## Implications of Regge Behavior for Processes Involving Photons

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(Received 16 January 1967)

Properties of the crossing matrix for helicity amplitudes are enumerated and then exploited in a simple Regge-pole model of photon-hadron interactions. We find that: (1) The vacuum or Pomeranchuk trajectory is absent in forward Compton scattering of real photons on nucleons or pions. Thus the total nuclear cross section for photons or nucleons should vanish asymptotically. (2) The amplitudes entering the forward spin-flip Compton scattering of virtual photons on protons, which are crucial in the proton structure contributions to the ground-state hyperfine splitting (hfs) in hydrogen, may be chosen so that they do not require a subtraction when writing dispersion relations for them in the energy variable. One must look elsewhere than to the high-energy behavior of the amplitudes for the erasure of the discrepancy between theory and experiment on the hfs. (3) In forward photoproduction of vector mesons on protons, the polarization of the mesons should be predominantly longitudinal at "high energy." (4) In the differential cross section  $d\sigma/dt$ for photoproduction of neutral pions, there should be a dip at  $t \approx -0.5$  BeV<sup>2</sup> because of a nonsense zero in all crossed-channel helicity amplitudes.

## I. INTRODUCTION

ROM the earliest days of Regge-pole theory,<sup>1</sup> the natural realm of application has been to scattering processes involving hadrons alone. There has been an occasional outcropping of activity in considerations of reactions also involving photons,<sup>2-4</sup> but nothing resembling the magnitude of intesest in pure hadron interactions. A significant step in the direction of investigating the consequences of the hypothesis of Regge behavior for photonic processes has been taken by Harari.<sup>5,6</sup> He demonstrates that the assumption that the high-energy behavoir of the forward non-spin-flip Compton scattering amplitudes for virtual photons on hadrons is governed by Regge-pole exchanges in the crossed channel offers a simple explanation for both the failure of calculations of  $\Delta I = 1$  electromagnetic mass splittings and the apparent success of the predictions of  $\Delta I = 2$  splittings. In the present work we continue this investigation and first further study Compton scattering of both real and massive photons on hadrons. We then consider the photoproduction of vector mesons and pions in the same context.

The essence of our analysis below, which does not attempt a detailed enumeration of the structure of photon-hadron processes, is the assumption of Regge behavior for scattering amplitudes at high energy. In

particular this includes the assumption that the zeroenergy intercept of the vacuum or Pomeranchuk trajectory,  $\alpha_p(0)$ , is 1. This, along with the crossing relations for helicity amplitudes, is our input. Our output is a series of statements of a kinematical nature about the behavior of the amplitudes in the forward direction; statements which are independent of the detailed dynamics underlying the situation.

Our most striking prediction, which we have subsequently discovered was found by Mur<sup>2</sup> in 1963, is that the Pomeranchuk trajectory, with  $\alpha(0)=1$ , does not contribute to the forward scattering of real photons on nucleons or pions, and thus, by the optical theorem, the nuclear part of the total cross sections for these will vanish at large energy. The noncontribution of the Pomeranchon is due to the presence of a nonsense zero<sup>7</sup> at  $\alpha(0) = 1$  in the *t*-channel helicity amplitudes which contribute to forward scattering in the direct channel. We next examine in more detail the forward Compton scattering of massive photons and conclude that one of the spin-flip amplitudes, as defined by Iddings,<sup>8</sup> requires a subtraction in writing for it is a dispersion relation in energy. It is possible, however, to define linear combinations of Iddings's amplitudes<sup>9</sup> which do not require subtractions. Since these spin-flip amplitudes represent the proton structure contributions to the ground-state hyperfine splitting (hfs) in hydrogen, we may say, within the context of the Regge model, that if one is judicious enough to use the amplitudes of Drell and Sullivan in Ref. 9, he cannot seek to explain the discrepancy between theory and experiment in the hfs problem by a subtraction constant.

Other predictions, which are probably more amenable to experimental test, are the asymptotic nonsuppression of the forward amplitude for photoproduction of

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<sup>\*</sup> National Science Foundation Postdoctoral Fellow.

Work supported by the U. S. Air Force Office of Research, Air Research and Development Command, under Contract No. AF 49(638)-1545.

