Gravitational Scattering of Light by Light*

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The photon-photon interaction through the creation and annihilation of a virtual graviton is investigated in the center-of-mass system, and is found to have eight times the Newtonian value plus a polarization-dependent repulsive contact interaction. The gravitational scattering cross sections for various states of photon polarizations are obtained.

I. INTRODUCTION

THE scattering of light by light through the creation and annihilation of virtual electron-positron pairs has been investigated by several authors.[1] We shall here consider the scattering of light by light through the creation and annihilation of a virtual graviton, and also show how the gravitational interaction of photons depends on their states of polarization.

Although the gravitational interaction is usually very weak compared with the electromagnetic interaction except at extremely high energies, the situation here is different, because at very low energies the gravitational interaction of photons predominates over the electromagnetic interaction. Physically this could be understood by observing that when the photon energy is very small compared with the electron rest energy, the probability of the creation of an electron-positron pair even in a virtual state is quite small.

The scattering of light by light not only represents a basic phenomenon in quantum electrodynamics, but it may also have astrophysical applications. The present investigation, for instance, may be of interest in Wheeler's theory of geons.[2]

II. GRAVITATIONAL PHOTON-PHOTON SCATTERING

We consider the gravitational scattering of two photons in the center-of-mass system, where the propagation four-vectors of the photons are p and p' in the initial state, and q and q' in the final state, so that

$$\mathbf{p}=-\mathbf{q}\,,\quad \mathbf{p}'=-\mathbf{q}'\,,\quad p_0=q_0=p_0'=q_0'\,,$$
$$\mathbf{k}=\mathbf{p}'-\mathbf{p}=-(\mathbf{q}'-\mathbf{q})\,,\quad \mathbf{s}=\mathbf{p}'+\mathbf{p}=-(\mathbf{q}'+\mathbf{q})\,. \quad (1)$$

The lowest-order diagrams for the process under consideration are shown in Fig. 1. The diagram (a'), which can be obtained from the diagram (a) by interchanging the roles of p' and q', represents the exchange effect, and its contribution will be included in the calculation of the scattering cross section but ignored in the derivation of the photon-photon potential. Moreover, since the diagram (b) remains unchanged on interchanging p' and q', we shall divide its contribution into two parts such that they can be obtained from each other by interchanging p' and q', and then drop the exchange part for the derivation of the photon-photon potential.

Following the earlier treatment[3] of the gravitational interaction of elementary particles, we find that the scattering matrix element for the diagram (a) is

$$\begin{aligned}S_a=&\,i(2\pi)^4\delta(p+q-p'-q')\\&\times a_i^\dagger(\mathbf{p}')a_j(\mathbf{p})a_l^\dagger(\mathbf{q}')a_m(\mathbf{q})(\kappa^2c\hbar/8p_0^2\mathbf{k}^2)\\&\times[(8p_0^2+\mathbf{k}^2)(\delta_{ij}k_lk_m+\delta_{lm}k_ik_j)\\&+4p_0^2(\delta_{il}k_jk_m+\delta_{jm}k_ik_l)\\&-(4p_0^2+\mathbf{k}^2)(\delta_{im}k_jk_l+\delta_{jl}k_ik_m)\\&+(16p_0^4-4p_0^2\mathbf{k}^2-\tfrac{1}{2}\mathbf{k}^4)\delta_{ij}\delta_{lm}\\&\qquad+\tfrac{1}{2}\mathbf{k}^4(\delta_{il}\delta_{jm}+\delta_{im}\delta_{jl})]\,,\end{aligned}\quad (2)$$

while the scattering matrix element for the diagram (b) is

$$\begin{aligned}S_b=&\,i(2\pi)^4\delta(p+q-p'-q')\\&\times a_i^\dagger(\mathbf{p}')a_j(\mathbf{p})a_l^\dagger(\mathbf{q}')a_m(\mathbf{q})(\kappa^2c\hbar/8p_0^4)\\&\times[2p_0^2(\delta_{il}k_jk_m+\delta_{jm}k_ik_l)\\&-(2p_0^4+p_0^2\mathbf{k}^2-\tfrac{1}{4}\mathbf{k}^4)\delta_{il}\delta_{jm}\\&\qquad+2p_0^4(\delta_{ij}\delta_{lm}+\delta_{im}\delta_{jl})]\,,\end{aligned}\quad (3)$$

