Potential Model Calculation of Proton-Proton Bremsstrahlung

M. I. SOBEL*
Brooklyn College of the City University of New York, Brooklyn, New York

AND

A. H. CROMER†
Northeastern University, Boston, Massachusetts

(Received 27 July 1966; revised manuscript received 16 March 1967)

The cross section $d\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$ for proton-proton bremsstrahlung is computed for various energies and geometries, using the Yale potential and the Hamada-Johnston potential. The results are compared with the experimental data at 200, 158, and 48 MeV. Good agreement is obtained with the 158-MeV data, though the calculated cross sections are somewhat too large in the forward direction (of the scattered photon) and somewhat too small in the backward direction. The integrated cross section $d\sigma/d\Omega_1 d\Omega_2$ is in fair agreement with the data at 200 MeV, but at 48 MeV the experimental cross section is three to four times smaller than the calculated values. This could be because of the acceptance of noncoplanar events in the experiment.

I. INTRODUCTION

IN a previous paper[1] we discussed the importance of the proton-proton bremsstrahlung reaction

$$p+p \to p+p+\gamma$$

as a method of obtaining information about the off-energy-shell behavior of the p-p interaction. Some results were presented based on the use of phenomenological potentials to compute the off-energy-shell matrix elements. Since that time, a number of experiments have been completed[2–6] at various energies. We have, therefore, extended our calculations, using the Yale potential[7] and the Hamada-Johnston potential,[8] both of which are quite successful in describing a large number of elastic-scattering data.[9]

A complete discussion of the method of computation has been published recently.[10] Basically, the electromagnetic interaction is treated to first order and the nuclear interaction is treated exactly. A nuclear-double-scattering term, which has a small effect on the total cross section[11] is neglected, and radiation from exchanged meson currents is omitted.

In I a factor of $\frac{1}{2}$ was omitted in the electromagnetic interaction and so the cross sections in that paper are all a factor of 2 too large. This regrettable error has resulted in some discussion of discrepancies between theory and experiment which are largely removed now. This error was brought to our attention by Marker.[12]

In Sec. II our results for the cross section $d\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$ are presented and are compared with available experimental data and with other theoretical results. Good agreement is obtained with the 158-MeV data, though the calculated cross sections are somewhat too large in the forward direction (of the scattered photon) and somewhat too small in the backward direction. The integrated cross section $d\sigma/d\Omega_1 d\Omega_2$ is in fair agreement with the data at 200 MeV, but at 48 MeV the experimental cross section is three to four times smaller than the calculated values. In Sec. III these results are discussed in terms of the off-energy-shell matrix elements of the p-p system.

II. RESULTS

To specify a p-p bremsstrahlung event, five parameters of the final state must be given, the remaining four then being determined by energy-momentum conservation. In I we consider a process in which two protons are observed to be scattered into solid angles Ω_1 and Ω_2, and the energy E_1 of one is measured. For convenience, the two protons in the final state were taken to be coplanar with the incident proton and to be scattered through polar energies θ_1 and θ_2 located symmetrically on either side of the incident beam ($\theta_1=\theta_2$, $\varphi_1=0$, $\varphi_2=\pi$). The energy E_2 of the other proton and the energy K and polar angle θ_γ of the photon are then determined. (Actually there are two kinematically possible values for

* Supported in part by a grant from the National Science Foundation.
† Supported in part by grants from the National Science Foundation and the Army Research Office (Durham).

[1] M. I. Sobel and A. H. Cromer, Phys. Rev. **132**, 2698 (1963). Hereafter this paper is referred to as I.
[2] B. Gottschalk, W. J. Shlaer, and K. H. Wang, Nucl. Phys. **75**, 549 (1966).
[3] B. Gottschalk, W. J. Shlaer, and K. H. Wang, Nucl. Phys. (to be published).
[4] R. E. Warner, Phys. Letters **18**, 289 (1965); Can. J. Phys. **44**, 1225 (1966).
[5] K. W. Rothe, P. F. M. Koehler, and E. H. Thorndike, Phys. Rev. Letters **16**, 1118 (1966).
[6] I. Slaus, J. W. Verba, J. R. Richardson, R. F. Carlson, W. T. H. van Oers, and L. S. August, Phys. Rev. Letters **17**, 536 (1966).
[7] K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. MacDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
[8] T. Hamada and I. D. Johnston, Nucl. Phys. **31**, 382 (1962).
[9] P. Signell and N. R. Yoder, Phys. Rev. **132**, 1707 (1963).
[10] A. H. Cromer and M. I. Sobel, Phys. Rev. **152**, 1351 (1966). Hereafter this paper is referred to as II. The factor of $\frac{1}{2}$ in Eq. (4.5) of this paper should be omitted.
[11] M. I. Sobel, Ph.D. thesis, Harvard University, 1964 (unpublished); Phys. Rev. **152**, 1385 (1966).
[12] D. Marker and P. Signell (to be published). We are very indebted to Dr. Marker for bringing this error to our attention.

