$K_2{}^0 \rightarrow \pi^0+2\gamma$ Decay*

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It is pointed out that if the experimentally reported large branching ratio of $\eta^0 \rightarrow \pi^0+2\gamma$ decay is indeed correct, it may also suggest a large branching ratio for the $K_2{}^0 \rightarrow \pi^0+2\gamma$ decay. The closeness of the masses of the K^0 meson and the η^0 meson, together with this unexpectedly large coupling constant of the $\eta^0 \rightarrow \pi^0+2\gamma$ decay implies that the diagram $K_2{}^0 \rightarrow \eta^0 \rightarrow \pi^0+2\gamma$ may lead to a relatively large branching ratio of the $K_2{}^0 \rightarrow \pi^0+2\gamma$ decay, contrary to our usual expectation. Simple calculation based on the pole model with the $SU(3)$ symmetry indicates that the frequency of this mode is at least comparable with or may even be larger than that of $K_2{}^0 \rightarrow 2\gamma$ decay. Furthermore, since both the $K_2{}^0 \rightarrow 2\pi^0$ and the $K_2{}^0 \rightarrow \pi^0+2\gamma$ decays are essentially 4γ processes, the decay $K_2{}^0 \rightarrow \pi^0+2\gamma$ could constitute an important background for the $K_2{}^0 \rightarrow 2\pi^0$ experiments if its rate is comparable with that of $K_2{}^0 \rightarrow 2\pi^0$ decay. Some kinematical features of this decay are also discussed.

RECENTLY, experiments[1-3] on the η decays revealed a surprisingly abundant occurrence of the decay mode, $\eta^0 \rightarrow \pi^0+2\gamma$. According to the compilation of Rosenfeld *et al.*,[4] the branching ratio is given by

$$R_\eta=\Gamma(\eta^0 \rightarrow \pi^0+2\gamma)/\Gamma(\eta^0 \rightarrow 2\gamma)\simeq 0.65. \qquad (1)$$

This large ratio seems to be rather hard to explain. The $SU(3)$ symmetry tells us that the matrix elements of the decays, $\pi^0 \rightarrow 2\gamma$ and $\eta^0 \rightarrow 2\gamma$ (neglecting X^0-η^0 mixing), are related by

$$M(\eta^0 \rightarrow 2\gamma)=(1/\sqrt{3})M(\pi^0 \rightarrow 2\gamma). \qquad (2)$$

Using[4] $\Gamma(\pi^0 \rightarrow 2\gamma)\simeq 7.4$ eV, we obtain $\Gamma(\eta^0 \rightarrow 2\gamma)\simeq 164$ eV and therefore from (1)

$$\Gamma(\eta^0 \rightarrow \pi^0+2\gamma)\simeq 74 \text{ eV}. \qquad (3)$$

Okubo and Sakita[5] made an estimate based on an effective decay Hamiltonian of the form

$$H=\xi F_{\mu\nu}F_{\mu\nu}\eta^0\pi^0. \qquad (4)$$

By assuming that the transition mass between η^0 and π^0 can be obtained by contracting the photon lines in Eq. (4), they obtained $\Gamma(\eta^0 \rightarrow \pi^0+2\gamma)\simeq 8$ eV. Since, as they stated, this estimate may easily be wrong by a factor $\simeq 10$, the large rate (3) suggested by the experiments might not be a very puzzling one.[6] The vector-meson-dominant model does not seem to provide a dominant mechanism. Namely, if we for instance choose the ρ meson as an intermediate state like $\eta \rightarrow \rho+\gamma \rightarrow (\pi+\gamma)+\gamma$ and $\eta \rightarrow \rho+\gamma \rightarrow (\pi^++\pi^-)+\gamma$, we obtain, crudely speaking,

$$R_\eta'=\Gamma(\eta^0 \rightarrow \pi^0+2\gamma)/\Gamma(\eta^0 \rightarrow \pi^++\pi^-+\gamma) \simeq |G_{\rho\pi\gamma}/G_{\rho\pi\pi}|^2<0.01,$$

contrary to the experimental ratio $\simeq 1$. In fact, detailed calculation[7] using the relevant coupling constants based on either static $SU(6)$ or $SU(3)+\omega-\phi$ mixing hypothesis gives $R_\eta'\simeq 10^{-2}$. Therefore, the contribution of vector-meson intermediate states may only be around 10–20% in the amplitude of $\eta^0 \rightarrow \pi^0+2\gamma$ decay if the ratio (1) is correct.

