# Noncompact Noninvariance Groups

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A sequence of noncompact intermediate-coupling groups has been constructed starting from the static strong-coupling group of Cook, Goebel, and Sakita. Our basic assumption involves the use of successive approximations to the meson currents in the inverse power of the strong-coupling constant. In particular, in the first approximation we obtain the intermediate-coupling group $ISL\ (n,C)$ from $SU\ (n)\otimes T_{n^2-1}$ as the basic strong-coupling group. Further, as a specific use of such groups we consider meson-baryon scattering.

## 1. INTRODUCTION

LET us assume that there is a symmetry group $G_0$ which is the idealized one-particle symmetry group when no interactions between particles is taken into account. If one thinks in terms of the quark model, where particles (baryons and mesons) arise as different bound-state levels of quarks and antiquarks or excitations of "quark matter," then the symmetry group $G_0$ describes the situation in the absence of interaction between excitation levels of "quark" matter. The interactions between particles destroys the symmetry $G_0$, the corresponding coupling currents $A_\alpha$ thus being a source of breaking.

We suppose that the currents $A_\alpha$ together with the generators of the symmetry group $G_0$ form an enlarged algebra of the noninvariance group $G$. The group $G$ could be responsible for the whole set of particle levels, while the symmetry group $G_0$ describes the properties of particles at each level. The group $G$ should then be noncompact in order to explain the infinite set of particle levels.[1]

It is most likely that the exact group $G$ is very complicated. Therefore, there is the problem of constructing the set of successive approximations to find it. Different steps of approximation should involve different numbers of particle interactions being taken into account. In order that such a group-theoretical approach be self-consistent, one has to require that at every step $(n)$ of the approximation, the generators of the symmetry group $G_0$ and the coupling currents $A_\alpha{}^n$ form a group $G_n$. (Adding some new currents should enable us then to go over the the next approximation $G_{n+1}$.) Each step $(n)$ of the approximation to $G$ is to be characterized by the group $G_n$, which contains among its generators those of the symmetry group $G_0$, and by the meson-baryon coupling matrices $A_\alpha{}^{(n)}$ corresponding to the approximation.

[1] A compact version of the noninvariance group $G$ is also possible if we consider only a finite set of particle levels, as discussed (and first pointed out) by E. C. G. Sudarshan and J. G. Kuriyan, Phys. Letters **21**, 106 (1966).

## 2. INTERMEDIATE-COUPLING GROUP OF THE STATIC THEORY

Let $gA_\alpha$ represent the Hermitian coupling matrix of meson $\Pi_\alpha$ interacting with a static baryon, $g$ being the coupling-strength parameter. Baryon masses differ by the second-order terms in $1/g$:

$$m_i=m_0+\Delta_i/g^2. \tag{1}$$

The perturbation expansion of the pole term of the amplitude for the meson-baryon scattering,

$$\Pi_\beta+B_j\rightarrow\Pi_\alpha+B_i, \tag{2}$$

is then given[2] in the strong-coupling approach[3,4] by

$$T_{\alpha\beta}{}^{ij}(\omega)\,|_{\text{pole}}=\frac{2\pi}{V\omega}t_{\alpha\beta}{}^{ij}(\omega)=\frac{2\pi}{V\omega}\times\left\{\frac{-g^2}{\omega}[A_\alpha,A_\beta]-\frac{1}{\omega^2}[A_\beta,[\Delta,A_\alpha]]\right\}, \tag{3}$$

where $\Delta$ is a diagonal matrix, $\Delta_{ij}=\delta_{ij}\Delta_i$.

The baryons $B_i$ and mesons $\Pi_\alpha$ are supposed to form multiplets of an internal symmetry group $G_0$ of the type $SU(n)$. Commutation relations between the Hermitian generators $M_\alpha$, $\alpha=1, 2, \cdots, n^2-1$ have the form

$$[M_\alpha,M_\beta]=iF_{\alpha\beta\gamma}M_\gamma. \tag{4}$$

It is already implied by the notation that the matrices $A_\alpha$ transform like mesons $\Pi_\alpha$, according to the lowest-dimensional real representation of $G_0$, or

$$[M_\alpha,A_\beta]=iF_{\alpha\beta\gamma}A_\gamma. \tag{5}$$

We assume that the following expansion is possible for

[2] C. J. Goebel, in *Proceedings of the 12th International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), Vol. I, p. 255.