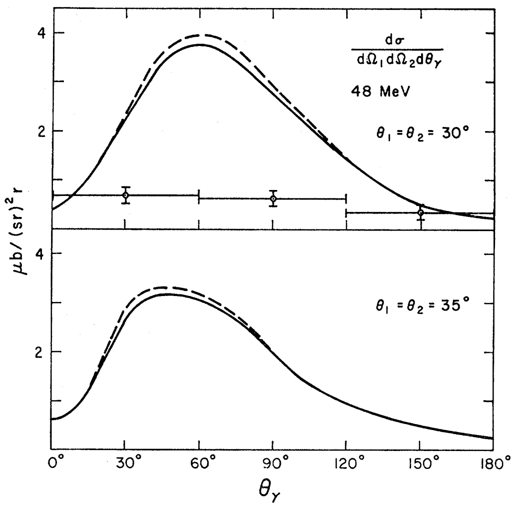

FIG. 1. Cross section (in $\mu b/sr^2$ rad) for p-p bremsstrahlung at 48 MeV calculated from the Yale potential (solid curve) and the Hamada-Johnston potential (dashed curve). The experimental points are from Slaus *et al.* (Ref. 6).

E_2 for each value of E_1. In practice this ambiguity is resolved by measuring E_2.) The cross section was then presented in the form $d\sigma/d\Omega_1 d\Omega_2 dE_1$ and plotted as a function of E_1. At the upper and lower ends of the energy spectrum the cross section diverges because of singularities in the phase-space factor [see II, Eq. (2.15)]. In this paper we consider the cross section $d\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$ as a function of θ_γ, for Ω_1 and Ω_2 fixed. This approach avoids the double-valued kinematics, since for each θ_γ there is only one set of values of E_1, E_2, K, and the cross section has no kinematic singularities. The two cross sections are related by a simple Jacobian [see II, Eqs. (2.12) through (2.15)].

Figure 1 shows the results at incident laboratory energy $E_0 = 48$ MeV, for $\theta_1 = \theta_2 = 30°$ and 35°, for the Yale and Hamada-Johnston potentials. The results are plotted for $0° \leqq \theta_\gamma \leqq 180°$, since in the symmetric case ($\theta_1 = \theta_2$) the same cross-section values occur for $180° \leqq \theta_\gamma < 360°$. In Table I the integrated cross sections[13] $d\sigma/d\Omega_1 d\Omega_2$ are given together with the experimental values of Warner[4] and Slaus *et al.*[6] and the latest theoretical result of Duck and Pearce[14] based on the Tabakin separable potential.[15] It should be pointed out that there are some disagreements about the calculations using the Tabakin potential.[16] However, our calculations are now in agreement with theirs, at least for the integrated cross sections at 158 and 200 MeV.

Figure 2 shows the results at $E_0 = 158$ MeV, for $\theta_1 = \theta_2 = 30°$, 35°, and 40°. The histogram and data points are from Gottschalk *et al.*[2,3] The data points[3] were taken with better azimuthal resolution but have somewhat poorer statistics than the data represented by the histogram.[2] The integrated cross sections are listed in Table II together with the results of Duck and Pearce.

TABLE I. The integrated proton-proton bremsstrahlung cross section $d\sigma/d\Omega_1 d\Omega_2$ in $\mu b/sr^2$, at $E_0 = 48$ MeV. The second and third columns give the values calculated in this paper using the Yale and the Hamada-Johnston potentials (Refs. 7 and 8). The fourth column gives the calculations of Duck and Pearce (Ref. 14) using the Tabakin potential (Ref. 15). The last two columns give the experimental results of Warner (Ref. 4) and of Slaus *et al.* (Ref. 6). All cross sections are in $\mu b/sr^2$.

