Gravitational Bremsstrahlung in Nonrelativistic Collisions*

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We first find the spectral resolution of the low-frequency gravitational radiation emitted by a system of colliding particles. This is done by applying Fourier analysis to Landau and Lifshitz's formula for the gravitational radiation intensity. The result is in accord with that recently obtained by Weinberg through quantum-gravidynamical methods, which predicts 6×10^{14} erg/sec for the solar gravitational radiation power. We then find the total gravitational radiation of the same scattering process for all frequencies and without resolution into a Fourier integral. The calculation is free from any cutoff process which is needed for the low-frequency power, and the result is one order of magnitude larger than the low-frequency power when applied to the solar system.

1. INTRODUCTION

THE problem of gravitational radiation from a system of interacting particles has recently been subjected to a new kind of treatment by Weinberg[1,2] who applied his method of quantum gravidynamics to derive a formula for the emission rate and spectrum of soft gravitons in arbitrary collision processes, from which he estimated the gravitational radiation emitted during thermal collisions in the sun. It was found that the solar thermal gravitational-radiation power due to soft-gravitational inner bremsstrahlung in an arbitrary nonrelativistic collision is $P\simeq6\times10^{14}$ erg/sec. This is larger than the gravitational radiation from classical sources, e.g., planetary motion such as that of the Jupiter-Sun system, which gives 7.6×10^{11} erg/sec, or the Venus-Sun or Earth-Sun systems, which radiate comparable amounts. Since other planets radiate considerably less, Weinberg concludes that "thermal gravitational radiation from the sun appears to be the dominant source of gravitational radiation from the solar system." Noticing, furthermore, that a binary star like Sirius A and B which gives off more classical radiation will also give off more thermal radiation, he arrives at the conclusion that "thermal collisions possibly may provide the most important source of gravitational radiation in the universe."

Based on the linearized classical general-relativistic theory, Landau and Lifshitz[3] have calculated the energy loss of a system of interacting particles per unit time by finding the outgoing energy flux in all directions, which they found from the energy-momentum pseudotensor of the gravitational field. Their calculation leads to the following expression for the energy loss:

$$-\frac{dE}{dt}=\frac{G}{45c^5}\left(\frac{d^3D_{ik}}{dt^3}\right)^2, \tag{1.1}$$

where D_{ik} is the quadrupole moment of the mass (see Sec. 3 below).

A material system acted upon by internal forces that are much larger than the gravitational forces was then considered by Peres and Rosen,[4] and the rate of work of such a system against its own gravitational field was computed. The result was shown to be equal to the rate of radiated energy obtained by Landau and Lifshitz, Eq. (1.1). Other general-relativistic methods which treat this problem without being confined to the linearized theory are also well known,[5–7] but general agreement on this has not been achieved (see Sec. 5).

In this paper we first find the spectral resolution of the gravitational radiation for a bounded system of masses by applying classical Fourier-integral methods and assuming that the total gravitational-radiation intensity of the system is given by Eq. (1.1). It is well known[8] that in the spectral distribution of the radiation accompanying a collision, the main part of the intensity is contained in frequencies $\omega\sim1/\tau$, where τ is the order of magnitude of the duration of the collision. For this interval of frequencies, however, one cannot obtain a general formula for the distribution. The "tail" of the distribution at low frequencies, satisfying the condition $\omega\tau\ll1$, is, however, easier to handle. This is in fact the case discussed by Weinberg through quantum-electrodynamical methods, and this part of our calculation might be considered as a classical version of Weinberg's treatment. So, assuming that the collisions occurring in the system of masses are nonrelativistic Coulomb collisions, the result for the spectral power of the gravitational radiation will be shown to be in accord with the result of Weinberg. This is shown in Sec. 3.

We then turn to the problem of finding the total gravitational radiation of all the frequencies of the

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[1] S. Weinberg, in *Lectures on Particles and Field Theory* (Prentice-Hall, Inc., Engelwood Cliffs, New Jersey, 1964), p. 405; other references to Weinberg's work on the quantum theory of massless particles are given on p. 484 of this book.