<sup>&</sup>lt;sup>1</sup>G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 5, 580

<sup>&</sup>lt;sup>1</sup>G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 5, 580 (1960); 8, 41 (1962).
<sup>2</sup>V. D. Mur, Zh. Eksperim. i. Teor. Fiz. 44, 2173 (1963); 45, 1051 (1963) [English transl.: Soviet Phys.—JETP 17, 1458 (1963); 18, 727 (1964)].
<sup>3</sup> J. Zmuidinas, Ann. Phys. (N. Y.) 27, 227 (1964).
<sup>4</sup> G. Zweig, Nuovo Cimento 32, 689 (1964); A. V. Berkov, E. D. Zhizzhin, V. D. Mur, and Yu. P. Nikitin, Zh. Eksperim. i Teor. Fiz. 45, 1585 (1964) [English transl.: Soviet Phys.—JETP 18, 1091 (1964)

<sup>&</sup>lt;sup>6</sup> H. Harari, Phys. Rev. Letters 17, 1303 (1966).
<sup>6</sup> T. L. Trueman and G. C. Wick Ann. Phys. (N. Y.) 26, 322 (1964); I. J. Muzinich J. Math. Phys. 5, 1481 (1964).

<sup>&</sup>lt;sup>7</sup> M. Gell-Mann, in Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN, edited by J. Prehter (CERN, Geneva, 1962), p. 533. <sup>8</sup> C. K. Iddings, Phys. Rev. 138, B446 (1965).

<sup>&</sup>lt;sup>9</sup>S. D. Drell and J. D. Sullivan, Phys. Rev. 154, 1477 (1967).

differential cross section  $d\sigma/dt$  in the process  $\gamma + p \rightarrow \pi^0 p$ . In the next section we present the properties of the helicity crossing relations that we shall need and discuss the derivation of some exact and asymptotic selection rules. The third section is concerned with the applications mentioned above, while the final section is devoted to a summary and discussion of the results and entertains some speculations.

#### II. THE CROSSING MATRIX

We consider the scattering of two particles of spins  $J_a$  and  $J_b$ , masses  $m_a$  and  $m_b$ , and helicities a and b into two particles of spins  $J_c$  and  $J_d$ , masses  $m_c$  and  $m_d$ , and helicities c and d. Our helicity amplitudes A are Lorentz invariants, the S matrix for the process

$$a+b \rightarrow c+d$$
 being

$$S(a+b \rightarrow c+d) = \delta_{cd;ab} + i(2\pi)^4 \delta^4(p_a+p_b-p_c-p_d) \\ \times N_a N_b N_c N_d A_{cd,ab}(s,t) ,$$

where  $N_i = (m_i/p_{0i})^{1/2}$  for fermions and  $1/(2p_{0i})^{1/2}$  for bosons. The variables *s* and *t* are as usual  $s = -(p_a + p_b)^2$ and  $t = -(p_a - p_c)^2$ . From Trueman and Wick or Muzinich<sup>6</sup> we find the relation between the s-channel helicity amplitudes  $A_{cd,ab}(s,t)$  and the helicity amplitudes  $M_{c\bar{a},\bar{d}b}$  in the *t* channel  $(\bar{d}+b\rightarrow\bar{a}+c)$ ,

$$A_{cd,ab}(s,t) = \sum_{i,j,k,l} d_{ic}{}^{J_c}(X_c) d_{ja}{}^{J_a}(X_a) M_{ij,kl}(s,t) \\ \times d_{kd}{}^{J_d}(X_d) d_{lb}{}^{J_b}(X_b).$$
(1)

The  $d_{m'm}^{j}(X)$  are the usual rotation matrices<sup>10</sup> with arguments

$$\cos X_{a} = \frac{\xi_{a}(s + m_{a}^{2} - m_{b}^{2})(t + m_{a}^{2} - m_{c}^{2}) - 2m_{a}^{2}(m_{c}^{2} - m_{a}^{2} + m_{b}^{2} - m_{d}^{2})}{\{[s - (m_{a} + m_{b})^{2}][s - (m_{a} - m_{b})^{2}][t - (m_{a} + m_{c})^{2}][t - (m_{a} - m_{c})^{2}]\}^{1/2}},$$
(2)

and similar expressions for the other crossing angles.<sup>6</sup>  $\xi_a = \pm 1$  depending on whether the particle in question is crossed or not in going from the s channel to the tchannel.

