ing $t_{\alpha\beta}+t_{\beta\alpha}$, the sum of the s- and u-channel amplitudes for meson-baryon scattering, employing the noninvariance group G_1 in the context of high-energy diffraction scattering.

One can now deduce from (13) that the difference between the uncrossed amplitude and the crossed one is proportional to the generators of the maximal compact subgroup of G_2, i.e., the invariance group G_0:

$$t_{\alpha\beta}(\omega)-t_{\beta\alpha}(\omega)\sim iF_{\alpha\beta\gamma}M_\gamma. \tag{15}$$

This is precisely the result of Sudarshan and Kuriyan,[5] who used the *compact* noninvariance group of the intermediate-coupling theory. It may be noted that in the strong-coupling theory of Cook, Goebel, and Sakita[4] the right-hand side of (15) is equal to zero.

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Nonleptonic Hyperon Decay and Baryon–Weak-Hamiltonian Commutators*

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The amplitude for the nonleptonic decay of hyperons is expressed in terms of an absorptive part and an equal-time commutator of the baryon field with the weak Hamiltonian. The assumption that the parity-conserving and parity-violating parts of the weak Hamiltonian transform like scalar (S_6) and pseudoscalar ($S_7{}^5$) quantities, respectively, enables us to evaluate the commutator. The absorptive parts are calculated under the assumption that they are dominated by single-particle states. The assumption that $S_7{}^5$ and the strong pseudoscalar vertex belong to the same octet when combined with the experimental observation that $A(\Sigma_+{}^+)=0$ enables the calculation of d/f for this octet. The value obtained is $\sqrt{3}$, which is in excellent agreement with the value obtained from leptonic decays. The results for p waves are not so definite, but indicate a dominant d-type coupling.

I. INTRODUCTION

MANY attempts have been made recently to explain weak interactions and especially nonleptonic decay of hyperons using the hypothesis of partially conserved axial-vector current (PCAC) and the algebra of $U(3)\times U(3)$ generated by integrated current components. All these attempts require in addition the assumption that the decay amplitudes do not change appreciably when we extrapolate the pion four-momentum from zero to its finite experimental value. Brown and Sommerfield[1] have shown for P waves that this is not always so and that there is an appreciable change in the amplitude as the pion four-momentum is taken to zero. Such large variation occurs also in the nonleptonic decay of K mesons, as has been pointed out by several authors.[2,3] It is therefore of interest to examine methods which do not need the assumption that the four-momentum of the pion goes to zero. Another motivation for our calculation is to study the implications of assumed commutation relations between two currents, one with zero baryon number and one with unit baryon number, and the "sidewise" dispersion relations natural to them.[4] Thus, we assume, in our approach, that SU_3 is a good symmetry and that the parity-conserving and parity-violating parts of the weak Hamiltonian transform like a scalar ($S_6\sim\frac{1}{2}\bar{q}\lambda_6 q$) and a pseudoscalar ($S_7{}^5\sim\frac{1}{2}i\bar{q}\lambda_7\gamma_5 q$), respectively.[3] This enables us to write down an expression for the part of

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[1] M. Suzuki, Phys. Rev. Letters 15, 986 (1965); H. Sugawara, *ibid.* 15, 870 (1965); *ibid.* 15, 997 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* 16, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* 16, 751 (1966).

[2] C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966); M. Suzuki, *ibid.* 16, 212 (1966).

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the Hamiltonian which couples to the baryons. The commutator of the weak Hamiltonian density and the baryon current density is then determined and both the S- and P-wave nonleptonic decays can be evaluated.

II. THE REDUCTION FORMULA, COMMUTATION RELATIONS, AND DECAY AMPLITUDES

We start with the expression for the amplitude for nonleptonic decay $[Y \to B(p)+\pi(k)]$ of hyperons:

$$\begin{aligned}(2k_0)^{1/2}\langle\pi B|H^W(0)|Y\rangle \\ &= -i\int d^4x\, \bar{u}_B e^{ip\cdot x}(\gamma_\mu\partial_\mu+m_B) \\ &\qquad \times[\theta(x_0)\langle\pi|[\psi_B(x),H^W(0)]|Y\rangle] \\ &= -i\bar{u}\left[\int d^4x\, e^{ip\cdot x}\theta(x_0)\langle\pi|[f_B(x),H^W(0)]|Y\rangle\right. \\ &\qquad \left.+\int d^4x\, e^{ip\cdot x}\delta(x_0)\langle\pi|[\gamma_0\psi_B(x),H^W(0)]|Y\rangle\right], \quad (1)\end{aligned}$$

where H^W is the weak Hamiltonian and $f_B(x) = (\gamma_\mu\partial_\mu+m_B)\psi_B(x)$. In view of our assumption about H^W, the part of it which couples to baryons has the form

$$\begin{aligned}H^W=&(2if_{ij6}F^c+2d_{ij6}D^c)\bar{\psi}_i(x)\psi_j(x) \\ &+(2if_{ij7}F^v+2d_{ij7}D^v)\bar{\psi}_i(x)\gamma_5\psi_j(x), \quad (2)\end{aligned}$$

where the f_{ijk} and d_{ijk} are the usual SU_3 structure constants of Gell-Mann.[5]

