mass-shell effect due to $m_\pi \to 0$ is negligibly small and standpoint (III) is also compatible with the β-decay experiments.

However, if we turn to problems dealing with the axial-vector charge operator, A_K, the situation becomes more involved. In addition to (1), we assume

$$\partial_\mu A_\mu{}^K = C_K \phi^K. \quad (4)$$

The generalized Goldberger-Treiman relation is given, for instance, by

$$C_K = (g_A)_\Lambda \frac{m_p + m_\Lambda}{G_{\Lambda pK}(m_K{}^2 = 0)} m_K{}^2, \quad (5)$$

where $(g_A)_\Lambda$ is the corresponding quantity to g_A in the $\Lambda \to p + e + \bar{\nu}$ decay. However, the value of the $G_{\Lambda pK}$ coupling constant is not very precisely known.[8] Furthermore from standpoint (II), the K-meson pole dominance for the matrix element of the operator $\partial_\mu A_\mu{}^K$ is now not so convincing. These considerations make it rather difficult to draw an unambiguous result for this case in standpoint (II), although one need deal only with on-mass-shell quantities.[9]

[4] W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

[5] B. Renner, Phys. Letters **20**, 72 (1966).

[6] This point of view was taken by S. Matsuda, S. Oneda, and J. Sucher, Phys. Rev. **159**, 1247 (1967).

[7] This point of view was stressed by K. Nishijima and S. Okubo. See Proceedings of the Argonne International Conference on Weak Interactions, Argonne National Laboratory Report No. ANL-7130, 1965, p. 418 (unpublished).

[8] The most recent estimate gives $(G_{\Lambda pK}{}^2/4\pi) \simeq 5$ [see M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966)]. This value is rather small compared with the values which have been often used in the past literature of the computations using the current algebra. If we use the best value of $(g_A)_\Lambda$ obtained by N. Brene *et al.* [Phys. Rev. **149**, 1228 (1966)] we obtain from (5) that $G_{\Lambda pK}{}^2(m_K{}^2 = 0)/4\pi \simeq 8$.

[9] Also in standpoint (I), the question "How good is the PCAC condition (4) with (5)?" will become more important.

We now come back to standpoint (III). Here we can bypass the above-mentioned difficulty, since C_K can be determined directly from the rate of $K \to \mu + \nu$ decay, if we assume the same Cabbibo angle for vector and axial-vector weak leptonic interaction.[10] However, in this approach we have to face up to the off-mass-shell correction, $m_K \to 0$, which looks like a considerable extrapolation. Therefore, in standpoint (III) an estimate of this effect is desired. Apart from this, from the theoretical point of view, it is certainly interesting to study, if feasible, the off-mass-shell effect.

In this note we point out that there is a relatively simple way to replace the off-mass-shell extrapolation $m_K \to 0$ by the one $m_\pi \to 0$, based on an approximation which seems reasonably good to us. This implies that one may study standpoint (III) with an extrapolation $m_\pi \to 0$ which is much smaller than $m_K \to 0$.

As one of the direct consequences, we compute the ratio of the coupling constants of the interaction $K^* \to K + \pi$ and $\rho \to \pi + \pi$ where one of the pions is off the mass shell.[11] This turns out to be very close to the corresponding one on the mass-shell obtained from the observed decay widths. Therefore, if this off-shell effect ($m_\pi \to 0$) is negligibly small, as indicated by Weisberger's calculation,[4] we seem to have been successful in explaining the observed branching ratio.

By using the same approximation we also compute the mixing angle of $I = 0$ 2^+ mesons, f and f', directly from experiment. We obtain a value which is close to the one predicted by the Gell-Mann–Okubo mass formula. These results seem encouraging and lend support to our approximation. For the sake of theoretical interest, we also present some estimates of the effect of the off-shell extrapolation $m_K \to 0$. We also compute the $\phi \to K + \bar{K}$ decay width using, however, the extrapolation $m_K \to 0$. Finally, we point out that one can derive a set of sum rules using specific charge commutators for the axial-vector coupling constants of the leptonic decays of hyperons which seems to give new insight to the Cabibbo theory of leptonic decays. All our arguments are based on only the charge algebras, not on relations involving the commutators between two currents.

