Fluctuations in Neutron Total Cross Sections

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The total cross sections of 18 elements from Mg to Bi were measured with energy spreads of 20 or 50 keV and a statistical accuracy of 1% as a function of neutron energy in the energy range from 4.5 to 7.5 MeV. No deviations from a smooth energy dependence were observed in the cross sections of the 10 elements from Co to Ta. For the remaining elements fluctuations were observed, the amplitudes of which decreased with neutron energy. For these elements, measurements were extended to higher energies until no fluctuations were observable. In addition, for Pb and Bi the energy interval from 2.5 to 4.5 MeV was investigated. The fluctuations could be accounted for by fluctuations in widths and spacings of compound-nucleus levels. It was not necessary to assume intermediate structure. Although the conditions for the Ericson theory of fluctuations may not be satisfied for all the measurements, this theory also accounts for the observed fluctuations at the higher energies.

I. INTRODUCTION

THE energy dependence of neutron total cross sections exhibits narrow peaks caused by compound-nucleus resonances and broad structure which can be described by the optical model. When the energy dependence is measured with an energy spread somewhat larger than the spacing of the resonances, fluctuations in the cross section will be caused by fluctuations in the number of levels or in the widths of the levels. In addition, Ericson[1] fluctuations may occur at energies high enough that the compound-nucleus levels overlap strongly. Ericson fluctuations have widths of the same magnitude as the compound-nucleus levels. They will, however, be relatively small in total cross sections because of the many available exit channels.

Recently structure in neutron total cross sections in the MeV region with widths and spacings as large as several hundred keV has been reported.[2–5] Such structure was observed in intermediate and heavy nuclei in several cases for which the experiment averaged over so many compound-nucleus levels that statistical fluctuations in the level widths and spacings could not explain the observations. This type of structure is usually called intermediate. It has been interpreted in terms of doorway states.[6,7]

In the present experiment details of the energy dependence of total cross sections were studied at energies around 6 MeV in elements ranging from Mg to Bi. The purpose was to investigate the presence of fluctuations or intermediate structure.

II. EXPERIMENTAL

The experimental procedure was similar to that used previously at this laboratory for measuring total neutron cross sections with good energy resolution.[8]

A. Source

Neutrons of energies below about 6 MeV were produced by bombarding tritium with protons accelerated by a tandem electrostatic accelerator. Above 6 MeV the *d-d* reaction served as the neutron source. Gas targets[9] 1.8 cm long with Ni entrance foils were used. Uncertainties in the foil thickness resulted in uncertainties in the neutron energy of about 20 keV.

Experiments were performed with neutron energy spreads of 20 and 50 keV (full width at half-maximum). For the 20-keV spread 0.8-μ-thick Ni foils were used with about $\frac{1}{4}$-atm target gas pressure. To obtain the 50-keV energy spread 1 atm of tritium was employed with 2.5-μ foils, or $\frac{1}{2}$ atm of deuterium with 1.3-μ foils.

The energy spread is determined primarily by straggling of the bombarding particles in the foil and by their energy loss in the gas target. A measurement of the sharp 4.93-MeV resonance in the C^{12} total cross section[8] yielded widths consistent with the calculated energy spreads.

B. Detector

Neutrons were detected with a stilbene scintillator 2.5 cm in diameter and 2.5 cm long. A discriminator bias was set to eliminate proton recoils from neutrons produced in the breakup of deuterons and to reduce the effect of background neutrons. Pulse-shape discrimination against electrons avoided the detection of γ rays.[10,11]

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[1] T. Ericson, Phys. Rev. Letters **5**, 430 (1960).

[2] D. W. Glasgow and D. G. Foster, Hanford Laboratory Report No. HW-SA-2875, 1963 (unpublished).

[3] K. K. Seth, Phys. Letters **16**, 306 (1965).

[4] F. Manero, Nucl. Phys. **65**, 419 (1965).

[5] A. J. Elwyn, J. E. Monahan, R. O. Lane, and A. Langsdorf, Jr., Nucl. Phys. **59**, 113 (1964).

[6] A. K. Kerman, L. S. Rodberg, and J. E. Young, Phys. Rev. Letters **11**, 422 (1963).

[7] H. Feshbach, A. K. Kerman, and R. H. Lemmer, in *Comptes Rendus du Congrès International de Physique Nucléaire II*, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, 1964), p. 693.

[8] D. B. Fossan, R. L. Walter, W. E. Wilson, and H. H. Barschall, Phys. Rev. **123**, 209 (1961).

[9] J. L. Fowler and J. E. Brolley, Rev. Mod. Phys. **28**, 103 (1956).

[10] F. D. Brooks, Progr. Nucl. Phys. **5**, 252 (1956).

[11] R. B. Owen, Nucleonics **17**, 92 (1959).

C. Procedure

Total cross sections were measured in a transmission experiment in good geometry with the samples placed about 13 cm from the source and the detector 40 cm from the source. Enough counts were taken at each energy to obtain a statistical accuracy of about 1%.

Corrections for inscattering were applied using the expression given by Bratenahl *et al.*,[12] taking into account neutrons detected after one or two elastic collisions. In the cases where the forward elastic-scattering cross section has not been measured, it was calculated under the assumption of diffraction by a black nucleus.[13] The correction varied from 2% for the lightest elements and the lowest energies to 7% for the heaviest nuclei at the highest energies and is estimated to have an uncertainty of 20%.

Corrections were also applied for room-scattered neutrons and for neutrons which did not originate in the target gas. The former correction was determined by inserting a long shadow bar between the source and detector, the latter by evacuating the target cell. The correction for background neutrons was less than 2%.

Finally, a correction was applied for counting losses. This correction of about 1% was calculated from the resolving time and checked experimentally by determining transmissions for various counting rates.

D. Samples

The following elements were studied: Mg, Al, Si, S, Ti, Fe, Co, Zn, Y, Zr, Nb, Ag, Sn, Sb, Pr, Ta, Pb, and Bi.[14] For Pb two isotopic compositions were used, normal lead and radiogenic lead, which contained 88% Pb^{206} and will hereafter be referred to as Pb^{206}. All the samples were in solid elemental form. They were cylinders about 2.5 cm in diameter and lengths to give a transmission of approximately 40%. For all the samples the measured densities agreed with tabulated values. Effects of impurities and nonuniformities in the samples are estimated to introduce less than 1% uncertainty in the measured cross sections.

III. RESULTS

The measured total neutron cross sections are plotted as a function of energy in Figs. 1–9. Measurements made with an energy spread of 20 keV are represented

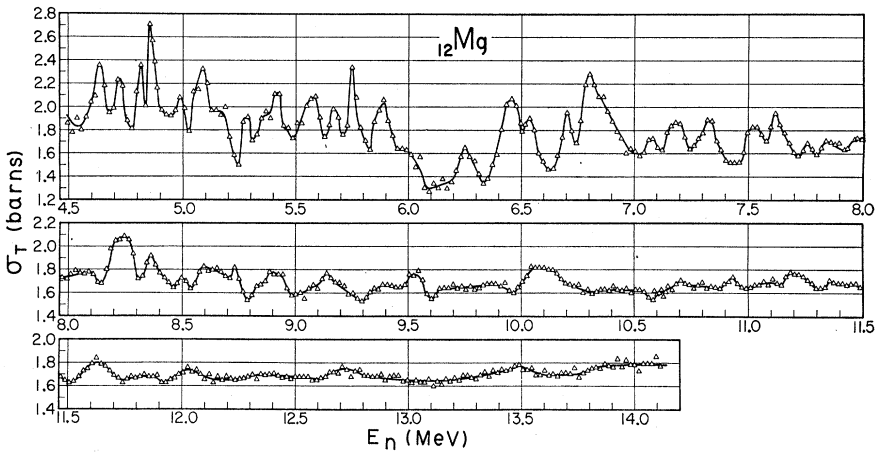

FIG. 1. The total neutron cross section of Mg measured with a neutron-energy spread of 20 keV and a statistical accuracy of 1%.

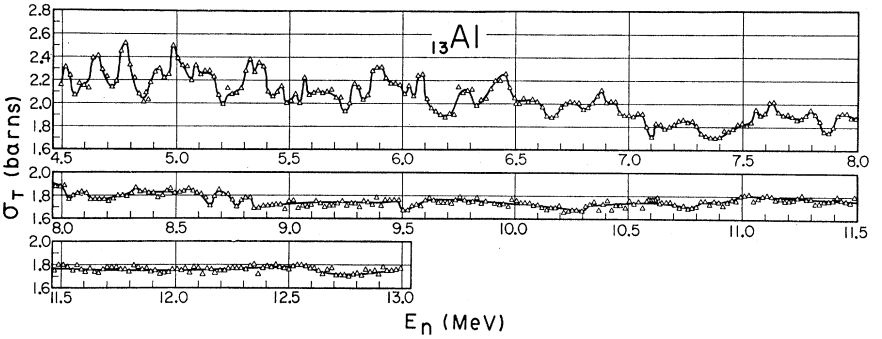

FIG. 2. The total neutron cross section of Al measured with a neutron-energy spread of 20 keV.

[12] A. Bratenahl, J. M. Peterson, and J. P. Stoering, Phys. Rev. **110**, 927 (1958).
[13] B. T. Feld, H. Feshbach, M. L. Goldberger, H. Goldstein, and V. F. Weisskopf, Atomic Energy Commission Report NYO-636, 1951 (unpublished).
[14] We wish to thank Professor R. Jensen of Marquette University for the loan of the Zn and Sn samples.