Using Eq. (2) and the usual anticommutation relation

$$\delta(x_0-y_0)\{\psi_i(x),\bar{\psi}_j(y)\}=\delta_{ij}\gamma_0\delta^4(x-y),$$

we can evaluate the commutator in Eq. (1). Specifically, we have

$$\begin{aligned}\delta(x_0)[\gamma_0\psi_k(x),H^W(0)] \\ &=\delta_{ik}\psi_j(0)(2if_{ij6}F^c+2d_{ij6}D^c)\delta^4(x), \\ \delta(x_0)[\gamma_0\psi_k(x),H^W(0)] \\ &=\delta_{ik}\gamma_5\psi_j(0)(2if_{ij7}F^v+2d_{ij7}D^v)\delta^4(x). \quad (3)\end{aligned}$$

We can then rewrite the last term of Eq. (1) as

$$\begin{aligned}\bar{u}\langle\pi|\psi_j(0)|Y\rangle &= \frac{\bar{u}_B[-i\gamma\cdot(q-k)+m_j]\langle\pi|f_j(0)|Y\rangle}{m_j^2-m_N^2} \\ &= g_{jY\pi}\bar{u}_B\gamma_5 u_Y/(m_j+m_N),\end{aligned}$$

and, similarly,

$$\bar{u}\langle\pi|\gamma_5\psi_j(0)|Y\rangle=g_{jY\pi}\bar{u}_B u_Y/(m_j-m_N). \quad (4)$$

Here m_N is the nucleon mass.

The first term on the right of Eq. (1) is the absorptive part of the hyperon decay amplitude. We assume an unsubtracted dispersion relation for it and saturate the intermediate states with octet baryons and pseudoscalar mesons. The absorptive part then gives two terms which together with Eq. (4) give us contributions which correspond to the two baryon poles and the K-meson pole in the conventional pole model. The absorptive part gives the s-channel $(Y+S\to B+\pi)$ and the t-channel $(Y+\bar{B}\to\pi+\bar{S})$ poles, while the equal-time commutator term gives terms equivalent to the u-channel $(Y+\bar{\pi}\to B+\bar{S})$ poles. The parity-violating S waves do not have any meson poles.

Defining

$$g_{jY\pi}=g_\pi(2iff_{jiY}+2dd_{jYi}), \quad (5)$$

and

$$\langle M_i|H^{W_c}|M_j\rangle=\phi_i(p')\phi_j(p)2D'd_{ij6},$$

where ϕ stands for mesons, we get the following expressions for the amplitudes:

$$\begin{aligned}
A(\Lambda_-^0)&=-g_\pi\sqrt{2}\left[\left(\frac{2}{3}\right)^{1/2}\frac{d(D^v-F^v)}{(m_\Sigma+m_N)}-\left(\frac{1}{6}\right)^{1/2}\frac{(f+d)(3F^v+D^v)}{(m_\Lambda+m_N)}\right],\\
A(\Xi_-^-)&=-g_\pi\sqrt{2}\left[\left(\frac{2}{3}\right)^{1/2}\frac{d(F^v+D^v)}{(m_\Xi+m_\Sigma)}+\left(\frac{1}{6}\right)^{1/2}\frac{(d-f)(3F^v-D^v)}{(m_\Xi+m_\Lambda)}\right],\\
A(\Sigma_+^+)&=-g_\pi\sqrt{2}\left[\frac{(2f+d)(D^v-F^v)}{(m_\Sigma+m_N)}-\frac{d(F^v+D^v/3)}{m_\Lambda+m_N}\right], \qquad (6)\\
A(\Sigma_0^+)&=-g_\pi\left[\frac{(D^v-F^v)(3f+d)}{(m_\Sigma+m_N)}\right],\\
A(\Sigma_-^-)&=-g_\pi\sqrt{2}\left[\frac{f(F^v-D^v)}{(m_\Sigma+m_N)}-\frac{d(F^v+D^v/3)}{(m_\Lambda+m_N)}\right],
\end{aligned}$$

