Kinematic Parameter for Estimating the Primary Energy and for Studying Nuclear Interactions from 17 to $\sim 10^3$ GeV*†

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The kinematic parameter $\eta = \text{arctanh}\,(\beta\cos\theta)$ for a secondary of a jet transforms from the laboratory system (LS) to η^* in a frame of reference moving with a velocity β_c with respect to the LS in the direction of the incident primary, according to the relation $\eta = \eta^* + \text{arctanh}\beta_c$. Then the velocity β_S of a "symmetric system," of a group of produced secondaries for which the mean value of η statistically vanishes, is obtained from the formula $\text{arctanh}\,\beta_S = \langle\eta\rangle$, which usually reduces to $\ln\gamma_S = -\langle\ln\tan\theta\rangle - \langle\ln[(1+x^2)^{1/2}\,x^{-1}]\rangle$, where $x = p_t/m$. For the usual situation where only the emission angles of a subset of *charged* secondaries of a jet is known, a parameter $\eta(\theta) \cong 0.46 - \ln\tan\theta$, which depends only on θ but which is consistent with the definition of η, is introduced, and the rather well-known distribution of the transverse momentum of pion secondaries is used in place of knowledge of values of β to calculate results based on the use of η. The $E(\theta)$ method, in which one substitutes $\langle\eta(\theta)\rangle$ for $\langle\eta\rangle$ to find the velocity of the symmetric system, β_S, is an improvement over the *spectrum-independent* formula by Castagnoli *et al.*, $\ln\gamma_{\text{Cast}} = -\langle\ln\tan\theta\rangle$, for estimating the primary energies of jets. This is shown with accelerator-produced jets with energies ranging from 17 to 30.9 GeV and cosmic-ray jets with energies around 10^3 GeV. Also studied are the magnitude of the statistical error in using the $E(\theta)$ method and various aspects of the problem of multiple production of particles which have been determined by examining the $\eta(\theta)$ distributions of jets and their dependence on the primary energy.

I. INTRODUCTION

FOR a jet, let β denote the velocity and θ the emission angle of a secondary in the laboratory system (LS) where the emission angle is measured with respect to the direction of the incident primary. Then the kinematic parameter, $\eta = \text{arctanh}(\beta\cos\theta)$, has a simple property with respect to Lorentz transformations,[1] namely, if β^* and θ^* are the secondary's velocity and emission angle, respectively, in a frame of reference which moves with a velocity β_c with respect to the LS in the direction of the primary, then $\eta^* = \text{arctanh}(\beta^*\cos\theta^*)$ is related to η by the relation

$$\eta = \text{arctanh}\beta_c + \eta^*. \tag{1}$$

[The above equation is proven in Appendix A, in which it is also shown how some simplification for the procedure of Lorentz transformations results from the use of Eq. (1).] It then follows that the distribution of η^* in *any* Lorentz system moving with a velocity β_c with respect to the LS in the direction of the primary can be obtained merely by shifting the origin along the η axis by an amount $\text{arctanh}\beta_c$. Equation (1) is also immediately useful for formulating backward and forward symmetry for the produced secondaries in the center-of-mass system (CMS) of a nucleon-nucleon or a nucleon–quasifree-nucleon collision. This assumption of "symmetry" in the CMS was also the basis of the median-angle[2] method and the Castagnoli[3] method of estimating the primary energy of the incident nucleon in the LS.

For a jet with n secondaries which are created in a single collision of two particles, it follows from Eq. (1) that

$$\langle\eta\rangle = \text{arctanh}\beta_c + \langle\eta^*\rangle. \tag{2}$$

For the "symmetric system," whose velocity β_S is defined by the relation

$$\langle\bar{\eta}\rangle = \frac{1}{n}\sum_i^n \text{arctanh}(\bar{\beta}_i\cos\bar{\theta}_i) = 0, \tag{3}$$

we have

$$\text{arctanh}\beta_S = \langle\eta\rangle. \tag{4}$$

In a nucleon-nucleon or a nucleon–quasifree-nucleon collision, β_S will coincide, *on the average*, with the velocity of the CMS of the two nucleons, $\beta_{\text{c.m.}}$, i.e.,

$$\langle\text{arctanh}\beta_S\rangle = \text{arctanh}\beta_{\text{c.m.}}, \tag{5}$$

which is equivalent to the relation

$$\text{arctanh}\beta_{\text{c.m.}} = \langle\langle\eta\rangle\rangle, \tag{5'}$$

because of Eq. (4). Since the initial system of the two nucleons has forward and backward symmetry in the CMS, there will be average forward and backward symmetry for any kinematic quantities related to the produced particles. In other words, it is assumed here that for each secondary with velocity $\bar{\beta}$ at $\bar{\theta}$ there exists one with velocity $\bar{\beta}$ at $180° - \bar{\theta}$.

This way of formulating the symmetry in the CMS is different from the median-angle[2] and Castagnoli[3] methods. Also another way of formulating the required

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[1] G. A. Milekhin, Zh. Eksperim. i Teor. Fiz. **35**, 1185 (1958) [English transl.: Soviet Phys.—JETP **8**, 829 (1959)].

[2] H. Bradt, M. F. Kaplon, and B. Peters, Helv. Phys. Acta **23**, 24 (1950).

[3] C. Castagnoli, T. Cortini, C. Franzinetti, A. Manfredini, and D. Moreno, Nuovo Cimento **10**, 1539 (1953); M. L. Shen and M. F. Kaplon, Ann. Phys. (N.Y.) **32**, 452 (1965); K. Imaeda and T. P. Shah, Nuovo Cimento **41**, 405 (1966).

symmetry is described in Appendix B. The present way of formulating forward and backward symmetry in the CMS is apparently less general than in Refs. 2 and 3, since we have to assume two kinematic quantities $\bar{\beta}$ and $\bar{\theta}$. But the advantage is in the fact that we do not need the basic assumption of $\bar{\beta}/\beta=1$, which is necessary to derive the spectrum-independent Castagnoli formula, $\ln\gamma_{\rm Cast}=-\langle\ln\tan\theta\rangle$, so we can easily achieve what the authors in Ref. 3 try to correct for the assumption.

The energy E_p of the incoming primary in the LS can be estimated from the following formula:

$$E_p=M_t(2\gamma_{\rm c.m.}{}^2-1)=M_t\cosh(2\langle\langle\eta\rangle\rangle)\,, \qquad (6)$$

where $\gamma_{\rm c.m.}=(1-\beta_{\rm c.m.}{}^2)^{-1/2}$, M_t is the mass of target (the mass M of a nucleon in this case) which is assumed to be at rest in the LS, and $\langle\langle\eta\rangle\rangle$ is meant to be an average taken over the individual values of $\langle\eta\rangle$ of jets whose primary energy in the LS is E_p.

In Sec. II a consistent parameter $\eta(\theta)$ is introduced to improve on the basic parameter, $(-\ln\tan\theta)$, of the Castagnoli method. In Sec. III the $E(\theta)$ method, an improved version of the spectrum-independent Castagnoli method, is tested using various jets for which the primary energy has been established by at least one method which is more accurate than the $E(\theta)$ method; a formula for an estimate of statistical errors in the $E(\theta)$ method of energy estimation is given. In Sec. IV, the dependence on the primary energy E_p of the standard deviation of the distribution of $\eta(\theta)$ is obtained, and this relation is correlated to the E_p dependence of the average CMS energy of secondaries. In Sec. V, for 30.9-GeV jets in nuclear emulsion, $\langle\langle\eta(\theta)\rangle\rangle$ has been found to depend strongly on the charged-particle multiplicity n_s and the number of heavily ionizing prongs N_h.

II. INTRODUCTION OF $\eta(\theta)$

It is convenient to introduce the parameter $x=p_t/m$, where p_t is the transverse momentum of the secondary and m its mass. Then

$$\begin{aligned}\eta(x,\theta)&=\operatorname{arctanh}[x\cos\theta(x^2+\sin^2\theta)^{-1/2}]\\&=(\pm)[\operatorname{arctanh}(1+v^2)^{-1/2}]\\&=(\pm)\{-\ln v+\ln[1+(1+v^2)^{1/2}]\}\,, \qquad (7)\end{aligned}$$

where $v^2=(1+x^2)x^{-2}\tan^2\theta$, $v\geq0$. The positive sign is used for $0°\leq\theta<90°$ and the negative sign for $90°<\theta\leq180°$.

When v is small compared with unity, which is observed to be the case in the LS with the majority of secondaries produced in high-energy jets,

$$\begin{aligned}\eta&\cong-\ln v+\ln2\\&=-\ln\tan\theta-\ln[(1+x^2)^{1/2}x^{-1}]+\ln2\,. \qquad (8)\end{aligned}$$

The rare case when a secondary has $\theta>90°$ is not considered. For pion secondaries the second term in the right-hand side of the above equation is usually small compared with the first term, since it is known that the majority of the pions have $p_t>m$, i.e., $x>1$. Also, $\operatorname{arctanh}\beta_S=\ln\gamma_S+\ln[1+(1-\gamma_S{}^{-2})^{1/2}]$, so that when $\gamma_S{}^2\gg1$, Eq. (4) reduces to

$$\ln\gamma_S=-\langle\ln\tan\theta\rangle-\langle\ln[(1+x^2)^{1/2}x^{-1}]\rangle\,. \qquad (9)$$

Thus, it can be seen that the spectrum-independent approximation of the Castagnoli method of energy estimation[3] is equivalent to neglecting the small second term in Eq. (9). It has been shown that the use of the spectrum-independent formula of the Castagnoli method generally causes one to overestimate the primary energy of jets.[4] (See also Sec. III.) This factor of overestimation is mainly represented by the second term in the right-hand side of Eq. (9).

On the other hand, $\eta\cong0$ when v is very large compared with unity, which condition can be achieved by $\theta\rightarrow90°$ and/or $x\ll1$. But the approximation embodied in the spectrum-independent Castagnoli formula reveals an inconsistency because $(-\ln\tan\theta)$ tends to $-\infty$ when θ tends to 90°. In using the formula, this sometimes results in $\gamma_{\rm Cast}$ being less than unity. In addition, the parameter $(-\ln\tan\theta)$ obviously cannot be used for those secondaries which are produced with $\theta>90°$.