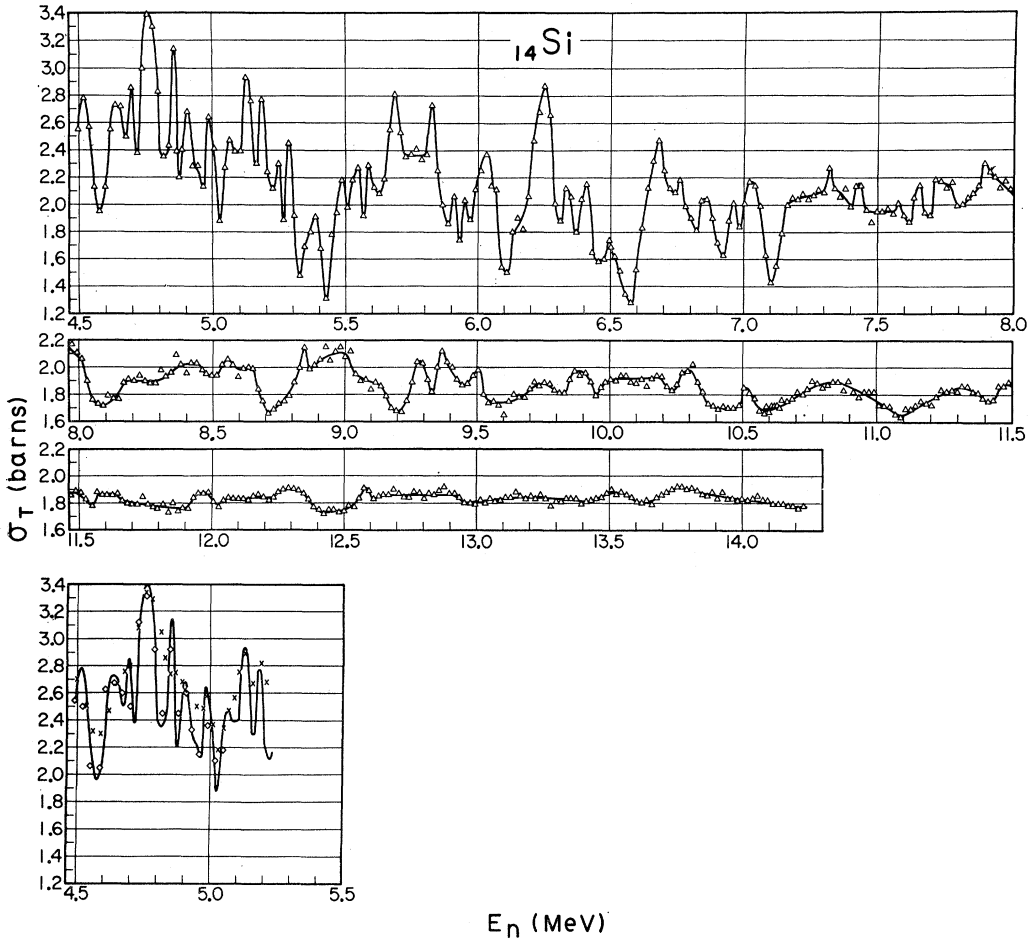

FIG. 3. The total neutron cross section of Si. The upper part of the figure shows the present measurements which were taken with an energy spread of 20 keV. In the lower part of the figure the crosses are the data of Calvi *et al.* (Ref. 16) which were taken with a 40-keV energy spread, and the diamonds are the data of Tsukada (Ref. 15) for which the energy spread was about 25 keV. The curve shown in the lower part of the figure represents the present data and is not drawn to fit the crosses and diamonds.

by triangles, and those with an energy spread of 50 keV by circles. The error bars shown are statistical standard errors. Where no error bars are indicated, the statistical uncertainty is smaller than the size of the symbols, except in the energy regions in which both triangles and circles are shown, where the statistical errors are given only for the measurements with the larger energy spread. In addition, there is a scale uncertainty of less than 2% caused by uncertainties in the inscattering correction and by possible sample impurities.

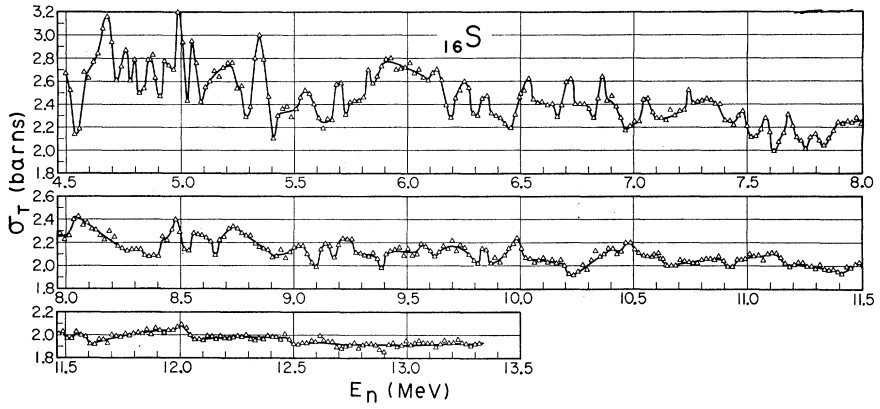

FIG. 4. The total neutron cross section of S.

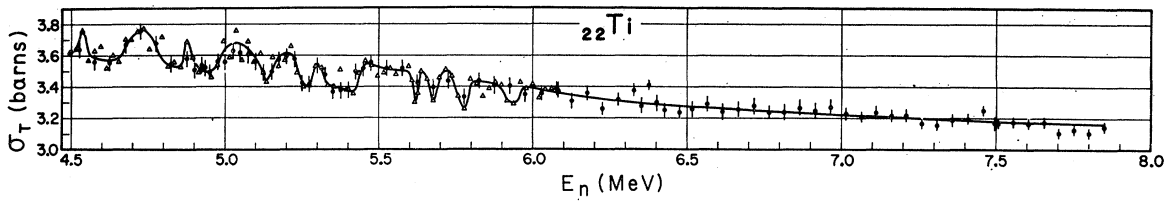

FIG. 5. The total neutron cross section of Ti. Triangles are data taken with an energy spread of 20 keV, dots are data taken with a 50-keV spread. All the measurements have a statistical uncertainty of 1%, but error bars are shown only for the 50-keV data.

Tabulations of the cross sections plotted in Figs. 1–9 are available from the Sigma Center, Brookhaven National Laboratory, Upton, New York, 11973.

Curves were drawn through the experimental data with as few inflection points as was consistent with the statistical errors, but no mathematical fit to the data was attempted. In the lower part of Figs. 3 and 8 data obtained at other laboratories are shown for comparison. The curves in the lower part of the figures are the same as those in the upper part and are not drawn through the data shown in the lower part of the figures.

Clear evidence for fluctuations in the cross sections of Mg, Al, Si, S, Ti, Fe, Pb^{206}, and Pb was found. For Bi the fluctuations were barely outside of statistics. For the remaining elements no evidence for fluctuations outside of statistics was found.

IV. COMPARISON WITH PREVIOUS MEASUREMENTS

The most extensive measurements with which the present data can be compared are those of Glasgow and Foster,[2] who covered the entire energy region and all the elements except Si, studied in the present experiment. The average cross sections agree within the uncertainty of the measurements in all cases although the present measurements are 2–5% higher for Co, Zn, Zr, Pr, and Ta.

Glasgow and Foster found some evidence for fluctuations in the cross sections outside of statistics for elements for which the present measurements are consistent with no fluctuations. A typical example is shown in Fig. 8. While the average cross sections are in good agreement, the present results show no indication of the fluctuations evident in the data shown in the lower part of Fig. 8.

For Mg, Al, S, Ti, Fe, Pb^{206}, and Pb fluctuations were observed by Glasgow and Foster and in the present experiment. The observed fluctuations are in fair agreement when the different energy spreads used in the two sets of measurements are taken into account.

In the lower part of Fig. 3 a comparison with data of Tsukada and Tanaka[15] and those of Calvi *et al.*[16] is presented. Tsukada and Tanaka used an energy spread of about 25 keV, Calvi *et al.* about 40 keV. The agreement of the present data with those of Tsukada and Tanaka is excellent both in the absolute values of the cross section and the details of the structure. The agreement with the measurements of Calvi *et al.* is also very good if the difference in resolution is considered. For S the agreement with the measurements of Tsukada and Tanaka is as good as for Si, and for Mg and Al there is good agreement with the measurements of Calvi *et al.*[16,17] and for S with those of Cuzzocrea *et al.*[18] in the common energy region.

Manero[4] reported structure in the cross section of Bi in the energy region from 3.2 to 5.2 MeV measured with a 60-keV energy spread. Although the present experiments show some evidence for fluctuations in the Bi cross section, the fluctuations observed by Manero, especially at the higher energies, are larger than those found in the present measurements. The fluctuations observed by Manero are, however, not much larger than might be expected from the statistical accuracy of the measurements.

The present measurements for Al and Si may be

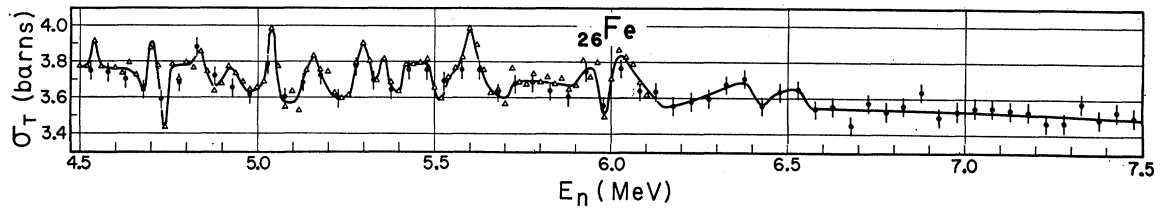

FIG. 6. The total neutron cross section of Fe.