For later reference one may mention another simple mechanism[8] which will be a natural one if the tadpole model of electromagnetic mass difference of hadrons is correct. Namely, we assume the existence of an $I=1$ normal 0^+ $(\eta\pi)$ resonance, $\epsilon(\epsilon^+,\epsilon^0,\epsilon^-)$, and consider the following intermediary of the ϵ meson:

$$\eta^0 \rightarrow \pi^0+\epsilon^0 \rightarrow \pi^0+(\gamma+\gamma). \qquad (5)$$

It may be mentioned that this mechanism leads to a simple effective Hamiltonian for the $\eta^0 \rightarrow \pi^0+2\gamma$ decay given by (4) whereas the vector-meson intermediate diagrams lead to a different form of effective Hamiltonian[7-8] from (4). In this note we should like to point out:

> Whatever the true mechanism of the $\eta^0 \rightarrow \pi^0+2\gamma$ decay may be, *if the branching ratio of this decay is indeed large as reported by experiments*, it may also suggest a large branching ratio for the $K_2{}^0 \rightarrow \pi^0+2\gamma$ decay contrary to our usual expectation.

Recently important experimental results on the CP-violating $K_2{}^0 \rightarrow \pi^0+\pi^0$ decay have been reported. A

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[2] M. A. Wahlig, E. Shibata, and I. Manelli, Phys. Rev. Letters **17**, 221 (1966).

[3] G. S. Strugalski *et al.*, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967).

[4] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Ross, Paul Soding, W. J. Willis, and C. G. Wohl, Rev. Mod. Phys. (to be published); Refs. 1, 2, and 3 give $R\simeq 0.9 \pm 0.1$, $R<0.5$, and $R=0.86\pm 0.40$, respectively.

[5] S. Okubo and B. Sakita, Phys. Rev. Letters, **11**, 50 (1963).

[6] $\eta^0 \rightarrow \pi^0+2\gamma$ and $\eta^0 \rightarrow 3\pi$ are A-allowed whereas $\eta^0 \rightarrow 2\gamma$ and $\pi^++\pi^-+\gamma$ are A-forbidden. J. B. Bronzan and F. Low, Phys. Rev. Letters **12**, 522 (1964).

[7] G. Oppo, University of Maryland, Department of Physics and Astronomy, Tech. Rept. No. 627, 1966 (unpublished). See also H. Pietchmann and W. Thirring, Phys. Letters **21**, 713 (1966); P. Möbius and H. Pietchmann, *ibid.* **22**, 684 (1966); W. Alles, A. Baracca, and A. T. Romos, Nuovo Cimento **45**, 272 (1966).

[8] G. Oppo and S. Oneda, Bull. Am. Phys. Soc. **12**, 127 (1967) (to be published).

CERN group obtained[9]

$$\Gamma(K_2{}^0 \to 2\pi^0)/\Gamma(K_2{}^0 \to \text{all}) \simeq (3.3_{-1.1}{}^{+1.8}) \times 10^{-3}, \quad (6)$$

and a Princeton group[10] reported

$$\Gamma(K_2{}^0 \to 2\pi^0)/\Gamma(K_2{}^0 \to \text{all}) \simeq (4.4 \pm 0.85) \times 10^{-3}. \quad (7)$$

The latter group also obtained a large $K_2{}^0 \to 2\gamma$ rate,

$$\Gamma(K_2{}^0 \to 2\gamma)/\Gamma(K_2{}^0 \to \text{all}) \simeq (7.4 \pm 1.6) \times 10^{-4}. \quad (8)$$

Since both the $K_2{}^0 \to 2\pi^0$ and the $K_2{}^0 \to \pi^0 + 2\gamma$ decays are essentially 4γ processes the decay $K_2{}^0 \to \pi^0 + 2\gamma$ may constitute an important background for the $K_2{}^0 \to 2\pi^0$ experiment if its decay rate is comparable with that of $K_2{}^0 \to 2\pi^0$ decay. Therefore, it may be worthwhile to look into this decay. Apart from this, the observation of this decay itself will provide useful information on the dynamics of nonleptonic weak interactions. In the following we present some estimates based on a simple model. The result seems to indicate that the frequency of this mode is at least comparable with or may even be larger than that of the $K_2{}^0 \to 2\gamma$ decay.