[2] S. Weinberg, Phys. Rev. **140**, B516 (1965).

[3] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon Press, London, 1962), Sec. 104, p. 363.

[4] A. Peres and N. Rosen, Ann. Phys. (N.Y.) **10**, 94 (1960).

[5] A. Trautman, *Lectures on General Relativity* (King's College, London, 1958); A. Peres, in *Recent Developments in General Relativity* (Pergamon Press, Warsaw, 1962), p. 361.

[6] M. Carmeli, Phys. Letters **9**, 132 (1964); Nuovo Cimento **37**, 842 (1965).

[7] P. C. Peters, Phys. Rev. **136**, B1224 (1964); S. F. Smith and P. Havas, Phys. Rev. **138**, B495 (1965).

[8] L. D. Landau and E. M. Lifshitz, Ref. 3, Secs. 66–71, pp. 195–218.

spectrum without confining ourselves to the "tail"; of course, we expect the result to be larger than that of the "tail" alone. The calculation is simple and free from the cutoff process needed for estimating the "tail" radiation alone (and also needed in the quantum version of the calculation). For the case of the sun it follows that the total radiation is only one order of magnitude larger than that of the "tail." This is shown in Sec. 4, whereas in Sec. 2 we review Weinberg's work very briefly for comparison and completeness. Section 5 is devoted to concluding remarks.

2. GRAVITON EMISSION FROM MASSLESS-PARTICLE LINES

For completeness, we review very briefly Weinberg's method on emission from massless-particle lines, confining ourselves to the graviton case. For the complete treatment, where the photon and graviton are discussed on the same footing, the reader is referred to Weinberg's paper[2] and lecture notes.[1]

An infrared virtual graviton is defined as one which connects two external lines and carries energy less than some convenient energy Λ chosen small enough to justify the approximation. In addition to the cutoff $|\mathbf{q}|\leq\Lambda$ which defines the infrared lines, there is another cutoff $|\mathbf{q}|\geq\lambda$ which displays the logarithmic divergences $\ln\lambda$, where λ is very small, $\lambda\ll\Lambda$. This last cutoff only affects the infrared lines, because only these lines give infrared divergences for $\lambda=0$. The term "connecting external lines" used above means that the infrared line may join onto a line that has already emitted soft real quanta or virtual infrared quanta, but not onto one which, by real or virtual emission, has acquired a momentum far off its mass shell.

The corrections due to the infrared virtual graviton for any process $\alpha\rightarrow\beta$ are then given by

$$S_{\beta\alpha}=S_{\beta\alpha}{}^0\exp\left[\frac{1}{2}\int_\lambda^\Lambda d^4q\, B(q)\right], \tag{2.1}$$

where $S_{\beta\alpha}{}^0$ is the S matrix without virtual infrared gravitons, and $B(q)$ is the result of joining a pair of factors of the form

$$(8\pi G)^{1/2}\textstyle\sum_n \eta_n p_n{}^\mu p_n{}^\nu/(p_n\cdot q-i\eta_n\epsilon) \tag{2.2}$$

with a graviton propagator. The effective virtual graviton propagator for an internal line carrying momentum q and joining a $(\mu\nu)$ vertex with a $(\rho\sigma)$ vertex has been shown by Weinberg to be given by

$$\frac{-i}{2(2\pi)^4}\frac{(g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}-g_{\mu\nu}g_{\rho\sigma})}{q^2-i\epsilon}. \tag{2.3}$$