[15] K. Tsukada and O. Tanaka, J. Phys. Soc. Japan **18**, 610 (1963).
[16] G. Calvi, R. Ricamo, A. Rubbino, and D. Zubke, Nucl. Phys. **48**, 408 (1963).
[17] G. Calvi, R. Potenza, R. Ricamo, and D. Vinciguerra, Nucl. Phys. **39**, 621 (1962).
[18] P. Cuzzocrea, S. Notarrigo, R. Ricamo, and F. Vinci, Nuovo Cimento **18**, 671 (1960).

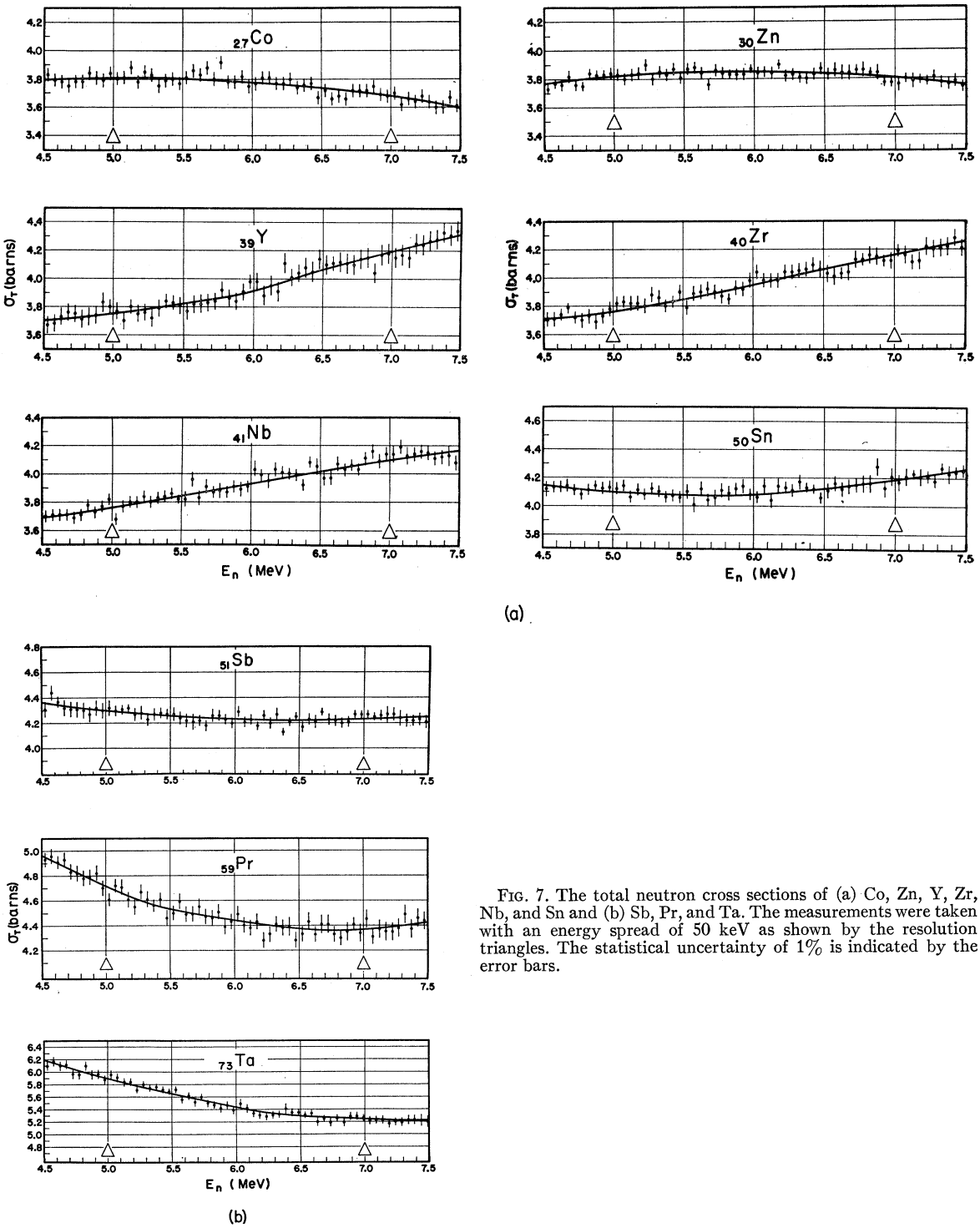

FIG. 7. The total neutron cross sections of (a) Co, Zn, Y, Zr, Nb, and Sn and (b) Sb, Pr, and Ta. The measurements were taken with an energy spread of 50 keV as shown by the resolution triangles. The statistical uncertainty of 1% is indicated by the error bars.

compared with measurements by Roessle and Tauber,[19] who investigated these elements with an energy spread of 20 keV in the energy region from 6.3 to 9.2 MeV. The two sets of measurements disagree in absolute value by about 20%. Although both sets of measurements show fluctuations of comparable widths and amplitudes, the fluctuations disagree in detail.

[19] E. Roessle and M. Tauber, in *Proceedings of the International Conference on the Study of Nuclear Structure with Neutrons, Antwerp, Belgium* (North-Holland Publishing Company, Amsterdam, 1966), p. 281.

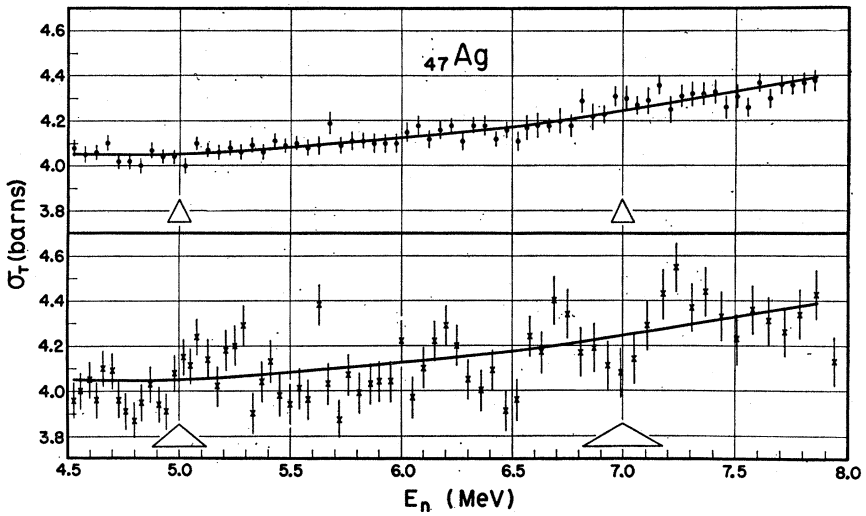

FIG. 8. The total cross section of Ag. The upper part of the figure shows the present measurements. For comparison the data of Glasgow and Foster (Ref. 2) are shown in the lower part of the figure. The curve shown in the lower part of the figure is the same as that drawn in the upper part of the figure through the present data. The resolution triangles show the energy spreads used in the two experiments.

Albergotti and Ferguson[20] have observed fluctuations in the cross section of Mg and Al between 12 and 14 MeV. These results are in good agreement with the present data except for a small difference in the Mg cross section near 13 MeV.

There are many previous measurements with which the average cross sections can be compared.[12,21–23] In all cases the results agree within the stated uncertainties except for a disagreement in the cross section of Co above 6 MeV measured by Bondelid *et al.*[23]

V. COMPARISON WITH OPTICAL-MODEL CALCULATIONS

Average cross sections obtained from the present experiment may be compared with optical-model results. In the cases in which large fluctuations were observed there is, however, considerable uncertainty in finding the average. Tabulated optical-model cross sections were available for three sets of calculations: (1) Results for Si, S, Ti, Fe, and Pb obtained by Wilmore and Hodgson,[24] who use a local potential without spin-orbit coupling with an energy dependence of the parameters to fit the nonlocal potential of Perey and Buck.[25] (2) Calculations by Emmerich,[26] who uses a local potential without spin-orbit coupling. (3) Calculations by Agee and Rosen,[27] who use a local potential with spin-orbit coupling with all parameters, except the depth of the real part of the potential, independent of energy.

Figure 10 shows a comparison between average cross sections obtained in the present study and the results of the three sets of calculations for three elements. The results for these elements are typical of all the elements. Generally, Wilmore and Hodgson's calculations are in best agreement with the measurements. Emmerich's calculations agree within a few percent although deviations as large as 10% occur. Agee and Rosen's calculations give in almost all cases cross sections typically 5% higher than the measurements with a few cases of as much as 15% difference.

As shown in Fig. 9, the average total cross sections of Pb, Pb^{206}, and Bi are almost independent of energy from 3.0 to 4.5 MeV. This feature is not reproduced by any of the available optical-model calculations. In this energy region the calculated cross sections are consistently smaller than the measured values.

VI. ANALYSIS

The present measurements show fluctuations in total cross sections outside the statistical uncertainty for Mg, Al, Si, S, Ti, Fe, Pb^{206}, and Pb. Three possible causes of such fluctuations will be discussed: (A) fluctuations in spacings and widths of compound-nucleus levels, (B) intermediate structure, and (C) Ericson fluctuations.