To incorporate the advantage that the second correction term in the right-hand side of Eq. (9) is usually *small* and to eliminate the minor inadequacies inherent in the spectrum-independent formula in the Castagnoli method, we introduce a new consistent parameter $\eta(\theta)$, which employs the average behavior of p_t for pions, as indicated in the following expression:

$$\eta(\theta)\equiv\int_0^R\eta(p_t/m_\pi,\theta)f(p_t)dp_t\Big/\int_0^R f(p_t)dp_t \quad \text{for}\quad\theta<90°\,, \qquad (10a)$$

and the relation

$$\eta(\theta)=-\eta(180°-\theta)\,. \qquad (10b)$$

The distribution of pion transverse momenta $f(p_t)dp_t$ is taken to be

$$f(p_t)=(1/p_0)^2p_t\exp(-p_t/p_0)\,, \qquad (11)$$

with $p_0=\langle p_t\rangle/2=0.17$ GeV/c. The mean value of p_t has been found experimentally to have very little variation throughout a wide range of the primary energy.[5] The upper limit R of the integration is taken

[4] E. Lohrmann, M. W. Teucher, and M. Schein, Phys. Rev. **122**, 672 (1961); P. L. Jain, E. Lohrmann, and M. W. Teucher, *ibid.* **115**, 643 (1959); H. Winzeler, B. Klaiber, W. Koch, M. Nikolic, and M. Schneeberger, Nuovo Cimento **17**, 8 (1960); A. Barbaro-Galtieri, A. Manfredini, C. Castagnoli, C. Lamborizio, and I. Ortalli, *ibid.* **20**, 487 (1961); H. H. Aly, C. M. Fisher, and A. Mason, *ibid.* **28**, 1117 (1963); H. Meyer, M. W. Teucher, and E. Lohrmann, *ibid.* **28**, 1399 (1963); R. D. Settles and R. W. Huggett, Phys. Rev. **133**, B1305 (1964).

[5] P. H. Fowler and D. H. Perkins, Proc. Roy. Soc. (London) **A278**, 401 (1964).

to be 1 GeV/c, just for convenience in performing the integration. The choice of R does not affect the value of $\eta(\theta)$ too much as long as $R/m\gg1$, since for $x\gg1$ the second term in the right-hand side of Eq. (8) becomes extremely small compared with the first term. With $R=1$ GeV/c, about 2% of the p_t distribution given by Eq. (11) is neglected.

In Fig. 1 the variation of $\eta(p_t/m_\pi=x,\theta)$ with $\tan\theta$ for $p_t=0.05$, 0.10, and 1.0 GeV/c are shown. The parameter $-\ln\tan\theta$, which is basic for calculating the primary energy with the Castagnoli method, is drawn in Fig. 1 for comparison. One can see that the curves for $-\ln\tan\theta$ are parallel for $\tan\theta\lesssim0.1$. In this region the approximate expression for η given by Eq. (8) is valid. When the distribution of transverse momenta for pion secondaries is assumed to be that given by Eq. (11), one has

$$\langle\ln[(1+x^2)^{1/2}x^{-1}]\rangle=0.233. \tag{12}$$

Therefore, in the region where $\tan\theta\lesssim0.1$,

$$\eta(\theta)=0.46-\ln\tan\theta. \tag{13}$$

The new parameter $\eta(\theta)$ is not linear to $(-\ln\tan\theta)$ as $\tan\theta$ increases from ~0.1. This is the range where the use of $\eta(\theta)$ becomes very significant.

With the use of the parameter $\eta(\theta)$, Eq. (4) can be approximated by

$$\operatorname{arctanh}\beta_S=\langle\eta(\theta)\rangle. \tag{14}$$

This formula must be used with caution when the number of produced particles in a nucleon-nucleon jet becomes so small that the surviving baryons among the secondaries have considerable influence. The transverse momentum of protons is known[6,7] to be ~0.5 GeV/c for jets with energies from 20 GeV to $\sim10^3$ GeV. This means that the second term in the right-hand side of Eq. (8) for a surviving proton is not as small as that shown in Eq. (12). In fact, using for a proton $p_t=0.5$ GeV/c, then $\ln[(1+x_p^2)^{1/2}x_p^{-1}]=0.7$. This indicates that instead of Eq. (14), Eq. (4) will be better approximated by

$$\begin{aligned}\operatorname{arctanh}\beta_S&=\langle\eta(\theta)\rangle-(2/n)\\&\quad\times\{\langle\ln[(1+x_p^2)^{1/2}x_p^{-1}]\rangle-0.233\}\\&\cong\langle\eta(\theta)\rangle-(2/n)0.5,\end{aligned} \tag{14'}$$

for nucleon jets. In Sec. V this is shown in detail to be the case for 30.9-GeV jets with small n_s.

III. ESTIMATION OF THE PRIMARY ENERGY OF JETS

Our study in the present section is for the usual situation in which only the emission angles of a subset of *charged* secondaries of a jet (which are the only ones readily observable using detectors such as nuclear

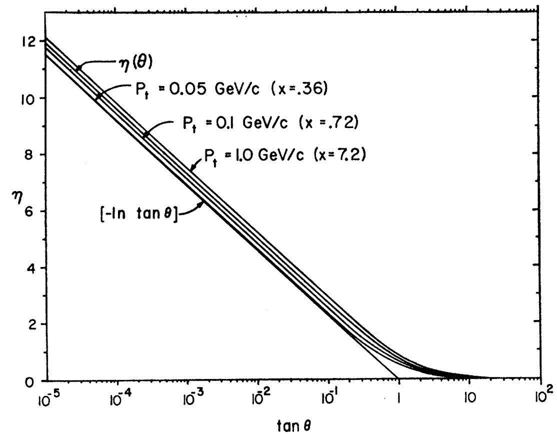


FIG. 1. The variation of $\eta(p_t,\theta)$ with $\tan\theta$ for several values of the pion transverse momentum and the variation of $\eta(\theta)$ with $\tan\theta$.

emulsion) are known. So in taking averages, only contributions of n_s charged secondaries will be taken into account. In addition to this we shall limit our study to (i) those jets with $n_s>5$, except for the Monte Carlo jets in Sec. III.D, and (ii) those jets with N_h as small as the number of jets in the samples permits. Conditions (i) and (ii) are imposed mainly to avoid a consistent overestimation and underestimation, respectively, of the primary energy which would occur for jets not satisfying these conditions. (This will be discussed in Sec. V.)

We shall define the energy estimate $E(\theta)$ to be given by

$$E(\theta)\equiv M[2\gamma_S^2(\theta)-1]=M\cosh[2\langle\eta(\theta)\rangle], \tag{15}$$

where M is the nucleon mass and the indicated average is over the n_s charged secondaries of the event. Similarly, E_{Cast} in the spectrum-independent formula in the Castagnoli method of energy estimation is defined by[8]

$$E_{\text{Cast}}\equiv M(2\gamma_{\text{Cast}}^2-1), \tag{16}$$

where $\ln\gamma_{\text{Cast}}=-\langle\ln\tan\theta\rangle$ with the indicated average also being taken over the charged particles of the event. Also, the E_{ch} method[9] will be studied. This E_{ch} method is based on the assumption that all the transverse momenta of the charged secondaries are constant and equal to 0.4 GeV/c, i.e.,

$$E_{\text{ch}}\equiv0.4\sum_i^{n_s}\csc\theta_i\cong\sum_{\text{ch}}E_i. \tag{17}$$

Thus $E_p=E_{\text{ch}}/K_{\text{ch}}$, where K_{ch} is the fraction of the available energy which is carried away by the charged secondaries. The fraction K_{ch} is estimated, on the average, to be 0.3 in Ref. 9. However, we have used $K_{\text{ch}}=1$ for our primary energy estimates using the E_{ch}

[6] D. R. O. Morrison, CERN Report No. CERN-TC-Physics 63-1, 1963, p. 1 (unpublished).
[7] C. O. Kim, Phys. Rev. 136, B515 (1964).

[8] For the rare examples in which emission angles of the secondaries are larger than 90° in the LS, it has been customary to omit the secondaries in the use of Eq. (16).
[9] ICEF collaboration, Nuovo Cimento Suppl. 1, 1039 (1964).

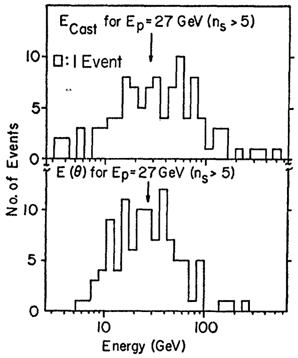


FIG. 2. The upper histogram is a distribution of Castagnoli energies for 27-GeV proton jets; the lower histogram is the corresponding distribution of $E(\theta)$ estimates.

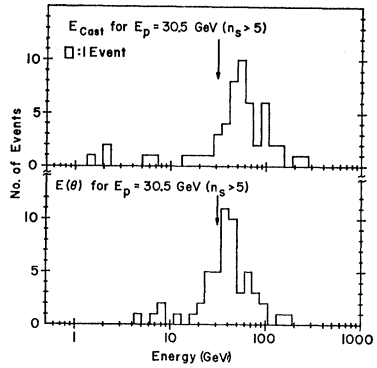


FIG. 3. The upper histogram is a distribution of Castagnoli energies for jets produced by 30.5-GeV protons; the lower histogram is the corresponding distribution of $E(\theta$ estimates.

method in Secs. III A–D. In Sec. III E, summarizing comments on the various methods of energy estimation which we studied are given .The statistical errors for the $E(\theta)$ method are discussed in Sec. III F.

A. Estimation of the Energy of Accelerator-Produced Jets

For estimating the primary energy of a jet, the $E(\theta)$ method is a considerable improvement over the E_{Cast} method. This will be shown by using jets produced in nuclear emulsion by accelerator protons and pions of known energy. Figures 2, 3, and 4 show the results of energy estimation for the three groups of jets produced by accelerator protons.[10–12] The jets which were included in the analysis had $N_h=0$ and 1, and also $n_s>5$. The plots in the upper halves of these figures are distributions of the primary energies as obtained by the E_{Cast} method and the plots in the lower halves of the figures are the distributions of energy estimates obtained by using the $E(\theta)$ method. Figure 5 shows the same type of plots for 73 jets with $N_h<3$ and $n_s>5$ produced by 17-GeV pions in emulsion.[13]

Table I lists $\langle\log_{10}(E/E_p)\rangle$ where E is the estimate of E_p for the $E(\theta)$, E_{Cast} and E_{ch} method. The quoted errors are determined from the standard deviations of the distribution involved. The average standard deviations for individual values of $\log_{10}(E/E_p)$ are given in the parentheses following the corresponding $\langle\log_{10}(E/E_p)\rangle$.

B. Jets Produced by Nucleon Fragments of a Primary Nucleus

A high-energy nucleus breaks gradually into fragments when it traverses a sufficiently thick block of nuclear emulsion. The fragments have approximately the same velocity in the LS as the primary nucleus and are emitted within a cone of very small angle about the primary's direction. This is due to the small velocity of the fragments with respect to the primary nucleus compared with the large velocity of the primary nucleus in the LS. In a sense, these fragments form a "pencil beam" around a core predominantly consisting of nucleons and nuclei with the same velocity as the primary nucleus. This is the reason why we chose to study a family of jets of the event No. 1115 of the Brawley stack.[14] Event No. 1115 involved a family of jets initiated by a nucleus with a charge $Z=15$. There were 26 *secondary* jets found inside a cone with its apex at the primary jet and having an opening angle of about 1 mrad with respect to the direction of the primary nucleus. Among these we chose for our study 9, 6, and 2 jets produced by singly charged, neutral, and multiply charged secondary particles, respectively. (Seven jets had $n_s\leq5$.) Jets No. 1115-6 and No. 1115-7, which did have $n_s>5$, were excluded from the sample because they were essentially superimposed and one could not be sure that the jet No. 1115-7 was not produced by a (tertiary) particle which was created in the immediately preceding jet No. 1115-6.

