Odd-Parity Baryon Resonances*

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A formalism is developed that is useful for discussing the spin dependence of collinear reaction amplitudes of hadrons. The formalism is applied to the potentials in meson-baryon states, where the mesons and baryons are members of the $SU(6)_W$ supermultiplets **35** and **56**. The simultaneous assumptions of $SU(6)_W$ symmetry and dominant low partial waves lead to predictions concerning odd-parity baryon resonances very similar to those of the quark model. If the potential corresponds to the $SU(6)_W$ representation **70**, the set of predicted resonances corresponds to the (**70**, **3**) representation of $SU(6)\otimes O(3)$. Effects are present that are analogous to spin-orbit splitting and configuration mixing in the quark model. These effects are calculable, so that the model leads to more predictions than the quark model. The predictions concerning the quantum numbers, masses, and branching ratios of the resonances are compared with experiment. The over-all agreement is good.

I. INTRODUCTION

If the nucleon octet and N^* decuplet are associated with the $SU(6)$ multiplet **56**, and the vector- and pseudoscalar-meson nonets are associated with the representation $\mathbf{35}\oplus\mathbf{1}$, the one-baryon-exchange forces in MB(meson-baryon) states are most attractive in the representation **56**, and the meson-exchange forces are most attractive in the representation **70**.[1] Furthermore, the baryon-exchange forces are particularly strong in P states, while meson-exchange forces are strong in states of both parities. Thus, the most attractive forces in odd-parity states are associated with the representation **70**. Although no theory exists that permits an accurate comparison of forces in different orbital-angular-momentum states, it is reasonable to suppose that S waves are important, since no centrifugal repulsion exists in these states. A deduction that has been made previously from the above considerations is that the meson-exchange forces may produce a 70-fold multiplet of odd-parity baryon resonances, associated with MB S states.[1,2]

If the M-exchange forces help produce such a multiplet, it is clear that the MMM interactions must exist. However, the only form of $SU(6)$ symmetry that allows a simple MMM interaction is the $SU(6)_W$ form, or an equivalent form.[3,4] Thus, the postulated resonances should be associated with the 70-fold MB multiplet of $SU(6)_W$, rather than $SU(6)$. In principle, such an interpretation is simple enough. The $SU(6)_W$ symmetry applies to the forward and backward amplitudes; the average of these two amplitudes corresponds to even orbital angular momentum. Furthermore, if the dominant even-l potentials involve S states, the behavior in the collinear directions is sufficient to determine the behavior in all directions.

One of the purposes of this paper is to point out that the $SU(6)_W$ interpretation implies that the spectrum of the postulated odd-parity resonance multiplet corresponds to the representation (**70**, **3**) of $SU(6)\otimes O(3)$, rather than with the **70** of $SU(6)$. The (**70**, **3**) assignment has been made previously on the basis of the quark model, and corresponds to adding a unit of orbital angular momentum to the 70-fold spin and internal-symmetry state of three quarks.[5] This assignment is consistent with the present experimental data.

The basic reason that this $SU(6)\otimes O(3)$ structure occurs in an $SU(6)_W$ model is simple. We illustrate the reason by considering a representation of $SU(3)\otimes SU(2)_W$, corresponding to W spin w. The MB states may be regarded as composites of four mathematical quarks and one antiquark. ["Quark" is used here to mean the fundamental sextet of $SU(6)$; this picture is used only as a simple means of extracting some of the properties of the assumed symmetry.] Since the spin and W-spin operators S^2 and W^2 differ only when operating on wave functions containing both quarks and antiquarks, the allowed spin values of the four-quark part of the MB wave function are $w+\frac{1}{2}$ and $w-\frac{1}{2}$.[6] In general, combination of these spin values with that of the antiquark may lead to the total spins $w+1$, w, and $w-1$. These are the total angular momenta that result from the presence of a unit orbital angular momentum in the quark model. This point is discussed in detail in Sec. IV B.

The above argument is not very significant for P-wave amplitudes, since the total angular momentum and total spin need not be the same for these amplitudes. In fact, previous work has shown that forces in P-wave MB states corresponding to the $SU(6)_W$ representation R may produce composites corresponding simply to

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[1] The meson-exchange forces are discussed by R. H. Capps, Phys. Rev. Letters **14**, 842 (1965).
[2] See, for example, J. G. Koerner, Phys. Rev. **152**, 1389 (1966); E. Golowich, *ibid.* **153**, 1466 (1967).
[3] The $SU(6)_W$ symmetric MMM interaction has been derived from the hypothesis of invariance under the group $\tilde{M}(12)$ by B. Sakita and K. C. Wali, Phys. Rev. **139**, B1355 (1965).
[4] R. H. Capps, Phys. Rev. **148**, 1332 (1966).
[5] A. N. Mitra and Marc Ross, Phys. Rev. **158**, 1630 (1967).
[6] The $SU(6)_W$ symmetry is defined and discussed by H. J. Lipkin and S. Meshkov, Phys. Rev. **143**, 1269 (1966).

the $SU(6)$ representation R.[7] It is seen that the roles of P-wave and S-wave states in an $SU(6)_W$-symmetric model of MB composites are the transpose of the corresponding roles in the three-quark model of baryon states.

Other purposes of this paper are to examine the predictions of the $SU(6)_W$-symmetric model in detail and to make comparisons with experiment. The predictions of the present paper differ from those of a previous work by the author in two important aspects.[8] First, one-meson-exchange forces are considered in Ref. 8, while no specific force mechanism is assumed here. The second difference is that the potential is assumed to correspond only to the $SU(6)_W$ representation **70** here, and to a superposition of representations in Ref. 8.

In Sec. II, a convenient formalism for analyzing the spin dependence of any relativistic, collinear, multiparticle amplitude is developed. The implications of $SU(6)_W$ symmetry and the nonrelativistic limit are discussed. In Sec. III, a dynamical model is described, in which the properties of two-hadron composites produced by $SU(6)_W$-invariant forces may be calculated. In Sec. IV, several theorems are proved concerning the implications of the assumption that the potential corresponds to a unique irreducible representation of $SU(6)_W$. It is shown that effects exist in the S-wave MB model that are analogous to spin-orbit splitting and configuration mixing in the quark model. The specific assumption that the potential corresponds to the representation **70** is made in Sec. V. A detailed comparison with experimental data is given in Sec. VI.

II. SPIN-EXCHANGE FORMALISM FOR COLLINEAR AMPLITUDES

A. General Formalism

In this section a formalism for analyzing the spin dependence of collinear amplitudes is developed, and some implications of exact $SU(6)_W$ symmetry are discussed.

We consider a general collinear amplitude involving n initial hadrons $\mu_1, \mu_2\cdots\mu_n$, and n' final hadrons $\mu_1\cdots\mu_{n'}$. The direction of interaction is taken as the z axis. We consider first the helicity representation, in which the z components of all the intrinsic spin vectors are specified. The matrix elements of the components S_x, S_y, and S_z of the total intrinsic spin operator are defined in the usual way. The wave functions may be rewritten in the representation in which S^2 is diagonal. It should be noted that this mathematical formulation is relativistically invariant, although much of the terminology used (including the identification of S_x and S_y with components of the total intrinsic spin) is nonrelativistic. In the S^2 representation, any component of the reaction amplitude T may be labeled $T_{\alpha s,\alpha' s'}{}^m$, where s and s' denote initial and final values of the total intrinsic spin, m is the (conserved) z component of this spin, and α and α' are all the other quantum numbers of the initial and final states.

Spin-exchange amplitudes X may be defined by the equation

$$X_{ss'\Delta}=\left(\frac{2\Delta+1}{2s+1}\right)^{1/2}\sum_m C(s'\Delta s;m0m)T_{s's}{}^m, \quad (2.1)$$

where the indices α and α' have been suppressed, and the C are Clebsch-Gordan coefficients, defined with the phase convention of Rose.[9] It follows from the symmetry of the C coefficients that X satisfies the symmetry condition

$$X_{s's\Delta}=(-1)^{s'-s}X_{ss'\Delta}. \quad (2.2)$$

One way of interpreting the spin-exchange Δ is to consider the crossed amplitude $T^{\rm cr}$, defined so that the vacuum is the final state, while the initial state consists of the n particles $\mu_1\cdots\mu_n$, and the n' antiparticles $\mu_1\cdots\mu_{n'}$ with their spin components reversed. The quantity Δ is the total intrinsic spin of the initial state described by $T^{\rm cr}$.

