Nuclear Structure Studies of Sr^{88} and Y^{89} Using 19-MeV Proton Scattering and the $Sr^{88}(He^3,d)$ Reaction*

M. M. STAUTBERG,† J. J. KRAUSHAAR, AND B. W. RIDLEY‡
Department of Physics and Astrophysics, University of Colorado, Boulder, Colorado
(Received 19 December 1966)

Elastic and inelastic angular distributions were obtained for 19-MeV protons scattered from Sr^{88} and Y^{89}. Deuteron angular distributions were also obtained from the $Sr^{88}(He^3,d)$ reaction and used to study the single-particle nature of the excited states of Y^{89}. Spectroscopic information was obtained by comparing the inelastic proton distributions with predictions of the distorted-wave theory using a real collective-model form factor. Proton angular distributions of the low-lying states of Y^{89} and Sr^{88} were compared to determine if the states of Y^{89} could be explained as the weak coupling of the $2p_{1/2}$ proton to the core-excited states of Sr^{88}. The states at 1.49 and 1.74 MeV in Y^{89} do not appear to be described adequately as core-excited states, while the states at 2.22 (or 2.87) and 2.52 MeV do. The proton angular distributions for those states which could be described in terms of simple shell-model configurations were also analyzed with the distorted-wave theory using shell-model form factors.

I. INTRODUCTION

THE level structure of odd-even nuclei has often been interpreted in terms of the weak coupling of a nucleon or nucleon hole to the collectively excited states of the even-even core.[1–7] Shafroth *et al.*[8] have suggested that several of the excited states in Y^{89} might be due to the weak coupling of the $2p_{1/2}$ proton to the core-excited states of Sr^{88}. They suggested that the $\frac{3}{2}^-$ state at 1.49 MeV and the $\frac{5}{2}^-$ state at 1.74 MeV in Y^{89} would correspond to the excitation of the 2^+ state at 1.84 MeV in Sr^{88}; the states at 2.22 and 2.53 MeV in Y^{89}, to the excitation of the 3^- state at 2.74 MeV in Sr^{88}; and the 2.84 and 3.1 MeV states in Y^{89}, to the 3.22-MeV state in Sr^{88}. In the present work, inelastic proton scattering from Sr^{88} and Y^{89} was used to investigate this hypothesis. A similar investigation using α particles was concurrently undertaken by Alster *et al.*[7] and their results have recently been reported.

Previous investigations of the core-excitation model show that it can be used with varying degrees of success. Among the first of such studies were the investigations of of Au^{197} and Cu^{63} nuclei. The low-lying states of Au^{197} have been well described[1] as the weak coupling of a $2d_{3/2}$ proton to the 2^+ excited state of the Pt^{196} core. Perey *et al.*[2] have studied the $Cu^{63}(p,p')$ reaction. Their results could be interpreted as the weak coupling of the $p_{3/2}$ proton to the Ni^{62} core provided the state at 1.41 MeV was $\frac{3}{2}^-$. However, the results of the $Ni^{62}(He^3,d)$ reaction[9] showed that this state is either $\frac{5}{2}^-$ or $\frac{7}{2}^-$. Thankappan and True[10] have made theoretical calculations using a strong core-to-particle interaction and predicted the proper sequence of spins, if the state at 1.41 MeV is $\frac{5}{2}^-$.

The N^{15} and Al^{27} nuclei have also been the subjects of core-excitation studies. The N^{15} states were investigated[4] by means of α-particle scattering from N^{15} and O^{16} and were well described by the weak coupling of the $p_{1/2}$ proton hole to the octupole state in O^{16}. Inelastic α-particle scattering from Al^{27} and Si^{28} showed[5] that states in Al^{27} might also be explained on the basis of the core-excitation model. Recently, inelastic α-particle scattering[6] from Pb^{207}, Pb^{208}, and Pb^{209} has been found to be in very good agreement with predictions of the weak coupling of a $3p_{1/2}$ neutron hole to the Pb^{208} 3^- core state for Pb^{207}, and of a $1g_{9/2}$ proton to the Pb^{208} 3^- core state for Bi^{209}.