Employing the method used by Lohrmann *et al.*,[15] a reliable estimate of the primary energies of the fragments was obtained by relative scattering measurements on the three singly charged tracks (Nos. 49, 50, and 51 of the jet No. 1115-6) which were bundled closely (separations $<10\ \mu$ in projection and $<4\ \mu$ in depth) between the He and Li fragments. The resut is

$$E_p=(0.57_{-0.06}^{+0.07})\times10^3\ \mathrm{GeV/nucleon}, \quad (18)$$

where the error is based on the propagation of errors estimated from the deviations of the average second differences resulting from each pairing of the three p tracks. The reading noise was eliminated using the standard method which employs the different dependence of noise and Coulomb scattering on the cell length.

[10] Barbaro-Galtieri *et al.*, in Ref. 4.

[11] M. Teranaka, J. Phys. Soc. Japan **20**, 1297 (1965). The author took the liberty of including ten *grey* tracks, which were among ten jets of $N_h=1$, as secondaries.

[12] J. J. Lord (private communication). The data have been used in a Ph.D. thesis submitted by E. R. Goza to the Department of Physics, University of Washington, Seattle, Washington, 1962 (unpublished); Ref. 9.

[13] H. H. Aly *et al.*, in Ref. 4.

[14] F. Abraham, R. Levi-Setti, C. H. Tsao, J. Gierula, K. Rybicki, W. Wolter, R. L. Fricken, and R. W. Huggett, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (Institute of Physics and the Physical Society, London, 1966), Vol. 2, p. 844; Enrico Fermi Institute Report Nos. EFINS 62-76 and 65-44 (unpublished).

[15] E. Lohrmann *et al.*, in Ref. 4.

TABLE I. Comparison of various methods energy estimation.[a]

Reference	Primary enery (GeV)	Number of jets	$\langle n_s \rangle$	$\langle \log_{10}[E(\theta)/E_p] \rangle$	$\langle \log_{10}(E_{Cast}/E_p) \rangle$	$\langle \log_{10}(E_{ch}/E_p) \rangle$
b	27	104	7.5	0.02±0.03 (0.34)	0.08±0.04 (0.44)	−0.13±0.02 (0.24)
c	30.5	54	7.7	0.09±0.04 (0.30)	0.17±0.06 (0.43)	0.00±0.04 (0.28)
d	30.9	126	7.9	0.01±0.03 (0.31)	0.15±0.03 (0.37)	−0.07±0.02 (0.27)
e	17	73	7.2	0.05±0.03 (0.27)	0.13±0.04 (0.33)	−0.02±0.03 (0.25)
f	570	13	18.2	0.03±0.10 (0.31)	0.17±0.10 (0.32)	0.05±0.13 (0.47)
g	3×10^3	10^3	9.8	0.09±0.01 (0.34)	0.29±0.01 (0.34)	−0.67±0.01 (0.31)
g	Using all the secondaries, including neutral secondaries		16	0.113±0.005 (0.15)	0.312±0.005 (0.15	−0.399±0.007 (0.23)

[a] Each number in parentheses is the standard deviation of the distribution of individual quantities whose mean is given directly above.
[b] Reference 10.
[c] Reference 11.
[d] Reference 12.
[e] Reference 13.
[f] Reference 14. (The two lowest estimates of energy were excluded for the tabulation, see Fig. 6.)
[g] Reference 21.

Also, by using the Kaplon formula[16] and the measured opening angle of the He fragment from the Li fragment, 0.97×10^{-4} rad, one obtains

$$E_p = 0.6\times10^3 \text{ GeV/nucleon}. \quad (19)$$

The results of the energy estimation from emission angles of jets are shown in Fig. 6; the top histogram is the energy distribution as obtained by using the E_{Cast} method and the lower histogram is the corresponding distribution of energy estimates obtained by the $E(\theta)$ method. The nucleon fragments show clustering *mostly* around 0.9×10^3 GeV in the top histogram and 0.6×10^3 GeV in the lower histogram. Table I lists $\langle \log_{10}(E/E_p) \rangle$ for the three methods of energy estimation, the $E(\theta)$, E_{Cast}, and E_{ch} methods. Here the two lowest energy estimates were deleted from the sample because they were interpreted as being associated with secondaries which were mixed with the fragments.

The two jets made by heavy fragments show a trend toward higher apparent energy when compared with jets made by nucleon fragments even after the ΔZ particles at the smallest angles in jets produced by heavy nuclei had been excluded when using Eq. (15) or Eq. (16). [$\Delta Z = Z_p - \sum_i Z_i'$, where the charge of the primary nucleus producing a jet is Z_p and that of a produced heavy fragment Z_i' (≥ 2).] This will be discussed further in conjunction with the cases in which the primary energy is underestimated by the $E(\theta)$ and E_{Cast} methods when they are applied to estimating the energy of jets produced in proton-nucleus collisions in Sec. V.

C. Jets Produced by Singly Charged or Neutral Cosmic-Ray Primaries

Selected for the present study were ten jets with $N_h \leq 4$ and $n_s > 5$, produced in nuclear emulsion by

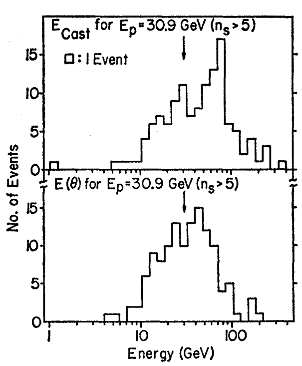


FIG. 4. The upper histogram is a distribution of Castagnoli energies for jets produced by 30.9-GeV protons; the lower histogram is the corresponding distribution of $E(\theta)$ estimates.

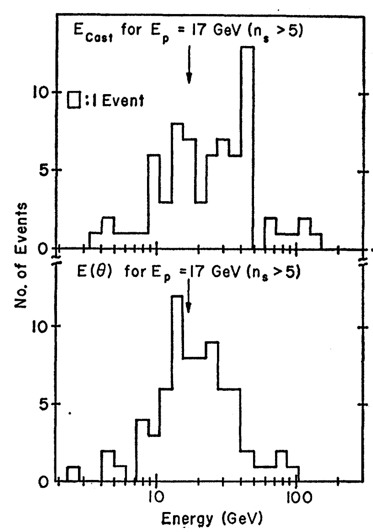


FIG. 5. The upper histogram is a distribution of Castagnoli energies for jets produced by 17-GeV pions; the lower histogram is the corresponding distribution of $E(\theta)$ estimates.

[16] M. F. Kaplon, B. Peters, H. L. Reynolds, and D. M. Ritson, Phys. Rev. 85, 295 (1952).

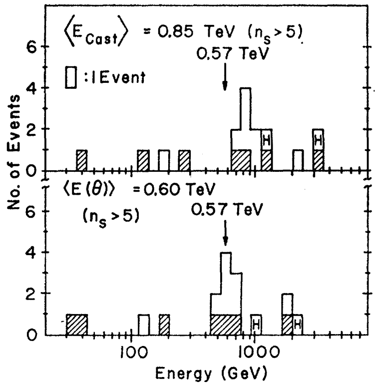


FIG. 6. The upper histogram is a distribution of energies estimated by the Castagnoli method for the 17 jets of Brawley event No. 1115; the lower histogram is the corresponding distribution of $E(\theta)$ estimates. The shaded boxes represent jets with $N_h > 5$ which were produced by nucleon fragments. The two jets produced by heavy primaries are identified by an H in the appropriate boxes.

singly charged or neutral primary particles with energies around 10^3 GeV, for which secondaries of the backward cone were well investigated by scattering measurements and grain counts. Table II lists the ten jets. The first two jets were investigated by Schein *et al.*,[17] and the next three jets were studied by the author.[7] The rest of the jets have been found in the Brawley stack and investigated by the author.[18]

The angular distributions ($\log_{10} \tan\theta$) in the LS of the ten jets are shown in Figs. 7(a)–(j) in the forms of the $\log_{10}(F/1-F)$ plots.[19] The closed circles represent points for the LS. When the Lorentz factor $\gamma_{\text{c.m.}}$, which defines the CMS is taken to be γ_{Cast}, the $\log_{10}[F/(1-F)]$ plot, shown by the triangles in the figure, would be the one which observers in the antilaboratory system (ALS)[20] would make for the jet. The transformations of angles of the indicated tracks in the figure from the LS to the ALS were possible without any unreasonable assumption about the LS velocities of those secondaries, since most of them had been measured. It can be seen in the figures that triangles do not match with the filled circles. For each jet, by choosing a Lorentz factor γ_K, one can achieve the closest match of the two $\log_{10}[F/(1-F)]$ plots, that for the LS and the other for the ALS. The details of this procedure are described in Appendix B. The squares show the closest match. Then an energy estimate E_K is obtained from the formula

$$E_K \equiv M(2\gamma_K^2 - 1)\,, \qquad (20)$$

where we have taken $\gamma_K = \gamma_{\text{c.m.}}$. This method of estimation of the primary energy of jets is equivalent to that developed so far, because a symmetry is required with respect to the forward and backward directions in the CMS, as shown in Appendix B.

TABLE II. Comparison of various methods of energy estimation for some well-measured cosmic-ray jets.[a]

Event No.	Type	Ref.	$E(\theta)$	E_{Cast}	E_{ch}	E_K
Schein p	$(0+20)_p$	b	3.8	5.7	2.1	3.0
Schein n	$(0+20)_n$	b	0.98	1.5	0.98	0.97
Texas No. 4	$(2+17)_p$	c	2.4	3.7	1.5	1.5
Texas No. 47	$(2+14)_p$	c	4.6	6.5	1.8	5.3
Texas No. 118	$(0+21)_p$	c	8.2	13.0	2.2	5.3
Brawley No. 1021	$(0+18)_p$	d	9.5	15.1	2.3	5.3
Brawley No. 1026	$(4+12)_p$	d	3.5	5.5	1.1	3.4
Brawley No. 1061	$(0+45)_p$	d	5.7	9.0	5.0	3.9
Brawley No. 1135	$(2+7)_p$	d	2.1	3.3	0.27	2.0
Brawley No. 1144	$(3+38)_p$	d	6.4	9.7	8.0	5.1
Averages	$[\langle n_s\rangle = 21.2]$		4.7	7.3	2.5	3.6
$\langle\log_{10}(E/E_p)\rangle$			0.10 ±0.03	0.29 ±0.04	−0.24 ±0.20	···
Standard deviation[e]			(0.09)	(0.11)	(0.63)	(···)

[a] All energies are in 10^3 GeV; in the lower part of the table we have used $E_p = E_K$.
[b] Reference 17.
[c] Reference 7.
[d] Reference 18.
[e] Each number in parentheses is the standard deviation of the distribution of individual $\log_{10}(E/E_p)$ whose mean is given above.