The following observations are crucial to the discussion: (1) If  $m_a = 0$ ,  $\cos X_a = \xi_a$  and  $X_a = 0$  or  $\pi$  as  $\xi_a = \pm 1$ . For these angles  $d_{\lambda\mu}{}^{J}(X)$  takes the simple forms,  $d_{\lambda\mu}{}^J = \delta_{\lambda\mu}$  and  $d_{\lambda\mu}{}^J(\pi) = \delta_{\lambda,-\mu}$ . The physical interpretation of this is quite straightforward. When the particle a is massless,  $\lambda_a$  may only be  $\pm J_a$ . If the crossing angle were anything but a multiple of  $\pi$ , it would mix in longitudinal components for  $J_a \ge 1$  and this is simply not allowed. The reduction of  $\cos X_a$  to  $\xi_a$  is, of course, independent of s or t.

(2) Suppose  $m_a=0$  and  $m_b=m_d$  (as, for example, in pion or vector-meson photoproduction on nucleons), then in the case that s is large and the scattering angle in the s channel is 0,  $\cos\theta_s = 1$ ,  $t = -(m_b^2 m_c^4 / s^2) + O(1/s^3)$ . The crossing angle  $X_c$  becomes

$$\cos X_{c} = -\xi_{c} [1 - (m_{b}^{2} + m_{c}^{2})^{2} / s^{2}) + O(1/s^{3})].$$

Under these circumstances, the element of the crossing matrix corresponding to particle c becomes

$$d_{c'c} J_{c}(X_{c}) = \delta_{c'-\xi_{c}c} + \alpha_{c'c} J_{c}/s + O(1/s^{2}).$$
(3)

 $\alpha_{c'c} J_c$  is essentially the matrix element  $\langle J_{c'} | J_{u} | J_c \rangle$ times some masses.

These two kinematic properties of the helicity crossing matrix will comprise the bulk of our input below. The other piece of information which we shall exploit is the nonsense property of the rotation matrices  $d_{m'm}^{J}(X)$ . This simply states that for integral J, say  $J_0$ , less than the maximum of m and m',  $d_{m'm}$  vanishes at least as  $(J-J_0)^{1/2}$ . Thus prepared we consider some applications.

#### **III. APPLICATIONS**

#### A. Forward Compton Scattering of Real Photons on Nucleons

For this process there are two independent helicity amplitudes. In the *s* channel where  $\gamma(k,\lambda_a) + N(p,\lambda_b) \rightarrow N(p,\lambda_b)$  $\gamma(k,\lambda_c) + N(p,\lambda_d)$  we write them as

$$A^{p} = A_{1,-1/2;1,-1/2}(s,t)$$
 and  $A^{a} = A_{1,1/2;1,1/2}(s,t)$ . (4)

The superscripts p and a refer to photon and nucleon spins parallel and antiparallel, respectively. From the first observation in the previous section we see that the only t-channel amplitudes which cross into these, for any values of s and t are, for

$$\bar{N}(\lambda_{d'}) + N(\lambda_{b'}) \to \gamma(\lambda_{a'}) + \gamma(\lambda_{b'}),$$

$$M_1 \equiv M_{1,-1;1/2,1/2} \quad \text{and} \quad M_2 \equiv M_{1,-1;1/2,-1/2}.$$
(5)

Upon writing partial-wave expressions for the M's, one finds a  $d_{02}{}^{J}(\theta_t)$  in the first and a  $d_{12}{}^{J}(\theta_t)$  in the second. After Reggeizing in the manner prescribed by Gell-Mann et al.,<sup>11</sup> namely, choosing that partial-wave helicity amplitude which has no fixed singularities in the complex J plane to contain the Regge poles, one observes an explicit  $\alpha(t)[\alpha(t)-1]$  in the Regge-pole contributions to the amplitude. At t=0, therefore,  $\alpha(0)=0$ is a nonsense-nonsense point in  $M_2$  and a sense-nonsense point in  $M_1$ .  $\alpha(0) = 1$  is a sense-nonsense point in  $M_2$  and a nonsense-nonsense point in  $M_1$  (since we are at t=0). A trajectory which passes through 0 or 1 at t=0, therefore, will not contribute to the amplitude. In particular, the vacuum or Pomeranchuk trajectory will have a