1. The method consists of the use of commutators typified by

$$A_K = [V_K, A_\pi]. \quad (6)$$

We take the matrix element of (6) between the state Q and P with four-momenta q_μ and p_μ.

$$\begin{aligned}\langle Q(q)|A_K|P(p)\rangle &= \sum_{Q'} \langle Q(q)|V_K|Q'(q')\rangle\langle Q'(q')|A_\pi|P(p)\rangle \\ &\quad - \sum_{P'} \langle Q(q)|A_\pi|P'(p')\rangle\langle P'(p')|V_K|P(p)\rangle.\end{aligned} \quad (7)$$

[10] With $\cos\theta_A = \cos\theta_V = 0.978$, we obtain $(C_K/m_K{}^2)^2 = 2.82 \times 10^4$ (MeV)^2, whereas from the pion decay, $(C_\pi/m_\pi{}^2)^2 = 1.73 \times 10^4$ (MeV)^2.

[11] Throughout this paper, we treat all the resonances as stable particles.

We now evaluate (7) at $|\mathbf{q}|=|\mathbf{p}|=\infty$.[12] Then, for instance, $\langle Q(q)|V_K|Q'(q')\rangle$ will be evaluated at $(q-q')^2=0$. In picking the state Q' we make the following approximation: We note that V_K is an $SU(3)$ generator in the symmetry limit. We propose to keep only the states Q' which have the same space-time quantum numbers as the state Q. Suppose Q and Q_α' belong to the same $SU(3)$ multiplet and can be connected by the operator V_K (like the case $Q=\pi$ and $Q_\alpha'=K$). If there are no other states which have identical quantum numbers to Q and Q_α', the value of the matrix element under consideration, $\langle Q|V_K|Q_\alpha'\rangle$, is known *to first order in the symmetry-breaking interaction*.[13] However, for instance, there may exist two states, Q_α' and Q_β', corresponding to the intermediate state Q' in question (e.g., the case $Q_\alpha'=\omega$ and $Q_\beta'=\phi$). In this case we write the states, Q_α' and Q_β', *to first order of symmetry breaking*, in terms of the two fields, $Q_\alpha^{(0)\prime}$ and $Q_\beta^{(0)\prime}$, which belong to different $SU(3)$ multiplets in the symmetry limit. If the masses of $Q_\alpha'^{(0)}$ and $Q_\beta'^{(0)}$ are close, there may be appreciable mixing and such terms should be kept. By the procedure described above, we can evaluate the matrix elements $\langle Q|V_K|Q_\alpha'\rangle$ and $\langle Q|V_K|Q_\beta'\rangle$, to first order, with a mixing parameter. We apply the same procedure whenever the mixing possibility exists. All other states Q_γ' will be dropped for the sake of simple calculation. We note that, since Q_γ' has different space-time quantum numbers from Q, and since V_K is essentially a scalar quantity, $\langle Q|V_K|Q_\gamma'\rangle$ must involve a higher-momentum barrier than the diagonal term $\langle Q|V_K|Q_\alpha'(\text{or }Q_\beta')\rangle$. Also, since Q and Q_γ' do not belong to the same $SU(3)$ multiplet, the mass difference between Q and Q_γ' will in general be large compared with that between Q and Q_α' (or Q_β'), which suppresses the importance of these Q_γ' terms. Experience with calculation of the type in question indicates that these factors tend to make these contributions of the omitted states small, and this seems plausible from the point of view that the contributions of the continuum are dominated by those of one-particle resonance states. (See, for instance, Ref. 22.) Therefore, even though the term $\langle Q|V_K|Q_\gamma'\rangle$ is formally of first order, the net contribution may be small, of comparable magnitude with the second-order term in the diagonal terms. Thus our computation may be good, in effect, to first order of symmetry breaking. Of course, we do not exclude the occurrence of some special circumstance. For instance, if the Q_γ' is a low-lying state which couples much more strongly than the corresponding Q_α' or Q_β', it might upset the argument. Needless to say, the accumulation of knowledge of resonances will make it possible to include estimates of $\langle Q|V_K|Q_\gamma'\rangle$ terms in the present approach.