D. Monte Carlo Jets

The various methods of energy estimation were applied to 1000 CKP Monte Carlo jets of $E_p = 3\times10^3$ GeV,[21] in which the total momentum of the produced kaons and pions (the total number of particles in a jet $n=16$) is strictly zero in the CMS. Table I lists $\langle\log_{10}(E/E_p)\rangle$ for various methods which use only the emission angles of secondaries.

Shown in Table III are the results obtained when the E_η method[22] is used with charged secondaries only and also when the E_η method is used with all the secondaries, including the neutral kaons and pions.

TABLE III. The E_η method of energy estimation[a] used for 1000 CKP Monte Carlo jets of $E_p = 3\times10^3$ GeV.[b]

	$\langle\log_{10}(E_\eta/E_p)\rangle$	$10^{\langle\log_{10}(E_\eta/E_p)\rangle}$	Std. dev.[c]
From charged secondaries	−0.01 ±0.01	0.98 ±0.02	(0.42)
From all secondaries	0.006±0.003	1.014±0.007	(0.07)

[a] Reference 22.
[b] Reference 21.
[c] Each number in parentheses is the standard deviation of the distribution of individual values of $\log_{10}(E_\eta/E_p)$.

[17] M. Schein, D. Haskin, E. Lohrmann, and M. W. Teucher, Phys. Rev. **116**, 1238 (1959).

[18] The study of these p jets and others is similar to that in Ref. 7, and is being prepared for publication.

[19] N. Duller and W. Walker, Phys. Rev. **93**, 215 (1954).

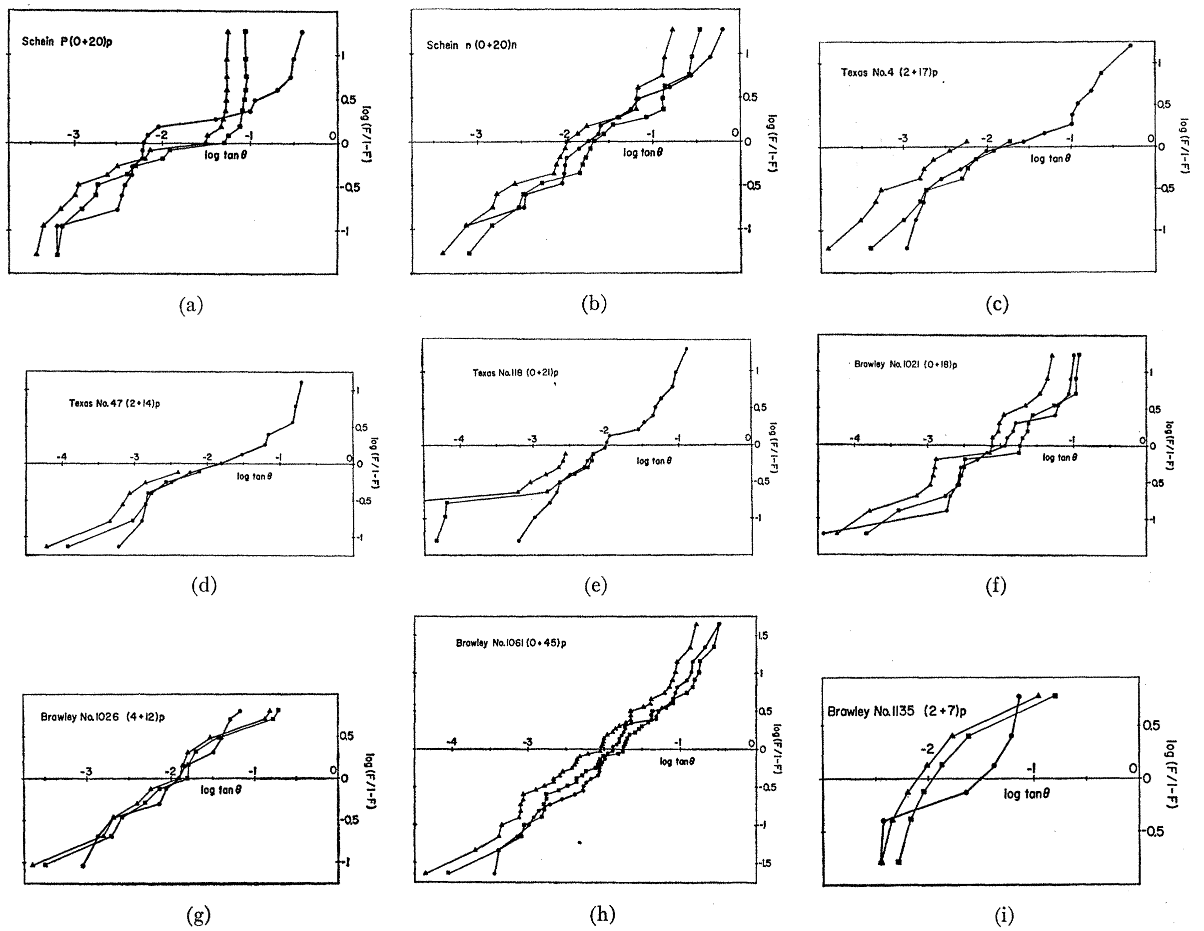
[20] This system is defined as the system which moves with respect to the CMS with the same speed as does the LS but in the opposite direction.

[21] R. W. Huggett, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (Institute of Physics and the Physical Society, London, 1966), Vol. 2, p. 898.

[22] The nominal energy estimate E_η is defined by

$$E_\eta \equiv M(2\gamma_S^2 - 1) = M\cosh(2\langle\eta\rangle)$$

in the E_η method.



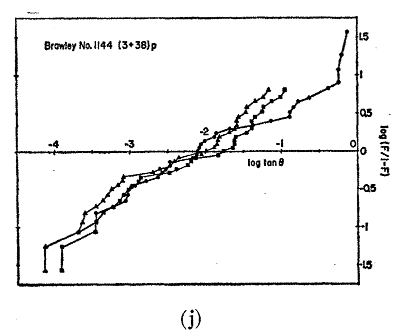


FIG. 7. Plots of $\log_{10}[F/(1-F)]$ versus $\log_{10}\tan\theta$. The closed circles designate points for the LS. The squares represent points for an ALS determined by $\gamma_{c.m.}=\gamma_K$, and the triangles represent points in an ALS determined by $\gamma_{c.m.}=\gamma_{Cast}$.

The means of $\log_{10}(E_\eta/E_p)$ are essentially zero and the standard deviation is reduced from 0.42 to 0.07 when neutral secondaries are included. This indicates the accuracy of Eq. (3) in expressing the condition of perfect forward-backward symmetry and zero total momentum of the produced particles in the CMS.

E. Various Methods of Energy Estimation

The $E(\theta)$ method has improved the spectrum-independent E_{Cast} method mainly in reducing the factor of overestimation (E/E_p) as summarized in Table IV. This is also shown by the two plots in Fig. 8; that in the upper half, the combined distribution of (E_{Cast}/E_p) for all 357 accelerator-produced jets and the other in the lower half, that of $[E(\theta)/E_p]$ for

TABLE IV. The average factor of overestimation of the primary energy.

Type of jets	$10^{\langle\log_{10}[E(\theta)/E_p]\rangle}$	$10^{\langle\log_{10}(E_{Cast}/E_p)\rangle}$
Accelerator jets	1.07 ± 0.05	$1.35_{-0.06}^{+0.07}$
Nucleon fragments	$1.1_{-0.2}^{+0.3}$	$1.5_{-0.3}^{+0.4}$
Cosmic-ray jets	1.3 (±0.1)[a]	2.0 (±0.02)[a]
Monte Carlo jets	1.23 ± 0.03	1.95 ± 0.04

[a] For these jets, we assumed that $E_p=E_K$.

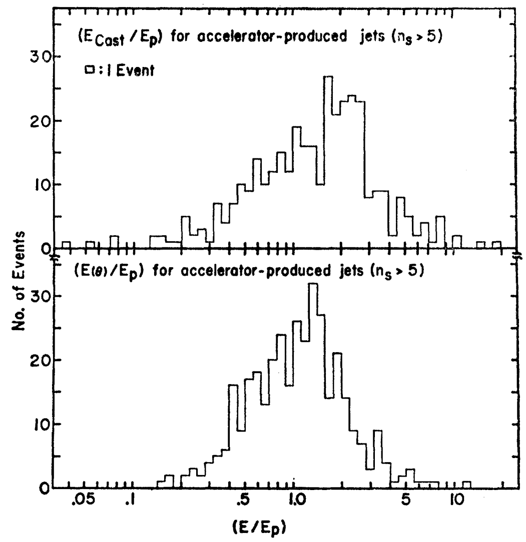


FIG. 8. The upper histogram is a composite distribution of (E_{Cast}/E_p) for 357 accelerator-produced jets; the lower histogram is the corresponding distribution of $E(\theta)/E_p$.

those jets. The standard deviation of the $\log_{10}[E(\theta)/E_p]$ distribution in Fig. 8, 0.31 ± 0.02, is less than that of the $\log_{10}(E_{\text{Cast}}/E_p)$ distribution, 0.39 ± 0.02. This implies that relative errors of estimating the primary energy from a single jet, $\Delta E/E$, are ${}_{-0.5}{}^{+1.0}$ and ${}_{-0.7}{}^{+1.5}$, respectively, for the $E(\theta)$ and E_{Cast} methods. [See detailed discussions for the standard error of $E(\theta)$ in Sec. III F].

The E_{ch} method has the smallest standard deviation of $\log_{10}(E_{\text{ch}}/E_p)$, 0.26 ± 0.01 for the accelerator jets, but it increases for the cosmic-ray jets of about 10^3 GeV. From our samples, we obtained $K_{\text{ch}}\cong0.8$. In the E_{ch} method, small-angle core tracks of a jet give the main contribution and the statistical fluctuations of a few such tracks of extreme emission angles in the forward cone must be responsible for the statistical fluctuations of E_{ch}, which may explain the large standard deviation of the E_{ch} method in Tables I and II. Of course, the determination of the primary direction in a singly charged or neutral jet, when it is defined from the centroid of the target diagram, introduces an additional experimental error which mostly affects the E_{ch} method.

TABLE V. Standard deviation of the distribution of $\log_{10}[E(\theta)/E_p]$.