<sup>&</sup>lt;sup>10</sup> M. E. Rose, *Elementary Theory of Angular Momentum*, (John Wiley & Sons, Inc., New York, 1957). <sup>11</sup> M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

contribution which vanishes at t=0. This is certainly the "leading" trajectory, being the unitarity bound, and its absence at t=0 implies that at high energy (wherever that may be) the differential cross section for Compton scattering on nucleons will show a dip as  $t \rightarrow 0$  and the nonleading trajectories, of which the prime candidate is that on which the  $f^0(1250)$  lies, will dominate. Alternatively using the optical theorem to relate  $\text{Im}A^{a(p)}(s,0)$ to  $\sigma^{a(p)}(s)$ , the total (nuclear) absorption cross section of photons on protons with spins antiparallel and parallel, respectively, we predict that both  $\sigma^{a}(s)$  and  $\sigma^{p}(s)$ vanish asymptotically as  $s^{\alpha_f \circ (0)-1} \approx s^{-0.5, 12}$ 

This prediction, which is contained in the papers of Mur<sup>2</sup> (though we feel this is a more transparent derivation), also holds for Compton scattering of real photons on spinless particles. Unfortunately these statements are, at best, difficult to test experimentally, particularly because of the competition of higher-order electromagnetic processes. These must be omitted since we make the Reggeization assumption only for the Compton amplitude to order  $e^2$ , that is, there are no intermediate states besides hadronic ones. Only this amplitude sufficiently resembles pure hadron scatterings so that an assumed Regge behavior for the latter may lead us to conjecture one for the former. Since, in particular our amplitude does not satisfy elastic unitarity the difficulty associated with  $\sigma_{\text{elastic}}(\gamma p) > \sigma_{\text{tot}}(\gamma p)$ ,<sup>2</sup> which arises from the contribution of the Pomeranchon to the elastic cross section, is simply not present.<sup>13</sup>

The curious asymmetry between hadron-nucleon asymptotic cross sections, all of which become constant (up to logs factors) at high energy, and the power law drop-off at the  $\gamma p$  total cross section can be removed by the dropping of almost any of our assumptions. For example, in a different context<sup>14</sup> it has been suggested that  $\alpha(0)$  for the Pomeranchuk trajectory is less than one; this of course restores its contribution to  $\sigma_{tot}(\gamma p)$ although it will be much smaller than expected on non-Regge grounds because of the  $\alpha(0)-1$  coefficient. Rather than dwell on other ways out of the striking prediction of vanishing  $\gamma p$  total cross sections, we proceed to more examples and leave any extended discussion to the last section.

## B. Forward Compton Scattering of Massive **Photons on Nucleons**

In the consideration of the proton structure contributions to the ground-state hyperfine splitting (hfs) in hydrogen<sup>8,9</sup> one encounters the forward spin-flip Compton scattering amplitudes for virtual photons on protons. An essential ingredient in the calculation of the

hfs is the assumption of unsubtracted dispersion relations in the energy for the scalar invariants in the decomposition of the Compton amplitude. We now examine the validity of this assumption within the bounds of Regge-pole model. Our main conclusion is that the Pomeranchuk trajectory can contribute here, in sharp distinction to the massless photon case, and that one of the amplitudes defined by Iddings in Ref. 8, namely  $D(\nu,k^2)$ , requires a subtraction. It is possible, however, to define linear combinations of Iddings' amplitudes<sup>9</sup> such that a subtraction is not necessary in either of them. Thus using the amplitudes of Drell and Sullivan one may safely (if he accepts the Regge-pole argument) use unsubtracted dispersion relations. The apparent discrepancy of  $\sim 40\pm 20$  parts per million between theory and experiment for the hfs cannot, therefore, be blamed on a subtraction constant. It is in fact quite possible that experiment may provide us with a changed value of the fine structure constant which will eliminate the hfs discrepancy and will then make the question of subtraction constants far less viable.

Let us now examine some of the details involved in this conclusion. We address ourselves to the forward Compton scattering shown in Fig. 1. The incident photon has momentum k,  $(mass)^2 = -k^2$ , and polarization helicity  $\epsilon_{\nu}(\lambda_b)$ . The target proton has momentum p, mass m, and helicity  $\lambda_a$ . The final state has a proton of momentum k,  $(mass)^2 = -k^2$ , and polarization helicity  $\epsilon_{\mu}'(\lambda_d)$  and a proton of momentum p, helicity  $\lambda_c$ . We choose the energy variable to be  $\nu = -p \cdot k/m$ and will be concerned with the large- $\nu$  behavior of the invariants. The S matrix for this process is

$$S[\gamma(k,\epsilon(\lambda_{b}))+N(p,\lambda_{a}) \rightarrow \gamma(k,\epsilon'(\lambda_{d}))+N(p,\lambda_{d})] = \delta_{cd;ab}$$