Returning to the terms involving A_π and A_K in (7), $\langle Q(q)|A_K|P(p)\rangle$ and $\langle Q'(q')|A_\pi|P(p)\rangle$ are related, by using PCAC, to the coupling constants of the processes $P\to Q+K$ $(m_K=0)$ and $P\to Q'+\pi$ $(m_\pi=0)$, respectively. Therefore, using (7), one can express the off-shell coupling constant for $P\to Q+K$ $(m_K=0)$ in terms of other coupling constants which involve only the pion off the mass shell, $m_\pi\to 0$. By using this procedure, one can replace the off-shell extrapolation $m_K\to 0$ with the more comfortable one $m_\pi\to 0$.

2. We now proceed to apply the procedure just described. We first take the matrix elements between $\pi^0(p')$ and $K^{*+}(p)$ with $|\mathbf{p}'|=|\mathbf{p}|=\infty$, of the following two-charge commutators:

$$\tfrac{1}{2}A_{K^-}=[V_{K^-},A_{\pi^0}],\quad A_{K^-}=[V_{\bar{K}^0},A_{\pi^-}]. \tag{8}$$

Using our approximation outlined above, we obtain in a symbolic notation

$$\tfrac{1}{2}\langle\pi^0|A_{K^-}|K^{*+}\rangle=\langle\pi^0|V_{K^-}|K^+\rangle\langle K^+|A_{\pi^0}|K^{*+}\rangle,$$
$$\langle\pi^0|A_{K^-}|K^{*+}\rangle=\langle\pi^0|V_{\bar{K}^0}|K^0\rangle\langle K^0|A_{\pi^-}|K^{*+}\rangle-\langle\pi^0|A_{\pi^-}|\rho^+\rangle\langle\rho^+|V_{\bar{K}^0}|K^{*+}\rangle.$$

One can eliminate the A_K term from the above two equations to obtain

$$\begin{aligned}\left(\frac{1}{2\pi}\right)^3\int d^3k\{&\langle\pi^0(p')|V_{K^-}|K^+(k)\rangle\langle K^+(k)|A_{\pi^0}|K^{*+}(p)\rangle\\&-\langle\pi^0(p')|V_{\bar{K}^0}|K^0(k)\rangle\langle K^0(k)|A_{\pi^-}|K^{+*}(p)\rangle\}\\=-\left(\frac{1}{2\pi}\right)^3\sum_{\text{spin}}\int d^3k&\langle\pi^0(p')|A_{\pi^-}|\rho^+(k)\rangle\times\langle\rho^+(k)|V_{\bar{K}^0}|K^*(p)\rangle.\end{aligned} \tag{9}$$

We define the relevant form factors as follows:

$$\langle\pi^0(p')|V_{\bar{K}^0}|K^0(k)\rangle=\frac{(2\pi)^3\delta(\mathbf{p}'-\mathbf{k})}{(2p_0'2k_0)^{1/2}}\left(\frac{1}{2}\right)^{1/2}\times[F_+(q^2)(k_0+p_0')+F_-(q^2)(k_0-p_0')], \tag{10a}$$

where

$q=p'-k$ and $F_+(0)=1$ and $F_-(0)=0$ in the $SU(3)$ limit.

$$\begin{aligned}\langle\rho^+(k)|V_{\bar{K}^0}|K^{*+}(p)\rangle=&\frac{(2\pi)^3\delta(\mathbf{k}-\mathbf{p})}{(2p_02k_0)^{1/2}}(-1)\\&\times\{(\epsilon^\rho\cdot\epsilon^{K^*})[f_1(q'^2)(p_0+k_0)+f_2(q'^2)(p_0-k_0)]\\&+(\epsilon^\rho\cdot p)\epsilon_0^{K^*}f_3(q'^2)+(\epsilon^{K^*}\cdot k)\epsilon_0^\rho f_4(q'^2)\\&+(\epsilon^\rho\cdot q')(\epsilon^{K^*}\cdot q')(1/m_\rho^2)\\&\times[f_5(q'^2)(p_0+k_0)+f_6(q'^2)(p_0-k_0)]\},\end{aligned} \tag{10b}$$

where $q'=p-k$, $f_1(0)=1$ and $f_2(0)=f_3(0)=\cdots=f_6(0)=0$ in the $SU(3)$ limit. ϵ^ρ and ϵ^{K^*} are the polarization vectors of the ρ and the K^* meson, respectively.