A. Fluctuations in Spacings and Widths of Compound-Nucleus Levels

In order to investigate the effect of fluctuations in widths and spacings an analysis of the data is carried out similar to that discussed by Agodi and Pappalardo.[28] These authors consider the case in which the experimental energy spread is larger than the width of the levels. It is not clear that this condition is satisfied for the present experiments. Therefore the experimental

[20] J. C. Albergotti and J. M. Ferguson, Nucl. Phys. **82**, 652 (1966).
[21] J. H. Coon, E. R. Graves, and H. H. Barschall, Phys. Rev. **88**, 562 (1952).
[22] J. L. Weil and K. W. Jones, Phys. Rev. **110**, 466 (1958).
[23] R. O. Bondelid, K. L. Dunning, and F. L. Talbott, Phys. Rev. **105**, 193 (1957).
[24] D. Wilmore and P. E. Hodgson, Nucl. Phys. **55**, 673 (1964).
[25] F. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962).
[26] W. S. Emmerich, *Fast Neutron Physics* (Interscience Publishers, Inc., New York, 1963), Part II, p. 1057.
[27] F. P. Agee and L. Rosen, Los Alamos Report LA3538MS, 1966 (unpublished).
[28] A. Agodi and G. Pappalardo, Nucl. Phys. **47**, 129 (1963).

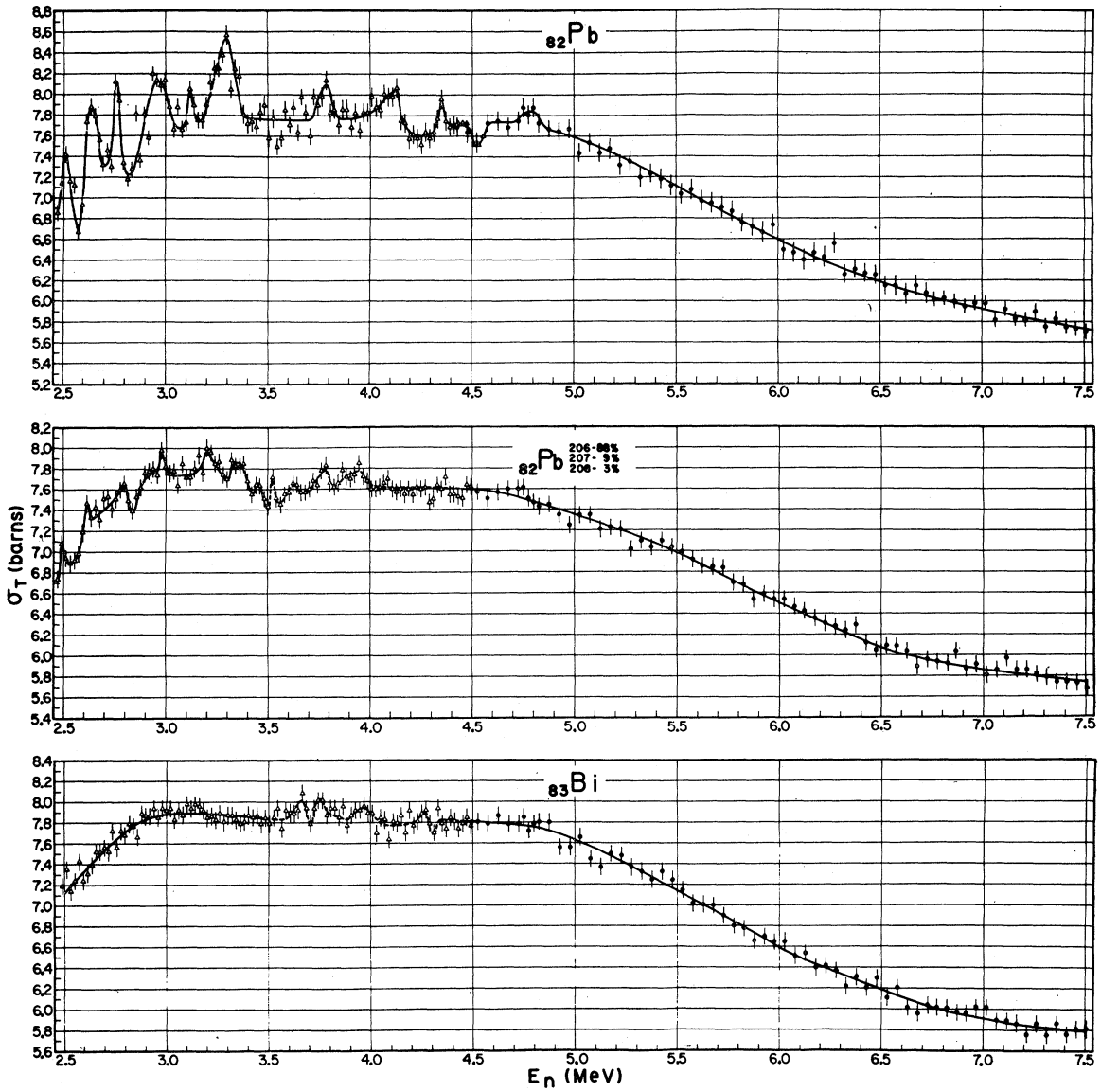

FIG. 9. The total neutron cross sections of natural lead, radiogenic lead, and Bi. The triangles represent data taken with an energy spread of 20 keV, dots are data taken with an energy spread of 50 keV.

data will be averaged over successive energy intervals Δ_n for which this condition should be satisfied.

In order to calculate the average cross section from resonance theory it is convenient to consider the total cross section as the sum of a slowly varying potential scattering cross section and a rapidly varying compound-nucleus-formation cross section. At the energies of the present measurements interference between potential and compound elastic scattering may be neglected because many partial waves contribute to the potential scattering and because the neutron width of the compound-nucleus levels is small compared to the total width.

The compound-nucleus-formation cross section averaged over an energy interval Δ_n, which is much larger than the spacings and total widths of the compound-nucleus levels, is given by[28]

$$\sigma_n=\frac{\pi\lambda^2}{\Delta_n}\sum_{J\Pi} g(J)\sum_{ls}\sum_{i=1}^{N^n{}_{J\Pi}} 2\pi\Gamma_i(ls|J\Pi)\,, \qquad (1)$$

where

$$g(J)=(2J+1)/2(2I+1)\,.$$

In this expression λ is the reduced wavelength, I is the spin of the target nucleus, $N^n{}_{J\Pi}$ is the number of compound-nucleus levels of spin J and parity Π in Δ_n, and $\Gamma_i(ls|J\Pi)$ is the partial width for neutron emission into

the entrance channel with relative orbital angular momentum l and channel spin s. The subscript i refers to the ith level having spin J and parity Π in Δ_n.

Although Eq. (1) holds strictly only if the levels do not overlap, Bethe[29] has shown that it applies even for overlapping levels if all the partial widths are small compared to the spacings. This condition is probably satisfied at the energies of the present experiment.

Since the nuclear-level density is a rapid function of energy, the number of levels over which the average is taken increases with energy if equal energy intervals are considered. In order to keep the number of levels over which the average is taken constant, Δ_n should be chosen to be inversely proportional to the average level density. In the analysis of the data it was assumed that the level density is given by the Fermi gas approximation in the form presented by Gilbert and Cameron.[30] It was found that the result of the analysis is quite insensitive to the assumed energy dependence of the level density.

The average cross sections σ_n will be different in different intervals Δ_n as a result of fluctuations in the number of levels in the intervals Δ_n or their neutron widths. A measure of the fluctuations of the cross section is the unnormalized variance F defined as

$$F=\langle(\sigma_n-\bar{\sigma})^2\rangle,$$

where σ_n is the average cross section in the interval Δ_n and $\bar{\sigma}$ is the average compound-nucleus-formation cross section which is here assumed to be independent of energy. This variance can be related[31] to the variances in the number and width of levels through Eq. (1). The result is

$$F=[2\pi^2\lambda\!\!\!^{-2}/\langle\Delta_n\rangle]^2\sum_{J\Pi}g^2(J)\times\{\sum_{ls}\langle N^n{}_{J\Pi}\rangle\,\mathrm{Var}[\Gamma_i(ls|J\Pi)]+[\mathrm{Var}N^n{}_{J\Pi}][\sum_{ls}\langle\Gamma_i(ls|J\Pi)\rangle]^2\},\quad(2)$$

where $\langle\Delta_n\rangle$ is the average size of the intervals Δ_n, $\lambda\!\!\!^{-}$ is the average reduced wavelength in the interval Δ which consists of the sum of the intervals Δ_n, and the Γ's are numbered consecutively throughout Δ. The variance (Var) and the average of the neutron widths is taken over the entire interval Δ. With the abbreviations

$$k_W=\frac{\mathrm{Var}[\Gamma_i(ls|J\Pi)]}{\langle\Gamma_i(ls|J\Pi)\rangle^2},$$