[3] B. Sakita, Lectures on Higher Symmetries of Hadrons, Dalhausie Summer School, 1965 (unpublished).

[4] T. Cook, C. J. Goebel, and B. Sakita, Phys. Rev. Letters **15**, 35 (1965).

the matrices $A_\alpha$ as functions of $1/g$:

$$A_\alpha=\sum_{n=0}^{\infty} A_\alpha{}^{(n)}(1/g)^n. \tag{6}$$

Here the leading term $A_\alpha{}^{(0)}$ is independent of $g$.

In the strong-coupling limit, $g\to\infty$, the pole term (3) should remain finite; hence, we must have

$$[A_\alpha{}^{(0)},A_\beta{}^{(0)}]=0, \tag{7a}$$

$$[A_\alpha{}^{(1)},A_\beta{}^{(0)}]+[A_\alpha{}^{(0)},A_\beta{}^{(1)}]=0. \tag{7b}$$

It follows from (5) and (6) that the matrices $A_\alpha{}^{(0)}$ have the properties of "momenta" $P_\alpha$ in $SU(n)$. Therefore, we shall denote $A_\alpha{}^{(0)}=P_\alpha$. Thus, the strong-coupling "noninvariance" group $G_1$ is the inhomogeneous group $ISU(n)$ or $SU(n)\otimes T_{n^2-1}$,[4] with generators $M_\alpha$ and $P_\alpha=A_\alpha{}^{(0)}$.

In order to find the noninvariance group in the next approximation[1,5] in $1/g$ (intermediate-coupling group $G_2$), let us consider the remaining terms in (6). We determine their properties from the condition that the amplitude (3) can depend on even powers of $1/g$ only. Then

$$[A_\alpha{}^{(n)},A_b{}^{(n')}]+[A_\alpha{}^{(n')},A_\beta{}^{(n)}]=0 \tag{8}$$

for even $n$ and odd $n'$.

In the frames of $G_1\sim SU(n)\otimes T_{n^2-1}$ one can satisfy the relation (7b) with the trivial choice $A_\beta{}^{(1)}=P_\beta$ only. Therefore, it is possible to get a nontrivial expansion (6) only by an extension of the group $G_1$ to some other group $G_2$. In the spirit of our program we want to interpret $A_\alpha{}^{(1)}$ as a quantity proportional to a generator of this extended group $G_2$. Now it is clear from (7b) that the commutator of $[P_\alpha,A_\beta{}^{(1)}]$ should be symmetric under interchange of indices $\alpha$ and $\beta$. A brief inspection of possible groups including $SU(n)\otimes T_{n^2-1}$ as a subgroup shows us that the enlarged group $G_2$ is the inhomogeneous group $ISL(n,C)$ with the generators $P_\alpha$, $P_0$, $M_\alpha$, and $N_\alpha=A_\alpha{}^{(1)}$. The generator $P_0$ is necessary for the closure of the algebra. The algebra of $ISL(n,C)$[6] includes, in addition to (4), the following relations:

$$\begin{aligned}
&[M_\alpha,N_\beta]=iF_{\alpha\beta\gamma}N_\gamma, && [M_\alpha,P_\beta]=iF_{\alpha\beta\gamma}P_\gamma,\\
&[N_\alpha,N_\beta]=iF_{\alpha\beta\gamma}M_\gamma, && [P_a,N_\beta]=D_{abc}P_c,\\
&[P_a,P_b]=0, && [P_0,M_\alpha]=0,
\end{aligned} \tag{9}$$

where $a=(\alpha,0)$ and $F_{\alpha\beta\gamma}$ and $D_{abc}$ are antisymmetric and symmetric structure constants of $ISL(n,C)$.