[12] S. Fubini and G. Furlan, Physics 1, 229 (1965).
[13] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

Using PCAC, one can relate, for instance, the quantity $\langle K^0(k)|A_{\pi^-}|K^{*+}(p)\rangle$ to the off-mass-shell coupling constant of the process $K^{*+}\to K^0+\pi^+$ $(m_\pi^2=0)$, as follows:

$$\langle K^0(k)|A_{\pi^-}|K^{*+}(p)\rangle|_{(p-k)^2=0}$$
$$=\frac{(2\pi)^3\delta(\mathbf{k}-\mathbf{p})}{(2k_0 2p_0)^{1/2}}\left(\frac{C_\pi}{m_\pi^2}\right)\frac{G_{K^{*+}K^0\pi^+}(m_\pi^2=0)}{i(k_0-p_0)}(\epsilon^{K^*}\cdot k). \quad (10c)$$

We now substitute expressions such as (10a), (10b), and (10c) into Eq. (9) and take a limit $|\mathbf{p}'|=|\mathbf{p}|=\infty$; make summations over ρ-meson spin states; and perform an integration over d^3k. For the spin summation we encounter the following four types of expressions:

$$a=\sum_{\text{spin}}(\epsilon^\rho\cdot p')(\epsilon^\rho\cdot\epsilon^{K^*})(p_0+k_0)f_1(0),$$
$$b=\sum_{\text{spin}}(\epsilon^\rho\cdot p')(\epsilon^\rho\cdot p)\epsilon_0^{K^*}f_3(0),$$
$$c=\sum_{\text{spin}}(\epsilon^\rho\cdot p')(\epsilon^{K^*}\cdot k)\epsilon_0^\rho f_4(0),$$
$$d=\sum_{\text{spin}}(\epsilon^\rho\cdot p')(\epsilon^\rho\cdot p)(\epsilon^{K^*}\cdot k)(p_0+p_0')f_5(0).$$

After simple manipulation in the limit corresponding to $|\mathbf{p}'|=|\mathbf{p}|\to\infty$, these expressions turn out to be

$$a=\frac{(m_\rho^2+m_{K^*}^2)(m_\rho^2-m_\pi^2)}{2m_\rho^2}\epsilon_0^{K^*}f_1(0)\simeq 0.8\ (\text{BeV})^2 f_1(0)\epsilon_0^{K^*},$$
$$b=\frac{(m_\rho^2-m_{K^*}^2)(m_\rho^2-m_\pi^2)}{4m_\rho^2}\epsilon_0^{K^*}f_3(0)\simeq -0.05\ (\text{BeV})^2 f_3(0)\epsilon_0^{K^*},$$
$$c=\frac{(m_{K^*}^2-m_\rho^2)(m_\rho^2-m_\pi^2)}{4m_\rho^2}\epsilon_0^{K^*}f_4(0)\simeq 0.05\ (\text{BeV})^2 f_4(0)\epsilon_0^{K^*},$$
$$d=-\frac{(m_{K^*}^2-m_\rho^2)^2(m_\rho^2-m_\pi^2)}{4m_\rho^4}\epsilon_0^{K^*}f_5(0)\simeq -0.018\ (\text{BeV})^2 f_5(0)\epsilon_0^{K^*}.$$

We notice that, whereas $f_1(0)=1$ in the symmetry limit, $f_3(0)$, $f_4(0)$, and $f_5(0)$ are all zero in the symmetry limit and are of the first order in the symmetry-breaking interaction. The b, c, and d terms also involve the expression $(m_\rho^2-m_{K^*}^2)$, which is zero in the symmetry limit, and therefore we can safely neglect these terms in comparison with a. This, in fact, corresponds to a realization of the Ademollo-Gatto theorem.[13] We therefore finally obtain[14–16] the ratio of the coupling constants for the $K^*\to K+\pi$ and $\rho\to\pi+\pi$ decays where one of the pions is off the mass-shell:

$$R=\frac{2G_{K^*K^-\pi^0}(m_{K^*}^2,m_K^2,m_\pi^2=0)}{G_{\rho^+\pi^-\pi^0}(m_\rho^2,m_\pi^2,m_\pi^2=0)}=\left(\frac{m_\rho^2+m_{K^*}^2}{2m_\rho^2}\right)\frac{f_1(0)}{F_+(0)}. \quad (11a)$$

Since, to first order in the symmetry-breaking interaction, $f_1(0)=F_+(0)=1$, we therefore obtain in our approximation,

$$R=\frac{m_\rho^2+m_{K^*}^2}{2m_\rho^2}=1.19. \quad (11b)$$

The corresponding ratio on the mass-shell determined from the experiment[17] is 1.15. That is, the value of (11b) predicts $\Gamma(K^{*+}\to\text{all})\simeq 52.5$ MeV from the experimental width, compared with the experimental value 50 ± 1.4 MeV.[17] In order to have some feeling about the effect of second-order symmetry breaking, we note that from the experimental K_{e3} decay rate, assuming that the form factor $F_+(q^2)$ is dominated by the K^* meson, we infer $F_+(0)\simeq 1.04$.[18] Since the mass difference between the K^* and the ρ meson is small, we may expect $f_1(0)$ is very close to 1.