The argument is based on the observation of the existence of the Feynman diagram shown in Fig. 1(a). The closeness of the masses of the K meson and the η meson, together with the unexpectedly large effective coupling constant of the $\eta^0 \to \pi^0 + 2\gamma$ decay, may enhance the importance of this diagram over other diagrams.

A. We compare the $K_2{}^0 \to \pi^0 + 2\gamma$ with the $K_2{}^0 \to 2\gamma$ decay. For the $K_2{}^0 \to 2\gamma$ decay, a similar diagram to Fig. 1(a), that is, Fig. 1(b), is also available. We write the relevant effective Lagrangian for the two-body weak transitions between pseudoscalar mesons, assuming the octet transformation properties, as follows[11,12]:

$$H_W = \sqrt{2} m_K{}^2 f_W K_2{}^0 (\pi^0 + 1/\sqrt{3}\, \eta^0). \quad (9)$$

Then from Fig. 1(b), the pion and η^0-meson pole diagrams lead to[1,2]

$$\Gamma(K_2{}^0 \to 2\gamma) \simeq 2 f_W{}^2 \left(\frac{a\sqrt{3} m_K{}^2}{m_\pi{}^2 - m_K{}^2} + \frac{1}{\sqrt{3}} \frac{m_K{}^2}{m_\eta{}^2 - m_K{}^2} \right)^2 \times \left(\frac{m_K}{m_\eta} \right)^3 \Gamma(\eta^0 \to 2\gamma), \quad (10)$$

where we have written instead of (2),

$$a M(\eta^0 \to 2\gamma) = 1/\sqrt{3}\, M(\pi^0 \to 2\gamma), \quad (11)$$

[9] J. M. Gaillard, F. Kriemen, W. Galbraith, A. Hussri, M. R. Jane, N. H. Lipman, G. Manning, T. Ratcliffe, P. Day, A. G. Parkam, B. T. Payne, A. C. Sherwood, H. Faissner, and H. Reithler, Phys. Rev. Letters **18**, 20 (1967).

[10] J. W. Cronin, P. F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters **18**, 25 (1967). Criegee *et al.* obtained $(1.3 \pm 0.6) \times 10^{-4}$ for the ratio (8). See L. Criegee, J. D. Fox, H. Frauenfelder A. O. Hanson, G. Moscati, C. F. Perdrisat, and J. Todoroff, Phys. Rev. Letters **17**, 150 (1966).

[11] S. Oneda and S. Hori, Phys. Rev. **132**, 1800 (1963).

[12] We neglect the contribution of the X^0 meson and the effect of X^0-η^0 mixing.

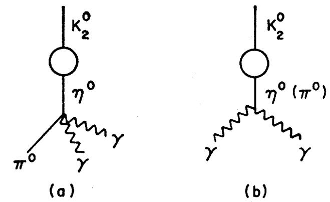

FIG. 1. The Feynman diagrams for the $K_2{}^0 \to \pi^0 + \gamma + \gamma$ decay and $K_2{}^0 \to \gamma + \gamma$ decay.

in order to indicate possible deviation from the prediction of the $SU(3)$ symmetry. In the symmetry limit, $a = 1$. Equation (10) shows that the π^0 and η^0 pole term tend to cancel each other in the symmetry limit. Therefore, for the $K_2{}^0 \to 2\gamma$ decay the pion pole term acts to suppress the contribution of the η^0 pole term. However, in the $K_2{}^0 \to \pi^0 + 2\gamma$ case this does not, in general, take place. Between the vertices, $\eta \to \pi + 2\gamma$ and $\pi \to \pi + 2\gamma$, there is no such simple relation as (2). Therefore, there is no *a priori* reason to believe that the π^0 pole term behaves in a similar way to the case of $K_2{}^0 \to 2\gamma$ decay. In fact, in the 0^+-meson intermediate model of the $PS \to PS + 2\gamma$ transition mentioned in (5), the strength of the $\eta^0 \to \pi^0 + 2\gamma$ amplitude is larger than the $\pi^0 \to \pi^0 + 2\gamma$ one, contrary to the relation (2) if we consider only the octet 0^+ meson.[13] Furthermore, we may even construct a model without difficulty in which both the pion and η^0-meson pole term contribute in a constructive way to the $K_2{}^0 \to \pi^0 + 2\gamma$ amplitude.[14] For the sake of simplicity, we here assume the probably most reasonable possibility, i.e., that the $K_2{}^0 \to \pi^0 + 2\gamma$ decay is reasonably well described by the η^0-meson pole term alone. By taking the form of interaction (4) [suggested by the model (5)] for the effective $\eta^0 \to \pi^0 + 2\gamma$ vertex, we obtain (in the rest system of the $K_2{}^0$ meson),