We now make use of one implication of $SU(6)_W$ symmetry, invariance to W spin rotations of 180° around the y axis. This invariance implies

$$\langle\psi_r'|T|\psi_r\rangle=\langle\psi'|T|\psi\rangle, \quad (2.3)$$

where $\psi_r=\exp(-i\pi W_y)\psi$. It has been shown that W-spin rotation is related to ordinary spin rotation by the equation[6]

$$\exp(-i\pi W_y)=\mathcal{P}\exp(-i\pi S_y), \quad (2.4)$$

where $\mathcal{P}$ is the intrinsic parity operator. This spin rotation changes the sign of m; our phase convention is such that

$$e^{-iS_y\pi}\psi_s{}^m=(-1)^{s-m}\psi_s{}^{-m}. \quad (2.5)$$

A simple way to establish Eq. (2.5) is to use Eq. (4.13) of Ref. 9 to verify the relation for $m=j$, and then to operate on both sides with S_x+iS_y, using the relation

$$e^ABe^{-A}=B+[A,B]+(2!)^{-1}[A,[A,B]]+\cdots.$$

Combination of Eqs. (2.3), (2.4), and (2.5) leads to the relation

$$T_{\alpha s,\alpha' s'}{}^{-m}=(-1)^{s-s'}\left(\prod_i\mathcal{P}_i\right)T_{\alpha s,\alpha' s'}{}^m, \quad (2.6)$$

where $\prod_i\mathcal{P}_i$ is the product of the intrinsic parities of all the particles. If one makes use of the Clebsch-Gordan symmetry condition,

$$C(s\Delta s';m0)=(-1)^{s-s'+\Delta}C(s\Delta s';-m0),$$

Eqs. (2.1) and (2.6) lead to the following limita-

[7] This follows from an extension to $SU(6)_W$ of the $SU(6)$ model of R. H. Capps, Phys. Rev. Letters **14**, 31 (1965), and of J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965). The extension to $SU(6)_W$ is trivial and is described by R. H. Capps, Phys. Rev. **144**, 1182 (1966); see especially p. 1188.

[8] R. H. Capps, Phys. Rev. **153**, 1503 (1967).

[9] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957), Chap. III.

tion on Δ:

$$(-1)^{\Delta}=\prod_i \mathcal{P}_i. \quad (2.7)$$

We now discuss the relation of this result to the nonrelativistic angular-momentum representation for scattering at arbitrary angles. In addition to the $n+n'$ spin variables, there are $n-1$ initial orbital angular momenta and $n'-1$ final orbital angular momenta in this representation. Again T^{cr} is used to denote the corresponding amplitude in which the final state is the vacuum; the initial state contains all the orbital angular momenta $l_1\cdots l_{n-1}\bar{l}_1\cdots\bar{l}_{n'-1}$, as well as the spin variables. The L-S coupling scheme for this initial state may be denoted by the symbol

$$(s_1\cdots\bar{s}_{n'})_{\Delta}(l_1\cdots\bar{l}_{n'-1})_{\Delta}, \quad (2.8)$$

where the subscript Δ denotes that the angular momenta within the parentheses couple to form angular momentum Δ.

The collinear amplitudes in the z direction correspond to the spin-wave function of those terms of the above expression in which the z components of all the orbital angular momenta are zero. It is a well-known property of Clebsch-Gordan coefficients that $(l_1l_2)_L$ contains a term corresponding to $l_1{}^0l_2{}^0$ only if $(-1)^L=(-1)^{l_1+l_2}$. It follows that $(l_1l_2\cdots\bar{l}_{n'-1})_{\Delta}$ contains a term corresponding to all zero components only if $(-1)^{\Delta}=(-1)^{\Sigma l_i}$. This is the product of the orbital parities. Parity conservation requires that $(-1)^{\Sigma l_i}=\prod_i \mathcal{P}_i$, and thus also leads to Eq. (2.7). Amplitudes in which the Δ of Eq. (2.8) do not satisfy Eq. (2.7) exist, but they do not contribute in the collinear directions.

B. $SU(6)_W$-Symmetric $B\rightarrow B'M$ and $BM\rightarrow B'M$ Amplitudes

We now limit attention to the cases $B\rightarrow B'M$ and $BM\rightarrow B'M$, where B and B' are two $SU(6)$ multiplets of the same parity. The most relevant case is that in which B and B' both denote the 56-fold baryon supermultiplet. The $B\rightarrow B'M$ amplitude is the interaction vertex. It is a straightforward procedure to write such a vertex in a manner satisfying $SU(6)_W$ symmetry. According to Eq. (2.7), the spin-exchange Δ is odd. In the nonrelativistic limit, the vertex is a P-wave vertex, so that the one orbital-angular momentum is unity.[10] Hence Δ is one.

In the case of the scattering amplitude $MB\rightarrow MB'$, Δ is even and is restricted to the values 0 and 2. This is obvious from the fact that the process $MB\rightarrow MB'$ may be regarded as proceeding through one-particle B'' intermediate states. One can use the techniques introduced in Sec. IV B of this paper to prove these limitations on Δ formally.

III. THE DYNAMICAL FRAMEWORK

We now restrict attention to the $MB\rightarrow MB$ amplitudes, where the B correspond to the multiplet **56**. It is assumed that the dominant singularities of the amplitudes are one-particle-exchange contributions to the left-hand cuts, and the two-particle (unitarity) right-hand cuts. The two-particle intermediate states associated with the unitarity cut do not preserve collinearity, and hence may not preserve exact $SU(6)_W$ symmetry. However, we will apply the symmetry only to the one-particle-exchange cuts. The Born-approximation amplitudes corresponding to these left-hand cuts are the potentials of the model; the terms potential and one-particle-exchange amplitudes will be used interchangeably. Meson degeneracy, baryon degeneracy, and exact symmetry are assumed.

In the nonrelativistic limit, the potentials may be written in the form of Eq. (2.8), i.e., $(s_M\bar{s}_{M'};s_B\bar{s}_{B'})_{\Delta}(l\bar{l}')_{\Delta}$. The $\Delta=0$ and 2 terms correspond to the central and tensor potentials discussed previously by the author.[10,11] For example, the tensor force theorem of Ref. 11 implies that the $\Delta=2$ terms can exist in MB states only of the $SU(3)$ representations **1**, **8**, and **10**. This theorem is relativistic, although the interpretation in terms of S and D waves is nonrelativistic.

The assumption of $SU(6)_W$ symmetry for the collinear amplitudes does not determine the entire amplitude. However, the two parities may be separated cleanly. The linear combinations $\frac{1}{2}(T_f\pm T_b)$ refer to even and odd orbital-angular momenta, respectively, where T_f and T_b denote the forward and backward amplitudes. For either parity, the relative importance of large and small orbital-angular momenta is not given by the symmetry, but depends on the details of the potentials. There is no accurate theory that can be used to predict this relative importance. However, we make the reasonable assumption that for each parity, the lowest partial waves are dominant, since the effect of centrifugal repulsion is least in these states. The amplitudes of the lowest partial waves are the P-wave amplitudes in the even-parity states, and the S-wave and S-D transition amplitudes in the odd-parity states.

In the P-wave case, $l=l'=1$, and Δ may be 0, 1, or 2. Since the $\Delta=1$ term does not contribute in the collinear directions, $SU(6)_W$ symmetry and rotational invariance are not sufficient to determine the amplitudes completely. One needs to know the type of particle exchange. The P-wave MM and MB states are important in the bootstrapping of the M and B; these problems are discussed in previous references.[12]

In the odd-parity case, the $\Delta=0$ and $\Delta=2$ amplitudes involving the lowest partial waves are the S-S and S-D amplitudes, respectively. We assume that the S-S and S-D amplitudes together satisfy $SU(6)_W$ symmetry. Since all S-S and S-D amplitudes contribute in

[10] The static limit of the $SU(6)_W$-symmetric MBB' interaction is discussed by R. H. Capps, Phys. Rev. **150**, 1263 (1966).

[11] R. H. Capps, Phys. Rev. Letters **16**, 1066 (1966).

[12] R. H. Capps, Phys. Rev. **148**, 1332 (1966); see also Ref. 4.

the collinear directions, this assumption, together with rotational invariance, is sufficient to determine the potential up to an over-all constant. Hence, the formalism of Sec. II is particularly useful for these waves of even orbital angular momentum. In the rest of the paper we will limit attention to even l. Only the average of the forward and backward amplitudes is considered, and denoted by T.