The factor given by Eq. (2.2) is the sum, over all external lines in the diagram, of the extra factors

$$(8\pi G)^{1/2}\eta p^\mu p^\nu/(p\cdot q-i\eta\epsilon), \tag{2.4}$$

where μ, ν are the graviton polarization indices, and $\eta=+1$ or -1 for an outgoing or incoming charged particle. These are the extra factors needed when we attach a soft-graviton line to an external line, and are obtained from

$$\begin{aligned}\tfrac{1}{2}i(2\pi)^4(8\pi G)^{1/2}(2p^\mu+\eta q^\mu)(2p^\nu+\eta q^\nu)[-i(2\pi)^{-4}]&\\ \times[(p+\eta q)^2+m^2-i\epsilon]^{-1},&\end{aligned} \tag{2.5}$$

in the limit $q\rightarrow 0$ (because $p^2+m^2=0$). Although the factor (2.5) holds only for the spin-zero external line to which we attach the graviton, its limit for $q\rightarrow 0$, Eq. (2.4), is valid for any spin. Also the dominance of the $1/(p\cdot q)$ pole in Eq. (2.4) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply the factor (2.2) over all external lines in the diagram.

The rate for a transition $\alpha\rightarrow\beta$ is given by the absolute square of (2.1):

$$\Gamma_{\beta\alpha}=\Gamma_{\beta\alpha}{}^0\exp\left\{\mathrm{Re}\int_\lambda^\Lambda d^4q\, B(q)\right\}. \tag{2.6}$$

A direct calculation then gives

$$\Gamma_{\beta\alpha}=\Gamma_{\beta\alpha}{}^0(\lambda/\Lambda)^B, \tag{2.7}$$

where B is a positive dimensionless constant

$$B=\frac{G}{2\pi}\sum_{n,m}\eta_n\eta_m m_n m_m\frac{1+\beta_{nm}{}^2}{\beta_{nm}(1-\beta_{nm}{}^2)^{1/2}}\ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right), \tag{2.8}$$

with β_{nm} the relative velocity defined by

$$\beta_{nm}=\left[1-\frac{m_n{}^2m_m{}^2}{(p_n\cdot p_m)^2}\right]^{1/2}. \tag{2.9}$$

The S-matrix element for emitting N *real* soft gravitons in a process $\alpha\rightarrow\beta$ is obtained by multiplying the nonradiative S matrix for $\alpha\rightarrow\beta$ by N factors of the form (2.2) and then contracting each of these factors with the appropriate graviton "wave function":

$$(2\pi)^{-3/2}(2|\mathbf{q}|)^{-1/2}\epsilon_\mu{}^*(\mathbf{q},\tfrac{1}{2}h)\epsilon_\nu{}^*(\mathbf{q},\tfrac{1}{2}h), \tag{2.10}$$

where $\mathbf{q}$ is the graviton momentum, $h=\pm 2$ is its helicity, and ϵ_μ is the corresponding polarization vector. It follows that the graviton-emission-matrix element is given by

$$\begin{aligned}S_{\beta\alpha}{}^{\mathrm{gr}}(12\cdots N)=S_{\beta\alpha}\prod_{r=1}^N(2\pi)^{-3/2}(2|\mathbf{q}_r|)^{-1/2}&\\ \times(8\pi G)^{1/2}\sum_n\frac{\eta_n[p_n\cdot\epsilon^*(q_r,\tfrac{1}{2}h_r)]^2}{[p_n\cdot q_r]}.&\end{aligned} \tag{2.11}$$

The rate for emission of N gravitons with energies near $\omega_1\cdots\omega_N$ is given by integrating the square of (2.11) over solid angles after summing over helicities and dividing by $N!$, since gravitons are bosons. The result is

$$\Gamma_{\beta\alpha}{}^{gr}(\omega_1\cdots\omega_N)d\omega_1\cdots d\omega_N=\frac{B^N}{N!}\Gamma_{\beta\alpha}\frac{d\omega_1}{\omega_1}\cdots\frac{d\omega_N}{\omega_N}, \quad (2.12)$$

where $\Gamma_{\beta\alpha}=|S_{\beta\alpha}|^2$. By using the representation of the step function, one finds for the rate of the transition $\alpha\rightarrow\beta$ accompanied by any number of soft gravitons with total energy less than E, and with each individual ω_r greater than the infrared cutoff λ,

$$\Gamma_{\beta\alpha}(\leqslant E)=\frac{1}{\pi}\sum_{N=0}^{\infty}\int_\lambda^E d\omega_1\cdots\int_\lambda^E d\omega_N\int_{-\infty}^{\infty}d\sigma\frac{\sin E\sigma}{\sigma}\times\exp(i\sigma\textstyle\sum_r\omega_r)\Gamma_{\beta\alpha}(\omega_1\cdots\omega_N). \quad (2.13)$$