[5] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964).

$$B(\Lambda_-^0)=-g_\pi\sqrt{2}\left[\left(\frac{2}{3}\right)^{1/2}\frac{d(F^c-D^c)}{(m_\Sigma-m_N)}-\left(\frac{1}{6}\right)^{1/2}\frac{(f+d)(3F^c+D^c)}{m_\Lambda-m_N}+\frac{(3f+d)}{\sqrt{6}}\frac{D'}{m_K{}^2-m_\pi{}^2}\right],$$

$$B(\Xi_-^-)=-g_\pi\sqrt{2}\left[\left(\frac{2}{3}\right)^{1/2}\frac{d(F^c+D^c)}{(m_\Xi-m_\Sigma)}-\frac{(d-f)}{\sqrt{6}}\frac{(3F^c-D^c)}{m_\Xi-m_\Lambda}-\frac{(3f-d)}{\sqrt{6}}\frac{D'}{m_K{}^2-m_\pi{}^2}\right],$$

$$B(\Sigma_+^+)=+g_\pi\sqrt{2}\left[\frac{d(F^c-D^c)}{m_\Sigma-m_N}-\frac{d(F^c+D^c/3)}{m_\Lambda-m_N}\right], \qquad (7)$$

$$B(\Sigma_0^+)=-g_\pi\left[\frac{(f-d)(F^c-D^c)}{m_\Sigma-m_N}-\frac{(f-d)D'}{m_K{}^2-m_\pi{}^2}\right],$$

$$B(\Sigma_-^-)=+g_\pi\sqrt{2}\left[\frac{f(F^c-D^c)}{m_\Sigma-m_N}-\frac{d(F^c+D^c/3)}{m_\Lambda-m_N}+\frac{(f-d)D'}{m_K{}^2-m_\pi{}^2}\right].$$

Note that the contributions from the two baryon poles change sign, relatively, in the A and B amplitudes.[6]

III. S-WAVE RESULTS

We assume that (D^v/F^v) for the parity-violating Hamiltonian which transforms as $S_7{}^5$ is the same as that for the strong coupling of pseudoscalar mesons to the baryons (d/f). Then it can be shown that the amplitudes satisfy the Lee-Sugawara sum rule.[7]

$$\Lambda_-^0+2\Xi_-^-=\sqrt{3}\Sigma_0^+,$$

where the sums of the two masses have been put equal to $2M$. If we next impose the experimentally observed requirement[8] that $A(\Sigma_+^+)=0$, we can solve for the common d/f from the equation

$$[2+(d/f)][(d/f)-1]-(d/f)(1+\tfrac{1}{3}d/f)=0, \qquad (8)$$

which gives

$$(d/f)^2=3 \quad \text{or} \quad d/f=\sqrt{3}. \qquad (9)$$

Willis *et al.*[9] have determined the ratio of D/F for the axial-vector current from semileptonic decays. Using PCAC, this is taken to be the same as that for the strong pseudoscalar vertex. This value is 1.7. The agreement with Eq. (9) is excellent. Our analysis thus provides an alternative derivation of the d/f ratio of the strong pseudoscalar vertex from the nonleptonic decay of hyperons.

Using this value of d/f, we can calculate the other amplitudes. We find

$$A(\Lambda_-^0)=-A(\Xi_-^-)=-(g_\pi\sqrt{2}/2M)F^v(1.5),$$
$$A(\Sigma_-^-)=-\sqrt{2}A(\Sigma_0^+)=(\sqrt{2}g_\pi/2M)F^v(1.25).$$

The agreement with observed values is very good except for $A(\Lambda_-^0)$, which is experimentally 25% smaller than $A(\Xi_-^-)$ (Table I).

IV. P-WAVE RESULTS

We define the strength of the mass-splitting spurion for baryons by F_M and D_M and for mesons by D_M' so that

$$F_M=(M_\Xi-M_N)/2,$$
$$D_M=\alpha_M F_M \quad (\alpha_M=-0.3),$$
$$D_M'=m_K{}^2-m_\pi{}^2.$$

Bludman[10] has shown that the contribution of K-meson pole terms can be absorbed into those of the baryon poles by defining an effective $\tilde{F}$ and $\tilde{D}$ for the parity-conserving weak spurion:

$$\tilde{F}=F^c-F_M D'/D_M',$$
$$\tilde{D}=D^c-D_M D'/D_M'.$$

The experimental amplitudes[8] can be fitted reasonably well with $\tilde{D}/\tilde{F}=-0.8$, as shown in Table I. To calculate D^c/F^c from this we need to know the strength D', i.e., the coupling of spurion to mesons. Our theory does not predict this parameter. To get some idea of its value

TABLE I. Comparison of theory and experiment for the P-wave amplitudes, for $\tilde{F}=6\times10^{-7}m_\pi$, $\tilde{D}/\tilde{F}=-0.8$.