Primary energy E_p(GeV)	Observed	Calculated from Eq. (22) using[a] $\langle\sigma\rangle$ and $\langle n_s\rangle$	$\Delta E(\theta)/E(\theta)$
17	0.27 ± 0.03	0.27	$({}_{-0.46}{}^{+0.86})$
27	0.34 ± 0.03	0.28	$({}_{-0.45}{}^{+0.90})$
30.5	0.30 ± 0.04	0.27	$({}_{-0.46}{}^{+0.86})$
30.9	0.31 ± 0.03	0.30	$({}_{-0.50}{}^{+0.99})$
0.57×10^3	0.31 ± 0.10	0.33	$({}_{-0.53}{}^{+1.14})$
3.7×10^3 [b]	(0.10 ± 0.03)	0.35	$({}_{-0.64}{}^{+1.24})$
3.0×10^3	0.34 ± 0.01	0.50	$({}_{-0.7}{}^{+2.2})$

[a] See the values of $\langle\sigma\rangle$ in Table VI and $\langle n_s\rangle$ in Tables I and II. For the Monte Carlo jets, $\langle\sigma\rangle=1.57\pm0.01$.
[b] This value is obtained using the E_K method, and hence the number in the second column reflects the correlation between the $E(\theta)$ and E_K methods.

F. Statistical Errors in the Estimation of $E(\theta)$

The statistical error in the $E(\theta)$ method of energy estimation arises essentially from the *finite* number of n_s charged secondaries. If one defines the standard error $\Delta\langle\eta(\theta)\rangle$ on the quantity $\langle\eta(\theta)\rangle$ by

$$\Delta\langle\eta(\theta)\rangle=\sigma/(n_s-1)^{1/2}, \qquad (21)$$

where σ is the standard deviation of the observed $\eta(\theta)$ distribution for an event, we obtain the form of standard error for $E(\theta)$ as[23]

$$\log_{10}\frac{E(\theta)\pm\Delta E(\theta)}{E_p}=(\pm)\frac{0.86\sigma}{(n_s-1)^{1/2}}. \qquad (22)$$

The errors calculated from Eq. (22) are compared with the corresponding experimental errors observed in Table V. The agreement is excellent, as can be seen in the table. This implies that the statistical fluctuation of $\eta(\theta)$ alone is responsible for the observed distribution of $\log_{10}[E(\theta)/E_p]$.[24]

Actually one can understand the general trends of the magnitude of the statistical errors of the $E(\theta)$ method of energy estimation. (The dependence of $\langle\sigma\rangle$ on E_p is shown and its significance is discussed in Sec. IV). The relative error, expressed in Eq. (22), will depend on the average multiplicity $\langle n_s\rangle$. If, on the average, $\langle n_s\rangle\propto E_p{}^{0.28}$, as concluded in Sec. IV, the relative error has a maximum around $E_p=60$ GeV and decreases on the average as E_p increases further.

[23] From Eq. (6), we have

$$\langle\langle\eta\rangle\rangle=\tfrac{1}{2}\,\text{arccosh}(E_p/M)\cong\tfrac{1}{2}\ln(2E_p/M),$$

and also from Eq. (15)

$$\langle\eta(\theta)\rangle=\tfrac{1}{2}\,\text{arccosh}[E(\theta)/M]\cong\tfrac{1}{2}\ln[2E(\theta)/M],$$

hence

$$\langle\eta(\theta)\rangle-\langle\langle\eta\rangle\rangle\cong\tfrac{1}{2}\ln[E(\theta)/E_p].$$

[24] The validity of the basic assumption about symmetry in the CMS has been one of the cardinal problems in investigating high-energy nuclear interactions. It has been reported that there exist certain classes of interactions of the primary energy $>10^2$ GeV for which this basic assumption does not hold: N. L. Grigorov, V. V. Guseva, N. A. Dobrotin, K. A. Kotelnikov, V. S. Murzin, S. V. Ryabikov, and S. A. Slavatinsky, in *Proceedings of the Moscow Cosmic-Ray Conference, 1959* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. I, p. 143; N. A. Dobrotin *et al.*, in *Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965* (Institute of Physics and the Physical Society, London, 1966), Vol. 2, p. 817. Particularly, these classes of interactions were identified as those with small n_s and σ: J. Gierula, Fortschr. Physik **11**, 109 (1963). Thus, to have consistency in $E(\theta)$, obtained from the angular distribution of a jet, one must always check $E(\theta)$ against another estimate of the primary energy which may be obtained from the magnitude of σ with the aid of Eq. (23), which we obtained experimentally. If these two estimates were widely apart beyond the *statistical* errors, we have to revise $E(\theta)$ for that particular event.

IV. DEPENDENCE OF THE AVERAGE STANDARD DEVIATION OF $\eta(\theta)$ ON E_p

For the samples of jets in Sec. III, the standard deviation σ of the $\eta(\theta)$ distribution for each individual jet has been calculated. Our observations of the average behavior of the parameter and their significance are discussed.

A. Dependence of $\langle\sigma\rangle$ on E_p and Average CMS Energies of Secondaries

The dependence of $\langle\sigma\rangle$ on E_p is shown in Table VI and is also displayed in Fig. 9. A similar increase has been noticed in terms of $\langle\log_{10}\tan\theta\rangle$ as the primary energy increases.[25] We observed also that $\langle\sigma\rangle$ for 30.9-GeV jets[12] is nearly independent of n_s and N_h when we consider only jets with $n_s<10$.

For our samples of jets, the result of a least-squares fit to an equation of the form $a+b\ln E_p$ is

$$\langle\sigma\rangle=(0.12\pm0.07)+(0.22\pm0.02)\ln E_p\,, \qquad (23)$$

where E_p is in units of GeV. The fit has its value of $\chi^2=8.0$, which indicates a confidence limit of $\sim10\%$, since the number of degrees of freedom for the fit is 4. The form of dependence of $\langle\sigma\rangle$ on E_p, $(a+b\ln E_p)^{1/2}$, which is derived in Ref. 1, seems to be in contradiction with our observations.

The significance of Eq. (23) can be best understood by the increase of the average CMS energies of secondaries as the primary energy increases.[5] From Eqs. (1) and (6), $\bar{\eta}$ in the CMS is related to η in the LS as

$$\begin{aligned}\bar{\eta}&=\eta-\operatorname{arctanh}\beta_{\text{c.m.}}\\&=[\eta(\theta)-\langle\eta(\theta)\rangle]+[\langle\eta(\theta)\rangle-\langle\langle\eta\rangle\rangle]\\&\approx\eta(\theta)-\langle\eta(\theta)\rangle\,,\end{aligned}$$

where we used the fact that

$$|\langle\eta(\theta)\rangle-\langle\langle\eta(\theta)\rangle\rangle|\ll|\eta(\theta)-\langle\eta(\theta)\rangle|$$

in the last step. $|\langle\eta(\theta)\rangle-\langle\langle\eta\rangle\rangle|\approx0.15$ for the accelerator-produced jets.[23] Therefore,

$$\sigma\approx(\langle\bar{\eta}^2\rangle)^{1/2}=\kappa\langle|\bar{\eta}|\rangle\,, \qquad (24)$$

where κ depends on the detailed form of distribution of $\bar{\eta}$. (For the, Gaussian distribution of $\bar{\eta}$, $\kappa\cong1$.) Now from Eq. (24) the average CMS energy $\langle\bar{E}\rangle$ of a secondary is obtained as

$$\begin{aligned}\langle\bar{E}\rangle&=\langle(m^2+p_t^2)^{1/2}\cosh\bar{\eta}\rangle\\&\cong(m^2+\langle p_t\rangle^2)^{1/2}\cosh(\langle\sigma\rangle/\kappa)\,.\end{aligned} \qquad (25)$$

[See Ref. 1 and also Appendix A for the first step of Eq. (25).] The average CMS energies of pions and kaons, calculated from Eq. (25) with the use of the $\langle\sigma\rangle$ values given in Table VI, are also listed in that table and seem to be in fair agreement with the directly measured values.[6,7,26] Here the average transverse momentum for pions used was 0.34 GeV/c and for kaons 0.39 GeV/c; $\kappa\equiv1$. Asymptotically the average CMS energy is a function of a simple power of E_p; $\langle\bar{E}\rangle\propto E_p^{0.22\pm0.02}$. This is close to the law $\langle\bar{E}\rangle\propto E_p^{1/4}$, which is a natural prediction of Fermi theory.[27]

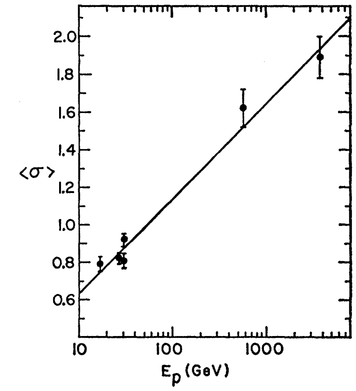


FIG. 9. The dependence of $\langle\sigma\rangle$ on the primary energy E_p, where $\sigma^2=(1/n_S)\times\sum_i[\eta(\theta)_i-\langle\eta(\theta)\rangle]^2$. The straight line in the figure represents the best logarithmic fit of the experimental points: $\langle\sigma\rangle=(0.12\pm0.07)+(0.22\pm0.02)\ln E_p$, where E_p is in units of GeV.

B. Dependence $\langle n_s\rangle$ on E_p

The dependence of the average multiplicity of jets on E_p may be predicted if the average CMS energy for secondaries is known and the inelasticity K does not depend drastically on the primary energy E_p. We have, from the law of conservation of energy in the CMS, for a collision between a nucleon of the primary energy E_p and a target nucleon at rest in the LS,

$$2KM\gamma_{\text{c.m.}}=\tfrac{3}{2}\langle n_s\rangle\langle\bar{E}\rangle\,, \qquad (26)$$

where all the secondaries produced are assumed as pions. Further, if we may assume the ratio of production of the number of charged kaons to charged pions[7] to be 0.2 and the K/π ratio is independent both on E_p and $\bar{\eta}$ in the CMS, we obtain, as a function of E_p, the average number of charged particles emerging from a proton-

TABLE VI. Dependence of $\langle\sigma\rangle$ on the primary energy and average CMS energies calculated from this dependence.

Primary energy E_p (GeV)	$\langle\sigma\rangle$	Av. CMS energy $\langle\bar{E}_\pi\rangle$ (GeV)	$\langle\bar{E}_K\rangle$ (GeV)
17	0.79±0.04	0.49±0.01	0.83±0.02
27	0.82±0.03	0.50±0.01	0.85±0.02
30.5	0.81±0.04	0.50±0.01	0.84±0.02
30.9	0.92±0.03	0.54±0.01	0.91±0.02
570	1.62±0.01	0.97±0.1	1.64±0.15
3.7×10^3	1.89±0.11	1.25±0.1	2.1 ±0.2

[25] D. H. Perkins, in *Progress in Elementary-Particle and Cosmic-Ray Physics* (North-Holland Publishing Company, Amsterdam, 1960), Vol. V, p. 257.