$$\Gamma(K_2{}^0 \to \pi^0 + 2\gamma) \simeq 2 f_W{}^2 \left(\frac{1}{\sqrt{3}} \frac{m_K{}^2}{m_\eta{}^2 - m_K{}^2} \right)^2 \times 0.55\, \Gamma(\eta^0 \to \pi^0 + 2\gamma). \quad (12)$$

Equations (10) and (12) lead to

$$R_K = \frac{\Gamma(K_2{}^0 \to 2\gamma)}{\Gamma(K_2{}^0 \to \pi^0 + 2\gamma)} \simeq \left(1 + \frac{3a(m_\eta{}^2 - m_K{}^2)}{m_\pi{}^2 - m_K{}^2} \right)^2 \times 1.35 \frac{\Gamma(\eta^0 \to 2\gamma)}{\Gamma(\eta^0 \to \pi^0 + 2\gamma)}$$

$$\simeq (1 - 0.70a)^2 \times 1.35 \frac{\Gamma(\eta^0 \to 2\gamma)}{\Gamma(\eta^0 \to \eta^0 + 2\gamma)}. \quad (13)$$

[13] In this model, for the $\pi^0 \to \pi^0 + 2\gamma$ transition, $I = 0$ member of the octet 0^+ meson will contribute. If we only count the product of the coupling constants, $M(\eta^0 \to \pi^0 + 2\gamma) = \sqrt{3} M(\pi^0 \to \pi^0 + 2\gamma)$.

[14] Note that for the $\pi^0 \to \pi^0 + 2\gamma$ transition $I = 0$ unitary singlet 0^+ meson can also contribute.

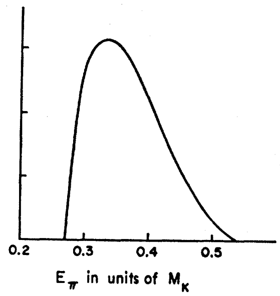

FIG. 2. Pion energy spectrum in the $K_2{}^0 \to \pi^0+2\gamma$ decay in the rest frame of $K_2{}^0$ meson computed on the basis of Fig. 1(a) using the $\eta^0 \to \pi^0+2\gamma$ vertex given by (4). The scale is arbitrary.

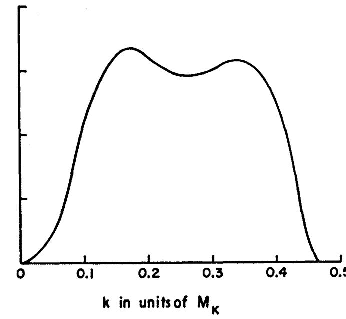

FIG. 3. Photon energy spectrum in the $K_2{}^0 \to \pi^0+2\gamma$ decay corresponding to the pion spectrum given in Fig. 2. The spectrum includes the photons coming from the decay of the secondary π^0-meson. The scale is arbitrary.

At present, our knowledge about the $SU(3)$ violation factor a is meager. If $|a| \ll 1$, i.e., $M(\eta \to 2\gamma)$ is much larger than the $SU(3)$ value, we obtain $R_K \simeq 2.1$ using the ratio (1). If a is close to the $SU(3)$ value 1, $R_K \simeq 0.18$. We therefore expect that unless deviation from the $SU(3)$ symmetry is very large, $R_K \lesssim 1$.

To summarize, if the branching ratio (1) is indeed correct, the rate $\Gamma(K_2{}^0 \to \pi^0+2\gamma)$ will at least be of the same order of magnitude or even larger than the rate $\Gamma(K_2{}^0 \to 2\gamma)$.[15] If the neglected pion pole term for the $K_2{}^0 \to \pi^0+2\gamma$ decay contributes in a constructive way, then the more drastic possibility that the rate $\Gamma(K_2{}^0 \to \pi^0+2\gamma)$ is comparable with the observed $\Gamma(K_2{}^0 \to \pi^0+\pi^0)$ may not be completely ruled out.