where the photon annihilation operators $a_j(\mathbf{p})$ and $a_m(\mathbf{q})$ and the creation operators $a_i^\dagger(\mathbf{p}')$ and $a_l^\dagger(\mathbf{q}')$ satisfy the relations

$$p_ja_j(\mathbf{p})=0\,,\quad q_ma_m(\mathbf{q})=0\,,$$
$$p_i'a_i^\dagger(\mathbf{p}')=0\,,\quad q_l'a_l^\dagger(\mathbf{q}')=0\,, \quad (4)$$

and the gravitational coupling constant κ is related to Newton's constant of gravitation G as

$$\kappa^2=16\pi G/c^4\,. \quad (5)$$

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[1] H. Euler, Ann. Physik **26**, 398 (1936); A. Achieser, Physik Z. Sowjetunion **11**, 263 (1937); R. Karplus and M. Neuman, Phys. Rev. **83**, 776 (1951). For a recent discussion, see A. A. Varfolomeev, Zh. Eksperim. i Teor. Fiz. **50**, 1024 (1966) [English transl.: Soviet Phys.—JETP **23**, 681 (1966)].

[2] J. A. Wheeler, Phys. Rev. **97**, 511 (1955).

[3] B. M. Barker, S. N. Gupta, and R. D. Haracz, Phys. Rev. **149**, 1027 (1966). See this paper also for the notation used here.

It is convenient to choose one of the polarization vectors of each photon in the plane of scattering and the other polarization vector perpendicular to this plane, and then carry out the transformation to the circularly polarized photons.[3] Applying this procedure to (2) and (3) and dropping the exchange part in (3), we obtain

$$\begin{aligned}S_a+S_b=&(-i/c\hbar)(2\pi)^4\delta(p+q-p'-q')\\&\times[V_{++}(\mathbf{k})a_+^\dagger(\mathbf{p}')a_+(\mathbf{p})a_+^\dagger(\mathbf{q}')a_+(\mathbf{q})\\&+V_{--}(\mathbf{k})a_-^\dagger(\mathbf{p}')a_-(\mathbf{p})a_-^\dagger(\mathbf{q}')a_-(\mathbf{q})\\&+V_{+-}(\mathbf{k})a_+^\dagger(\mathbf{p}')a_+(\mathbf{p})a_-^\dagger(\mathbf{q}')a_-(\mathbf{q})\\&\quad+V_{-+}(\mathbf{k})a_-^\dagger(\mathbf{p}')a_-(\mathbf{p})a_+^\dagger(\mathbf{q}')a_+(\mathbf{q})],\end{aligned}\tag{6}$$

with

$$V_{++}(\mathbf{k})=V_{--}(\mathbf{k})=-(2c^2\hbar^2p_0^2\kappa^2/\mathbf{k}^2),\tag{7}$$
$$\begin{aligned}V_{+-}(\mathbf{k})=V_{-+}(\mathbf{k})=&-(2c^2\hbar^2p_0^2\kappa^2/\mathbf{k}^2)\\&\times[1-(\mathbf{k}^2/4p_0^2)-(\mathbf{k}^4/16p_0^4)+(\mathbf{k}^6/64p_0^6)],\end{aligned}$$

where a_+ and $a_+^\dagger$ denote the annihilation and creation operators for photons with their spin axes parallel to their directions of motion, while a_- and $a_-^\dagger$ denote those for photons with their spin axes antiparallel to their directions of motion.

The gravitational potential for photon-photon interaction can be obtained from (7) by the relation

$$V(\mathbf{r})=(2\pi)^{-3}\int d\mathbf{k}\,e^{i\mathbf{k}\cdot\mathbf{r}}V(\mathbf{k}),\tag{8}$$

which gives, on denoting the relativistic mass of the photon as μ,

$$V_{++}(\mathbf{r})=V_{--}(\mathbf{r})=-8G\mu^2/r,\tag{9}$$

while $V_{+-}(\mathbf{r})$ and $V_{-+}(\mathbf{r})$ differ from (9) only by repulsive contact terms.