The assumption that only S-S and S-D terms occur in the potential (Born-approximation amplitude) is justified rigorously in the threshold limit, since these are the only terms that can occur to order k^2. The S-S, S-D assumption is more general than the k^2 assumption, however. For example, in the discussion of the meson-exchange potential in Ref. 10, it was shown that the relative strength of any two S-D terms, or of any two S-S terms, is independent of energy, provided that the spin-wave functions of the scattering particles are treated nonrelativistically.

The potential matrix of the model is a large matrix in the space of many S-wave and many D-wave channels. The D-D elements are zero. We do not attempt to calculate the relative importance of the S-S and S-D terms. Rather, the aim is to reduce the potential to a set of disconnected 2×2 matrices, each involving one S-wave and one D-wave channel. As a first step, we define the potential elements by the symbols $U_{\alpha s,\alpha' s',\Delta}$, where (s,α) and (s',α') are the total intrinsic spin and other quantum numbers of the states, and Δ is equal to 0 and 2 for the S-S and S-D elements, respectively. The first pair of indices of the S-D elements refers to the S-wave states. The U are defined in terms of the X of Eq. (2.1) by the relation $U_{ss'\Delta}=(2s+1)^{-1/2}X_{ss'\Delta}$, i.e.,

$$U_{\alpha s,\alpha' s',\Delta}=\frac{(2\Delta+1)^{1/2}}{2s+1}\sum_m C(s'\Delta s;m0m)T_{\alpha' s',\alpha s}{}^m. \quad (3.1)$$

It may be shown that the S-S and S-D elements of U are related to elements of the conventional S matrix in the partial-wave representation by the equation $U=(S-1)/(2i\rho)$, where ρ is a function of energy common to all the amplitudes.

Since the two spin indices of the S-S amplitudes must be the same, we shorten the symbols $U_{\alpha s,\alpha' s,0}$ and $U_{\alpha s,\alpha' s',2}$ to $U_{\alpha s,\alpha'}$ and $U_{\alpha s,\alpha' s'}$, i.e., only the first spin index is given for the S-S amplitudes, and the values of Δ are omitted. Since the coupling of the orbital angular momentum in the S states is trivial, we use the symbols $\psi_{\alpha s}$ to denote either states of spin and internal symmetry only, or the corresponding S states. The symbol $\varphi_{\alpha j s}$ denotes a D-wave state of internal symmetry α, total angular momentum j, and total intrinsic spin s.

The calculational procedure is as follows: One determines the collinear, odd-parity amplitudes $T_{\alpha s,\alpha' s'}{}^m$ from the assumption that a particular $SU(6)_W$ representation is involved, and computes the U from Eq. (3.1). The S-S potential is diagonalized; the eigenvalue corresponding to the eigenvector $\psi_{\alpha j}$ is denoted by $U_{\alpha j}{}^S$. The normalized D-wave state vector $\varphi_{\alpha j}{}^D$ that is coupled to a particular $\psi_{\alpha j}$ may be defined by the equation

$$\varphi_{\alpha j}{}^D=\sum_{\beta s} U_{\alpha j,\beta s}\varphi_{\beta j s}/U_{\alpha j}{}^D, \quad (3.2)$$

$$U_{\alpha j}{}^D=(\sum_{\beta s} U_{\alpha j,\beta s}{}^2)^{1/2}. \quad (3.3)$$

The $U_{\alpha j}{}^D$ is the S-D potential element connecting $\psi_{\alpha j}$ and $\varphi_{\alpha j}{}^D$.

In Sec. IV A it is proved that the D-state wave functions coupled to orthogonal, S-state eigenvectors are orthogonal. This theorem enhances the usefulness of the approach described above, because it implies that in the representation of the $\psi_{\alpha j}$ and $\varphi_{\alpha j}{}^D$, the potential separates into disconnected 2×2 S-D-state potential matrices. We do not attempt to calculate the over-all S-state/D-state branching fraction of the resonances. However, the S- and D-state eigenvectors permit the calculation of the relative amplitudes for S-state decays and for D-state decays, and the relative sizes of the $U_{\alpha j}{}^S$ and $(U_{\alpha j}{}^D)^2$ associated with different composites lead to predictions concerning mass splitting of the supermultiplet. These results are not dependent on the relative importance of $U_{\alpha j}{}^S$ and $(U_{\alpha j}{}^D)^2$.

IV. THE PARAMETERS OF THE POTENTIAL

A. Simplicity Theorems

In order to prove three theorems concerning the simplicity of the potentials when only one $SU(6)_W$ irreducible representation is involved, we consider again the odd-parity collinear potential T. The theorems depend on the fact that the elements of U are related to certain elements of T in a particular representation, and on the fact that T is proportional to a projection operator.

In the (S^2,S_z) representation of T, the basis vectors are $\psi_{\alpha s}{}^m$. We define a new orthonormal basis χ by the equation

$$\chi_{\alpha s\Delta}=\left(\frac{2\Delta+1}{2s+1}\right)^{1/2}\sum_m C(s\Delta s;m0m)\psi_{\alpha s}{}^m, \quad (4.1)$$

where Δ takes on the integral values from 0 to $2s$, and $[(2\Delta+1)/(2s+1)]^{1/2}$ is a normalization factor. In the χ representation, only those elements of T that involve at least one state of the type $\chi_{\alpha s0}$ are of interest. If the two spin values are the same, these elements are related to the U of Eq. (3.1) by the following simple rules:

$$T_{\alpha s0,\alpha' s0}=U_{\alpha s,\alpha'}, \quad (4.2)$$

$$T_{\alpha s0,\alpha' s2}=U_{\alpha s,\alpha' s}, \quad (4.3)$$

$$T_{\alpha s0,\alpha' si}=0, \quad i\neq 0 \text{ or } 2. \quad (4.4)$$

The T matrix is symmetric. It is a simple matter to verify these rules by using Eqs. (3.1) and (4.1) and the condition $C(s0s;m0m)=1$. The last rule, Eq. (4.4), follows from the result of Sec. II B, that $U_{\alpha,\beta,\Delta}$ vanishes unless Δ is equal to 0 or 2.

If the spin values s and s' of two states are different, we are concerned only with quadratic sums of the type $\sum_i T_{\alpha s0,\alpha' s' i} T_{\beta s0,\alpha' s' i}$. Such a sum is invariant as to the choice of spin basis in the $\alpha' s'$ state; if one uses the S_z basis, it is easy to establish the relation

$$\sum_i T_{\alpha s0,\alpha' s' i} T_{\beta s0,\alpha' s' i} = (2s+1)^{-1} \sum_m T_{\alpha s,\alpha' s'}{}^m T_{\beta s,\alpha' s'}{}^m . \quad (4.5)$$

It is pointed out in Sec. II B that for $s \neq s'$, only potential terms corresponding to $\Delta=2$ exist. This implies the proportionality relation $T_{\alpha s,\alpha' s'}{}^m = K_{\alpha\alpha'} C(s2s';m0m)$, where the $K_{\alpha\alpha'}$ are constants. If this relation is used in both Eq. (3.1) and Eq. (4.5), and the symmetry properties of the Clebsch-Gordan coefficients are used,[9] the following equation results:

$$\sum_i T_{\alpha s0,\alpha' s' i} T_{\beta s0,\alpha' s' i} = U_{\alpha s,\alpha' s'} U_{\beta s,\alpha' s'} . \quad (s \neq s') \quad (4.6)$$

This equation is the analog of Eq. (4.3) for the $s \neq s'$ case.

Next, we show that T is proportional to a projection operator by considering the basis (rn), where r and n denote an irreducible representation of $SU(6)_W$ and the state within the representation. In this basis, T is given by $T_{rn,r'n'} = \mathcal{C}_r \delta_{rr'} \delta_{nn'}$, where the $\mathcal{C}_r$ are constants. If only one irreducible representation contributes, the subscripts on $\mathcal{C}$ may be dropped, and $(T/\mathcal{C})$ is a projection operator, i.e.,

$$(T/\mathcal{C}) = (T/\mathcal{C})^2 . \quad (4.7)$$

In our model, where T is a potential, the constant $\mathcal{C}$ is real. If the $\alpha s0$, $\beta s0$ element of Eq. (4.7) is considered, and Eqs. (4.2), (4.3), and (4.6) are used to write the T elements in terms of U elements, the results are

$$\mathcal{C}^{-1} U_{\alpha s,\beta} = \mathcal{C}^{-2} \Big(\sum_{\alpha'} U_{\alpha s,\alpha'} U_{\beta s,\alpha'} + \sum_{\alpha'} U_{\alpha s,\alpha' s} U_{\beta s,\alpha' s} + \sum_{s' \neq s} \sum_{\alpha'} U_{\alpha s,\alpha' s'} U_{\beta s,\alpha' s'} \Big) . \quad (4.8)$$

The three theorems follow from this equation.