3. We determine the mixing angle between the $I=Y=0$ 2^+ mesons, $f(1254)$ and $f'(1500)$. We again take the matrix elements of the commutators (8), now between the π^0 and $K^{**+}(1415)$, and proceed as above. We write, to first order of symmetry breaking, for the physical f and f' in terms of the unitary singlet f_1 and the $I=Y=0$ member of the octet, f_8,

$$f=\cos\theta\, f_1+\sin\theta\, f_8,\quad f'=\cos\theta\, f_8-\sin\theta\, f_1. \quad (12)$$

Experimentally the rate $f'\to\pi+\pi$ is very small[19] so that we assume $G_{f'\pi\pi}=0$. Then we obtain,[20] in our approximation,

$$\frac{G_{K^{**+}K^0\pi^-}(m_{K^{**}}^2,m_K^2,m_\pi^2=0)}{G_{f\pi^0\pi^0}(m_f^2,m_\pi^2,m_\pi^2=0)}=\frac{(m_f^2-m_\pi^2)(m_f^4+m_{K^{**}}^4+4m_f^2m_{K^{**}}^2)}{6(m_{K^{**}}^2-m_K^2)m_f^4}(\sqrt{6})\sin\theta. \quad (13)$$

[14] Numerically similar results have been obtained by H. T. Nieh, Phys. Rev. Letters **15**, 902 (1965); Phys. Rev. **146**, 1012 (1966); Riazuddin and Fayyazuddin, *ibid.* **147**, 1071 (1966); R. J. Rivers, *ibid.* **152**, 1263 (1966).

[15] We note that the results (11), (13), and (14) do not depend on C_π and C_K. Therefore, they also follow from standpoint (I). However, the results are approximate to the extent that the relations (1) and (4) are approximate.

[16] The effective coupling constants for $S(0^+)$, $V(1^-)$, and $T(2^+)$ meson decays into pseudoscalar mesons P and P' are defined by writing the corresponding effective Lagrangians in the form $G_{SPP'}SPP'$, $iG_{VPP'}V_\mu[P\cdot\partial_\mu P'-(\partial_\mu P)\cdot P']$ and $G_{TPP'}T_{\mu\nu}(\partial_\mu P)\times(\partial_\nu P')$.

[17] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

[18] S. Oneda and J. Sucher, Phys. Rev. Letters **15**, 927 (1965); **15**, 1049E (1965).

[19] $\Gamma(f'\to K+\bar{K})/\Gamma(f'\to\text{all})\simeq 0.6$. Since phase space favors $f'\to\pi+\pi$ compared with $f'\to K+K$, this is a good approximation.

[20] $$\Gamma(K^{**0}\to K^+\pi^-,K^0\pi^0)=\left(\frac{G_{K^{**+}K^0\pi^-}{}^2}{30\pi}\right)\frac{p^5}{m_K^2}\frac{3}{2},$$
$$\Gamma(f^0\to\pi^0\pi^0,\pi^+\pi^-)=\left(\frac{G_{f^0\pi^0\pi^0}{}^2}{15\pi}\right)\frac{p^5}{m_f^2}\frac{3}{2},$$
where p is the momentum of the secondary particle.