B. We add a few remarks on the kinematical features of the $K_2{}^0 \to \pi^0+2\gamma$ decay. In (A) we have discussed the rate of this decay based on a model where the $\eta^0 \to \pi^0+2\gamma$ vertex is predominantly given by the simple form (4) with an approximately constant coupling constant ξ. As mentioned before, however, (4) is not a unique form of the $\eta^0 \to \pi^0+2\gamma$ vertex. In fact the diagrams involving vector meson give rise to a different form of $\eta^0 \to \pi^0+2\gamma$ vertex although we expect their contribution to the amplitude of $K_2{}^0 \to \pi^0+2\gamma$ decay to be around 10 to 20 percent. If we use only the simplest $\eta^0 \to \pi^0+2\gamma$ vertex given by (4), the spectrum for the secondary pion with energy E_π (and momentum p_π) and the photon with energy k in the rest system of the $K_2{}^0$ meson will take a simple form

$$\int_{m_\pi}^{(E_\pi)_{\max}} dE_\pi \int_{\frac{1}{2}(m_K-E_\pi-p_\pi)}^{\frac{1}{2}(m_K-E_\pi+p_\pi)} dk(m_K{}^2+m_\pi{}^2-2m_KE_\pi)^2$$

$$= \int dE_\pi(E_\pi{}^2-m_\pi{}^2)^{1/2}(m_K{}^2+m_\pi{}^2-2m_KE_\pi)^2. \quad (14)$$

Figure 2 shows the pion energy spectrum which indicates a broad maximum around $E_\pi \simeq 0.35 m_K$. Therefore, the effective mass of the directly emitted two-photon system has a broad peak around $2m_\pi$.[16] This can serve to distinguish the decay $K_2{}^0 \to \pi^0+2\gamma$ from the one $K_2{}^0 \to \pi^0+\pi^0$ in an experiment, such as Ref. 9, where all four γ rays are detected. In the same model, the spectrum of the directly emitted photon is given by

$$\int dk(m_K{}^2-m_\pi{}^2-2m_Kk)^3 \frac{k^3}{(m_K-2k)^3}. \quad (15)$$

In Fig. 3 we have shown a very crude photon spectrum (computed by a slide rule) which includes the photons coming from the decay of the secondary π^0 meson using the π^0 energy spectrum given by (14). The background from $K_2{}^0 \to \pi^0+2\gamma$ decay to events called $K_2{}^0 \to \pi^0+\pi^0$, in the experiment of Ref. 10, is in the region $180<k<225$ MeV. Whereas the $K_2{}^0 \to 2\pi^0$ gamma rays give a flat energy spectrum, the spectrum shown in Fig. 3 illustrate some appreciable fall-off as the upper limit is approached.[16] It would be interesting to see whether there is any evidence for $K_2{}^0 \to \pi^0+2\gamma$ decays in the experiments that have measured the $K_2{}^0 \to \pi^0+\pi^0$ decays.

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[15] A crude estimate has been made by using a vector-meson dominant model including the effect of ω-ϕ mixing [S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. **136**, B1064 (1964)]. An estimate of the value of f_W can be obtained by using the current commutation relation and the hypothesis of partially conserved axial-vector current. Direct extension of the method of Callan and Treiman [C. G. Callan and S. B. Trieman, Phys. Rev. Letters **16**, 153 (1966)] gives $f_W=1.2\times10^{-7}$ whereas the use of Hara and Nambu's prescription for the extrapolated amplitude [Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966)] gives $f_W=4.2\times10^{-8}$. Using $\Gamma(\pi^0 \to 2\gamma)=7.4$ eV, these estimates give

$$\Gamma(K_2{}^0 \to \gamma+\gamma)/\Gamma(K_2{}^0 \to \text{all})=1.8\times10^{-3}$$

and 2.2×10^{-4}, respectively, compared with the experimental one, Eq. (8). See S. Oneda and J. C. Pati, Phys. Rev. **155**, 1621 (1967).

[16] Different form of $\eta^0 \to \pi^0+2\gamma$ vertex such as obtained from the intermediate vector-meson model gives a very different spectrum (Refs. 7 and 8). For instance, the pion energy spectrum increases continuously from zero at $E_\pi=m_\pi$ to maximum at $E_\pi=(E_\pi)_{\max}$. The existence of these diagrams will affect the upper ends of the pion and photon spectra to some extent.