In the frames of $ISL(n,C)$ the relations (8) can be satisfied by the choice

$$\begin{aligned}
A_\alpha{}^{(n)}&=\lambda^{(n)}P_\alpha \quad \text{for even } n,\\
A_\alpha{}^{(n)}&=\lambda^{(n)}N_\alpha \quad \text{for odd } n,
\end{aligned} \tag{10}$$

where $\lambda^{(n)}$ are constants.

Thus, insofar as we limit ourselves by $ISL(n,C)$ the expansion (6) can be written in a closed form

$$A_\alpha=f_1(1/g^2)P_\alpha+f_2(1/g^2)N_\alpha, \tag{11}$$

with the finite limits $f_1(0)$ and $f_2(0)$ if $g^2\to\infty$. The result may be summarized in the statement that in the strong-coupling approach to the static theory, the second-order noninvariance group $G_2$ (intermediate-coupling group) is $ISL(n,C)$, and meson-baryon coupling matrices are given by (11). The sequence of the approximate groups $G_0$, $G_1$, and $G_2$ is the following:

$$G_0=SU(n)\to G_1=SU(n)\otimes T_{n^2-1}\to G_2=ISL(n,C).$$

With every step of the approximation the (noninvariance) group $G$ is enlarged, including more and more noninvariance generators.

## 3. APPLICATION TO MESON-BARYON SCATTERING

As an application of the present "intermediate-coupling" group, we consider the meson-baryon scattering amplitude, when the symmetry group $G_0$ is $SU(3)$. In this case the first-order noninvariance group is the strong-coupling group $SU(3)\otimes T_8$ of Cook, Goebel, and Sakita.[4] If we repeat our arguments of Sec. 2 we shall be led to $ISL(3,C)$ as the second-order noninvariance group. The sequence of approximation groups in our case is

$$G_0=SU(3)\to G_1=SU(3)\otimes T_8\to G_2=ISL(3,C).$$

With (6) one gets, for $T_{\alpha\beta}$ as a matrix in baryon isobar (unitary) space in a pole approximation,

$$\frac{V\omega}{2\pi}T_{\alpha\beta}\equiv t_{\alpha\beta}(\omega)=-\frac{a^2}{\omega}iF_{\alpha\beta\gamma}M_\gamma-\frac{1}{\omega^2}[P_\beta,[\Delta,P_\alpha]], \tag{12}$$

where $\alpha$, $\beta$, $\gamma$ are $SU(3)$ indices and $a$ is a parameter. We have retained in (12) only terms which do not depend explicitly on $g^2$. The crossing relation $t_{\alpha\beta}(\omega)=t_{\beta\alpha}(-\omega)$ is evidently satisfied by (12).

If we write the amplitude $t_{\alpha\beta}(\omega)$ in an explicitly crossing-symmetric form,

$$t_{\alpha\beta}=\frac{g_{\alpha\beta}+\omega R_{\alpha\beta}}{-\mu-iq},\quad g_{\alpha\beta}=g_{\beta\alpha},\quad R_{\alpha\beta}=-R_{\beta\alpha}, \tag{13}$$

then a comparison with (12) at $\omega\approx0$ gives

$$g_{\alpha\beta}=2\mu[P_\beta,[\Delta,P_\alpha]],\quad R_{\alpha\beta}=2\mu a^2 iF_{\alpha\beta\gamma}M_\gamma. \tag{14}$$

It may be remarked that the term with $g_{\alpha\beta}$ contributes only to the sum of $s$- and $u$-channel amplitudes, $t_{\alpha\beta}+t_{\beta\alpha}$, of the meson-baryon scattering. Actual calculation of $g_{\alpha\beta}$ involves additional approximations, such as saturating the complete set of intermediate states by the one-particle states only. Further this sum would be a characteristic contribution for the first-order noninvariance group $G_1$ (the group of strong-coupling theory). We have elsewhere discussed the importance of calculat-

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[5] J. G. Kuriyan and E. C. G. Sudarshan, Phys. Rev. Letters **16**, 825 (1965).

[6] Yu. V. Novozhilov and I. A. Terentjev, Yadern. Fiz. **3**, 1138 (1966) [English transl.: Soviet J. Nucl. Phys. **3**, 827 (1966).

ing $t_{\alpha\beta}+t_{\beta\alpha}$, the sum of the $s$- and $u$-channel amplitudes for meson-baryon scattering, employing the noninvariance group $G_1$ in the context of high-energy diffraction scattering.