Using[17] $\Gamma(K^{**+}\to K+\pi)/\Gamma(K^{**+}\to \text{all})\simeq 0.5\pm 0.1$, $\Gamma\times(K^{**+}\to \text{all})\simeq(96\pm 7)$ MeV and $\Gamma(f\to 2\pi)=(112\pm 8)$ MeV, we obtain $\theta\simeq 33°$. It is interesting to notice that this value of θ is close to the value $\theta\simeq 30°$ obtained by using the first-order Gell-Mann–Okubo mass formula.[21] Considering the experimental uncertainty in the width, the result seems to be satisfactory. For possible future usefulness,[6] we write a similar sum rule for the hypothetical 0^+ nonets. We introduce K' $(I=\frac{1}{2}, Y=\frac{1}{2})$, ξ $(I=0, Y=0)$, and σ $(I=0, Y=0)$. The σ and the ξ are related to the unitary singlet σ_1 and the octet ξ_8 in a similar way to (12) by an angle θ'. We obtain

$$\frac{G_{K'^+K^0\pi^-}}{m_{K'}^2-m_K^2}=(\sqrt{6})\sin\theta'\frac{G_{\sigma^0\pi^0\pi^0}}{m_\sigma^2-m_\pi^2}+(\sqrt{6})\cos\theta'\frac{G_{\xi^0\pi^0\pi^0}}{m_{\xi'}^2-m_\pi^2},\quad(14)$$

where, in each of the expressions for G, one of the pions is on the zero mass-shell. We note that in deriving Eq. (14), we neglect the contribution of vector-meson states. Since the vector mesons are the low-lying and strongly coupling particles, the status of the relation (14) [and correspondingly (17)] might be more debatable than that of (11), (13), (15), and (16).

4. The commutators (8) can also be used to replace the off-mass-shell coupling constant $(m_K=0)$ with the one $(m_\pi=0)$. From (9) alone, we get

$$\frac{G_{K^*K\pi}(m_K^2=0)}{G_{K^*K\pi}(m_\pi^2=0)}=\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)=0.78.\quad(15)$$

In a similar way, we get, using (8),

$$\frac{2G_{\rho^0K^-K^+}(m_K^2=0)}{G_{\rho^0\pi^-\pi^+}(m_\pi^2=0)}=\left(1-\frac{(m_{K^*}^2-m_\rho^2)^2}{4m_{K^*}^2m_\rho^2}\right)\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)=0.76,\quad(16)$$

$$\frac{G_{K'K\pi}(m_K^2=0)}{G_{K'K\pi}(m_\pi^2=0)}=\left(\frac{m_{K'}^2-m_\pi^2}{m_{K'}^2-m_K^2}\right)\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)=1.4$$

$$(\text{for } m_K'=725\text{ MeV}).\quad(17)$$

5. By taking the matrix element of the first expression of (8) between the K^- and the ϕ^0 meson, we obtain, in our approximation (θ is the ω-ϕ mixing angle),

$$\frac{G_{\phi^0K^-K^+}(m_\phi^2,m_K^2,m_K^2=0)}{G_{K^{*+}K^-\pi^0}(m_{K^*}^2,m_K^2,m_\pi^2=0)}=-\sqrt{3}(\cos\theta)\alpha,$$

$$\alpha=\frac{m_{K^*}^2+m_\phi^2}{2m_{K^*}^2}\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)\simeq 0.90.\quad(18)$$

[21] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 325 (1965).

The factor α does not appear in the usual $SU(3)$ calculation. If we take[17] $\Gamma(\phi\to\text{all})=3.6$ MeV and $\Gamma(\phi\to K^+ + K^-)/\Gamma(\phi\to\text{all})=0.47$, we obtain from (18) $\theta\simeq 35°$, compared with a value $\theta\simeq 39°$ predicted by the first-order mass formula. However, uncertainty in the experimental ϕ width and branching ratio is, at the moment, still uncomfortably large. Moreover, Eq. (18) involves off-mass-shell $(m_K\to 0)$ coupling. Therefore, the above evaluation of the value of θ is not very useful at present.[22]

6. We now discuss the Cabibbo theory from a point of view of the present approach. We notice that (particularly in the quark model of hadrons) the currents we are dealing with, V_μ^π, A_μ^π, V_μ^K, and A_μ^K, are exactly the Cabibbo currents which govern the weak interactions. We point out that, in the presence of the $SU(3)$ symmetry-breaking interaction, the following approach to the semileptonic interactions based on the commutation relations of type (6) satisfied by these basic weak currents gives an instructive insight into the analysis of semileptonic processes. Take the matrix element of the adjoint of the second expression of (8) between the proton and the Λ with infinite momenta.[23] In our approximation, we get

$$-\langle p|A_{K^+}|\Lambda\rangle=\langle p|V_{K^0}|\Sigma^+\rangle\langle\Sigma^+|A_{\pi^+}|\Lambda\rangle-\langle p|A_{\pi^+}|n\rangle\langle n|V_{K^0}|\Lambda\rangle.\quad(19)$$