[26] H. Filthuth, in *Proceedings of the Aix en Provence International Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires de Saclay, Seine et Oise, 1961), Vol. 1, p. 93.

[27] E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950); Phys. Rev. **81**, 683 (1951); **92**, 452 (1953); **93**, 1434 (1954).

TABLE VII. Dependence of $\langle\langle\eta(\theta)\rangle\rangle$ on n_s and N_h of 30.9-GeV jets.[a]

$n_s\backslash N_h$	0	1	2,3	4,5	6–8	9–15	>16
1	···	4.5±0.2	3.8±0.2	3.8±0.3	3.3±0.6	3.3±0.9	4.4
2	2.8±0.2	2.7±0.3	2.5±0.2	2.4±0.3	3.0±0.5	2.4±0.3	2.2
3	3.1±0.2	2.7±0.1	2.6±0.2	2.6±0.2	2.0±0.3	1.6±0.2	1.8±0.5
4	2.3±0.1	2.4±0.2	2.0±0.1	2.4±0.2	2.3±0.3	1.9±0.3	1.9±0.3
5	2.3±0.1	2.5±0.1	2.3±0.2	2.0±0.1	2.0±0.1	2.2±0.1	1.5±0.2
6	2.2±0.1	2.1±0.1	2.0±0.1	1.9±0.1	1.9±0.2	1.8±0.1	1.5±0.2
7	2.1±0.1	2.1±0.1	1.9±0.1	2.0±0.1	1.8±0.1	1.6±0.1	1.6±0.1
8	2.3±0.1	2.0±0.2	2.0±0.1	2.0±0.1	1.9±0.4	1.7±0.1	1.5±0.1
9	2.1±0.1	1.7±0.2	1.9±0.1	2.0±0.1	1.8±0.1	1.7±0.1	1.6±0.7
10	2.1±0.3	2.0±0.1	1.8±0.1	1.8±0.1	1.4±0.1	1.4±0.1	1.3±0.1
11	1.9±0.4	1.9±0.4	1.7±0.1	1.8±0.1	1.6±0.1	1.5±0.1	1.4±0.1
12	2.0±0.2	1.8±0.2	1.9±0.1	1.6±0.2	1.5±0.2	1.5±0.1	1.4±0.1
13	2.3	1.8	1.7	2.2±0.04	1.4±0.1	1.4±0.1	1.3±0.04
14	···	2.0	1.7	1.6	1.6±0.1	1.2±0.2	1.3±0.1
15	1.7	1.2	1.8	1.7±0.1	···	1.4±0.6	1.3±0.1
16	···	···	···	···	1.7	1.4±0.1	1.2±0.1
17	···	···	···	···	···	1.3±0.1	1.2±0.1
18	···	···	···	···	···	1.1	0.9±0.2
19	···	···	···	1.6	···	···	1.1±0.1
20	···	···	···	···	···	1.4	1.3±0.1
21	···	···	···	···	···	···	1.1±0.1
22	···	···	···	···	1.6	1.4±0.1	1.0
23	···	···	···	···	···	1.3	···
25	···	···	···	···	···	···	0.9
26	···	···	···	···	···	···	0.6

[a] See Ref. 12.

proton collision as

$$\langle n_s\rangle=\frac{3.0K[M(M+E_p)]^{1/2}}{\cosh(0.12+0.22\ \ln E_p)}, \qquad (26')$$

where we assumed explicitly that

(i) $\kappa=1$ [See Eq. (25)]; for pions $\langle p_t\rangle=0.34$ GeV/c and for kaons $\langle p_t\rangle=0.39$ GeV/c,

(ii) the two original protons re-emerge as surviving protons and also no particles other than pions and kaons are produced.

The predicted dependence of $\langle n_s\rangle$ on E_p based on Eq. (26′) is shown in Fig. 10 for three assumed values of K. Some experimental observations about $\langle n_s\rangle$ for jets (with $N_h\leq 5$ in nuclear emulsion) produced by by nucleons of $E_p>20$ GeV are shown in the figure.[12,14,15,28–30] The experimental observations of the

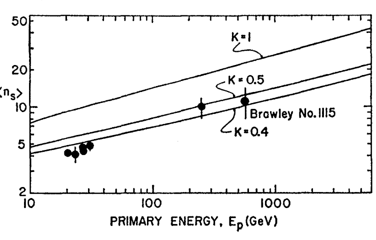


FIG. 10. The average number of charged secondaries versus the primary energy E_p, predicted by Eq. (26′). The inelasticity K is a variable parameter. The filled circles are the experimental points (Refs. 12, 14, 14, 28–30) in nuclear emulsion ($N_h\leq 5$).

[28] H. Meyer *et al.*, in Ref. 4.

[29] G. Cvijanovich *et al.*, Nuovo Cimento **20**, 1012 (1961).

[30] Y. Baudinet-Robinet, M. Morand, Tsai-Chü, C. Castagnoli, G. Dascola, S. Mora, A. Barbaro-Galtieri, G. Baroni, and A. Manfredini, Nucl. Phys. **32**, 452 (1962).

dependence agree with our predictions when $K\equiv 0.4$ or 0.5. The asymptotic behavior of $\langle n_s\rangle$ is predicted by Eq. (26) as a function of a simple power of E_p: $\langle n_s\rangle \propto E_p^{0.28\pm0.02}$. This is very close to the law of $\langle n_s\rangle \propto E_p^{1/4}$, first suggested by Fermi.[27]

V. DEPENDENCE OF $\langle\langle\eta(\theta)\rangle\rangle$ OF 30.9-GeV JETS ON n_s AND N_h

The averages of $\langle\eta(\theta)\rangle$, $\langle\langle\eta(\theta)\rangle\rangle$, as a function of n_s and N_h, are shown in Table VII for 1271 jets in nuclear emulsion produced by 30.9-GeV protons.[12] Each grouping according to N_h contains almost the same number of jets. The number of fragmentation prongs N_h is related to the target nucleus of a proton-nucleus collision. If $N_h>8$, we can be certain that the target nucleus is one of the heaviest nuclei (Ag, Br) among the constituent nucleus, (H, C, N, O, Ag, and Br), of nuclear emulsion. For comparison, $\langle\langle\eta\rangle\rangle$ should be 2.094 if the symmetric system has the velocity of the CMS, $\beta_{\text{c.m.}}{}^0$, for a collision of a 30.9-GeV proton with a target *proton* which is at rest in the LS.

The trends shown in Table VII are

(i) $\langle\langle\eta(\theta)\rangle\rangle$ becomes very large as n_s decreases.

(ii) $\langle\langle\eta(\theta)\rangle\rangle$ becomes smaller as N_h increases.

Trend (i) in terms of $-\langle\log\tan\theta\rangle$ or E_{Cast} has been already noticed by various authors,[10,28,31] but no successful account of its significance has been given. [For instance, arctanh $\beta_{\text{c.m.}}{}^0=4$ and 3 means that the apparent energy $E(\theta)=1.4\times10^3$ and 1.9×10^2 GeV, respectively.] Trend (ii) has been attributed to the target

[31] H. Winzeler *et al.*, in Ref. 4.

mass becoming greater than the mass of a proton [used in Eq. (16)] as N_h increases.[32]

Our present study of the above two trends will be seriously attempted only for the jets with even number of $n_s \leq 8$. The study of the trends for the jets with odd number of $n_s \leq 7$ is given in Appendix C. By limiting the study in this way to jets with a small number of produced particles, our analysis will be mainly confined to *single* proton-nucleus collisions, since successive collisions, if any ever occur in the same nucleus, are expected to increase the number of particles produced. For the group of jets with $n_s > 10$, one could expect to have effects of complex collisions.

A. Transverse Momentum of Two Surviving Protons Deduced from Jets with Even Number of n_s

The three following plausible assumptions are made for the analysis of the 30.9-GeV jets with even n_s:

(i) The target is a proton.

(ii) The incident and target protons did not lose their charges and the two surviving baryons were included among the n_s charged secondaries.

(iii) The rest of (n_s-2) particles are pions.

From charge conservation, condition (i) is valid if no secondaries of a large grain density are frequently identified as being fragmentation products (of number N_h).[33] Condition (iii) is necessary along with (ii) to deduce the total number of produced particles from the number of charged secondaries using the relation

$$n=\tfrac{3}{2}(n_s-2)+2=\tfrac{3}{2}(n_s-\tfrac{2}{3}) \quad \text{for even } n_s. \qquad (27)$$

Here we neglect any production of nucleon pairs or kaons. If the K/π ratio is known, the factor $\frac{3}{2}$ in Eq. (27) will need to be slightly revised. From Eqs. (14′) and (27), the dependence of $\langle\langle\eta(\theta)\rangle\rangle$ on n_s will be given by the formula

$$\langle\langle\eta(\theta)\rangle\rangle=\langle\operatorname{arctanh}\beta_S\rangle+[4/3(n_s-\tfrac{2}{3})] \times\{\langle\ln[(1+x_p^2)^{1/2}x_p^{-1}]\rangle-0.233\}, \quad \text{for even } n_s. \qquad (28)$$

[32] E. M. Friedländer, Nuovo Cimento **14**, 796 (1959); H. H. Aly, J. G. M. Duthie, and C. M. Fisher, Phil. Mag. **4**, 993 (1959); A. Barbaro-Galtieri, A. Manfredini, B. Quassiati, C. Castagnoli, A. Gainotti, and I. Ortalli, Nuovo Cimento **21**, 469 (1961).

[33] The data used in the present analysis have an inherent bias, due to the standard criterion adopted for identifying a "secondary" track of a jet from a fragmentation track, namely, that the secondary should have a grain density less than 1.4 times the plateau grain density. This standard criterion automatically causes those backward surviving protons which emerge with a kinetic energy $T \leq 0.38$ GeV [or $\sqrt{(-t)}=\sqrt{(2MT)} \leq 0.84$ GeV] to be classified not as secondaries, but as "fragmentation products." According to investigation by Damgaard *et al.* [Nuovo Cimento (to be published)] this bias is a very serious one with elastic or quasi-elastic events having one grey proton track in the backward cone. However, for all the jets with one grey proton track and more than one pion track in the backward cone, events having protons with $T < 0.4$ GeV comprise less than 7% of all the events with identified grey proton tracks. In this sense we must be cautious in our analysis of events with $n_s=1$ and 2. Otherwise most of our conclusions should not be altered seriously because of the bias.