Applying Eq. (2.13) to Eq. (2.12) one finds for the graviton-emission rate

$$\Gamma_{\beta\alpha}{}^{gr}(\leqslant E)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin E\sigma}{\sigma}\exp\left\{B\int_\lambda^E\frac{d\omega}{\omega}e^{i\omega\sigma}\right\}d\sigma. \quad (2.14)$$

For $\lambda\rightarrow 0$, Eq. (2.14) gives

$$\Gamma_{\beta\alpha}{}^{gr}(\leqslant E)=(E/\lambda)^B b(B)\Gamma_{\beta\alpha}, \quad (2.15)$$

where $b(x)$ is some real function:

$$b(x)=\frac{1}{\pi}\int_{-\infty}^{\infty}d\sigma\frac{\sin\sigma}{\sigma}\exp\left(x\int_0^1\frac{d\omega}{\omega}(e^{i\omega\sigma}-1)\right)$$
$$\simeq 1-\frac{1}{12}\pi^2x^2+\cdots. \quad (2.16)$$

We notice that for $\lambda\rightarrow 0$ the factor $(E/\lambda)^B$ becomes infinite, since B is a positive constant.

Using Eq. (2.7) in Eq. (2.15), one obtains

$$\Gamma_{\beta\alpha}{}^{gr}(\leqslant E)=(E/\Lambda)^B b(B)\Gamma_{\beta\alpha}{}^0, \quad (2.17)$$

which shows no dependence on the infrared cutoff λ. The factor E^B in (2.17) represents the shape of the energy spectrum for $0\leqslant E\leqslant\Lambda$.

It should be mentioned that it is crucial in Weinberg's work that the infrared divergences arise only from diagrams in which the soft real or virtual graviton is attached to an external line, with "external line" not including the soft real gravitons themselves. While this can be justified in electrodynamics and graviton theory, it is not applicable to theories involving charged massless particles, such as the Yang-Mills theory.

Also to the lowest order in G, Eq. (2.17) gives the power spectrum of soft gravitons accompanying a reaction $\alpha\rightarrow\beta$ as

$$Ed\Gamma_{\beta\alpha}(\leqslant E)\simeq B\Gamma_{\beta\alpha}{}^0 dE. \quad (2.18)$$

The rate of emission of energy (the power) in soft gravitational radiation during collision is given by

$$P(\leqslant\Lambda)=\int_0^\Lambda E d\Lambda(\leqslant E). \quad (2.19)$$

Using (2.17), we obtain

$$P(\leqslant\Lambda)=\frac{B}{1+B}b(B)\Lambda\Gamma_0$$
$$\simeq B\Lambda\Gamma_0, \quad (2.20)$$

since $B\simeq 10^{-38}$ and $b(B)\simeq 1$. For an elastic nonrelativistic collision, it follows that B is

$$B=(16G/5\pi)Q_{ij}Q_{ij}, \quad (2.21)$$

where

$$Q_{ij}=\tfrac{1}{2}\textstyle\sum_n \eta_n m_n v_{ni}v_{nj}, \quad (2.22)$$

and repeated Latin indices are summed over 1, 2, 3. For nonrelativistic elastic two-body scattering, for example, one obtains

$$Q_{ij}Q_{ij}=\tfrac{1}{2}\mu^2v^4\sin^2\theta_c, \quad (2.23)$$

where μ is the reduced mass, $v=|\mathbf{v}_1-\mathbf{v}_2|$ is the relative velocity, and θ_c is the scattering angle in the center-of-mass system. Thus we obtain