We note, for example, $\langle p|A_\mu^{K+}|\Lambda\rangle\propto g_{p\Lambda}\bar{u}_p\gamma_5\gamma_\mu u_\Lambda$, where $Gg_{p\Lambda}\sin\theta_A$ [$g_{p\Lambda}\equiv(g_A)_\Lambda$ of Eq. (5)] is the observed axial-vector coupling constant for the $\Lambda\to p+e+\bar{\nu}$ decay at zero momentum transfer. G is the coupling constant of muon decay. For the sake of generality, assume that there exists another $I=Y=0$ $\frac{1}{2}^+$ baryon Y^0 and write the physical Λ as $\Lambda=\cos\phi\,\Lambda_8-\sin\phi\,Y_1$, where the Λ_8 and Y_1 denote the octet and singlet. Then from (19), a sum rule follows to our approximation,

$$g_{p\Lambda}=g_{\Sigma^+\Lambda}-(\sqrt{\tfrac{3}{2}})g_{pn}\cos\phi.\quad(20)$$

In the same way we also obtain a set of sum rules

$$\begin{aligned}
g_{n\Sigma^-}&=-(1/\sqrt{2})g_{\Sigma^0\Sigma^-}+(\sqrt{\tfrac{3}{2}})g_{\Lambda\Sigma^-}\cos\phi,\\
g_{n\Sigma^-}&=\sqrt{2}g_{\Sigma^+\Sigma^0}+g_{pn},\\
g_{\Lambda\Xi^-}&=(-\sqrt{\tfrac{3}{2}})g_{\Xi^0\Xi^-}\cos\phi+g_{\Lambda\Sigma^-},\\
g_{\Sigma^+\Xi^0}&=-(1/\sqrt{2})g_{\Sigma^+\Sigma^0}+(\sqrt{\tfrac{3}{2}})g_{\Lambda\Sigma^-}\cos\phi,\\
g_{\Sigma^+\Xi^0}&=g_{\Xi^0\Xi^-}+\sqrt{2}g_{\Sigma^0\Sigma^-}.
\end{aligned}\quad(21)$$

Note that in deriving these sum rules *we do not use*

[22] If the experimental numbers become very precise, one could try to include the off-mass-shell correction based on the feeling obtained from the estimates, (15) and (16). Since $m_\phi>m_\rho$, the off-shell correction $(m_K\to 0)$ for the $\phi\to K+\bar{K}$ coupling may be small compared with that for $\rho\to K+\bar{K}$ coupling.

[23] Dr. J. Sucher called our attention to the work of E. R. McCliment (unpublished). He used a similar approach to ours for this problem. The main difference is that we are dealing, in the spirit of the current-commutator calculation, with the directly observable quantities such as $g_{p\Lambda}$, etc. He made an estimate of the Q_γ' term which we dropped. Namely, he kept the decuplet states and concluded that their effects are small.

PCAC. Therefore, a test of the Cabibbo theory (i.e., the determination of the angle θ_A) in the broken $SU(3)$ symmetry is provided by looking at these sum rules. However, we note that the same sum rules are also obtained in the original pure $SU(3)$ analysis due to Cabibbo if $\phi=0$ (by eliminating the ratio of d-type and f-type coupling of weak currents, which we do not need to introduce in our approach). This means that the sum rules, which are independent of the d/f ratio in the pure $SU(3)$ analysis, may still be valid, effectively to the first order in the symmetry-breaking interaction, if our approximation is valid.[23] The validity of these sum rules has already been tested by experiments to a rather satisfactory extent. However, in order to really study the pertinent question whether we have one angle $\theta_A=\theta_V$ or not, we seem to need more accurate experiments.[24]

Finally, we remark about the sum rules for the strong $\bar{B}BP$ coupling and the generalized Goldberger-Treiman relation. If we use PCAC in (19), we obtain a sum rule similar to (20) for the strong $\bar{B}BP$ coupling (in our approximation):

$$G_{p\Lambda K}(m_K^2=0)=\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)[G_{\Sigma^+\Lambda\pi^-}(m_\pi^2=0) - (\sqrt{\tfrac{3}{2}})G_{pn\pi^-}(m_\pi^2=0)\cos\phi]. \quad (22a)$$