θ_1	Yale	Hamada-Johnston	Duck and Pearce	Warner	Slaus *et al.*
30°	11.6	12.0	2.02	2.12±0.36	3.3±1.4
35°	10.0	10.2		3.04±0.44	

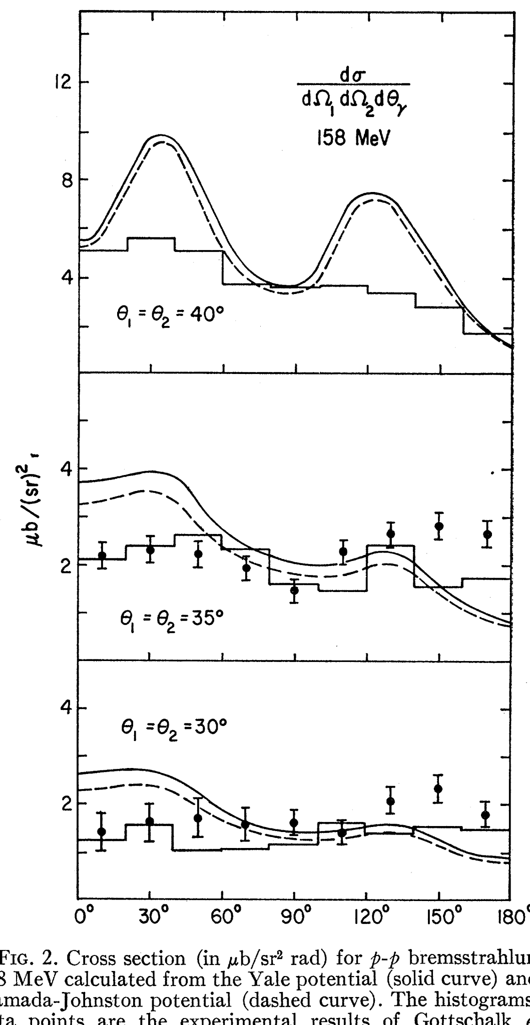

FIG. 2. Cross section (in $\mu b/sr^2$ rad) for p-p bremsstrahlung at 158 MeV calculated from the Yale potential (solid curve) and the Hamada-Johnston potential (dashed curve). The histograms and data points are the experimental results of Gottschalk *et al.* (Refs. 2 and 3, respectively).

[13] The cross section is integrated from 0° to 360°.

[14] I. Duck and W. A. Pearce, Phys. Letters **21**, 669 (1966); W. A. Pearce and I. Duck, Bull. Am. Phys. Soc. **12**, 16 (1967). We wish to thank Dr. Pearce for sending us his latest results prior to publication.

[15] F. Tabakin, Ann. Phys. (N. Y.) **30**, 51 (1964).

[16] P. Signell and D. Marker, in Proceedings of the Williamsburg Conference on Intermediate Energy Physics, The College of William and Mary, Williamsburg, Virginia, 1966, Vol. 2, p. 667 (unpublished).

Table III gives the integrated cross section at 200 MeV for $\theta_1=\theta_2=30°$, 35°, and 40°. The experimental values are those of Rothe *et al.*[5]

In order to get some idea of the dependence on E_0 we have calculated cross sections at $\theta_1=\theta_2=35°$, for $E_0=20$, 90, 120, and 200 MeV. These results are presented in Fig. 3 for the same two potentials. In Fig. 4 we have plotted $d\sigma/d\Omega_1 d\Omega_2$ for $\theta_1=\theta_2=35°$ as a function of E_0.

III. DISCUSSION

From the figures it is seen that the two potentials give very similar results at all energies and angles. This is not surprising since these potentials have many general features in common, and, in fact, their off-energy-shell behavior is more or less the same over the relevant range of center-of-mass momenta.[17] The Tabakin potential has a similar off-energy-shell dependence and so the discrepancy between our results and those of Duck and Pearce is disturbing.