and

$$k_N=\frac{\mathrm{Var}N^n{}_{J\Pi}}{\langle N^n{}_{J\Pi}\rangle},$$

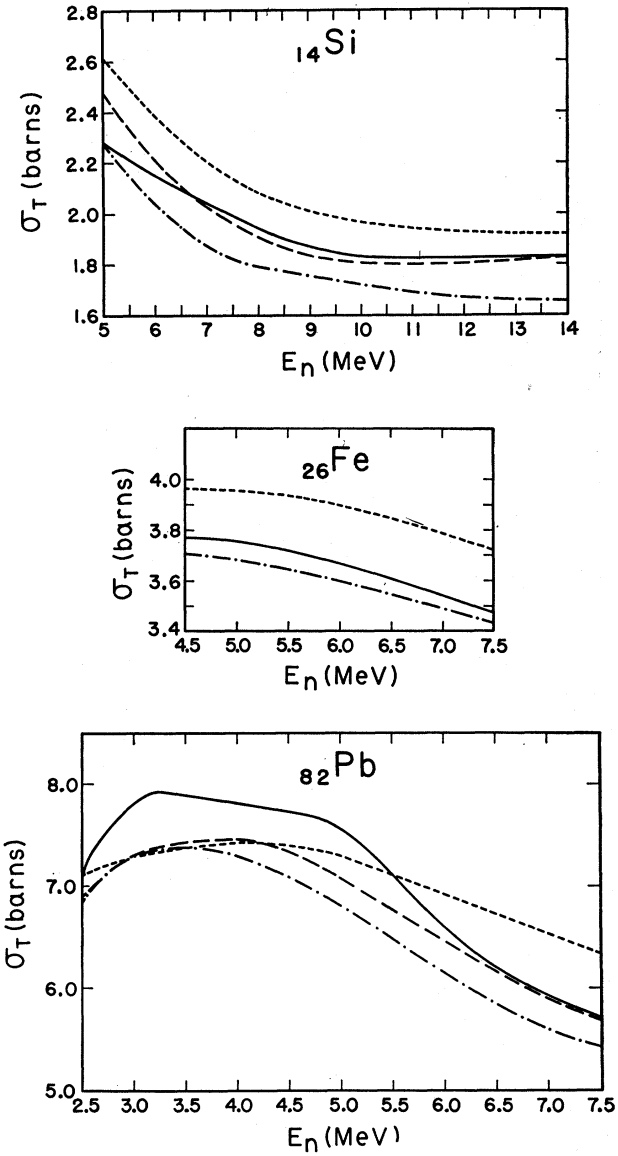

FIG. 10. Comparison of the measurements of the total cross sections of Si, Fe, and Pb with several optical-model calculations. The present experimental results are shown by the solid curves, the calculations of Wilmore and Hodgson (Ref. 24) by the long dashes, the calculations of Agee and Rosen (Ref. 27) by the short dashes, and Emmerich's calculations (Ref. 26) by the dot-dash curves. For Fe the calculations of Wilmore and Hodgson agree almost exactly with the present measurements and are not shown.

Eq. (2) may be written

$$F=(\pi\lambda\!\!\!^{-2})^2\sum_{J\Pi}g^2(J)\frac{1}{\langle N^n{}_{J\Pi}\rangle}\times[k_W\sum_{ls}(T_{ls}{}^J)^2+k_N(\sum_{ls}T_{ls}{}^J)^2],\quad(3)$$

where $T_{ls}{}^J$ are the transmission coefficients. The transmission coefficients are related to the average spacings

[29] H. A. Bethe, Rev. Mod. Phys. 9, 69 (1937).
[30] A. Gilbert and A. G. W. Cameron, Can. J. Phys. 43, 1446 (1965).
[31] A. D. Carlson, Ph.D. thesis, University of Wisconsin, Madison, Wisconsin, 1967 (unpublished).

of levels $D_{J\Pi}$ and the average widths by

$$T_{ls}{}^{J}=2\pi\langle\Gamma_i(ls|J\Pi)\rangle/D_{J\Pi}.$$

There is, however, some question regarding the validity of this relationship for overlapping levels.[32,33]

In order to obtain Eq. (3) certain simplifying assumptions have to be made, e.g., that the shape of the distributions over which the variance is taken is the same for the different quantum numbers, that for a given level partial widths for different quantum numbers are not correlated, and that the spacings of levels with the same quantum numbers are not correlated. This last assumption has been questioned particularly by Dyson,[34] who has proposed a statistical model for which correlations between successive level spacings occur. The occurrence of such correlations would decrease k_N, since according to Dyson's theory

$$\mathrm{Var}N^n{}_{J\Pi}\propto \ln\langle N^n{}_{J\Pi}\rangle,$$

while for uncorrelated spacings

$$\mathrm{Var}N^n{}_{J\Pi}\propto \langle N^n{}_{J\Pi}\rangle.$$

Equation (3) agrees with the expresssion given in Ref. 28 if k_N is unity, i.e., the spacings follow an exponential distribution, and if it is assumed that the spin of the target nucleus is large enough that the first sum inside the brackets in Eq. (3) is small compared to the second.

In the special case of spin-zero targets only one l value leads to a given compound state, so that Eq. (3) simplifies to the form

$$F=(\pi\lambda^2)^2\sum_{J\Pi}g^2(J)\frac{(T^J)^2}{\langle N^n{}_{J\Pi}\rangle}[k_W+k_N]. \quad (4)$$

If the neutron-width and level-spacing distributions are known, the quantities k_W and k_N may be calculated. From studies of slow neutron resonances it is known[35] that the width distribution agrees with that proposed by Porter and Thomas,[36] and the distribution of the level spacings is consistent with that proposed by Wigner.[37] For the Porter-Thomas distribution k_W is 2, for the Wigner distribution k_N has the value 0.27. At very high excitation energies where the levels overlap ($\Gamma/D\gg1$), the distributions of both widths and spacings are expected to be exponentials for which both k_W and k_N are unity.

For the case of spin-zero target nuclei the sum k_W+k_N, which appears in Eq. (4), has under these assumptions the value 2.27 for low bombarding energies, while at high bombarding energies the sum is 2. The sum may therefore be expected to be near 2 over a large range of energies for spin-zero target nuclei. The observed fluctuations will be analyzed using Eq. (3) or (4). In these equations F is obtained from the experiments, the transmission coefficients are known from optical-model calculations,[26] the quantities k_W and k_N are deduced from the width and level distributions, and $\langle N^n{}_{J\Pi}\rangle$ which is essentially the level density is treated as an unknown. In order to evaluate the level density ρ it will be assumed that it is a product of an energy-dependent factor $\omega(E)$ and a spin-dependent factor $H(J\Pi)$. The level-density expression given by Gilbert and Cameron[30] is of this form, although they use a very slow dependence of $H(J\Pi)$ on energy. Gilbert and Cameron's values of $H(J\Pi)$ were used in the analysis. Equations (3) and (4) can then be written

$$F\Delta_n=(\pi\lambda^2)^2\frac{1}{\omega(E_n)}\sum_{J\Pi}\frac{g^2(J)}{H(J\Pi)}\times[k_W\sum_{ls}(T_{ls}{}^J)^2+k_N(\sum_{ls}T_{ls}{}^J)^2] \quad (5)$$

and

$$F\Delta_n=(\pi\lambda^2)^2\frac{1}{\omega(E_n)}\sum_{J\Pi}\frac{g^2(J)(T^J)^2}{H(J\Pi)}[k_W+k_N]. \quad (6)$$

In the analysis the energy-dependent part of the level density $\omega(E_n)$ was evaluated at an energy E_L corresponding to the middle of the energy interval Δ_L at the highest energy in Δ.

If the target sample consists of several isotopes, the variance in the cross section must be considered separately for each isotope. The variance for the isotopic mixture can be found by adding the variances for each isotope weighted by the square of the isotopic abundance. In the present investigation either the elements for which fluctuations were analyzed are monoisotopic, or one isotope was sufficiently abundant that the contributions to the variance from the other isotopes were negligible.

The applicability of Eq. (1) requires that each Δ_n is much larger than the widths and spacings of the compound-nucleus levels. Since it was not known how small a Δ_n would satisfy this condition, a given energy interval Δ within which fluctuations were analyzed was divided into several different sets of intervals Δ_{nL}. Within each set L the Δ_{nL} were chosen so that each Δ_{nL} contained on the average the same number of levels according to the level-density formula of Gilbert and Cameron. As an example, the interval $\Delta=2$ MeV from 4.5 to 6.5 MeV in the cross section of S was divided into nine sets L ranging from 4 intervals Δ_{n4} to 35 intervals Δ_{n35}. These different sets will be denoted by the size of the interval $\Delta_L=\Delta_{LL}$ at the highest energy within Δ. In the example just discussed the set which

[32] P. A. Moldauer, Phys. Rev. **135**, B 642 (1964).
[33] P. A. Moldauer, Rev. Mod. Phys. **36**, 1079 (1964).
[34] F. J. Dyson and M. L. Mehta, J. Math. Phys. **4**, 701 (1963).
[35] F. W. K. Firk, J. E. Lynn, and M. C. Moxon, in Proceedings of the International Conference on Nuclear Structure, Kingston, Canada (University of Toronto Press, Toronto, Canada, 1960), p. 757.
[36] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[37] E. P. Wigner, Oak Ridge National Laboratory Report No. ORNL-2309, 59, 1957 (unpublished).

contains four Δ_{n4} has $\Delta_L=0.34$ MeV. When the condition on Δ_n is satisfied, the quantity $F\Delta_L$ should be independent of Δ_L.

The calculation of the variance of the cross section requires a knowledge of the average compound-nucleus-formation cross section $\bar{\sigma}$. Although $\bar{\sigma}$ could be calculated from the optical model, more reliable values could be obtained from the measured cross sections by the following procedure: A potential scattering cross section taken from an optical-model calculation[26] was subtracted from the measured cross sections which were analyzed. In spite of the relatively good fit of the calculations of the total cross sections according to this model (see Fig. 10), small differences between calculations and measurements as well as the effect of a slow energy dependence of $\bar{\sigma}$ were evaluated by averaging the experimental results over increasing energy intervals of size δ. It should be noted that this analysis involves two different energy intervals over which the cross section is averaged. The energy intervals Δ_n are used to determine the variance, while the intervals δ serve to estimate the average compound-nucleus-formation cross section.

In the example of the sulfur cross section, values of δ were chosen ranging from 0.5 to 1.5 MeV. At each energy E_M at which $\bar{\sigma}$ was needed, the measured cross section minus the potential scattering cross section was averaged in the energy interval from $E_M-\frac{1}{2}\delta$ to $E_M+\frac{1}{2}\delta$. This average will be called $\bar{\sigma}_\delta(E_M)$. The variance of the cross section was calculated from

$$F_L(\delta)=\langle(\sigma_{nL}-\bar{\sigma}_\delta)^2\rangle,$$

where σ_{nL} is the average of the difference between the measured and the potential cross section in the interval Δ_{nL}, and $\bar{\sigma}_\delta$ the average value of $\bar{\sigma}_\delta(E_M)$ for all E_M within Δ_{nL}.