$$B=(8G/5\pi)\mu^2v^4\sin^2\theta_c. \quad (2.24)$$

In applying Eq. (2.20) to actual calculations, Γ_0 might be taken as the collision rate ignoring gravitation altogether. The rate for such collisions per cm^3 per sec is then given by

$$\Gamma_0=vn_1n_2(d\sigma/d\Omega), \quad (2.25)$$

where n_1 and n_2 are the number densities of particles 1 and 2. Hence Eq. (2.20) gives for the total power emitted in soft gravitational radiation attributable to 1–2 collisions:

$$P(\leqslant\Lambda)=\frac{8G\mu^2}{5\pi\hbar c^5}v^5n_1n_2V\Lambda\int\left(\frac{d\sigma}{d\Omega}\right)\sin^2\theta_c d\Omega, \quad (2.26)$$

where V is the volume of the source, and the factor $(\hbar c^5)^{-1}$ has been inserted in order to convert the equation from units with $c=\hbar=1$ to cgs units, since the latter will be used throughout the rest of this paper. Equation (2.26) may be used directly to calculate the thermal gravitational radiation from any hot body, since everything in the universe is transparent to gravitons.

3. FOURIER SPECTRAL RESOLUTION

A. The Spectral Resolution of the Intensity of Dipole and Quadrupole Radiation

In discussing[8] the spectral distribution of the intensity of radiation, one distinguishes between expansions into Fourier series and Fourier integrals. In the case of the collision of charged particles one deals with the expansion into a Fourier integral; the quantity of interest is the total energy radiated during the collision in the form of waves with frequencies in an interval (ω, $\omega+d\omega$). This part of the total radiation lying in the frequency interval $d\omega$ is obtained from the usual formula for the intensity by replacing the square of the field with the square modulus of its Fourier component and multiplying by 4π.

The intensity of dipole radiation, for example, is well known to be given by[8]

$$I=(2/3c^3)\ddot{\mathbf{s}}^2,$$
$$\mathbf{s}=\sum e\mathbf{x}. \tag{3.1}$$

Thus the energy radiated throughout the time of collision in the form of waves with frequencies in the interval $d\omega$ is given by

$$d\mathcal{E}(\omega)=(8\pi/3c^3)[\ddot{\mathbf{s}}(\omega)]^2 d\omega, \tag{3.2}$$

where $\ddot{\mathbf{s}}(\omega)$ is given by

$$\ddot{\mathbf{s}}(\omega)=(1/2\pi)\int_{-\infty}^{\infty}\ddot{\mathbf{s}}e^{i\omega t}dt. \tag{3.3}$$

There is no dipole gravitational radiation and the lowest multipole radiation comes from the quadrupole. The intensity for the gravitational quadrupole radiation is given by[3]

$$I=(G/45c^5)(d^3D_{ik}/dt^3)^2, \tag{3.4}$$

where G is the Newton gravitational constant and D_{ik} is the quadrupole moment of the mass:

$$D_{ik}=\int\rho(3x^ix^k-\delta^{ik}x_sx_s)d^3x. \tag{3.5}$$

Thus the formula for the spectral resolution of the intensity of gravitational quadrupole radiation is

$$d\mathcal{E}(\omega)=(4\pi G/45c^5)[\Delta_{ik}(\omega)]^2 d\omega, \tag{3.6}$$

where

$$\Delta_{ik}(\omega)=(1/2\pi)\int_{-\infty}^{\infty}\frac{d^3D_{ik}}{dt^3}e^{i\omega t}dt. \tag{3.7}$$

B. Radiation of Low Frequencies in Collision

The major part of the intensity of radiation accompanying a collision is contained in frequencies ω of order $1/\tau$, where τ is a characteristic time of the collision.[8] There is no general formula for the spectrum in this region. Instead, let us consider the "tail" of the distribution at low frequencies with the condition

$$\omega\tau\ll 1. \tag{3.8}$$

In the integrals (3.3) and (3.7) the field functions of radiation, i.e., $\ddot{\mathbf{s}}$ and d^3D_{ik}/dt^3, are significantly different from zero only during a time interval of the order of τ. Therefore, we can replace $\exp(i\omega t)$ in these integrals with unity and obtain

$$\ddot{\mathbf{s}}(\omega)=(1/2\pi)[\dot{\mathbf{s}}(f)-\dot{\mathbf{s}}(i)], \tag{3.9}$$

$$\Delta_{ik}(\omega)=(1/2\pi)\left[\frac{d^2D_{ik}(f)}{dt^2}-\frac{d^2D_{ik}(i)}{dt^2}\right], \tag{3.10}$$

where f and i mean after and before the collision.