$$+i(2\pi)^{4} \left(\frac{m^{2}}{p_{0}^{2}4k_{0}^{2}}\right)^{1/2} \delta^{4}(p+k-p-k)\bar{u}(p,\lambda_{c})\epsilon_{\mu'}(\lambda_{d})$$

$$\times F_{\mu\nu}(k^{2},\nu)\epsilon_{\nu}(\lambda_{b})u(p,\lambda_{a}) = \delta_{cd;ab} + i(2\pi)^{4} \left(\frac{m^{2}}{p_{0}^{2}4k_{0}^{2}}\right)^{1/2}$$

$$\times \delta^{4}(p+k-p-k)A_{\lambda_{c}\lambda_{d},\lambda_{a}\lambda_{b}}(k^{2},\nu). \quad (6)$$

 $F_{\mu\nu}$  may be written as<sup>5,9,15</sup>

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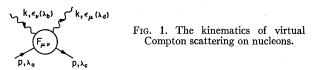
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$$F_{\mu\nu}(k^{2},\nu) = t_{1}(k^{2},\nu)(k^{2}\delta_{\mu\nu}-k_{\mu}k_{\nu})+t_{2}(k^{2},\nu)$$

$$\times \left(\nu^{2}\delta_{\mu\nu}+k^{2}\frac{p_{\mu}p_{\nu}}{m^{2}}+\frac{\nu}{m}(p_{\mu}k_{\nu}+p_{\nu}p_{\mu})\right)+\frac{1}{2}G_{1}(k^{2},\nu)$$

$$\times \left\{\left[\gamma_{\nu},\gamma\cdot k\right]p_{\mu}-\left[\gamma_{\mu},\gamma\cdot k\right]p_{\nu}-\left[\gamma_{\mu},\gamma_{\nu}\right]m\nu\right\}+\frac{1}{2}G_{2}(k^{2},\nu)$$

$$\prime \qquad \times \left\{\left[\gamma_{\nu},\gamma\cdot k\right]k_{\mu}-\left[\gamma_{\mu},\gamma\cdot k\right]k_{\nu}+\left[\gamma_{\mu},\gamma_{\nu}\right]k^{2}\right\}.$$
(7)



<sup>15</sup> J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

<sup>&</sup>lt;sup>12</sup> S. C. Frautschi (unpublished). <sup>13</sup> We wish to thank W. I. Weisberger for a discussion about this point. Incidentally, in a model in which the full Compton amplitude Reggeizes, the paradox  $\sigma_{\text{elastic}} > \sigma_{\text{tot}}$  only arises at enormous energies. Since  $\sigma_{\text{tot}} \sim \alpha(s/m_p^2)^{-0.5}$  and  $\sigma_{\text{elastic}} \sim \alpha^2/\ln(s/m_p^2)$ ,  $S \sim m_p^2/\alpha^2$  or  $^3 \sim 10^{4-6}$  (BeV)<sup>2</sup> is needed.

<sup>&</sup>lt;sup>14</sup> N. Cabibbo, J. J. Kokedee, L. Hurwitz, and Y. Ne'eman, Nuovo Cimento 45, 245 (1966).

and

In the forward direction there are four nonvanishing helicity amplitudes which we choose to be

$$\begin{aligned} A_1(k^2,\nu) &\equiv A_{1/2,-1;1/2,-1}(k^2,\nu), \ A_2(k^2,\nu) &\equiv A_{1/2,0;1/2,0}(k^2,\nu), \\ &A_3(k^2,\nu) &\equiv A_{1/2,1;-1/2,0}(k^2,\nu), \end{aligned}$$

and

$$A_4(k^2,\nu) \equiv A_{1/2,1;1/2,1}(k^2,\nu).$$
(8)

At  $k^2=0$  only  $A_1$  and  $A_4$  and  $t_2$  and  $G_1$  survive. The connection between the  $A_i$  and the invariant amplitudes is given by

$$A_{1}(k^{2},\nu) = [\nu^{2}t_{2}(k^{2},\nu) + k^{2}t_{1}(k^{2},\nu)] + [m\nu G_{1}(k^{2},\nu) - k^{2}G_{2}(k^{2},\nu)], \quad (9)$$

$$A_{2}(k^{2},\nu) = k^{2} [t_{1}(k^{2},\nu) - t_{2}(k^{2},\nu)], \qquad (10)$$

$$A_{3}(k^{2},\nu) = \sqrt{2}(-k^{2})^{1/2} [mG_{1}(k^{2},\nu) - \nu G_{2}(k^{2},\nu)], \qquad (11)$$

and

$$A_{4}(k^{2},\nu) = \left[\nu^{2}t_{2}(k^{2},\nu) + k^{2}t_{1}(k^{2},\nu)\right] - \left[m\nu G_{1}(k^{2},\nu) - k^{2}G_{2}(k^{2},\nu)\right].$$
(12)