TABLE VIII. Results of least-squares fits of $\langle\langle\eta(\theta)\rangle\rangle$ of even n_s to Eq. (28).[a]

N_h	$\langle\operatorname{arctanh}\beta_S\rangle$	$\frac{4}{3}\{\langle\ln[(1+x_p^2)^{1/2}x_p^{-1}]\rangle -0.233\}$	χ^2 of fits	(M_t/M)[b]
0	2.08±0.09 (2.22±0.16)	0.9±0.3 (1.5±0.8)	1.8 (0.8)	$1.0_{-0.1}^{+0.2}$
1	1.89±0.16 (1.58±0.32)	1.2±0.5 (2.8±1.5)	1.3 (0.1)	$1.5_{-0.4}^{+0.6}$
2,3	1.89±0.09 (1.94±0.18)	0.9±0.4 (0.4±0.9)	0.5 (0.1)	$1.7_{-0.3}^{+0.4}$
4,5	1.81±0.14 (1.55±0.23)	1.1±0.6 (2.5±1.2)	4.9 (3.0)	$1.8_{-0.5}^{+0.6}$
6–8	1.69±0.16 (1.09±0.28)	1.7±0.7 (1.7±1.6)	0.3 (0.3)	$2.3_{-0.7}^{+0.9}$
9–15	1.54±0.11 (1.50±0.28)	1.2±0.5 (1.5±1.8)	0.2 (0.2)	$3.2_{-0.7}^{+0.8}$
(>16)	··· (1.21±0.35)	··· (2.0±2.0)	··· (0.2)	···
Average (without >16) (all N_h)		(1.04±0.18 (1.52±0.46)		

[a] Numbers inside parentheses are those corresponding to fits with $n_s=4, 6, 8$; others $n_s=2, 4, 6, 8$.
[b] The mass of target M_t is obtained from the formulas

$$E(\theta)=M_t(2\gamma_{\text{c.m.}}^2-1),$$

$\langle\beta_S\rangle=\beta_{\text{c.m.}}$ and given in units of a proton mass M.

Table VIII lists the values of two variables, $\langle\operatorname{arctanh}\beta_S\rangle$ and the second term in the right-hand side of Eq. (28). These were obtained by least-squares fits to Eq. (28) of the values of $\langle\langle\eta(\theta)\rangle\rangle$ given in Table VII for even n_s, one group for $n_s=2$, 4, 6, and 8, and, inside the parentheses, for $n_s=4$, 6, and 8. Reasonable fits were indeed obtained, as can be seen from the χ^2 values for the fits, which are also listed in Table VIII.

It is striking to see that a single number can consistently represent the second term in the right-hand side of Eq. (28), whose average is

$$\tfrac{4}{3}\{\langle\ln[(1+x_p^2)^{1/2}x_p^{-1}]\rangle-0.233\}=1.5\pm0.5. \qquad (29)$$

Our choice of the value shown in Eq. (29) from the two corresponding values in Table VIII is due to the consideration of inherent bias discussed in Ref. 33. The most probable transverse momentum $\tilde{p}_t$ of either of the two baryons can now be obtained from Eq. (29). We find

$$\tilde{p}_t=0.24_{-0.07}^{+0.13}\ \text{GeV}/c. \qquad (30)$$

This is a value close to the most probable value of p_t obtained by direct measurements[6,7]; hence our explanation of extreme overestimation of the primary energy of jets of small n_s is achieved. Equation (29) may be used as the basis for a correction factor when $E(\theta)$ is used to estimate the primary energy of jets with a small number of secondaries, since the transverse momentum of protons seems to stay rather constant as the primary energy varies up to around 10^3 GeV.[7]

B. The Target Mass

It is clear that $\langle \text{arctanh}\beta_S \rangle$ has a dependence on N_h, as in Ref. 32. When Eq. (6) is assumed in addition to Eq. (5), values of the target mass can be assigned for each value of $\langle \text{arctanh}\beta_S \rangle$. The last column of Table VIII gives values for the target masses found in this way for the 30.9-GeV jets. From these values it appears that an interaction may be considered to be a nucleon-nucleon collision only for those jets with $N_h=0$ and possibly $N_h=1$. The other jets with $N_h>1$ have target masses larger than the proton mass, as has been noticed in previous investigations.[32] Nevertheless, our present analysis differs from those described in Ref. 32 in that we have deduced the same effect primarily through the use of jets with small n_s (≤ 8) which should be expected with single proton-nucleus collisions.

The present trends for jets with a large number of N_h that $\langle \text{arctanh}\beta_S \rangle \ll 2.094$ imply that an energy estimate $E(\theta)$, when Eq. (15) should be used, would consistently be smaller than 30.9 GeV for those jets produced by 30.9-GeV protons. This trend of under-estimation of the primary energy by the $E(\theta)$ method for the use with jets from a proton-nucleus collision as well as the trend for nucleus-nucleon jets to give higher apparent energies with the $E(\theta)$ method is used, (as has been mentioned at the end of Sec. III B), has been predicted in Ref. 1.

VI. CONCLUSIONS

We have reformulated forward and backward symmetry for the produced secondaries in the CMS of a nucleon-nucleon or a nucleon–quasifree-nucleon collision with the aid of a convenient property of the kinematic parameter $\eta=\text{arctanh}(\beta\cos\theta)$ in Lorentz transformations. Thus we have found the formula to obtain the velocity of the "symmetric system,"

$$\text{arctanh}\beta_S=\langle\eta\rangle. \qquad (4)$$

This allowed us to accomplish rather easily what the authors in Ref. 3 studied to correct for the assumption of $\bar{\beta}/\beta=1$, which was basic and necessary to obtain the spectrum-independent formula

$$\ln\gamma_{\text{Cast}}=-\langle\ln\tan\theta\rangle,$$

in the Castagnoli method of energy estimation. Moreover, we have obtained and tested the formula

$$\ln\gamma_S=-\langle\ln\tan\theta\rangle-\langle\ln[(1+x^2)^{1/2}x^{-1}]\rangle, \qquad (9)$$

which could also be derived from the E_K method in Appendix B.

By using the various jets found in nuclear emulsion, it is shown that the $E(\theta)$ method, which is based on Eq. (4), has improved the E_{Cast} method, which used the spectrum-independent formula of the Castagnoli method,[3] mainly in reducing the factor of overestimation and the magnitude of relative error for estimating the primary energy from the angular distribution of charged secondaries of a jet. Also it appears that the statistical error, which essentially arises from the *finiteness* of the number of charged secondaries in a single jet, can alone account for the relative error of $E(\theta)$ for the accelerator-produced jets.[24] By employing the $E(\theta)$ method, one will not be able to increase the accuracy of the method even for high-energy cosmic-ray jets beyond a relative error $_{-0.5}{}^{+1.0}$—even for a nucleon-nucleon jet.

By noticing that the standard deviation σ of $\eta(\theta)$ distribution of a jet is, in fact, on the average $|\bar{\eta}|$ in the CMS and also from its simple dependence on the primary energy E_p,

$$\langle\sigma\rangle=(0.12\pm0.07)+(0.22\pm0.02)\ln E_p, \qquad (23)$$

which could be obtained from the samples of jets which we used to test the $E(\theta)$ method, the average CMS energies $\langle\bar{E}\rangle$ of secondaries were correlated with E_p. We deduced that

$$\langle\bar{E}\rangle \propto E_p^{0.22\pm0.02}$$

and also

$$\langle n_s\rangle \propto E_p^{0.28\pm0.02}$$

asymptotically.

Finally, from a detailed study of $\langle\langle\eta(\theta)\rangle\rangle$, as a function of n_s and N_h, about 1271 jets in nuclear emulsion produced by 30.9-GeV protons, the following have been learned:

(i) The large factor of overestimation of the primary energy, which would occur for the classes of jets with small n_s, should the $E(\theta)$ method be applied, can be accounted for as being due to a small value of $x_p=p_t/M$ for surviving baryons from Eq. (9). A by-product of the above study is the knowledge about the most probable transverse momentum of the surviving baryons; $\tilde{p}_t=0.24_{-0.07}{}^{+0.13}$ GeV/c.

(ii) It appears that in the LS the average velocity of the symmetric system of a proton-nucleus collision is smaller than the velocity of the CMS of a proton-proton collision. The effective mass of the target increases as N_h increases.

This study helps to emphasize that the many principal characteristics of inelastic processes in the high-energy nuclear interactions may be deduced *statistically* from a relatively simple technique of studying the angular distributions of secondaries in a jet, because of the small and constant average transverse momenta of secondaries.

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APPENDIX A: THE PARAMETER η AND LORENTZ TRANSFORMATION OF A FOUR-VECTOR

A four-vector $(\boldsymbol{\alpha},\alpha_4)$ in the LS transforms to $(\boldsymbol{\alpha}^*,\alpha_4^*)$ in a frame of reference S^*, moving with a velocity β_c with respect to the LS as follows:

$$\alpha_t=\alpha_t^*, \quad \text{(A1a)}$$

$$\alpha_l=\gamma_c(\alpha_l^*+\beta_c\alpha_4^*), \quad \text{(A1b)}$$

$$\alpha_4=\gamma_c(\alpha_4^*+\beta_c\alpha_l^*), \quad \text{(A1c)}$$

where $\gamma_c=(1-\beta_c^2)^{-1/2}$, and $\alpha_4^2-\boldsymbol{\alpha}^2=\alpha_4^{*2}-\boldsymbol{\alpha}^{*2}=\alpha^2$ is the invariant length of the four-vector; α_t and α_l are transverse and parallel components, respectively, of the three-vector $\boldsymbol{\alpha}$ with respect to the direction of motion of S^*. From Eqs. (A1b) and (A1c), we define η as

$$\tanh\eta\equiv\alpha_l/\alpha_4=(\alpha_l^*+\beta_c\alpha_4^*)/(\alpha_4^*+\beta_c\alpha_l^*). \quad \text{(A2)}$$