THEOREM I. *If the S-S part of the potential is diagonal, the D-wave state vectors connected to two orthogonal S-wave eigenvectors are themselves orthogonal.*

This is the theorem mentioned in Sec. III. It is obvious if the spins of the two S-wave eigenvectors are different, so we consider only the case in which the spins are the same. The two S-states are identified with the αs and βs in Eq. (4.8). Since the S-S part of U is diagonal, the left side of Eq. (4.8) and the first summation on the right side vanish. The resulting relation may be written $\sum_{s'\alpha'} U_{\alpha s,\alpha' s'} U_{\beta s,\alpha' s'} = 0$. This is the required orthogonality relation, since the D-wave state vectors $\varphi_{\alpha s}{}^D$ are defined by Eq. (3.2).

THEOREM II. *The nonzero eigenvalues of the S-S potentials are all of the same sign.*

In order to prove this, we identify both αs and βs in Eq. (4.8) with any one of the eigenvectors $\psi_{\gamma j}$ of the S-S potential. The right-hand side of the equation is non-negative definite, so the sign of the $U_{\gamma j,\gamma}$ on the left depends only on the sign of $\mathcal{C}$.

THEOREM III. *The S-S and S-D elements of the 2×2 U matrix associated with a particular S-state $\psi_{\gamma j}$ are related simply. This relation may be expressed in terms of a parameter $\lambda_{\gamma j}$ in the following way:*

$$\mathcal{C}^{-1} U_{\gamma j}{}^S = \tfrac{1}{3}(1+\lambda_{\gamma j}) , \qquad \mathcal{C}^{-2} |U_{\gamma j}{}^D|^2 = (2/9)(1+\tfrac{1}{2}\lambda_{\gamma j} - \tfrac{1}{2}\lambda_{\gamma j}{}^2) . \quad (4.9)$$

This theorem also follows from identifying both αs and βs in Eq. (4.8) with the state in question. One assumes that the S-S part of U is diagonalized and uses the definition of $U_{\gamma j}{}^D$, Eq. (3.3). The significance of the parameter λ becomes clear in part B of this section.

Theorems I and III are both important, because they imply that the predictions of the model are not dependent on the relative strengths of the S-S and S-D potentials. The theorems depend on the projection-operator property of $(T/\mathcal{C})$, and do not apply in the model of Ref. 8, in which different representations of $SU(6)_W$ are superposed with different strengths.

B. Effective Spin-Orbit Coupling and Configuration Mixing

In this section, the potential corresponding to a particular $SU(3)\otimes SU(2)_W$ submultiplet of an $SU(6)_W$ multiplet is considered. The W spin is denoted by w. It is pointed out in the introduction that the spin values that may be associated with this multiplet are $w+1$, w, and (unless $w=\frac{1}{2}$) $w-1$. We now show that if the potential is parametrized by the $\mathcal{C}$ and λ of Eq. (4.9), the three λ values are given in terms of one constant.

In this analysis, each MB state is pictured as a composite of a four-quark state Q and a single antiquark A. The S-state wave function of any MB state of W spin w is a sum of two parts, in which the W spin (and spin) of the Q states are $w+\frac{1}{2}$ and $w-\frac{1}{2}$. The wave functions corresponding to $S_z=m$ may be written

$$\psi^m = a_+ \psi^m(Q_{w+1/2}A) + a_- \psi^m(Q_{w-1/2}A) , \quad (4.10)$$

where $a_+{}^2 + a_-{}^2 = 1$. One may use ordinary Clebsch-Gordan coefficients to write out the two parts, i.e.,[9]

$$\psi^m(Q_{w+1/2}A) = -\left(\frac{w+1-m}{2w+2}\right)^{1/2} Q^{m-1/2} A^{1/2} - \left(\frac{w+1+m}{2w+2}\right)^{1/2} Q^{m+1/2} A^{-1/2} , \quad (4.11a)$$

$$\psi^m(Q_{w-1/2}A) = \left(\frac{w+m}{2w}\right)^{1/2} Q^{m-1/2} A^{1/2} - \left(\frac{w-m}{2w}\right)^{1/2} Q^{m+1/2} A^{-1/2} . \quad (4.11b)$$

The relative signs of the two terms in Eq. (4.11a) and in Eq. (4.11b) are opposite to those that would occur if the spin value of each state were w. This results from the fact that the matrix elements of the W-spin and S-spin lowering operators are equal for the Q states and opposite for the A states.[6]

One may use Eqs. (4.11) to write Eq. (4.10) in the S^2 representation, i.e.,

$$\psi^m = Z_{w+1}{}^m \psi_{w+1}{}^m (Q_{w+1/2}A) + Z_w{}^m \psi_w{}^m + Z_{w-1}{}^m \psi_{w-1}{}^m (Q_{w-1/2}A),$$

where the subscript is the spin, and the various quantities are given by the relations

$$Z_{w+1}{}^m = -a_+[(w+1)^2 - m^2]^{1/2}/(w+1), \quad (4.12a)$$

$$Z_{w-1}{}^m = -a_-[w^2 - m^2]^{1/2}/w, \quad (4.12b)$$

$$Z_w{}^m = m\bar{Z},\ \bar{Z} = \{[a_+/(w+1)]^2 + [a_-/w]^2\}^{1/2}, \quad (4.12c)$$

$$\psi_w{}^m = \bar{Z}^{-1}\left[-\frac{a_+}{w+1}\psi_w{}^m(Q_{w+1/2}A) + \frac{a_-}{w}\psi_w{}^m(Q_{w-1/2}A)\right]. \quad (4.13)$$

All the ψ are normalized to unity.

In the representation of the $\psi_{w+1}{}^m$, $\psi_w{}^m$, and $\psi_{w-1}{}^m$, the odd-parity collinear amplitudes T are given by the simple equation

$$T_{ss'}{}^m = \mathcal{C} Z_s{}^m Z_{s'}{}^m, \quad (4.14)$$

where $\mathcal{C}$ is the over-all strength constant defined in Sec. IVA.

It is seen from Eq. (4.9) that it is sufficient to consider the S-S potential elements in order to determine the values of λ_j associated with the different angular momenta. The S-S amplitudes may be computed from Eqs. (3.1) and (4.14). One needs Eqs. (4.12a) through (4.13) and the algebraic identity

$$\sum_{m=1/2}^{w} m^2 = \tfrac{1}{6}w(w+1)(2w+1),$$

where the index ranges through all half-odd-integral values from $\frac{1}{2}$ to w. The results of the computation are

$$\mathcal{C}^{-1}U_{w+1} = \tfrac{1}{3}a_+{}^2(2w+1)/(w+1),$$
$$\mathcal{C}^{-1}U_{w-1} = \tfrac{1}{3}a_-{}^2(2w+1)/w, \quad (w \geq \tfrac{3}{2}) \quad (4.15)$$
$$\mathcal{C}^{-1}U_w = \tfrac{1}{3}[a_+{}^2 w/(w+1)] + \tfrac{1}{3}[a_-{}^2(w+1)/w].$$

It is convenient to define a parameter λ that is characteristic of the $SU(3)\otimes SU(2)_W$ multiplets by the equations

$$a_+{}^2 = \frac{w+1}{2w+1}[1+w\lambda],\ a_-{}^2 = \frac{w}{2w+1}[1-(w+1)\lambda]. \quad (4.16)$$

The S-S potentials for the three spin values are functions of the two parameters $\mathcal{C}$ and λ. It is seen from Eqs. (4.15) and (4.16) that the S-S potentials may be written in the form of Eqs. (4.9), i.e., $\mathcal{C}^{-1}U_s = \frac{1}{3}(1+\lambda_s)$, where the λ_s are given in terms of λ by the equation

$$\lambda_s = \tfrac{1}{2}\lambda[s(s+1) - w(w+1) - 2]. \quad (4.17)$$

This is just the relation that would be obtained in a quark model, in which a multiplet of spin w is combined with a unit orbital angular momentum, and the degeneracy of the potentials is split by a spin-orbit term.[13] However, the λ coefficients for different $SU(3)\otimes SU(2)_W$ multiplets are calculable in the $SU(6)_W$ model, as is shown in Sec. V. It should be noted that if the splitting parameter λ is zero, the relative probability of the $Q_{w+1/2}$ and $Q_{w-1/2}$ states in Eq. (4.10) is the statistical value.