For example, the spectral distribution of the total resolution emitted by a charge when it is accelerated from zero velocity to v can be obtained from (3.1), (3.2), and (3.9) as

$$d\mathcal{E}(\omega)=(2e^2v^2/3\pi c^3)d\omega, \tag{3.11}$$

a result which can also be obtained by different methods.[8]

C. Gravitational Radiation in Nonrelativistic Collisions

To find the spectral resolution of the gravitational radiation we notice that[3] for a bounded system of masses, d^2D_{ik}/dt^2 can be given in the form

$$\frac{d^2D_{ik}}{dt^2}=2\int(3\tau_{ik}-\delta_{ik}\tau_{ss})d^3x, \tag{3.12}$$

where τ_{ik} is

$$\tau_{ik}=\rho v^iv^k.$$

For a system of two particles we have therefore

$$\frac{d^2D_{ik}}{dt^2}=2\sum m(3v^iv^k-\delta^{ik}v^sv^s). \tag{3.13}$$

Let us introduce the quantity Q_{ik}:

$$Q_{ik}=(1/12)\left[\frac{d^2D_{ik}(f)}{dt^2}-\frac{d^2D_{ik}(i)}{dt^2}\right], \tag{3.14}$$

or more explicitly,

$$Q_{ik}=\tfrac{1}{2}\sum_n\eta_nm_n(v_n{}^iv_n{}^k-\tfrac{1}{3}\delta^{ik}v_n{}^sv_n{}^s), \tag{3.15}$$

where $\eta_n=+1$ or -1 if n is a final or an initial particle. Using the conservation law of energy in its nonrelativistic form,

$$\sum_n\eta_nm_n(1+\tfrac{1}{2}\mathrm{v}_n{}^2)=0,$$

we get for the last term on the right-hand side of Eq. (3.15),

$$+\tfrac{1}{3}\delta^{ik}\sum_n\eta_nm_n.$$

For nonrelativistic elastic scattering, this term is zero and we obtain

$$Q_{ik}=\tfrac{1}{2}\sum_n\eta_nm_nv_n{}^iv_n{}^k. \tag{3.16}$$

But this is just the same function Q_{ik} introduced by

Weinberg and given by Eq. (2.22). We thus obtain for (3.6), using (3.10) and (3.16),

$$d\mathcal{E}(\omega)=(16G/5\pi c^5)Q_{ik}Q_{ik}d\omega. \qquad (3.17)$$

For two-body scattering we have, according to Eq. (2.23),

$$Q_{ij}Q_{ij}=\tfrac{1}{2}\mu^2v^4\sin^2\theta_c, \qquad (3.18)$$

where μ is the reduced mass, $v=|\mathbf{v}_1-\mathbf{v}_2|$ is the relative velocity, and θ_c is the scattering angle in the center-of-mass system. Hence we get for the spectral distribution of the gravitational radiation of such a scattering

$$d\mathcal{E}(\omega)=(8G\mu^2v^4/5\pi c^5)\sin^2\theta_c d\omega, \qquad (3.19)$$

or, using the relation $dE=\hbar d\omega$,

$$d\mathcal{E}(\omega)=(8G\mu^2v^4/5\pi\hbar c^5)\sin^2\theta_c dE. \qquad (3.20)$$