Then

$$\begin{aligned}\eta&=\operatorname{arctanh}(\alpha_l/\alpha_4)\\&=\tfrac{1}{2}\ln[(1+\alpha_l/\alpha_4)/(1-\alpha_l/\alpha_4)]\\&=\tfrac{1}{2}\ln\frac{(1+\beta_c)(1+\alpha_l^*/\alpha_4^*)}{(1-\beta_c)(1-\alpha_l^*/\alpha_4^*)}\\&=\tfrac{1}{2}\ln\frac{1+\beta_c}{1-\beta_c}+\tfrac{1}{2}\ln\frac{1+\alpha_l^*/\alpha_4^*}{1-\alpha_l^*/\alpha_4^*}\\&=\operatorname{arctanh}\beta_c+\eta^*, \quad \text{(1)}\end{aligned}$$

where $\eta^*=\operatorname{arctanh}(\alpha_l^*/\alpha_4^*)$. This proves Eq. (1). Now generally,

$$\alpha_l=(\alpha^2+\alpha_t^2)^{1/2}\sinh\eta, \quad \text{(A3a)}$$

$$\alpha_4=(\alpha^2+\alpha_t^2)^{1/2}\cosh\eta, \quad \text{(A3b)}$$

where α^2 and α_t^2 are invariants. When $(\boldsymbol{\alpha},\alpha_4)$ is the energy-momentum four-vector $(\mathbf{p},E)$, Eqs. (A3a) and (A3b) become

$$p_l=(m^2+p_t^2)^{1/2}\sinh\eta,$$
$$E=(m^2+p_t^2)^{1/2}\cosh\eta.$$

APPENDIX B: THE E_K METHOD OF ESTIMATING THE PRIMARY ENERGY OF JETS

When the LS velocity β and the emission angle θ of a jet secondary are known, the emission angle of the secondary in the ALS, θ^{-1}, can be obtained from the formula

$$\tan(\theta^{-1})=-\frac{\beta\sin\theta}{\gamma_{\text{c.m.}}^2[2\gamma_{\text{c.m.}}-(1+\beta_{\text{c.m.}}^2)\beta\cos\theta]}, \quad \text{(A4)}$$

where $\gamma_{\text{c.m.}}$ is the Lorentz factor defining the CMS. The Lorentz factor γ_K is obtained by finding a best match between the $\log_{10}[F/(1-F)]$ plot[19] in the LS and the corresponding plot in the ALS. This is performed with the aid of the computer. First the Castagnoli energy estimate E_{Cast} in the LS for a jet is computed from Eq. (16). Then the 13 $\log_{10}[F/(1-F)]$ plots in the ALS are constructed, by using the detailed information about β and θ of secondaries in the LS (mainly for the backward-cone tracks of the jet), for $\gamma_{\text{c.m.}}$, corresponding the $E_p/E_{\text{Cast}}=2$, 1.5, 1.0, 0.9, 0.8. 0.7, 0.6, 0.5. 0.4, 0.3, 0.2, and 0.1. Among the 13 E_p values, the one which gives the best match between the $\log_{10}[F/(1-F)]$ plots of the LS and ALS is designated as E_K. (Because of an error in the computer program, the Castagnoli energy E_{Cast} actually used for the process was $1.16\times E_{\text{Cast}}$. This does not affect the final choice of E_K.) For the jets which have secondaries with $\theta>45°$, where $-\ln\tan\theta+\ln 2$ becomes a bad approximation to η, the E_K method can be used after elimination of equal number of extremely backward tracks both in the LS and in the ALS.

Let us prove that the E_η method[22] and E_K method are equivalent. From Eq. (A4),

$$\frac{\tan(180°-\theta^{-1})}{\tan\theta}=\frac{\beta_{\text{c.m.}}(1+v^2)^{1/2}+[1-1/(2\gamma_{\text{c.m.}}^2)]}{2\gamma_{\text{c.m.}}^2v^2-2v^2/\gamma_{\text{c.m.}}^2-1/(2\gamma_{\text{c.m.}}^2)}, \quad \text{(A5)}$$

where $v^2=(1+x^2)x^{-2}\tan^2\theta$. When the following two plausible conditions are satisfied for all the secondaries both in the LS and in the ALS (or after a few tracks which do not satisfy the conditions are eliminated),

(i) $1/\gamma_{\text{c.m.}}^2\ll 1$,
(ii) $1>v^2>1/[2\gamma_{\text{c.m.}}^2(\gamma_{\text{c.m.}}^2-1)]$,

then Eq. (A5) reduces to the following:

$$2\gamma_{\text{c.m.}}^2\tan(180°-\theta^{-1})/\tan\theta\cong[1+(1+v^2)^{1/2}]/v^2. \quad \text{(A6)}$$

Taking the logarithm of both sides of Eq. (A6) and formulating the condition of "symmetry" by

$$\langle\ln\tan(180°-\theta^{-1})\rangle=\langle\ln\tan\theta\rangle, \quad \text{(A7)}$$

we obtain the relation

$$\begin{aligned}\ln\gamma_S&\cong-\frac{1}{n}\sum_i\{\ln v_i-\tfrac{1}{2}\ln[(1+(1+v_i^2)^{1/2})/2]\}\\&\approx-\langle\ln\tan\theta\rangle-\langle\ln(1+x^2)^{1/2}x^{-1}\rangle, \quad \text{(A8)}\end{aligned}$$

which is equivalent to Eq. (9). For the failure of part of the condition (ii), $v^2 > 1/[2\gamma_{c.m.}^2(\gamma_{c.m.}^2 - 1)]$, for a secondary it is sufficient that the emission angle of the secondary be larger than 90° either in the LS or in the ALS.

APPENDIX C: THE n_s DEPENDENCE OF $\langle\langle\eta(\theta)\rangle\rangle$ WITH ODD n_s AND THE PROPERTIES OF THE SURVIVING TARGET NEUTRON

To understand the dependence of $\langle\langle\eta(\theta)\rangle\rangle$ of odd n_s, as a function of n_s and N_h, the following plausible assumption, similar to those made in Sec. V.A are made:

(i) The target is a neutron.

(ii) The incident proton did not lose its charge and the target neutron did not acquire a charge in the interaction.

(iii) Of the shower particles, $n_s - 1$ are pions.

From these, we have

$$n = \tfrac{3}{2}(n_s - 1) + 2 = \tfrac{3}{2}(n_s + \tfrac{1}{3}) \quad \text{for odd } n_s. \qquad \text{(A9)}$$

It seems necessary to include the effect of the surviving target neutron with emission angle θ_n, because of the dominant role played by the surviving baryons and their possible correlation with the number of produced particles. Now

$$\frac{n_s\langle\langle\eta(\theta)\rangle\rangle + \eta(\tilde{\theta}_n)}{n_s+1} = \langle \operatorname{arctanh}\beta_S\rangle + \frac{4}{3(n_s+\frac{1}{3})}\{\langle \ln[(1+x_p^2)^{1/2}x_p^{-1}]\rangle - 0.233\} \quad \text{for odd } n_s, \qquad \text{(A10)}$$

where $\eta(\tilde{\theta}_n)$ is the most probable value of $\eta(\theta)$ for the emission angle of the target neutron, θ_n.

Thus from Eqs. (28) and (A10), we obtain $\eta(\tilde{\theta}_n)$ conveniently from the relation

$$\frac{\eta(\tilde{\theta}_n)}{n_s+1} = \langle\langle\eta(\theta)\rangle\rangle_{2\lambda} - \frac{n_s}{n_s+1}\langle\langle\eta(\theta)\rangle\rangle_{2\lambda-1}, \qquad \text{(A11)}$$

where $\langle\langle\eta(\theta)\rangle\rangle_{2\lambda-1}$ is the value of the average of $\langle\eta(\theta)\rangle$ for events with odd $n_s (n_s = 2\lambda - 1, \lambda = 1, 2, 3, \text{and } 4)$, and $\langle\langle\eta(\theta)\rangle\rangle_{2\lambda}$ is the corresponding average for events with even n_s ($n_s = 2\lambda$, $\lambda = 1$, 2, 3, and 4), where all events used for an average are in the same grouping on the basis of N_h. (See Table VII.) Table IX lists the values of $\eta(\tilde{\theta}_n)/2\lambda$ ($\lambda = 1$, 2, 3, and 4), thus obtained through the use of Eq. (A11). The result obtained by combining the two values of $\langle\langle\eta(\theta)\rangle\rangle$ for the $N_h = 0$ events, with $n_s = 4$ and events with $n_s = 3$, is omitted from the table, since the type of a jet (0+3) seems partly due to production of electron pairs in the Coulomb field of nuclei.[30]

The averages of the most probable values of the emission angle of the surviving backward neutron, $\tan\tilde{\theta}_n$, were obtained and they are listed in Table IX according to the multiplicities $n_s = 2\lambda$ and $n_s = 2\lambda - 1$ ($\lambda = 1$, 2, 3, and 4) of the groups of events used. If the inelasticity K and p_t for the surviving baryons do not have any correlation, the surviving baryon becomes more energetic and more frequently emitted with smaller emission angle in the LS as K approaches unity. In other words, the preliminary trend shown in Table IX indicates that *the inelasticity K approaches unity as production of particles (soft mesons) becomes more numerous in inelastic nuclear interactions*. Under this assumption of no correlation between K and p_t, preliminary values of the inelasticity K and four-momentum transfer, $\sqrt{(-t)}$, for the surviving backward neutron which could be deduced from $\tilde{p}_t = 0.24$ GeV/c and $\langle\tan\tilde{\theta}_n\rangle$ are calculated and listed in Table IX. The values of K and $\sqrt{(-t)}$ agree with the results from the direct measurements.[7,34] The preliminary indication of an n_s dependence of K and $-t$, which is in accord with the observations by Damgaard *et al.*,[35] might need to be investigated further in view of a recent theory by Kastrup *et al.*[36]

[34] R. W. Huggett, K. Mori, C. O. Kim, and R. Levi-Setti, in *Proceedings of the 1963 International Cosmic-Ray Conference, Jaipur, India* (Commercial Printing Press, Ltd., Bombay, India, 1963), Vol. 5. p. 3.

[35] G. Damgaard, K. H. Hansen, J. E. Hooper, T. N. Rengarajan, and P. Voss, Nuovo Cimento (to be published).

[36] H. A. Kastrup and G. Mack, Phys. Letters **22**, 331 (1966).

TABLE IX. $\eta(\tilde{\theta}_n)/(2\lambda)$ deduced from "even-odd" differences of neighboring $\langle\langle\eta(\theta)\rangle\rangle$ and their kinematic parameters.[a]

Combinations of n_s	N_h						Inelasticity		
(2λ)–$(2\lambda-1)$	0	1	2,3	4,5	6–8	9–15	$\langle\tan\tilde{\theta}_n\rangle$	(K)	$\sqrt{(-t)}$
(2–1)	···	0.5±0.3	0.7±0.2	0.5±0.4	1.3±0.5	0.8±0.5	$0.5_{-0.1}^{+0.2}$	0.3±0.1	0.5±0.1
(4–3)	···[b]	0.4±0.2	0.1±0.2	0.4±0.2	0.8±0.3	0.7±0.4	$0.4_{-0.1}^{+0.2}$	0.4±0.1	$0.6_{-0.1}^{+0.2}$
(6–5)	0.3±0.1	0.0±0.2	0.2±0.2	0.2±0.2	0.3±0.2	0.0±0.2	$0.7_{-0.3}^{+0.6}$	$0.3_{-0.2}^{+0.02}$	$0.4_{-0.02}^{+0.2}$
(8–7)	0.5±0.1	0.2±0.2	0.3±0.1	0.3±0.1	0.4±0.1	0.3±0.1	$0.12_{-0.05}^{+0.08}$	$0.7_{-0.2}^{+0.05}$	$1.6_{-0.9}^{+0.6}$

[a] The two kinematic parameters, inelasticity, and $\sqrt{(-t)}$ were calculated assuming that the surviving backward neutron had the most probable transverse momentum, $\tilde{p}_t = 0.24$ GeV/c [See Eq. (30)].

[b] The combination of (4−3) for $N_h = 0$ group has been omitted, since jets of the type (0+3) seem partly due to production of electron pairs in the Coulomb field of nuclei (See Ref. 30).