In the cases considered in this article, the sign of $\mathcal{C}$ is such that the S-S potentials are attractive, and the composites are assumed to exist. It is seen from Eq. (4.9) that to first order in λ_j, the deviations from the mean of the S-D elements $|U_j{}^D|^2$ are proportional to λ_j, and a large $|U_j{}^D|^2$ is associated with a large $U_j{}^S$. Hence, if the λ_j are not large, the presence of a spin-orbit-type mass-splitting term is not dependent on the relative importance of the S-S and S-D potentials.

If more than one $SU(3)\otimes SU(2)_W$ multiplet of a particular $SU(3)$ representation exists in the $SU(6)_W$ multiplet, the S-state wave functions of a particular spin corresponding to the different multiplets may not be orthogonal. In such a case, an effect similar to configuration mixing of the quark model occurs. In the $SU(6)_W$ model, it is straightforward to calculate the effects of the mixing. The potential elements corresponding to the different $SU(3)\otimes SU(2)_W$ elements must be superposed, and the S- and D-wave eigenvectors found. The λ parametrization of Eq. (4.9) remains valid, but the simple spin-orbit splitting relation of Eq. (4.17) does not remain valid.

V. CALCULATED RESULTS FOR $SU(6)_W$ MULTIPLET 70

The formulas contained in this section are necessary for readers interested in duplicating, extending, or understanding thoroughly the results of the model. Other readers may skip to the next section.

The simplest mechanism known to provide strong forces in the odd-parity MB states is the one-meson-exchange mechanism. As pointed out before, the M-exchange force is most attractive in the $SU(6)_W$ representation **70**.[14] Hence, we assume that the potential corresponds to the **70**, and that the S-state potentials

[13] For a recent review of the quark model, see R. H. Dalitz, in Proceedings of the Second Annual Tokyo Institute of Theoretical Physics at Oiso, Japan, 1966 (to be published). For an earlier review, see R. H. Dalitz, *High Energy Physics* (Gordon and Breach Science Publishers, Inc., New York, 1965), pp. 251–323.

[14] The potentials are discussed in Ref. 1. In configuration space, each S-S potential contains a Yukawa term and a contact-interaction term of opposite sign. The contact term contributes in a region where the nonrelativistic approximation is not justified, so the sign of the potential has been equated with that of the Yukawa term.

in the eigenstates are attractive. (See theorem II of Sec. IVA.) The results will depend only on this assumption, and not on the detailed exchange mechanism that provides the forces. The convention is used that a positive $U_{\alpha j,\alpha}$ denotes attraction; thus the over-all strength constant $\mathcal{C}$ of Eq. (4.9) is positive. Since we do not calculate the absolute strength of the potential, it is convenient to set $\mathcal{C}=1$ for the remainder of the paper.

The $SU(3)\otimes SU(2)_W$ structure of the representation **70** is $(1,2_W)\oplus(10,2_W)\oplus(8,2_W)\oplus(8,4_W)$, where the numbers indicate the multiplicities. Thus, the resonance multiplets expected are

$$(1,2)+(1,4)+(10,2)+(10,4)+(8,2)+(8,2)+(8,4)+(8,4)+(8,6)\,.$$

We present the results by first neglecting the configuration mixing that exists in the octet states, and listing the S-state wave functions (in terms of the M and B) and the λ parameters of Eq. (4.17). It is easy to compute these by making use of a table of $SU(6)$ Clebsch-Gordan coefficients.[15] The symbol ψ_i is used to denote an S-wave state vector of angular momentum $\frac{1}{2}i$, while P, P_1, V, V_1, N, and D denote the P-meson octet, P singlet, V octet, V singlet, nucleon octet, and N^* decuplet, respectively. The equations are:

$(1,2_W)$ Multiplet $(\lambda=0)$

$$\psi_3=(VN)\,, \tag{5.1}$$

$$\psi_1=(\tfrac{1}{4})^{1/2}(VN)-(\tfrac{3}{4})^{1/2}(PN)\,. \tag{5.2}$$

$(10,2_W)$ Multiplet $(\lambda=\frac{1}{3})$

$$\psi_3=(10/21)^{1/2}(VD)+(5/42)^{1/2}(V_1D)-(2/7)^{1/2}(PD)-(1/14)^{1/2}(P_1D)-(1/21)^{1/2}(VN)\,, \tag{5.3}$$

$$\psi_1=(1/3)^{1/2}(VD)+(1/12)^{1/2}(V_1D)-(25/48)^{1/2}(VN)+(1/16)^{1/2}(PN)\,. \tag{5.4}$$

$(8,2_W)$ Multiplet $(\lambda=\frac{1}{6})$

$$\psi_3=(1/13)^{1/2}[-(25/6)^{1/2}(VD)+(5/2)^{1/2}(PD)+(10/3)^{1/2}(VN)_d+(8/3)^{1/2}(VN)_f+(1/3)^{1/2}(V_1N)]\,, \tag{5.5}$$

$$\psi_1=(1/6)^{1/2}(VD)-(1/48)^{1/2}(VN)_d+(49/240)^{1/2}(VN)_f+(2/15)^{1/2}(V_1N)-(1/16)^{1/2}(PN)_d-(5/16)^{1/2}(PN)_f-(1/10)^{1/2}(P_1N)\,. \tag{5.6}$$

$(8,4_W)$ Multiplet $(\lambda=\frac{1}{6})$

$$\psi_5=(VD)\,, \tag{5.7}$$

$$\psi_3=(1/24)^{1/2}(VD)+(5/8)^{1/2}(PD)-(5/24)^{1/2}(VN)_d+(1/24)^{1/2}(VN)_f+(1/12)^{1/2}(V_1N)\,, \tag{5.8}$$

$$\psi_1=(1/7)^{1/2}[-(5/3)^{1/2}(VD)-(5/6)^{1/2}(VN)_d+(1/6)^{1/2}(VN)_f+(1/3)^{1/2}(V_1N)-(5/2)^{1/2}(PN)_d+(1/2)^{1/2}(PN)_f+(P_1N)]\,. \tag{5.9}$$

The symbols in parentheses denote normalized state vectors of the proper spins and internal symmetry, and the subscripts d and f denote symmetric (d-type) and antisymmetric (f-type) octet-octet–state vectors. The conventional f/d ratio is given in terms of the coefficients C_d and C_f of the two types of octet terms by the formula $f/d=(5/9)^{1/2}C_f/C_d$. It must be emphasized that exact $SU(6)_W$ symmetry has been assumed, and that the nonzero values of the mass-splitting parameters result entirely from $SU(6)$ Clebsch-Gordan coefficients.

If no configuration mixing is present, the spin-wave functions associated with the D-state wave functions are also the ψ_i of Eqs. (5.1)–(5.9). However, states of different total intrinsic spin contribute to the D-state wave function $\varphi_{\gamma_j}{}^D$ associated with a composite γ_j. One can find these state vectors from Eq. (3.2) and the following formulas for the S-D elements of the U matrix associated with a particular $SU(3)\otimes SU(2)_W$ multiplet. The two indices are twice the total angular momentum, and twice the total spin of the D-wave state.