One can now deduce from (13) that the difference between the uncrossed amplitude and the crossed one is proportional to the generators of the maximal compact subgroup of $G_2$, i.e., the invariance group $G_0$:

$$t_{\alpha\beta}(\omega)-t_{\beta\alpha}(\omega)\sim iF_{\alpha\beta\gamma}M_\gamma. \qquad (15)$$

This is precisely the result of Sudarshan and Kuriyan,[5] who used the *compact* noninvariance group of the intermediate-coupling theory. It may be noted that in the strong-coupling theory of Cook, Goebel, and Sakita[4] the right-hand side of (15) is equal to zero.

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# Nonleptonic Hyperon Decay and Baryon–Weak-Hamiltonian Commutators*

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The amplitude for the nonleptonic decay of hyperons is expressed in terms of an absorptive part and an equal-time commutator of the baryon field with the weak Hamiltonian. The assumption that the parity-conserving and parity-violating parts of the weak Hamiltonian transform like scalar ($S_6$) and pseudoscalar ($S_7^5$) quantities, respectively, enables us to evaluate the commutator. The absorptive parts are calculated under the assumption that they are dominated by single-particle states. The assumption that $S_7^5$ and the strong pseudoscalar vertex belong to the same octet when combined with the experimental observation that $A(\Sigma_+^+)=0$ enables the calculation of $d/f$ for this octet. The value obtained is $\sqrt{3}$, which is in excellent agreement with the value obtained from leptonic decays. The results for $p$ waves are not so definite, but indicate a dominant $d$-type coupling.

## I. INTRODUCTION

Many attempts have been made recently to explain weak interactions and especially nonleptonic decay of hyperons using the hypothesis of partially conserved axial-vector current (PCAC) and the algebra of $U(3)\times U(3)$ generated by integrated current components. All these attempts require in addition the assumption that the decay amplitudes do not change appreciably when we extrapolate the pion four-momentum from zero to its finite experimental value. Brown and Sommerfield[1] have shown for $P$ waves that this is not always so and that there is an appreciable change in the amplitude as the pion four-momentum is taken to zero. Such large variation occurs also in the nonleptonic decay of $K$ mesons, as has been pointed out by several authors.[2,3] It is therefore of interest to examine methods which do not need the assumption that the four-momentum of the pion goes to zero. Another motivation for our calculation is to study the implications of assumed commutation relations between two currents, one with zero baryon number and one with unit baryon number, and the "sidewise" dispersion relations natural to them.[4] Thus, we assume, in our approach, that $SU_3$ is a good symmetry and that the parity-conserving and parity-violating parts of the weak Hamiltonian transform like a scalar $(S_6\sim\frac{1}{2}\bar{q}\lambda_6 q)$ and a pseudoscalar $(S_7^5\sim\frac{1}{2}i\bar{q}\lambda_7\gamma_5 q)$, respectively.[3] This enables us to write down an expression for the part of

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[1] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965); H. Sugawara, *ibid.* **15**, 870 (1965); *ibid.* **15**, 997 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); L. S. Brown and C. M. Sommerfield, *ibid.* **16**, 751 (1966).

[2] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); M. Suzuki, *ibid.* **16**, 212 (1966).

[3] Y. Hara and Y. Nambu, Phys. Rev. Letter **16**, 875 (1966).

[4] A. M. Bincer, Phys. Rev. **118**, 855 (1960); S. D. Drell and H. R. Pagels, *ibid.* **140**, B397 (1965); H. Suura and L. M. Simmons, Jr., *ibid.* **148**, 1579 (1966); K. Bardakci, *ibid.* **155**, 1788 (1967); G. Mohan, H. S. Mani, and L. K. Pande, Nuovo Cimento **44**, 265 (1966); S. Fenster and N. Panchapakesan, Phys. Rev. **154**, 1326 (1967); R. Delbourgo, A. Salam, and J. Strathdee, Phys. Letters **22**, 680 (1966); R. C. Hwa and J. Nuyts, Phys. Rev. **151**, 1215 (1966).