	Experiment[a]	Theory
$B(\Lambda_-^0)$	2.2×10^{-6}	2.3×10^{-6}
$B(\Xi_-^-)$	1.6×10^{-6}	1.7×10^{-6}
$B(\Sigma_+^+)$	4.0×10^{-6}	2.8×10^{-6}
$B(\Sigma_0^+)$	2.3×10^{-6}	2.0×10^{-6}

[a] Reference 8.

[6] The same S-wave results have been obtained on the basis of a pole model by Riazuddin, Fayyazuddin, and A. H. Zimmerman, Phys. Rev. **137**, 1556 (1965).

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we make three alternative assumptions for the value of D'.

(1) $D'/D=-(D'/D)_{\text{mass}}$, $D^c/F^c=1.25$, $|D'|=7.5\times10^{-6}m_\pi^2$,

(2) $(D'/F)=-(D'/F)_{\text{mass}}$, $D^c/F^c=-1.3$, $|D'|=2.7\times10^{-6}m_\pi^2$,

(3) $D'/D=+2(D'/F)_{\text{mass}}$, $F^c=0$, $|D'|=5.4\times10^{-6}m_\pi^2$.

If we extrapolate the four-momentum of the pion to zero, we can relate[11] D' to the decay $K_1^0\rightarrow 2\pi$ using PCAC and current commutation relations. In this case we get for D' a value of $4\times10^{-6}m_\pi^2$, to be compared with the other values quoted above.

From our results, it seems quite likely that not only S_7[5] but S_6 also has a dominant D-type coupling to baryons. In the $U(3)\times U(3)$ scheme of Gell-Mann,[5] both the scalar and pseudoscalar octets do have a D-type coupling to baryons when the baryons are assigned to (3,3*). The amount of F-type coupling developed by symmetry breaking could be calculated if we had a reliable estimate of D', the strength of K-π spurion. It is thus desirable to get a relation between D' and the $K_1^0\rightarrow 2\pi$ amplitude which does not require the four-momentum of the pion to go to zero.

[11] Riazuddin and K. T. Mahanthappa, Phys. Rev. **147**, 972 (1966).

V. CONCLUSIONS

We have found another example of the usefulness of assumed commutators involving baryon fields.[4] In our application a simplification results from the fact that the weak spurion carries no four-momentum. The method avoids pion-mass extrapolation at the cost of requiring commutation relations among current densities. Our good numerical results indicate that the intervening commutators do in fact lead to *unsubtracted* dispersion relations, but we have no proof that subtractions are absent.

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Mass Splitting and the Quark Theory. I

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We calculate the masses of the decuplet and the octet baryons by using a spin- and $SU(3)$-spin-dependent quark-quark force. We compare it with the required quark-antiquark force used in a previous paper in order to explain the mass splitting of the P and the V nonet mesons. The result is consistent with the assumption that these two forces are related to each other by charge conjugation. We further observe that the ratio of radial integrals for quark-quark and quark-antiquark symmetry-breaking forces is the same as that for the symmetric forces, if quarks are very heavy.

I. INTRODUCTION

ALMOST immediately after the discovery of the baryons, certain regularities were noticed among the masses of the baryons and the mesons. For example, Nambu[1] remarked that the baryons and the mesons discovered up to that time had masses which are integer or half-integer multiples of the unit $137mc^2$ (70 MeV). However, it was found that some of the baryons and mesons discovered later did not follow this rule.[2] It has been pointed out by the author[3] that the masses of the well-established multiplet baryons, namely the octet ($j=\frac{1}{2}$) and the decuplet ($j=\frac{3}{2}$) baryons, follow from the Gell-Mann–Okubo mass formula[4] with integer and half-integer coefficients of the units of 70 MeV. The Gell-Mann–Okubo mass formula cannot be applied to the mesons directly. However, it has been pointed out by the author[5] that the mass splitting of the P (pseudoscalar) and V (vector) nonet mesons can be explained well by introducing a certain linear mass operator in addition to the quadratic one introduced by Okubo.[4] Then the required mass operators for the mesons have coefficients which are simple fractional multiples of $137mc^2$. Such a trend in both the baryons and the mesons seems to indicate that we should not discuss the baryon masses and the meson masses separately, but should try to deduce them from a common basis.

Now the success of the $SU(3)$ theory has been so

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