$w=\frac{1}{2}$ case

$$U_{33}=-c_+^2\,,\quad U_{31}=-\tfrac{1}{2}c_+(c_+^2+c_-^2)^{1/2}\,,\quad c_+^2=\tfrac{1}{3}(1+\tfrac{1}{2}\lambda)\,,\quad c_-^2=1-\tfrac{3}{2}\lambda\,. \tag{5.10}$$

$w=\frac{3}{2}$ case

$$U_{55}=-\tfrac{2}{3}(14)^{1/2}c_+^2\,,\quad U_{53}=-(\tfrac{7}{2})^{1/2}c_+(c_+^2+c_-^2)^{1/2}\,,\quad U_{51}=2c_+c_-\,,\quad U_{33}=c_+^2+c_-^2\,,\quad U_{31}=-\tfrac{1}{2}c_-(c_+^2+c_-^2)^{1/2}\,,\quad c_+^2=\tfrac{1}{10}(1+\tfrac{3}{2}\lambda)\,,\quad c_-^2=\tfrac{1}{6}(1-\tfrac{5}{2}\lambda)\,. \tag{5.11}$$

Those elements not given may be determined from the formula

$$U_{ij}=(-1)^{i-j}[(2j+1)/(2i+1)]^{1/2}U_{ji}\,. \tag{5.12}$$

The octet state vectors $\psi_1(8,2_W)$ and $\psi_1(8,4_W)$ are not orthogonal, and the vectors $\psi_3(8,2_W)$ and $\psi_3(8,4_W)$ are not orthogonal. Thus, the configuration mixing effect discussed in the last part of Sec. IV B occurs. This mixing, for either $j=\frac{1}{2}$ or $\frac{3}{2}$, leaves unchanged the sum of the two λ_i values for the spin in question, but increases the difference between them. The subscripts a and b are used to denote the composite of larger and smaller λ_i values, respectively. It is a straightforward procedure to diagonalize the S-S parts of the superposition of the $(8,2_W)$ and $(8,4_W)$ U matrices, and to compute the eigenvectors. These eigenvectors are linear combinations of the corresponding eigenvectors of the two $SU(3)\otimes SU(2)_W$ eigenvectors. The coeffi-

[15] Convenient tables of $SU(6)$ Clebsch-Gordan coefficients, and of $SU(3)$ Clebsch-Gordan coefficients, are given, respectively, by C. L. Cook and G. Murtaza, Nuovo Cimento **39**, 531 (1965), and by P. McNamee, S. J. Chilton, and Frank Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

cients of these combinations, and the λ_i values, are listed below.

$$\lambda_{3a}=0.14\,,\quad \lambda_{3b}=-0.22\,,$$
$$\psi_{3a}=0.90\psi_3(8,2_W)+0.34\psi_3(8,4_W)\,,\qquad (5.13)$$
$$\psi_{3b}=-0.47\psi_3(8,2_W)+0.95\psi_3(8,4_W)\,.$$
$$\lambda_{1a}=-0.14\,,\quad \lambda_{1b}=-0.44\,,$$
$$\psi_{1a}=-0.94\psi_1(8,2_W)+0.25\psi_1(8,4_W)\,,\qquad (5.14)$$
$$\psi_{1b}=0.37\psi_1(8,2_W)+0.98\psi_1(8,4_W)\,.$$

In order to compute the D-wave state vectors in the configuration-mixing cases, one first computes the S-D elements of the U associated with the unmixed state vectors, and then uses the superposition equations, Eqs. (5.13) and (5.14), together with the basic definition of $\varphi_{\gamma j}{}^D$, Eq. (3.2).

VI. COMPARISON WITH EXPERIMENT

A. Resonance Quantum Numbers and Branching Ratios

The calculations of Sec. V lead to a predicted supermultiplet of odd-parity baryon resonances that correspond to the **(70, 3)** representation of $SU(6)\otimes O(3)$. In this section, we compare the experimental data with the predicted set of resonances, and with the predicted S-wave and D-wave branching ratios. Mass splitting is discussed in Sec. VI B.

The experimental information concerning baryon resonances changes rapidly with time. Except where otherwise noted, we will take as an experimental standard the recent compilation of Rosenfeld *et al.*[16] All of the odd-parity baryons listed in Ref. 16 may be associated either with members of the **(70, 3)** multiplet, or with Regge recurrences of these particles. The particles (but not the recurrences) are listed in Table I, along with their $SU(3)\otimes SU(2)$ assignments. The four multiplets (1,2), (1,4), $(8,4)_a$, and (8,6) are nearly complete. In addition, strangeness-zero particles have been found that may be members of the $(8,2)_a$, $(8,2)_b$, and (10,2) multiplets. The $SU(3)$ assignments of these latter three particles are speculative, but any other assignment would require a representation other than **1**, **8**, or **10**. Hence, of the predicted nine multiplets, only the (10,4) and $(8,4)_b$ are absent. Furthermore, some tentative evidence for the (10,4) exists, as is discussed later in this section. In our model, there is no ambiguity in assignment resulting from the presence of two (8,4) and two (8,2) multiplets. The predicted forces are most attractive in the $(8,4)_a$ and $(8,2)_a$ multiplets, so these are identified with the observed (8,4) and the lighter of the observed (8,2) multiplets.

[16] A. H. Rosenfeld, A. Barbaro-Galtieri, J. Kirz, W. J. Podolsky, M. Roos, W. J. Willis, and C. Wohl, University of California Radiation Laboratory Report No. UCRL-8030, August 1966 revision (unpublished).

TABLE I. Calculated and experimental partial widths (in MeV) of the odd-parity baryon resonances.

Particle	Mode	S- or D-state probability (%)	Calc. partial width	Expt. partial width
(1,2) multiplet				
$Y_0^*(1405)$	$(\Sigma\pi)_S$	28	35 (input)	≈ 35
(1,4) multiplet				
$Y_0^*(1520)$	$N\bar{K}$	9.4	2.9	4.6
	$\Sigma\pi$	14.1	8	9
(8,4)$_a$ multiplet				
$N^*(1518)$	$N\pi$	8.6	69	≈ 40
	$(N^*\pi)_S$	35	69	$\lesssim 40_{(S+D)}$
	$(N^*\pi)_D$	30.5	10	
$Y_1^*(1660)$	$N\bar{K}$	0.07	0.2	~ 7.5
	$\Lambda\pi$	0.8	3.7	~ 2.5
	$\Sigma\pi$	7.0	17	~ 15
	$(Y^*\pi)_S$	5.8	10.5	$\lesssim 10_{(S+D)}$
	$(Y^*\pi)_D$	5.1	0.8	
$\Xi^*(1820)$	$\Lambda\bar{K}$	1.4[a]	2.8[a]	≈ 10
	$\Sigma\bar{K}$	0.31[a]	0.2[a]	Unseen
	$\Xi\pi$	0.31[a]	0.8[a]	~ 1
(8,6) multiplet				
$N^*(1688)$	$N\pi$	1.42	24	≈ 35
	$N^*\pi$	20.0	39	$(\Gamma_T\approx 100)$
$Y_1^*(1765)$	$N\bar{K}$	3.8	26	≈ 42
	$\Lambda\pi$	1.42	12 (input)	12
	$\Sigma\pi$	0.95	5	<3
	$Y_1^*\pi$	3.3	2.5	≈ 8
(8,2)$_a$ multiplet				
$N^*(1570)$	$(N\pi)_S$	12.8	52	≈ 39
	$(N\eta)_S$	7.5(11.6)[b]	15(23)[b]	Seen
	$N^*\pi$	10.3	5	$(\Gamma_T\approx 130)$
$Y_0^*(1670)$	$(N\bar{K})_S$	20.0	67	Seen $(\Gamma_T\approx 18)$
	$(\Sigma\pi)_S$	0.52	2	Unseen
	$(\Lambda\eta)_S$		<1	Seen
(8,2)$_b$ multiplet				
$N^*(1700)$	$(N\pi)_S$	17.6	84	$\lesssim 220$
	$(N\eta)_S$	3.3(5.1)[b]	11(17)[b]	$(\Gamma_T\approx 240)$
	$N^*\pi$	8.1	17	
(10,2) multiplet				
$N^*(1670)$	$(N\pi)_S$	3.13	14.5	≈ 80
	$N^*\pi$	14.2	25	$(\Gamma_T\approx 180)$

[a] Calculated numbers for Ξ^* computed on basis of decuplet-octet mixing [see Eq. (6.4)].
[b] The η-mode parameters outside and inside parentheses are calculated on the basis of η-X mixing angles of 0° and $-10\frac{1}{2}$°, respectively.

The S-wave and D-wave partial widths are assumed proportional to the products of the phase-space factors and the probabilities of the states in the S-wave and D-wave state vectors. No dependence of the decay amplitudes on the λ factors has been assumed. The predicted probabilities and partial widths are shown in Table I, and compared with the experimental partial-width data. The S-wave modes are distinguished with the subscript S. The subscript T indicates an experimental total width. The calculations were made by taking the $Y_0^*(1405)\to\pi\Sigma$ and $Y_1^*(1765)\to\pi\Lambda$ as the input S- and D-state decays, and using the following phase-space factors, $\rho_S=k$, $\rho_D=k^5/M^4$: where M is the mass of the composite, and k is the decay momentum. A correction for a finite radius of interaction may be appropriate, especially for the D-wave decays. However, since the data are tentative and the effects of symmetry breaking are not small, such a correction would depend sensitively on the modes chosen as input, and so is not made in this paper.