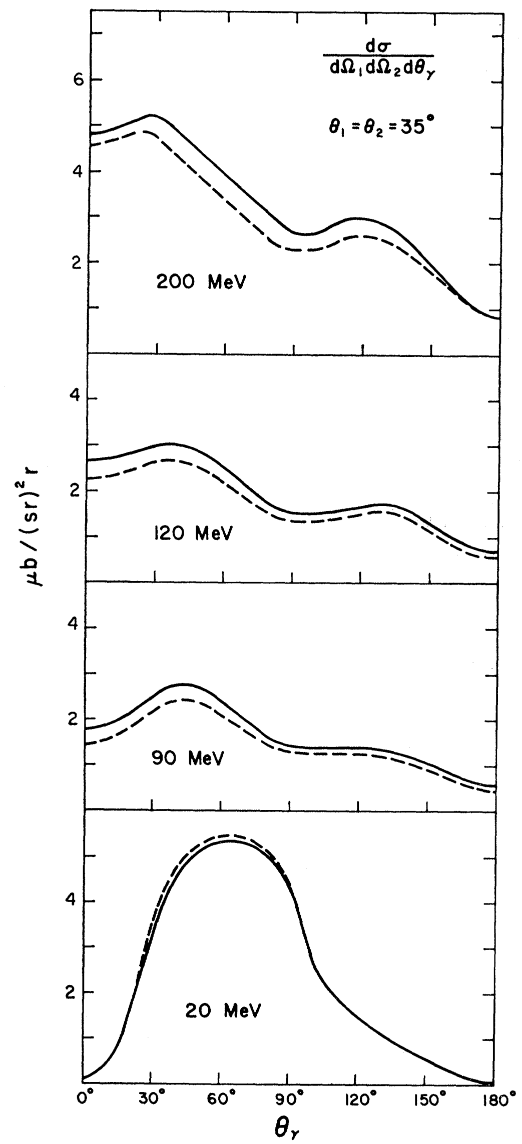

FIG. 3. Cross section for p-p bremsstrahlung at $\theta_1=\theta_2=35°$ calculated from the Yale potential (solid curve) and the Hamada-Johnston potential (dashed curve).

TABLE II. The integrated proton-proton bremsstrahlung cross section $d\sigma/d\Omega_1 d\Omega_2$ in μb/sr^2, at $E_0=158$ MeV. The second and third columns give the values calculated in this paper using the Yale and the Hamada-Johnston potentials (Refs. 7 and 8). The fourth column gives the calculations of Duck and Pearce (Ref. 14) using the Tabakin potential (Ref. 15). The last column are the experimental results of Gottschalk, Shlaer, and Wang (Refs. 2 and 3). The values in parentheses are from Ref. 3. All cross sections are in μb/sr^2.

θ_1	Yale	Hamada-Johnston	Duck and Pearce	Gottschalk *et al.*
30°	11.0	9.6	10.9	7.8±1.5 (10.6±2.1)
32.5°	12.8	11.2	11.5	10.1±2.0
35°	15.6	13.6	13.4	12.4±2.5 (14.0±2.8)
37.5°	19.8	17.6	16.3	13.3±2.7
40°	36.6	33.6	24.6	23.8±4.8

TABLE III. The integrated proton-proton bremsstrahlung cross section $d\sigma/d\Omega_1 d\Omega_2$ in μb/sr^2, at $E_0=200$ MeV. The second and third columns give the values calculated in this paper using the Yale and the Hamada-Johnston potentials (Refs. 7 and 8). The fourth column gives the calculations of Duck and Pearce (Ref. 14) using the Tabakin potential (Ref. 15). The last column gives the experimental results of Rothe *et al.* (Ref. 5). All cross sections are in μb/sr^2.

θ_1	Yale	Hamada-Johnston	Duck and Pearce	Rothe *et al.*
30°	13.4	12.2	12.8	13±3
35°	20.2	18.0		14±3
40°	50.4	50.4		29±6

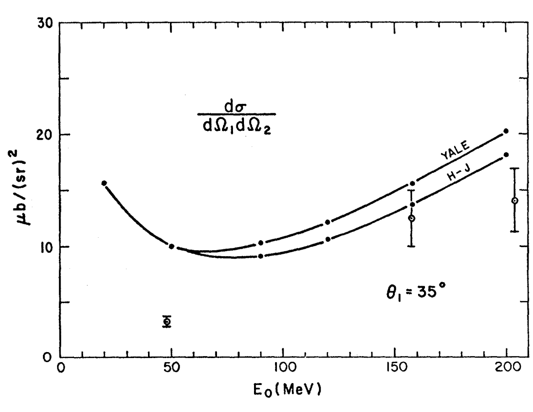

FIG. 4. Integrated cross section $d\sigma/d\Omega_1 d\Omega_2$ at $\theta_1=\theta_2=35°$ as a function of incident energy E_0 calculated from the Yale potential (solid curve) and the Hamada-Johnston potential (dashed curve). The data points at 48, 158, and 200 MeV are from Refs. 4, 2, and 5, respectively.