In Fig. 11(a), 11(b), 11(c), and 11(d), the quantity $F_L(\delta)\Delta_L$ is plotted as a function of Δ_L for different values of δ for the measurements on S in the energy range from 4.5 to 6.5 MeV. For each value of δ it is assumed that $F_L(\delta)\Delta_L$ approaches a value which is independent of Δ_L. This value, which we denote as $[F_L(\delta)\Delta_L]$, is indicated by the horizontal lines. The error bars which are shown are determined by the finite number of intervals into which Δ is divided. In Fig. 11(e) the values of $[F_L(\delta)\Delta_L]$ are plotted as a function of δ. Above 1 MeV this quantity does not depend much on δ. This value is then substituted into Eq. (6) to calculate $\omega(E_L)$ after application of a correction for fluctuations caused by the statistical uncertainty in the cross-section measurements. In the calculation of $\omega(E_L)$ the isotopic abundance of S^{32} is taken into account.

The same procedure was repeated for the neutron energy intervals 6.5 to 8.5 MeV and 8.5 to 10.5 MeV. Above this energy the fluctuations in cross section became comparable to the statistical uncertainty in the measurements so that no reliable analysis could be performed.

For Mg and Si the same analysis as for S was carried out except that the additional interval from 10.5 to 12.5 MeV was included, while for Ti, Fe, and Pb the analysis could be carried out only for one interval.

In the case of Al, Eq. (5) has to be used in the analysis, since Al^{27} has spin $\frac{5}{2}$. The quantity $\omega(E_L)$ was calculated first with $k_W=2$ and $k_N=0.27$ and then with $k_W=k_N=1$. For both energy intervals which were considered, i.e., 4.5–6.5 MeV and 6.5–8.5 MeV the values of ω which were obtained with the two different assumptions differed by about 30%, and the average was taken.

In Fig. 12 the level densities deduced in the present study are shown. These are compared with the level densities given by Gilbert and Cameron which are shown in the figure as solid lines. The numbers shown in parentheses are the ratios of $\omega(E)$ found in the present analysis to the $\omega(E)$ given in Ref. 30. The error bars are determined primarily by the effect of the finite number of intervals. For Ti, Fe, and Pb, the ratios of the $\omega(E)$ from the analysis to that of Ref. 30 are given in Table I. For Pb^{206} and Bi and for other energies the fluctuations were not large enough to permit an analysis.

Considering the uncertainties both in the results of the present analysis and in the level-density formula the agreement between them is satisfactory, although there is a tendency for the present results to be higher. Such a difference could be the result of the presence of correlations between successive level spacings.

The expected variance in the cross section was calculated from Eq. (5) for the elements heavier than Fe except Pb. It was found to be smaller than that caused by counting statistics for the elements from Co to Ta. The two variances were comparable for Pb^{206} and Bi.

B. Intermediate Structure

As has been pointed out in Sec. III, the cross sections of the elements from Co to Ta are smooth functions of neutron energy and show no evidence for intermediate structure, in contrast to the findings of some other authors. The fluctuations observed for the other elements could be accounted for in terms of fluctuations in level widths and spacings as shown in Sec. VI A. It is nevertheless possible that intermediate structure may also be present. Although some authors have deduced the presence of intermediate structure from an inspec-

TABLE I. Level densities.

Compound nucleus	Excitation energy (MeV)	Ratio of level density from present analysis to calculated values
Ti^{49}	14.0	2.9
Fe^{57}	13.7	1.8
Pb^{209}	8.4	2.0

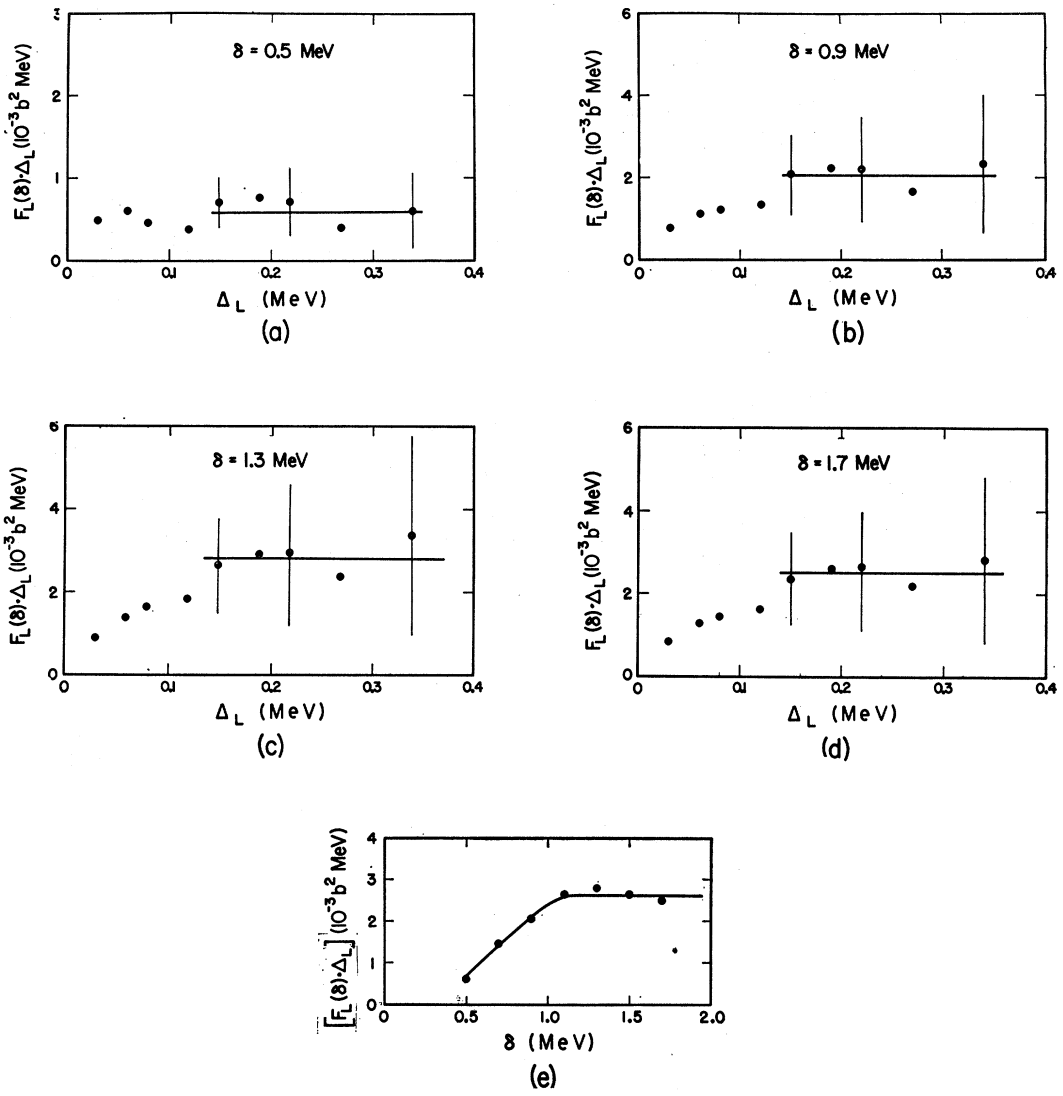

FIG. 11. Determination of the variance of the total cross section for the example of sulfur in the neutron-energy range from 4.5 to 6.5 MeV. The variance is calculated for various subdivisions of this interval characterized by Δ_L and for various averaging energies δ. The quantity $F_L(\delta)\Delta_L$ should approach a constant value for large Δ_L. These limiting values are plotted in part (e) as a function of δ. Error bars shown on some points are estimated from the finite range of data.

tion of cross-section curves, this procedure may lead to inconclusive results as has been shown by Singh.[38] With resonance parameters chosen at random from expected distributions Singh *et al.* found a 30% chance that in an interval $\Delta=25\Gamma$ a peak 6Γ wide occurs, where Γ is the average width of compound-nucleus levels. For this reason a more quantitative analysis for intermediate structure was attempted, following the procedure proposed by Pappalardo.[39]

In this analysis the following correlation function is used:

$$F(\epsilon,\delta)=\frac{1}{N}\sum_{k=1}^{N}[\sigma_T(E_k)-\bar{\sigma}_{T\delta}(E_k)]\times[\sigma_T(E_k+\epsilon)-\bar{\sigma}_{T\delta}(E_k+\epsilon)]. \quad (7)$$

Here $\sigma_T(E)$ is the measured total cross section at the energy E, $\bar{\sigma}_{T\delta}(E)$ is the total cross section averaged over the energy range from $E-\frac{1}{2}\delta$ to $E+\frac{1}{2}\delta$, and N is the number of measurements. $F(0,\delta)$ increases with δ until $\delta\gg\Gamma$. For large δ, $F(0,\delta)$ becomes independent of δ. In this flat region $F(0,\delta)$ is the mean square fluctuation in the cross section. The characteristic width Γ of the fluctuations can then be obtained from the dependence of $F(\epsilon,\delta)$ on ϵ. When the condition $\delta\gg\Gamma$ is satisfied,

[38] P. P. Singh, P. Hoffman-Pinther, and D. W. Lang, Phys. Letters 23, 255 (1966).
[39] G. Pappalardo, Phys. Letters 13, 320 (1964).

the symbol δ may be omitted in the correlation function, and it will be denoted by $F(\epsilon)$.