These may be solved for the invariants to find

$$(k^{2}+\nu^{2})t_{1}(k^{2},\nu) = \frac{1}{2} \left[ A_{1}(k^{2},\nu) + A_{4}(k^{2},\nu) \right] + \frac{\nu^{2}}{k^{2}} A_{2}(k^{2},\nu), \quad (13)$$

$$(k^{2}+\nu^{2})t_{2}(k^{2},\nu) = \frac{1}{2} \left[ A_{1}(k^{2},\nu) + A_{4}(k^{2},\nu) \right] -A_{2}(k^{2},\nu), \quad (14)$$

$$mG_{1}(k^{2},\nu)(k^{2}-\nu^{2}) = \frac{1}{2}\nu \left[A_{4}(k^{2},\nu)-A_{1}(k^{2},\nu)\right] + \frac{(-k^{2})^{1/2}}{\sqrt{2}}A_{3}(k^{2},\nu), \quad (15)$$

and

$$(k^{2}-\nu^{2})G_{2}(k^{2},\nu) = \frac{1}{2} \left[ A_{4}(k^{2},\nu) - A_{1}(k^{2},\nu) \right] - \frac{\nu A_{3}(k^{2},\nu)}{\sqrt{2}(-k^{2})^{1/2}}.$$
 (16)

If the leading behavior of the  $A_i$  is  $\nu^{\alpha}$ , then the behavior of the invariants is

$$t_1 \rightarrow \nu^{\alpha}, \quad t_2 \rightarrow \nu^{\alpha-2}, \quad G_1 \rightarrow \nu^{\alpha-1}, \quad \text{and} \quad G_2 \rightarrow \nu^{\alpha-1}.$$
 (17)

The question to answer, naturally, is what is the largest  $\alpha$  allowed within the context of the Regge model we have been following. We answer this question by considering the helicity amplitudes between states of definite parity and appropriate symmetry for the process  $\gamma + \gamma \rightarrow N + \overline{N}$ .<sup>16</sup> Then we cross to the direct channel using Eq. (1) and ask whether at t=0, any of the eight *t*-channel helicity amplitudes which contain the Pomeranchuk trajectory contribute to the  $A_i$ . The answer to this question is that each of the *s*-channel helicity ampli-

tudes  $A_1 \cdots A_4$  receives a contribution from one or more of the eight *t*-channel amplitudes in which the Pomeranchuk trajectory appears. In the combination  $A_1(k^2,\nu) - A_4(k^2,\nu)$ , however, the contributions from the Pomeranchon cancel out and this combination behaves at large  $\nu$  as  $\nu^{\alpha}$ ,  $\alpha < 1$ . This combination of amplitudes is just  $A^p - A^a$  of Eq. (2) for  $k^2 \neq 0$ .

The spin-flip amplitudes of Iddings<sup>8</sup> are called  $D(k^2,\nu)$ and  $G(k^2,\nu)$  and are related to  $G_1$  and  $G_2$  by (m is set equal to 1)

 $G(k^2,\nu) = -G_2(k^2,\nu)$ 

$$D(k^2,\nu) = G_1(k^2,\nu) - \nu G_2(k^2,\nu),$$

and to the  $A_i$  by (16) and

$$D(k^{2},\nu) = -\frac{A_{3}(k^{2},\nu)}{\sqrt{2}(-k^{2})^{1/2}}.$$
(19)

 $G(k^{2},\nu)$  behaves as  $\nu^{\alpha_{p}(0)-1} = \text{constant}$  for large  $\nu$  and, since it is an odd function of  $\nu$ , requires no subtraction.  $D(k^{2},\nu)$  behaves as  $\nu^{\alpha_{p}(0)} = \nu$  and requires one subtraction.