[17] M. I. Sobel, Phys. Rev. **138**, B1517 (1965).

The agreement with the angular distribution at 158 MeV (Fig. 2) is fair at 30° and 35° though the theoretical cross section is too large in the forward direction and too small in the backward direction. In fact, from Fig. 3 we see that for all energies above about 100 MeV the cross section decreases by a factor of 3 or 4 as θ_γ varies from 0° to 180°. The over-all phase-space factor (called $\mathfrak{F}$ in II) exhibits quantitatively this same decrease and the p-p matrix elements remain approximately constant as θ_γ varies.[18] However, K, the photon momentum, decreases by a factor of around 2 as one goes from forward to backward scattering, and the cross section goes roughly as K^{-3} (a factor of K^{-2} coming from the energy denominators and a factor of K^{-1} coming from the electromagnetic vertex). Thus, on the basis of order-of-magnitude estimates, the cross section would be expected to increase with θ_γ. The actual decrease is a consequence of an increasing amount of cancellation among the four amplitudes. It was observed in the early work of Marshak and Ashkin[19] that a large amount of cancellation should occur, such that in the dipole approximation (and Born approximation) the bremsstrahlung cross section is zero. This cancellation is more and more effective, to the extent of a factor of around 8 as θ_γ increases from 0° to 180°.

At the higher energies there is a tendency for the theoretical cross section to peak at $\theta_\gamma \sim 40°$ and 140°, and this becomes more pronounced as θ_1 increases from 30° to 40°, corresponding to becoming closer to the energy shell. There is some indication of this behavior in the 158-MeV data. As the incident energy decreases, the theoretical shape changes with $d\sigma/d\Omega_1 d\Omega_2 d\theta_\gamma$ becoming quite small at $\theta_\gamma \sim 0°$ just as at 180°.

The calculated integrated cross sections $d\sigma/d\Omega_1 d\Omega_2$ at 158 and 200 MeV are in very good agreement with the experimental values except at 40°. Gottschalk *et al.*[3] have pointed out that their 40° data could be only 60% of the true coplanar cross section because of the acceptance of noncoplanar events in their original experiment.[2]

At 48 MeV the theoretical integrated cross section is three to four times larger than the experimental values. Several factors may be responsible for this discrepancy. It has been argued[3,5] that the experimental results at this energy may be too small by a factor of 1.5 to 2 because of the fact that noncoplanar events (that is, events in which the incident proton and the two final protons are not coplanar) were accepted. Experimental studies of noncoplanarity at high energy[3,5] indicate that the cross sections fall off rapidly out of the plane so that a somewhat uncertain correction must be applied to experimental data taken with finite counter heights.

In describing the basic p-p interaction, the Coulomb force has been neglected. This approximation is justified at high energies, but at 48 MeV it may lead to a significant error in the predicted cross section. Another effect is due to the neglect of the double scattering term. This effect is expected to be largest for θ_γ near 180°. It has been estimated[11] for this case at 158 MeV to be less than about 7%. Finally, there is an unknown possible contribution because of the interaction of exchange meson currents with the electromagnetic field.

In conclusion we point out that there is a large group of bremsstrahlung experiments which needs to be done, including measurements at other energies, with unequal proton angles and with noncoplanar geometry. The experimental data is already approaching a point where it may be analyzed phenomenologically using quasi-phase parameters[17] in analogy to conventional phase-shift analyses. The level of agreement with experiment indicates that a search for improved potential parameters to fit observed off-energy-shell effects will be useful. Further improvements in the theory to include double scattering effects and the Coulomb interaction should also be made.

ACKNOWLEDGMENTS

We wish to thank Dr. L. Heller for correcting a mistake in sign in our calculation.

Parts of this work were done at the computation centers at Brooklyn College, Massachusetts Institute of Technology, New York University, and Northeastern University, and we acknowledge the cooperation of the staffs at these institutions. We wish to thank Marc Friedlander for assistance with the calculations.

[18] The approximate constancy of the p-p matrix elements as they are evaluated further and further from the energy shell is a significant property of the potentials. It holds at all energies considered.

[19] J. Ashkin and R. E. Marshak, Phys. Rev. **76**, 989 (1949); **76**, 58 (1949).