If structure is present which is much broader than Γ, $F(0,\delta)$ will increase again as δ becomes comparable to the width of this structure. The absence of a second rise is taken as evidence that such intermediate structure need not be assumed.

This analysis was performed for Mg, Al, Si, S, Ti, Fe, and Pb over the energy range in which data with 20-keV energy spread had been taken. For the four lightest elements the analysis was also performed separately for the two halves and four quarters of the range. The results for Si are shown in Fig. 13. The error bars shown are deduced from the finite number of fluctuations that are analyzed and are larger than the variation of $F(0,\delta)$ at the higher values of δ. Results obtained for the other elements are similar to those shown for Si except those for Fe which are shown in Fig. 13 (h). The oscillations exhibited by the correlation function for Fe are, however, also within the uncertainties caused by the number of fluctuations.

Shown as an insert to Fig. 13 (g) is a plot of $F(\epsilon)$ versus ϵ for the Si data from 11.7 to 14.2 MeV. The dots are the results obtained using Eq. (7). The uncorrected value of Γ is found as the value of ϵ for which $F(\epsilon,\delta)$ has half of the value of this function at $\epsilon=0$. The curve that is drawn follows the theoretical dependence[40] of $F(\epsilon)$ on ϵ, i.e.,

$$F(\epsilon)=F(0)\Gamma^2/(\epsilon^2+\Gamma^2).$$

Deviations from this dependence are expected for several reasons: the finite number of fluctuations analyzed, counting statistics, energy spread, and the fact that this relationship is only exact when $\Gamma/D\gg1$.

The corrected values of Γ obtained from the analyses of the Mg, Al, Si, and S data are shown in Table II. These values have been corrected for the effect of energy spread using the method proposed by Lang[41] and for counting statistics. For the remaining elements which were analyzed the corrections are too large to give reliable results. In all cases including Ti and Fe the widths are consistent with the order-of-magnitude calculations of Corti *et al.*[42] and are much smaller than the hundreds of keV expected for intermediate-structure resonances.

C. Ericson Fluctuations

If a total cross section is measured with an energy spread small compared to the width of compound-nucleus levels, fluctuations should be observable even if a large number of levels overlap, as a result of the interference of levels of the same spin and parity. These fluctuations should have widths of the order of the width of the compound-nucleus levels. The theory of these fluctuations was developed by Ericson.[1,40] In this theory it is assumed that all the levels have the same total width Γ and that fluctuations caused by fluctuations in level spacings may be neglected.

The expression obtained by Ericson for the correlation function for the total cross section at energies at which a large number of levels overlap is, according to Eq. (36)

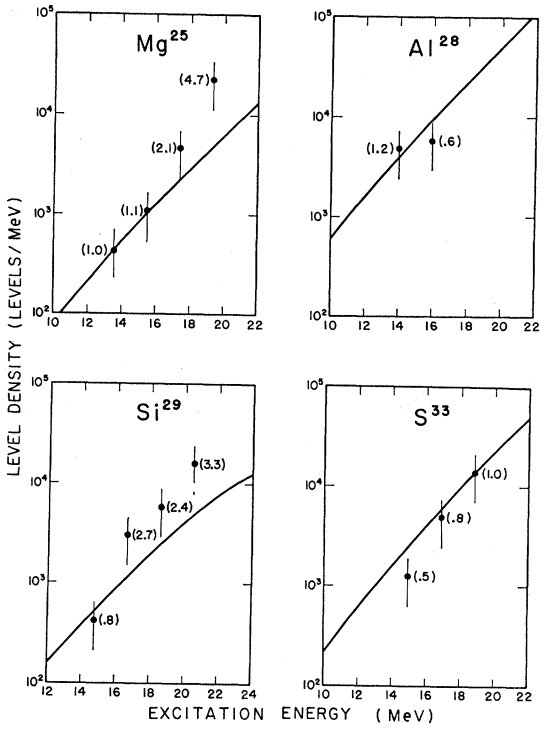

FIG. 12. Level densities of Mg^{25}, Al^{28}, Si^{29}, and S^{33}. The curves are calculated from the formula given by Gilbert and Cameron (Ref. 30). The points are determined from the fluctuation analysis of the present data. In parentheses near each point is the ratio of the level density determined from the analysis to the calculated one shown by the curve. The error bars show the uncertainty in the analysis which is estimated to be about 50%.

TABLE II. Statistical properties of energy levels.

Compound nucleus	Excitation energy (MeV)	ρ_{Ex}/ρ_G	ρ/ρ_G using Eq. (6)	Γ (keV)	$\Gamma/D_{J\pi}$	$F(0)/2\pi\lambda\!\!\!^{-2}\alpha^2$ (barns)
Mg^{25}	12.8	1.2		19	1	0.27
Mg^{25}	15.1	1.2		27	2	0.13
Mg^{25}	17.4	1.1	1.1	38	10	0.04
Mg^{25}	18.6	1.5	1.8	38	15	0.02
Al^{28}	13.1	0.4		16	4	0.07
Al^{28}	15.1	0.2	0.25	42	24	0.025
Si^{29}	14.0	0.3		24	1	0.71
Si^{29}	16.3	0.5		34	4	0.18
Si^{29}	18.7	0.7	1.2	41	11	0.06
Si^{29}	21.0	1.2	1.6	44	24	0.02
S^{33}	14.1	0.5		11	2	0.25
S^{33}	16.2	0.3	0.3	30	12	0.08
S^{33}	18.3	0.4	0.4	29	29	0.04

[40] T. Ericson, Ann. Phys. (N.Y.) **23**, 390 (1963).
[41] D. W. Lang, Nucl. Phys. **72**, 461 (1965) and private communication.
[42] M. Corti, M. G. Marcazzan, L. Milazzo Colli, and M. Milazzo, Energia Nucl. (Milan) **13**, 312 (1966).

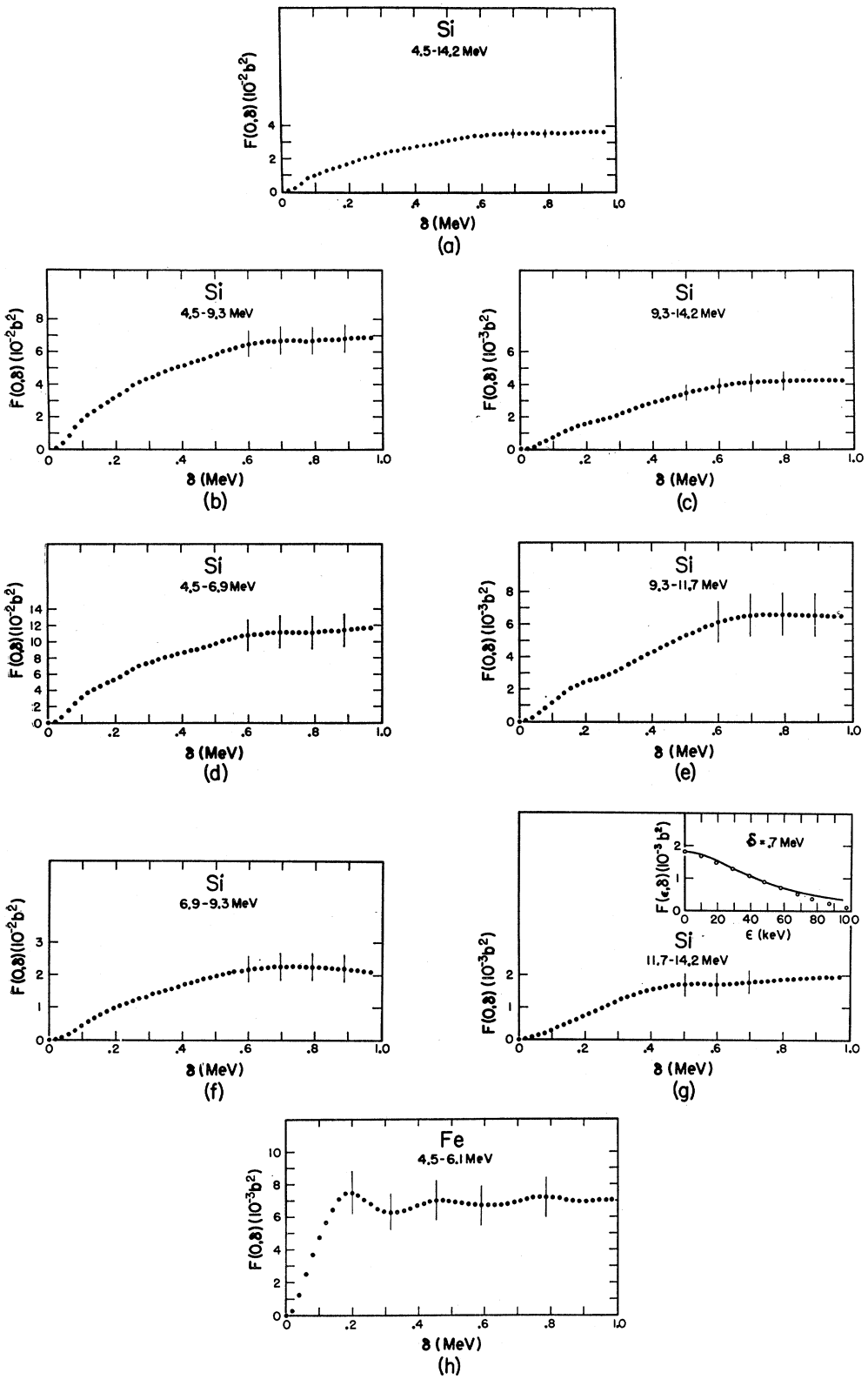

FIG. 13. Examples of the self-correlation function as a function of the averaging interval δ for several energy intervals for Si and for Fe. Error bars shown on some of the points are uncertainties caused by the finite range of data. In the insert in part (g) the correlation function is shown as a function of the correlation energy ϵ for a fixed averaging interval in the highest-energy quarter of the Si data.

of Ref. 40,

$$F(\epsilon)=\frac{\Gamma^2}{\epsilon^2+\Gamma^2}\frac{k_W}{\pi\Gamma}(\pi\lambda\!\!\!^{-2})^2\sum_{J\Pi}[g^2(J)]D_{J\Pi}\sum_{ls}(T_{ls}{}^J)^2. \quad (8)$$

Moldauer[32] has found that there should be an additional term in Eq. (8) if the target nucleus has spin, but the magnitude of this term is not known.