We may, however, choose as our invariants the  $H_i$  of Drell and Sullivan:

$$H_1(k^2,\nu) = \frac{1}{2}G_1(k^2,\nu) \text{ and } H_2(k^2,\nu) = -[G_2(k^2,\nu)/2\nu].$$
 (20)

Since the large- $\nu$  behavior of  $H_1$  is determined by  $(A_4-A_1)/\nu$  which behaves as  $\nu^{\alpha-1}$  with  $\alpha < 1$  because the Pomeranchon does not enter, it requires no subtraction. Also  $H_2$  does not need a subtraction since it behaves as  $\nu^{\alpha_p(0)-2}=1/\nu$ , its high-energy behavior being determined by  $A_3/\nu^2$ .

#### C. Forward Photoproduction of Vector Mesons

The production of high-energy photons at the CEA and DESY has made possible the gathering of experimental information on the photoproduction of vector mesons from proton and other targets.<sup>17</sup> The incident photon energies do not yet exceed 6 BeV, which may be too low for predictions of a Regge-pole model to be true. We present this example then to anticipate the results of higher-energy experiments and to provide a contact with the standard models<sup>17–19</sup> which are typically variants of diffraction mechanisms.

In Sec. II we have chosen observation (2) to correspond to the kinematics of photoproduction, with  $m_a=0=$  the mass of the photon,  $m_c=$  the mass of the vector meson,  $m_V$ , and  $m_b=m_d=$  the mass of the proton,  $m_p$ . The argument contained there tells us that for  $S \gg (m_p^2 + m_V^2)$  or lab energies large compared to a few BeV, the vector meson is behaving at  $\theta_s \approx 0$  as follows:

$$A(1\lambda_a 1\lambda_c) \approx \sum d_{\lambda_c \lambda_c'} d_{\lambda_a \lambda_{a'}} M_{11,\lambda_a' \lambda_c'}, A(1\lambda_a 0\lambda_c) \approx \sum d_{\lambda_c \lambda_c'} d_{\lambda_a \lambda_{a'}} M_{10,\lambda_{a'} \lambda_{c'}}.$$

<sup>17</sup> H. Harari, Phys. Rev. 155, 1565 (1967), and references therein.

<sup>&</sup>lt;sup>16</sup> For an indication of the lengthy details involved one may see M. Gell-Mann, M. L. Goldberger, and F. E. Low, Rev. Mod. Phys. **36**, 640 (1964).

 <sup>&</sup>lt;sup>18</sup> S. M. Berman and S. D. Drell, Phys. Rev. 133, B791 (1964).
 <sup>19</sup> M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966).

This implies that—unlike the case of Compton scattering—no nonsense zero appears in the contribution of the Pomeranchon<sup>20</sup> to the photoproduction of transverse vector mesons. The *t*-channel amplitude for photoproduction of longitudinally polarized mesons contains a kinematic factor which is proportional to *s* except at the forward cone. Thus a suppression of photoproduction of longitudinally polarized vector mesons may occur in the forward direction. The last observation is due to Dr. Harvey K. Sheppard, to whom we are also indebted for pointing out a sign error in the original manuscript.

### D. Photoproduction of Neutral Pions on Protons

As another, by now almost trivial, example of a nonsense zero we consider  $\gamma + p \rightarrow \pi^0 p$ . We choose neutral outgoing mesons so that for the small values of t with which we shall be concerned, the pion pole in the t channel will be absent. It is, of course, necessary for gauge invariance when charged pions are produced. Because the pion is spinless, *all* helicity amplitudes in the t channel are of the form

# $M(N(\lambda_a) + \bar{N}(\lambda_b) \rightarrow \gamma \pi) = M_{10,\lambda_a\lambda_b},$

which has, upon performing a partial-wave expansion and Reggeizing, a nonsense zero when the trajectory function  $\alpha(t)$  passes through zero. The obvious (and perhaps only) candidates available for Regge trajectories in the t channel are the  $\rho$  and  $\omega$  trajectories. These two trajectories, in fact, all trajectories belonging to the 1<sup>-</sup> nonet, have  $\alpha(0) \approx 0.5$  and slopes such that near  $t \approx -0.5$  (BeV)<sup>2</sup>,  $\alpha(t) = 0.12$  We expect, therefore, that near  $t \approx -0.5$  (BeV)<sup>2</sup> the differential cross section  $d\sigma(\gamma p \rightarrow \pi^0 p)/dt$  should exhibit at high energy a dip characteristic of a nonsense zero. The physical explanation for our nonsense zero here is that at  $\alpha(0) = 0$  we are trying to induce an electromagnetic 0-to-0 transition between the Regge pole and the pion. If the  $\rho$  and  $\omega$ trajectories are sufficiently separated at this value of t, by virtue, for example of having different slopes, and couple to  $\gamma \pi$  and NN with approximately equal strength, one might hope to see a double dip. Realistically, one should expect only a broad dip near  $\alpha_{\rho,\omega}(t) \approx 0$ . To the authors' knowledge there is not sufficient data on  $\pi^0$  photoproduction at high energy to let us say whether or not there is the expected dip.