The off-the-mass-shell extrapolation is indicated explicitly. Sum rules corresponding to (21) are obvious. The study of these sum rules is interesting, although we have to be aware of the extrapolation $m_K\to 0$ for the $G_{p\Lambda K}$ coupling. We notice, however, some suggestive features of the sum rule (22a). In the usual pure $SU(3)$ analysis, by eliminating the ratio of F-type to D-type coupling, an analogous sum rule on the mass-shell to (22a) is obtained,

$$G_{p\Lambda K}=[G_{\Sigma^+\Lambda\pi^-}-(\sqrt{\tfrac{3}{2}})G_{pn\pi^-}]. \quad (22b)$$

The factor, $(C_\pi/m_\pi^2)(m_K^2/C_K)\simeq 0.61<1$, which stands in front of the right-hand side of Eq. (22a) makes the value of $G_{p\Lambda K}(m_K^2=0)$ smaller than the one predicted by the pure $SU(3)$ calculation, (22b). This is, at least, in the right direction to agree with the experimentally indicated small value[8] of $G_{p\Lambda K}$.

From standpoint (III), (2) and (5) imply, for instance,

$$\left(\frac{g_{pn}}{G_{pn\pi^-}(m_\pi^2=0)}\right)\bigg/\left(\frac{g_{p\Lambda}}{G_{p\Lambda K^-}(m_K^2=0)}\right) =\left(\frac{C_\pi}{m_\pi^2}\right)\left(\frac{m_K^2}{C_K}\right)\frac{m_n+m_\Lambda}{2m_p}\simeq 0.85. \quad (23)$$

We note again that $G_{p\Lambda K^-}(m_K^2=0)$ may differ from its on-mass-shell value by an appreciable amount. (One might say at least $\simeq 20\%$ from the experience indicated by (15) and (16). We also do not know the sign of this correction.) Therefore, the corresponding value of the ratio (23) *on the mass shell* could deviate from unity to some extent, although the possibility that the off-mass-shell effect acts in such a way as to restore the value obtained in (23) to unity cannot also be ruled out. From these considerations, we should keep in mind the possibility that the ratio of F-type to D-type coupling (in the usual $SU(3)$ analysis terminology, which we do not need to use in our approach) may differ to some extent (probably at most 30–40%) between the strong $\bar{B}BP$ interaction and the weak axial-vector coupling. Therefore, it is not surprising even if we observe some deviation from the generalized Goldberg-Treiman relation. If we know the rather precise value of $G_{\Sigma^+\Lambda\pi^-}$ coupling, using (22a) we may check the validity of the Eq. (23) by measuring the value of $g_{p\Lambda}$.

Note added in proof. We remark here the following interesting possibility which will also favor our approximation adopted in discussing the broken $SU(3)$ symmetry: The degree of the $SU(3)$ breaking in the matrix elements of the $SU(3)$ generator, V_K, at zero momentum transfer may be considerably smaller than the corresponding $SU(3)$ breaking in the matrix elements of the axial-vector charges. We know that at least in the K_{e3} decay, the value of the matrix element $\langle\pi(p')|V_K|K(p)\rangle$ at zero momentum transfer [see, for instance, Eq. (10a)] is very close to the $SU(3)$ value. If this is generally the case, our approximation will be good since we do not make approximations on the axial-vector coupling constants. In particular, if we assume small renormalization of the vector coupling constants, we are justified, in the broken $SU(3)$ symmetry, to determine the axial-vector Cabibbo angle θ_A from the sum rules (20) and (21). We note that if we use the presently available experimental rates of β, $\Lambda\to p+e+\nu$ and $\Sigma\to\Lambda+e+\nu$ decays, the sum rule (20) is compatible with having $\theta_V=\theta_A$ within the experimental error. Much more accurate data on these decays is required to settle the question of whether we have $\theta_V=\theta_A$ or not.

We would like to thank Professor J. Sucher for his important remarks and constructive discussions. We also thank Professor G. A. Snow for his helpful conversations.

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[24] At the moment there is no convincing evidence for the existence of the Y^0. If we discard the effect of Y^0, we expect, under our approximation, that by checking the sum rules (20) and (21), we obtain for the observed value $\theta_A\simeq\theta_V$ within, say, 10% if we have the equality in the bare coupling.