Equation (8) is similar to Eq. (3), and it is again convenient to express the level spacing in terms of an energy-dependent factor $\omega(E)$ and a spin-dependent factor $H(J\Pi)$ as explained in Sec. VI A. Equation (8) can then be written

$$F(0)=\frac{k_W}{\pi\Gamma}(\pi\lambda\!\!\!^{-2})^2\frac{1}{\omega(E)}\sum_{J\Pi}\frac{g^2(J)}{H(J\Pi)}\sum_{ls}(T_{ls}{}^J)^2, \quad (9)$$

where $\omega(E)$ is taken at the average excitation energy in the interval analyzed. In the analysis $\omega(E)$ is again treated as the unknown. Γ and $F(0)$ are determined from the experiment. The transmission coefficients are taken from optical-model calculations,[26] and k_W is assumed to be unity at the high energies at which Eq. (9) is applicable, as discussed in Sec. VI A. For $H(J\Pi)$ the values given in Ref. 30 were used.

It may be noted that Eqs. (5) and (9) are similar. This means that Ericson fluctuations produce very similar effects in the total cross sections as fluctuations in neutron widths. The assumption of equally spaced levels used in deriving Eq. (9) is equivalent to setting k_N equal to zero. Equation (9) applies only if the energy spread is much smaller than Γ. If the opposite is the case, Eq. (9) goes over into Eq. (5), as can be shown using the calculations of Gibbs.[43] Therefore the level densities obtained from Eq. (9) should be the same as those obtained from Eq. (5) for $k_N=0$ provided $\Gamma/D_J\gg1$.

The values of $F(0)$ and Γ which are needed in the analysis were obtained by the procedure discussed in Sec. VI B. These values were corrected for the effects of the experimental neutron-energy spread,[41] for the effect of counting statistics, and for the isotopic abundance. Using Eq. (9) values of $\omega(E)$ were calculated for Mg, Al, Si, and S for energy intervals of lengths one-quarter of the entire range of measurements. Only those results were used for which the correction for counting statistics was less than 25%. For the remaining elements for which the cross sections showed fluctuations, the corrections for counting statistics or energy spread were too large to give reliable results.

The results of the analysis are presented in Table II. In the third column the ratio of the level density obtained using Eq. (9) (denoted ρ_{Er}) to that given in Ref. 30 (denoted ρ_G) is shown. In the fourth column the corresponding ratios obtained using Eq. (5) with $k_N=0$ and $k_W=1$ are presented. The latter ratios are given only at excitation energies which are high enough that the assumption $k_W=1$ appears justified. It should be noted that these ratios are lower than those shown in Fig. 12, since in the analysis from which Fig. 12 was obtained the assumption $k_N=1$ was made. The ratios obtained by the two methods of analysis agree fairly well. This may be taken as evidence that the condition for the Ericson-type analysis is satisfied, i.e., $\Gamma/D_{J\Pi}\gg1$. Although it is possible to deduce from the experimental data values of ρ with an uncertainty of the order of 30%, the uncertainty in the assumptions regarding the value of k_N introduces a much greater uncertainty. Considering the uncertainty in both the theoretical level-density formula and in the experiment as well as in the analysis, the deviations of the ratios shown in Table II from 1 are not unreasonable.

The values of Γ shown in the fifth column were obtained from the correlation functions as explained at the end of Sec. VI B. It is estimated that the uncertainty in Γ is ±5 keV.

For Si^{29} widths Γ have previously also been obtained by several authors using fluctuation analyses of charged-particle reactions. The experiments which were performed with good energy resolution yielded results in excellent agreement with the present results; they are the studies of the $Al^{27}(d,\alpha)$ and the $Al^{27}(d,p)$ reactions by Gadioli *et al.*[44] and of the $Al^{27}(d,\alpha)$ reaction by Cassagnou *et al.*[45] On the other hand, the neutron total cross-section measurements of Roessle and Tauber[19] yielded values two to three times smaller than those obtained in the present study for Al^{28} and Si^{29}.

In the sixth column estimates of $\Gamma/D_{J\Pi}$ are given. These numbers are averages for the three lowest values of J and are obtained by dividing the widths shown in column five by the level spacings calculated from Ref. 30.

In the last column of Table II $F(0)/2\pi\lambda\!\!\!^{-2}\alpha^2$ is tabulated, where $F(0)$ is the mean square fluctuation in the cross section and α the isotopic abundance of the target nuclide. As has been shown by Moldauer [Eq. (112) in Ref. 32], $F(0)/2\pi\lambda\!\!\!^{-2}\alpha^2$ is related to the average of the fluctuating part of the cross section for elastic scattering.

VII. DISCUSSION

Individual resonances occur in neutron total cross sections for all intermediate and heavy nuclides at sufficiently low energies. As the energy is increased, resonances begin to overlap and fluctuations in cross sections occur which are caused by the effect of several resonances. At higher energies yet, the level density is so large that the cross sections become smooth. The energy at which this occurs depends on the nuclide.

[43] W. R. Gibbs, Los Alamos Scientific Laboratory Report No. LA-3266, 1965 (unpublished); P. Fessenden, W. R. Gibbs, and R. B. Leachman, Phys. Rev. Letters **15**, 796 (1965).

[44] E. Gadioli, G. M. Marcazzan, and G. Pappalardo, Phys. Letters **11**, 130 (1964).

[45] Y. Cassagnou, I. Yori, C. Levi, T. Mayer-Kuckuk, M. Mermaz, and L. Papineau, Phys. Letters **6**, 209 (1963).

Experiments carried out at this laboratory many years ago[46] showed that the transition to a smooth cross section occurs at an energy which generally decreases with atomic weight except that the closed-shell nuclides behave like lighter nuclides, an effect which was particularly pronounced[47] for the isotopes of Pb. In Fe, for example, the early measurements which extended to 3 MeV exhibited fluctuations up to the highest energies; in Sr and Zr large fluctuations were found up to 1 MeV,[46,48] and in Ce up to 0.2 MeV.[46]

The present experiments give information about the fluctuations in total cross sections at higher energies. The experiments show that the energies at which fluctuations become unobservable with a statistical accuracy of 1% and an energy spread of 20 keV are 12 MeV or above for Mg, Al, Si, and S, between 6 and 7 MeV for Ti and Fe, and between 4 and 5 MeV for two isotopes of Pb and for Bi. For all the elements from Co to Ta which were investigated, the transition takes place below 4.5 MeV. It appears that the observed presence or absence of fluctuations, and the energy of the transition to a smooth dependence are consistent with the usually accepted level densities and the usually employed resonance-parameter distributions. No additional hypothesis about the presence of intermediate structure was needed.

The present results are in disagreement with some reports[2,4] on fluctuations in neutron total cross sections which have been interpreted as evidence for intermediate structure. There are additional observations[3,5] of fluctuations in neutron total cross sections at energies lower than those investigated in the present experiment. These fluctuations have been taken as proof of the presence of intermediate structure.[49] Fluctuations in the total cross sections of heavy nuclides up to 0.6 MeV have also been found recently by Smith[50] although these measurements do not appear to agree with those of Ref. 3. It is not clear what an analysis similar to that presented here would have yielded for the measurements at lower energies, but it is possible that the fluctuations could be due to fluctuations in neutron widths.

Actually there is some doubt whether intermediate resonances caused by doorway states should be observable in the energy dependence of total cross sections. It is not generally realized that resonances associated with the single-particle states predicted by the optical model are only in exceptional cases observable in the total cross section. The observed broad maxima in the total cross sections are in almost all cases nonresonant optical-interference phenomena. This has been emphasized by McVoy,[51] who has also pointed out that a doorway state will not be observable against a smooth background if $\Gamma_\uparrow/(\Gamma_\uparrow+\Gamma_\downarrow)\ll 1$, where $\Gamma_\uparrow$ is the width for the decay of the doorway state into the entrance channel and $\Gamma_\downarrow$ its width for forming more complicated configurations.

According to Feshbach[49] doorway states will be most readily observable if the number of open channels which are detected experimentally is small, so that the total neutron cross section would be least sensitive to the presence of doorway states. The best chance of detection in the total cross section would be for light nuclei at low energies, and this is the condition of the experiment of Ref. 5, while the opposite is the case for most of the present measurements. No published neutron experiment gives, however, as convincing evidence for intermediate structure as does the observation of charged-particle scattering near isobaric analog states.[52] In particular, neutron experiments have so far not shown the expected spreading of a doorway state over many compound-nuclear states to form a "microgiant" resonance,[53] as would be expected for intermediate and heavy nuclei.

ACKNOWLEDGMENTS

We wish to thank Dr. P. Moldauer for several helpful discussions and Miss Cheryle Evans for her assistance in the evaluation of the data.

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[47] H. H. Barschall, C. K. Bockelman, R. E. Peterson, and R. K. Adair, Phys. Rev. **76**, 1146 (1949).
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