To obtain the gravitational scattering cross section for two circularly polarized photons from (6) and (7), we consider the following two cases separately:

1. When both the photon spins are parallel to the directions of motion or both are antiparallel, we obtain for the differential scattering cross section after including the exchange effect

$$d\sigma_A/d\Omega=(p_0/4\pi c\hbar)^2(2c^2\hbar^2p_0^2\kappa^2)^2[(1/\mathbf{k}^2)+(1/\mathbf{s}^2)]^2,\tag{10}$$

with

$$\mathbf{k}^2=2p_0^2(1-\cos\theta),\quad \mathbf{s}^2=2p_0^2(1+\cos\theta),\tag{11}$$

which gives, on denoting the photon energy as $h\nu$,

$$d\sigma_A/d\Omega=[64G^2(h\nu)^2/c^8\sin^4\theta].\tag{12}$$

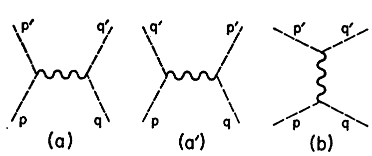

FIG. 1. Lowest-order diagrams for the gravitational scattering of light by light. A dashed line denotes a photon, while a wavy line denotes a graviton.

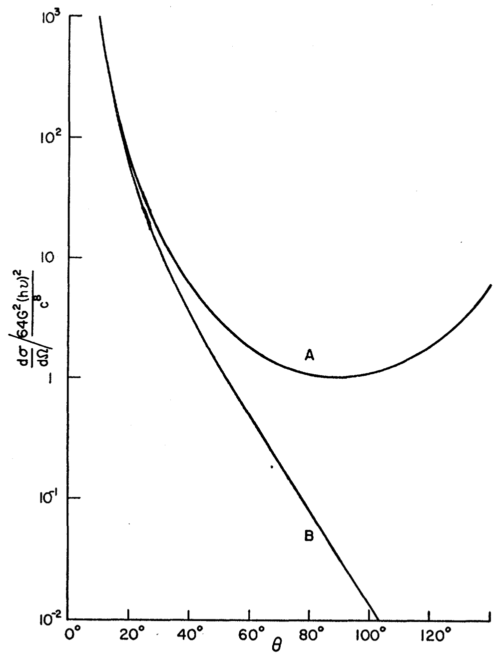

FIG. 2. Differential cross sections on the logarithmic scale for the gravitational scattering of two circularly polarized photons. Curve A refers to the case when the photons are similarly polarized, and curve B refers to the case when they are dissimilarly polarized.

2. When one of the photon spins is parallel to the direction of motion while the other one is antiparallel, we find

$$\begin{aligned}d\sigma_B/d\Omega=&(p_0/4\pi c\hbar)^2(2c^2\hbar^2p_0^2\kappa^2)^2\\&\times[(1/\mathbf{k}^2)-(1/4p_0^2)-(\mathbf{k}^2/16p_0^4)+(\mathbf{k}^4/64p_0^6)]^2\end{aligned}\tag{13}$$

or

$$\begin{aligned}d\sigma_B/d\Omega=&[64G^2(h\nu)^2/c^8\sin^4\theta]\\&\times[1+\sin^2(\tfrac{1}{2}\theta)]^2\cos^{12}(\tfrac{1}{2}\theta),\end{aligned}\tag{14}$$

where θ denotes the angle between the initial and final directions of the photon with its spin parallel to the direction of motion.

The differential cross sections (12) and (14) are shown in Fig. 2, which shows that the gravitational photon-photon scattering is polarization independent for small scattering angles, but depends markedly on the photon polarizations for large scattering angles. It follows from (12) and (14) that the scattering cross section for unpolarized photons[4] is given by

$$\begin{aligned}d\sigma/d\Omega=&[32G^2(h\nu)^2/c^8\sin^4\theta]\\&\times[1+(1+\sin^2\tfrac{1}{2}\theta)^2\cos^{12}(\tfrac{1}{2}\theta)\\&\quad+(1+\cos^2\tfrac{1}{2}\theta)^2\sin^{12}(\tfrac{1}{2}\theta)].\end{aligned}\tag{15}$$

[4] This result presumably has a correspondence-principle connection with the formula for the deflection of one pencil of light by another, an analyzed by R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Clarendon Press, Oxford, England, 1934), but no attempt has been made here to trace out this connection.