It will be noted [see Eq. (2.24)] that the expression $(8G\mu^2v^4/5\pi\hbar c^5)\sin^2\theta_c$, appearing in Eq. (3.20), is just Weinberg's positive dimensionless constant B. Thus we have

$$d\mathcal{E}(\omega)=BdE \qquad (3.21)$$

for a scattering of two particles. The rate for such collisions per cm^3 per sec [compare Eq. (2.25)] is $vn_1n_2\times(d\sigma/d\Omega)$, where n_1 and n_2 are the number densities of particles 1 and 2. Hence we get for the total power emitted in soft-gravitational radiation attributable to 1–2 collisions

$$P(\leqslant\Lambda)=\frac{8G\mu^2}{5\pi\hbar c^5}v^5n_1n_2V\int_0^\Lambda dE\int\left(\frac{d\sigma}{d\Omega}\right)\sin^2\theta_c d\Omega, \qquad (3.22)$$

with V the volume of the source and Λ an upper limit for the energy. Equation (3.22) is identical with that obtained by Weinberg, Eq. (2.26).

To conclude this section, we mention that in applying Eq. (3.22), one encounters two cutoff processes. The first is the upper cutoff energy Λ which is taken by Weinberg as half the kinetic energy,

$$\Lambda\simeq\tfrac{1}{4}\mu v^2. \qquad (3.23)$$

The second comes from the integral over the differential cross section. For the sun, assuming that the most frequent collisions are the Coulomb collisions between electrons and protons or electrons, we may take[2]

$$\begin{aligned}\mu&=m_e,\\ v&=(3KT/m_e)^{1/2},\\ n_1&=n_e,\\ n_2&=n_e+n_p=2n_e.\end{aligned} \qquad (3.24)$$

The cross-section integral in Eq. (3.22) is then proportional to the diffusion coefficient and is estimated as[9]

$$\int\left(\frac{d\sigma}{d\Omega}\right)\sin^2\theta_c d\Omega=\int 2\pi\rho d\rho\,\sin^2\theta_c$$

$$=\frac{8\pi e^4}{(3KT)^2}\ln\Lambda_D, \qquad (3.25)$$

where Λ_D is the ratio of the Debye shielding radius (used to cut off the integral) to the average impact parameter.

In the next section we will use another method to estimate the total gravitational radiation power without referring to any of these cutoff processes.

4. TOTAL GRAVITATIONAL RADIATION

We now find the total gravitational radiation of the scattering process described in Sec. 3, for all possible frequencies, and without resolution into Fourier integrals. The result of the previous sections cannot be used to estimate the total gravitational radiation, since Eq. (3.22) shows that the power spectrum as a function of the frequency is constant and would therefore lead to infinity.

Let us denote the gravitational radiation accompanying a collision of two charged particles by $\Delta\mathcal{E}$. This is the total energy radiated throughout the time of the collision in the form of gravitational waves including all possible frequencies. $\Delta\mathcal{E}$ is obtained from the intensity I of Eq. (3.4) by integrating the latter over the time interval $(-\infty,\infty)$:

$$\Delta\mathcal{E}=\int_{-\infty}^{\infty}Idt, \qquad (4.1)$$

with

$$I=(G/45c^5)(d^3D_{kl}/dt^3)^2.$$

Since the rate for such collisions per cm^3 per sec is [see Eq. (2.25)] $vn_1n_2(d\sigma/d\Omega)$, we therefore obtain for the total power radiated for any two particles 1 and 2

$$P=\int V\Delta\mathcal{E}vn_1n_2(d\sigma/d\Omega)d\Omega$$

or

$$P=Vvn_1n_2\int_0^\infty\Delta\mathcal{E}2\pi\rho d\rho. \qquad (4.2)$$

But the integral in (4.2) is the familiar effective radiation.[8] Its value for quadrupole gravitational radiation

[9] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), Chap. 5.