Note added in proof. F. J. Gilman has been kind enough to point out to us that the nonsense zero in neutral pion photoproduction is exhibited in the work of M. P. Locher and H. Rollnik [Phys. Letters 22, 696 (1966)]. They only consider the  $\omega$  Regge-pole contribution to  $\gamma p \to \pi^0 p$ ; our treatment is both simpler and more general. They present data for  $\gamma p \to \pi^0 n$  at values of  $s \sim 5$  and 7 (BeV)<sup>2</sup> in which an optimist does see the nonsense dip at  $t \approx -0.6$  (BeV)<sup>2</sup>.

### IV. SUMMARY AND DISCUSSION

We have demonstrated above a few simple consequences of a rather naive Regge-pole picture of processes involving photon-hadron interactions. The only prediction experimentally verifiable in a reasonably straightforward manner is the dip in  $d\sigma(\gamma p \rightarrow \pi^0 p)/dt$  near  $t\approx -0.5$  (BeV)<sup>2</sup> at high energies due to the nonsense zero in all helicity amplitudes at  $\alpha(t)=0$ . Our other two results are probably of theoretical interest only: (1') the vanishing of  $\sigma_{tot}(\gamma p)$  for real photons at large energies, and (2') the absence of a subtraction in the spin-flip massive photon-proton amplitudes arising in the hydrogen hfs calculation, when those amplitudes are properly chosen. References 8 and 9 have discussed (2') at some length so we shall content ourselves with dwelling on (1') here.

In particular, we would like to inquire about ways out of the vanishing of the contribution of the Pomeranchon to  $\sigma_{tot}(\gamma p)$ . First we note that if moving Regge cuts give important contributions asymptotically, the result is considerably modified and weakened. For example, the selection rule against J=1 exchanges at t=0 reduces the contribution of the cut, associated with the Pomeranchon from  $\int^1 p(J)s^J dJ \sim s(\ln)^\beta$  to  $\int^1 p(J)(J-1)s^J dJ \sim s/(\ln s)^{\beta-1}$ . Thus the Regge-cut part of the amplitude will be more important than the pole part and no dip is expected. Furthermore, the vanishing of the total nuclear cross section, if it occurs at all, will be only logarithmic.

Another possibility is that the residue function associated with the  $\gamma\gamma$  Pomeranchon vertex develop a  $t^{-1}$  singularity near  $t=0.^{21}$  This will restore a finite contribution of the Pomeranchuk trajectory to forward Compton scattering. The contribution will be proportional to  $d\alpha_{p}(t)/dt|_{t=0}$ .

## ACKNOWLEDGMENTS

The friendly criticism and advice of M. L. Goldberger, J. Schwarz, S. B. Treiman, and W. I. Weisberger have been enormously helpful. Also some extensive discussions with S. D. Drell and J. D. Sullivan were invaluable in fostering Sec. IIIB.

<sup>&</sup>lt;sup>20</sup> We are assuming here that Regge behavior holds despite the fact that in the case where  $m_a \neq m_c$  the cosine of the scattering angle in the *t* channel does not become large when *s* becomes large. This problem has been thoroughly treated by M. L. Goldberger and C. E. Jones, Phys. Rev. Letters **17**, 105 (1966); Phys. Rev. **150**, 1269 (1966); and by D. Z. Freedman and J. M. Wong, Phys. Rev. Letters **17**, 569 (1966). The conclusion of these authors is that the imposition of Mandelstam analyticity on the Regge representation is sufficient to produce the usual Regge asymptotic behavior. This will be taken as "justification" for our assumption in this case.

<sup>&</sup>lt;sup>21</sup> Since the writing of this paper this possibility has been thoroughly investigated by H. Abarbanel, F. E. Low, I. Muzinich, S. Nussinov, and J. H. Schwarz. These authors now have a report of their investigations in preparation.