However, α-particle scattering from Sr^{88} and Y^{89} indicates that the excited states of Y^{89} are not well described in terms of the core-excitation model. Their results show that the alternative single-particle description of the 1.49- and 1.74-MeV states (the elevation of a $2p_{3/2}$ proton and a $1f_{5/2}$ proton, respectively, to the $2p_{1/2}$ orbit) is probably a better one than core excitation. The results from the $Zr^{90}(d,He^3)$ reaction[11] give further evidence for the single-particle interpretation for these states. Further discussion and comparison of proton scattering and α-particle scattering data will be presented later.

In addition to the investigation of the core-excitation model, the distorted-wave analysis of the proton angular distributions using a collective-model form factor has been helpful in adding to the existing level-structure information. In both Sr^{88} and Y^{89} only a few unambiguous

* Work supported in part by the U. S. Atomic Energy Commission.
† Present address: University of Kentucky, Lexington, Kentucky.
‡ Permanent address: Atomic Energy Research Establishment, Harwell, England.

[1] A. Braunstein and A. de-Shalit, Phys. Letters **1**, 264 (1962).
[2] F. Perey, R. J. Silva, and G. R. Satchler, Phys. Letters **4**, 25 (1963).
[3] J. Vervier, Nuovo Cimento **6**, 1412 (1963).
[4] A. Bussiere, N. K. Glendenning, B. G. Harvey, J. Mahoney, and J. R. Meriwether, Phys. Letters **16**, 296 (1956).
[5] J. Kokame, K. Fukunaga, and H. Nakamura, Phys. Letters **14**, 234 (1965).
[6] J. Alster, Phys. Rev. **141**, 1138 (1966).
[7] J. Alster, D. C. Shreve, and R. J. Peterson, Phys. Rev. **144**, 999 (1966).
[8] S. M. Shafroth, P. N. Trehan, and D. M. Van Patter, Phys. Rev. **129**, 704 (1963).
[9] A. G. Blair, Phys. Letters **9**, 37 (1964).
[10] V. K. Thankappan and William W. True, Phys. Rev. **137**, B793 (1965).
[11] D. C. Shreve, C. D. Kavaloski, J. S. Lilley, and Nelson Stein, Bull. Am. Phys. Soc. **11**, 118 (1966); J. S. Lilley (private communication).

spin and parity assignments have been made.[12,13] The distorted-wave analysis has been used primarily to analyze the inelastic scattering from even-even nuclei. However, since Y^{89} has a ground-state spin and parity of $\frac{1}{2}^-$, only one value of the angular-momentum transfer l can contribute, giving a final-state spin of $J=l\pm\frac{1}{2}$ and a parity change $(-)^l$, neglecting any contribution from spin flip.

Recently the inelastic proton angular distributions of Zr^{90}, Zr^{92}, and Zr^{94} have been successfully analyzed[14,15] using the distorted-wave theory with a shell-model form factor for the interaction. The initial and final nuclear states were described by single-particle wave functions and the interaction was a sum of two-body interactions between the incident proton and target nucleons. The Y^{89} state at 0.91 MeV is known to arise from the elevation of the $2p_{1/2}$ proton to the $1g_{9/2}$ orbit.[13] Therefore, it would be interesting to see if the shell-model analysis using the same interaction as in the zirconium isotopes will predict the correct magnitude and shape of the angular distribution for this state. If so, this would add confidence to the use of this same interaction in an investigation of other states using a single-particle description.

In order to investigate the single-particle nature of states of Y^{89}, the $Sr^{88}(He^3,d)$ reaction was also studied. Since the $1g_{9/2}$ orbit closes the shell at 50 protons, this experiment will be of particular interest in studying the higher-lying shell-model states of Y^{89}.