can easily be found to be[10]

$$\chi=(32\pi G/9c^5)\mu e^2v^3. \tag{4.3}$$

Accordingly, we get for the total power

$$P=(32\pi G/9c^5)\mu e^2v^4n_1n_2V, \tag{4.4}$$

or, using Eq. (3.24), we obtain for the total radiation due to nonrelativistic collisions in the sun

$$P=(64\pi G/9c^5)m_e^{-1}e^2n_e^2(3KT)^2V. \tag{4.5}$$

Using the same data taken by Weinberg[2] for the sun's core, namely

$$T\simeq 10^7\ {}^\circ\text{K},\quad n_e\simeq 3\times 10^{25}\ \text{cm}^{-3},\quad V\simeq 2\times 10^{31}\ \text{cm}^3. \tag{4.6}$$

Eq. (4.5) gives for the solar gravitational radiation power

$$P\simeq 5\times 10^{15}\ \text{erg/sec}. \tag{4.7}$$

This result is about 10 times the one obtained when Eq. (3.22) is applied. It has also been obtained without referring to any cutoff process.

5. CONCLUDING REMARKS

We have compared the results obtained for the gravitational radiation emitted by a system of particles by a "quantum-gravidynamical" calculation developed by Weinberg, with the results obtained by an application of the classical "Landau-Lifshitz" formula that is based on the linear approximation. We have seen that under the same physical assumptions concerning the scattering process the two formalisms predict the same result. Using the linearized theory, however, enables us to get the total power without referring to the cutoff process that was needed for the low-frequency power by both methods.

Let us finally mention that the great amount of work on both classical and quantum theory of gravitational radiation in the last years shows that both these domains contain basic problems far more difficult to resolve than had been believed before. The linearized theory of gravitation has long been considered as suspect. Even the existence of gravitational radiation as a legitimate solution of Einstein's field equations with sources has been questioned by some workers.[11]

On the other hand, after the initial enthusiasm first for quantization of the linearized theory, then for the quantization of the full theory (implying quantization of geometry), there has been fairly general disappointment and a widespread realization that neither the physical nor the mathematical problems are fully understood, much less overcome.[12]

It thus seems that our calculations should not be interpreted as a demonstration of agreement of two well-established theories. The numerical results obtained cannot be considered as a definite experimental prediction of the general theory of relativity. Even if these results were to be found in disagreement with experiment, this could not be taken as a reflection on general relativity.

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[10] The effective radiation for the quadrupole electromagnetic radiation is given by Landau and Lifshitz (Ref. 3, p. 217). For the gravitational case the calculation is very similar. Using the same notation as that of Landau and Lifshitz, we have for the gravitatonal quadrupole moment

$$D_{kl}=\mu(3x_kx_l-r^2\delta_{kl}).$$

Also we get

$$dx_k/dt=v_k,$$
$$d^2x_k/dt^2=(e^2/\mu r^3)x_k,$$
$$d^3x_k/dt^3=(e^2/\mu r^4)(v_kr-3x_kv_r).$$

The intensity is then found to be

$$I=(G/45c^5)(d^3D_{kl}/dt^3)^2 =(8Ge^4/15c^5r^4)(v^2+11v_\phi^2).$$

The effective radiation is obtained by integrating I over t and ρ as outlined in the above reference:

$$\chi=\int_0^\infty\int_{-\infty}^{+\infty} Idt2\pi\rho d\rho =(32\pi G/9c^5)\mu e^2v_0^3.$$

Compare also V. V. Batygin and I. N. Toptygin, *Problems in Electrodynamics* (Academic Press Inc., London, 1964), p. 442.

[11] For an excellent review of the problem of gravitational radiation see: A. Trautman, in Proceedings of the London Conference on Relativistic Theories of Gravitation, 1965, Vol. 1 (unpublished). See also *Proceedings of the Florence Meeting on General Relativity Problems of Energy and Gravitational Waves*, edited by G. Barbera (Pubblicazinoni del Comitato Nazionale per le Manifestazioni Celebrative, Firenze, 1965).

[12] For a recent review see B. S. DeWitt, University of North Carolina report, 1966 (unpublished). For a concise discussion of the theoretical problem see L. Rosenfeld, Nucl. Phys. **40**, 353 (1963).