The f/d ratios of the (PN) decay modes of the octet resonances are especially interesting. The D-wave, PN wave functions of the predicted spin-$\frac{5}{2}$ and -$\frac{3}{2}$ octets are listed below:

$$\varphi^D(8,6)=(4/35)^{1/2}[(5/8)^{1/2}(PN)_d - (1/8)^{1/2}(PN)_f-(1/4)^{1/2}(P_1N)], \quad (6.1)$$

$$\varphi^D(8,4)_a=0.415[0.472(PN)_d +0.783(PN)_f+0.405(P_1N)], \quad (6.2)$$

$$\varphi^D(8,4)_b=0.221[0.129(PN)_d -0.790(PN)_f-0.600(P_1N)], \quad (6.3)$$

where the numbers outside the brackets are included so that the entire D-state wave functions, rather than the (PN) parts, are normalized to unity. The predicted f/d ratios of the observed $(8,6)$ and $(8,4)_a$ multiplets are $(-\frac{1}{3})$ and 1.24, respectively.

The Y_1^* particles of the $(8,6)$ and $(8,4)_a$ multiplets are ideal for measuring f/d ratios, since the k^5 phase-space factors corresponding to any two of the $\pi\Lambda$, $\pi\Sigma$, and $\bar{K}N$ modes of either of these particles are within a factor of two of each other. It is seen from Table I that the measured $\pi\Lambda/\pi\Sigma$ and $\bar{K}N/(\pi\Lambda+\pi\Sigma)$ branching ratios are quite different for the $Y_1^*(1660)$ and $Y_1^*(1765)$, and that these differences are in the same directions as the theoretical predictions.[17,18] Our predictions are also in accord with the phase-sign analysis of $K^-n\rightarrow\Lambda\pi^-$ amplitudes by Kernan and Smart, which indicates that f/d is less than one for the $Y^*(1765)$ and more than one for the $Y^*(1660)$.[18,19]

The $\Xi^*(1820)$ is a bit of a problem, because of the experimental fact that the $\bar{K}\Lambda$ decay rate is much greater than that of either the $\pi\Xi$ or $\bar{K}\Sigma$ mode. It may be seen from an $SU(3)$ Clebsch-Gordan table that this behavior is inconsistent with an octet assignment and any f/d ratio, and is also inconsistent with a pure decuplet assignment.[15] One possible way out of the dilemma would be to assume that the Ξ^* is associated with the representation **27**, but there is no experimental evidence for any of the other members of the multiplet. An alternative possibility, not in conflict with the model of this paper, is that the particle is a mixture of an octet and decuplet particle.[20] In order to illustrate this possibility, we have computed the branching ratios that would result from the following admixture of $(10,4)$ and $(8,4)_a$ states:

$$\psi(\Xi^*)=(\cos 24.9°)\psi(10,4)+(\sin 24.9°)\psi(8,4)_a. \quad (6.4)$$

The mixing angle was chosen so that the $\pi\Xi$ and $\bar{K}\Sigma$ amplitudes of the f-type octet state would cancel those of the decuplet. It is seen from the calculated branching ratios of Table I that this mechanism can explain the $\bar{K}\Lambda$ dominance. If this octet-decuplet mixing hypothesis is correct, the other decuplet states, and another spin-$\frac{3}{2}\Xi^*$ should be found.[21]

It is instructive to trace the origin of some of the predicted f/d ratios. The Clebsch-Gordan coefficients of $SU(6)$ are such that the f/d ratios associated with the PN states of the $(8,4)$ and $(8,2)$ multiplets of the representation **70** are $(-\frac{1}{3})$ and $\frac{5}{3}$, respectively.[15] The predicted f/d ratio for the physical spin-$\frac{5}{2}$ octet is the same in the quark model and $SU(6)_W$ model, since this multiplet arises from the $(8,4)$ states of the **70** in both models. On the other hand, the quark model does not lead to a definite prediction for the f/d ratio of the physical spin-$\frac{3}{2}$ octet, unless a specific additional assumption concerning configuration mixing is made.

The $(8,2)$ and $(10,2)$ multiplets are interesting, although the experimental data concerning these are meager. The experimental widths of the $N_{1/2}^*(1570)$, $N_{1/2}^*(1700)$, and $N_{3/2}^*(1670)$ are determined from phase-shift analyses and are not known accurately. However, our calculations agree with the experimental facts that the $N^*(1700)$ is the most elastic of these particles,[22] and that the πN^* mode of the $N^*(1670)$ decay appears to dominate the πN mode.[23]

Frequently, it has been conjectured that the η-baryon octet decay modes of all particles of the $(8,2)_a$ multiplet will stand out experimentally. If this occurs, and if the η-X mixing is not extremely large, some strong $SU(3)$-breaking mechanism must be present. A candidate for such a mechanism has been proposed by Mitra and Ross.[5] We do not assume an $SU(3)$-breaking mechanism, but have calculated the ηN decays of the $N^*(1570)$ and $N^*(1700)$ from the S-wave state vectors determined from Eqs. (5.6), (5.9), and (5.14). If these

[17] Decay data from the $Y_1^*(1765)$ recently have been analyzed by R. P. Uhlig, G. R. Charlton, P. E. Condon, R. G. Glasser, G. B. Yodh, and N. Seeman, Phys. Rev. **155**, 1448 (1967). These authors conclude that the best value of f/d for this multiplet is in the range $-0.25<f/d<-0.1$.

[18] The relation between f/d and the parameter α used by many authors is $f/d=\alpha/(1-\alpha)$.

[19] Anne Kernan and Wesley M. Smart, Phys. Rev. Letters **17**, 832 (1966).

[20] This possibility has been suggested by Mitra and Ross, Ref. 5, and by the author, Ref. 8.

[21] A new Y_1^* of mass about 1680 MeV has been discovered by M. Derrick, T. Fields, J. Loken, R. Ammar, R. E. P. Davis, W. Kropac, J. Mott, and F. Schweingruber, Phys. Rev. Letters **18**, 266 (1967). The spin and parity of this particle have not been measured. The $\bar{K}^0p/\pi^+\Lambda$ branching ratio is 0.3 ± 0.2. The upper limit on the $\pi^+\Sigma^0$ mode is given by $\pi^+\Sigma^0/\pi^+\Lambda<0.25$. If this resonance is associated with the (10, 4) multiplet of the present model, and k^5 phase-space factors are used, the predicted branching ratios are $\pi^+\Lambda:\bar{K}^0p:\pi^+\Sigma^0=1:0.44:0.18$. Thus, this assignment is consistent with the data.

[22] C. Michael, Phys. Letters **21**, 93 (1966).

[23] P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys Letters **18**, 342 (1965).

ψ are normalized to unity, the PN parts are given by

$$\psi(8,2)_a=0.713[0.131(PN)_d+0.828(PN)_f+0.546(P_1N)+\cdots],$$

$$\psi(8,2)_b=0.724[-0.931(PN)_d+0.087(PN)_f+0.357(P_1N)+\cdots].$$

The ηN decay amplitude depends on the η-X mixing angle θ, and is given by the formula $\frac{1}{2}(\cos\theta)[(\frac{1}{5})^{1/2}C_d-C_f]+(\sin\theta)C_1$, where the C are the coefficients of the d-type, f-type, and P_1 terms in the S-wave state vectors. The assumption that the Gell-Mann–Okubo sum rule for the squares of the P-meson masses is exact leads to the condition $|\theta|\sim10\frac{1}{2}°$. We choose the sign of θ from the quark model, i.e., the mixing is such as to decrease the sum of the probabilities of the Λ quarks and antiquarks in the wave function of the lighter meson. This leads to the choice $\theta=-10\frac{1}{2}°$; the sign is opposite to that of the V-meson case because the octet probability exceeds the singlet probability in the wave function of the lighter P meson. The calculated ηN partial widths corresponding both to $\theta=0$ and $\theta=-10\frac{1}{2}°$ are shown in Table I.

The existence of the $Y_0^*(1670)$ is somewhat uncertain. However, if the particle exists, future measurements concerning the branching ratios will be extremely interesting. The $\pi\Sigma$ and $\bar{K}N$ modes are of comparable phase space, and are of pure d type and predominant f type, respectively. The $\bar{K}N/\pi\Sigma$ ratio is a good measure of f/d. Furthermore, the ηN mode is also pure d type in the absence of η-X mixing, so the $\eta N/\pi\Sigma$ ratio may provide evidence concerning the mixing.

B. Average Masses of the Multiplets

We now consider the mass-splitting of the (**70**, **3**) supermultiplet, placing particular emphasis on the possible effects of the mass-splitting parameters λ_i. Since $SU(3)$ breaking is not considered in this paper, we discuss only the average masses corresponding to the nine $SU(3)\otimes SU(2)$ multiplets. Only one or two members of four of the octets and decuplets have been identified; in these cases the average experimental mass is taken to be 130 MeV heavier than that of the strangeness-zero member.

It is clear that the mass splitting should not depend on λ_i alone, for in the $SU(6)_W$-symmetric bootstrap models of the meson and baryon supermultiplets no parameter analogous to λ_i appears, yet appreciable splitting occurs. In baryon bootstrap models, an assumption that has been quite successful in fitting the experimental masses is the assumption that the mass (or mass squared) is a linear function of the average meson mass and average baryon mass in the wave function.[24] Using this as a guide, we try the following formula for the average masses of the $SU(3)\otimes SU(2)$, odd-parity baryon multiplets:

$$M_i=a+b(\langle\mu\rangle_i-\mu_P)+c(\langle m\rangle_i-m_N)-d\lambda_i, \quad (6.5)$$

where $\langle\mu\rangle_i$ and $\langle m\rangle_i$ are the rms meson and baryon masses of the S-wave state vectors corresponding to the multiplet i, and μ_P and m_N are taken for convenience to be 410 MeV and 1159 MeV, the rms masses of the P octet and N octet. The parameters λ_i are those of Eqs. (4.9). The constants a, b, c, and d are to be determined phenomenologically; if Eq. (6.5) is sensible they should all be positive.

TABLE II. Contributions to the formula for the average masses of the $SU(3)\otimes SU(2)$ multiplets, in MeV.

Multiplet	b term	c term	Sum of (abc) terms	λ_i	Calc. mass	Expt. mass
(1,2)	39	0	1405	0	1405[a]	1405
(1,4)	154	0	1520	0	1520[a]	1520
$(8,4)_a$	87	314	1767	0.14	1660[a]	1660
$(8,2)_a$	103	124	1593	−0.14	1700[a]	~1700
(8,6)	154	498	2018	0.25	1820	~1820
$(8,2)_b$	85	56	1507	−0.44	1845	~1830
$(8,4)_b$	107	256	1729	−0.22	1901	Unseen
(10,2)	147	208	1720	−0.33	1976	~1800
(10,4)	113	475	1954	0.17	1826	?

[a] Input masses used to determine the parameters.

In Table II the experimental masses of the multiplets are compared with those calculated from Eq. (6.5), and also to the masses calculated when the d term is omitted. Since the c and d terms both vanish for the two $SU(3)$ singlets, these multiplets are used to determine a and b. The average of the λ parameters for the $(8,4)_a$ and $(8,2)_a$ multiplets is nearly zero, so the average of these masses is used to determine c, and the $(8,4)_a$ mass is then used to determine d. The values of the constants are

$$a=1366\text{ MeV},\quad b=0.34,\quad c=2.15,\quad d=770\text{ MeV}.$$

The predicted masses are of the right order, though that of the (10,2) multiplet is somewhat too high. It is seen from those rows of the table after the $(8,4)_a$ row that the sign of the difference between the experimental mass and that calculated from the a, b, and c terms alone is always opposite to the sign of λ, as it should be. Thus, there is some evidence for the effect of the λ term. This "spin-orbit" splitting term helps explain the fact that the 1405-MeV Y_0^* stands out experimentally more than any of the other spin-$\frac{1}{2}$ particles, since the splitting parameters vanish for the $SU(3)$ singlets, and favor the larger spins for the other multiplets.

VII. CONCLUSIONS

The $SU(6)_W$-symmetric composite model of the hadrons, in which the constituent particles are the lighter mesons and baryons, is an attempt to understand

[24] See R. H. Capps, Phys. Rev. Letters **14**, 456 (1965). This paper contains references to pre-$SU(6)$ papers in which a similar assumption is made.

$SU(6)$ symmetry. The quark model also may be regarded as an attempt to understand $SU(6)$ symmetry. It is natural that results that depend directly on the symmetry are common to the two models. One of the main points of this paper is that many predictions that go beyond $SU(6)$ symmetry are also common to the two models. One such prediction is the limitation of meson resonances to the $SU(3)$ representations **1** and **8**, and of baryon resonances to the representations **1**, **8**, and **10**. It was pointed out previously that the spin-dependent potentials in an $SU(6)_W$-symmetric theory are limited to these representations, so that the experimental absence of other representations may be explained by the assumption that spin-dependent potentials are crucial.[11] In the present paper we have elucidated three other features of an $SU(6)_W$-symmetric model of baryon resonances that exist in the quark model: a spectrum classifiable by the group $SU(6)\otimes O(3)$, a mass-splitting term with a spin-orbit-type j dependence, and configuration mixing. The existence of approximately $SU(6)_W$-symmetric MBB and MMM interactions may be one of the reasons that the quark model works so well.

The $SU(6)_W$ model leads to more predictions concerning these baryon resonances than does the quark model, since the several spin-orbit-type splitting and configuration-mixing parameters are calculable. It is shown in Sec. VI that the extra predictions, as well as those common to the quark model, are in satisfactory agreement with experiment.

It is worthwhile discussing briefly the intrinsic symmetry-breaking that is present in this model. The potential (Born approximation) satisfies exact $SU(6)_W$ symmetry in the collinear directions. However, because of the nonzero λ parameters, the potentials in the different states are not all the same. These potentials are expected to lead to composites of different masses. It is clear that if we had calculated scattering amplitudes, in which poles associated with the nondegenerate composites existed, the amplitudes would not satisfy the exact symmetry. This is not a contradiction, because the right-hand, unitarity cut must be included in a calculation in which the poles appear, and the momentum of each virtual particle associated with this cut need be in the direction of the collinear amplitude. Hence, the cut violates collinearity, and also $SU(6)_W$ symmetry.

The assumptions of this paper are different from those of a previous paper by the author on odd-parity baryon resonances.[8] The principal difference is that the potential is taken to correspond to the $SU(6)_W$ representation **70** in the present paper, and to a superposition of representations in Ref. 8, the coefficients of the superposition being determined from the assumption that meson exchange supplies the forces. One motivation for the modification of this paper is the fact that other forces may be important. For example, the forces resulting from exchange of the resonances themselves may be important. If the resonances correspond to the **70**, these forces are such as to increase the attraction in the $SU(6)_W$ state **70**, and decrease it in the state **56**.[25]

The experimental data favor the present model over that of Ref. 8. In Ref. 8 the strongest potentials correspond to the representations **70** and **56**; for this reason it is not surprising that the spectrum corresponds to the sum of the $SU(6)\otimes O(3)$ representations (**70**, **3**) and (**56**, **3**). Experimentally, there is no evidence for the (**56**, **3**) resonances.

Many undiscovered particles are predicted by the model. If the assumption that the $\Xi^*(1820)$ is an octet-decuplet mixture is correct, another nearby Ξ^*, and the other decuplet members should be observed. In particular, $N_{3/2}^*$ and Y_1^* particle should be found with masses on the order of 1600 and 1700 MeV.[21] The $N^*(1570)$ and $N^*(1700)$ particle should belong to octets with (PN) couplings that are predominantly f type and d type, respectively. Another spin-$\frac{3}{2}$ octet should be found, with (PN) coupling predominantly of the f type. It is interesting to note that the predicted coupling of the undiscovered $(8,4)_b$ multiplet to PN states is comparitively small, as seen from Eq. (6.3).

The techniques of this paper may be applied to other systems, such as MM and $B(MM)$ systems, where (MM) denotes a multiplet of even-parity meson-meson resonances. We shall make a few comments about even-parity MM systems. If the MM potential is taken to correspond to the representation **35**, the theorems of Sec. IV A apply. However, the arguments concerning the spectrum and spin-orbit interaction of Sec. IVB do not apply, since the MM states contain two mathematical antiquarks. One can show that an MM, $SU(3)\otimes SU(2)_W$ multiplet of w spin 1 contains spin-1 and -2 states, in general, while a w-spin-0 multiplet contains spin-0 and -2 states. Thus, the assumption of a **35** potential will lead to a set of particles similar to that predicted from the quark model, except that the $SU(3)$ singlet states are limited to spin-1 and spin-2 states.

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[25] The **70**-exchange column of the $56\otimes35$, $SU(6)$ crossing matrix is given by V. Singh and B. M. Udgaonkar, Phys. Rev. **139**, B1585 (1965).