II. EXPERIMENTAL PROCEDURES

The proton and He^3 beams were obtained from the University of Colorado 52-in. fixed-field alternating-gradient cyclotron.[16] The beam-handling system and scattering chamber have been described previously[14] with the exception that the beam-handling system has been modified so that an intermediate image is formed in the vault in an effort to reduce the energy spread. The energy of the proton beam was determined by use of the cross-over method.[17]

A. The (p,p') Experiment

The Sr^{88} proton angular distributions were obtained with a self-supporting natural strontium foil 3.4 mg/cm^2 thick. Since strontium oxidizes very readily in air, the target was transferred to the scattering chamber in a vacuum cell.[18]

Natural strontium has an abundance of 82.56% of the Sr^{88} isotope. Therefore, strongly excited states from another isotope of strontium might be visible in the spectra. In order to identify states of Sr^{88}, $Sr^{88}O_2$ enriched to 99.84% was obtained from Oak Ridge National Laboratory. A target was made by imbedding the strontium oxide powder in a film of polystyrene.[19]

The thickness of the natural metal foil was determined by observing proton scattering from it and the strontium oxide target. Since polystyrene does not contain oxygen, the known cross sections for oxygen[20] were used to extract the cross section of strontium assuming the chosen composition SrO_2. The thickness of 3.4 mg/cm^2 was determined by comparison of cross sections using the metal foil and the strontium oxide target.

As a check on the thickness measurement the best-fit optical-model parameters (Potential 1 and 1′) determined from the 19.4-MeV proton scattering[15] on Zr^{92} and Zr^{94} were used in the optical-model search program of Perey.[21] The unnormalized strontium elastic cross sections were fitted by varying the normalization only. The results of this method gave a target thickness of 3.3 and 3.4 mg/cm^2. The Y^{89} target was a self-supporting foil having a thickness of 0.62 mg/cm^2 and isotopically pure.

A particle-identification system was not used and the setup of the detection system was similar to that described elsewhere.[14] In the proton energy spectrum of interest there were no deuterons. Any α-particle groups could have been readily discernible by their kinematic shift, but none were seen.

For the scattering from Sr^{88} a lithium-drifted silicon solid-state detector approximately 2500-μ thick was placed 8.86 in. from the target with a collimator $\frac{3}{16}$ in. in diam placed directly in front of the detector. The detector was cooled to the temperature of dry ice. Measurements were made from 20–160° in 5° steps with an incident proton energy of 19.5 MeV. The energy resolution was 120 keV. An energy spectrum at 65.3° is shown in the upper part of Fig. 1. In the lower portion a spectrum from $Sr^{88}O_2$ is shown. A peak at 2.50 MeV is just discernible. The angular distributions of the 2.74- and 2.50-MeV states showed that both had the same characteristic shape. The large amount of background in the spectrum along with the statistical fluctuations prevented a clear definition of the number of counts in the peak. The ratio of the number of counts in the 2.50-MeV peak to that in the 2.74-MeV peak was very much less than the corresponding ratio for the natural strontium target. Therefore, the 2.50-MeV state is iden-

[12] *Nuclear Data Sheets*, complied by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C.), NRC 60-3-67.

[13] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences—National Research Council, Washington 25, D. C., 1960), NRC 60-3-78.

[14] W. S. Gray, R. A. Kenefick, J. J. Kraushaar, and G. R. Satchler, Phys. Rev. **142**, 735 (1966).

[15] M. M. Stautberg and J. J. Kraushaar, Phys. Rev. **151**, 969 (1966).

[16] Rodman Smythe, Nucl. Instr. Methods **18**, **19**, 582 (1962); D. A. Lind, J. J. Kraushaar, Rodman Smythe, and M. E. Rickey, *ibid.* **18**, **19**, 62 (1962); D. A. Lind, M. E. Rickey, and B. M. Bardin, *ibid.* **18**, **19**, 129 (1962).

[17] B. M. Bardin and M. E. Rickey, Rev. Sci. Instr. **35**, 902 (1964); Rodman Smythe, *ibid.* **35**, 1197 (1964).

[18] M. M. Stautberg, thesis, University of Colorado, 1965 (unpublished).

[19] N. S. Wall and J. W. Irvine, Jr., Rev. Sci. Instr. **24**, 1146 (1953).

[20] W. W. Daehnick, Phys. Rev. **135**, B1168 (1964).

[21] F. G. Perey, Phys. Rev. **131**, 745 (1963).

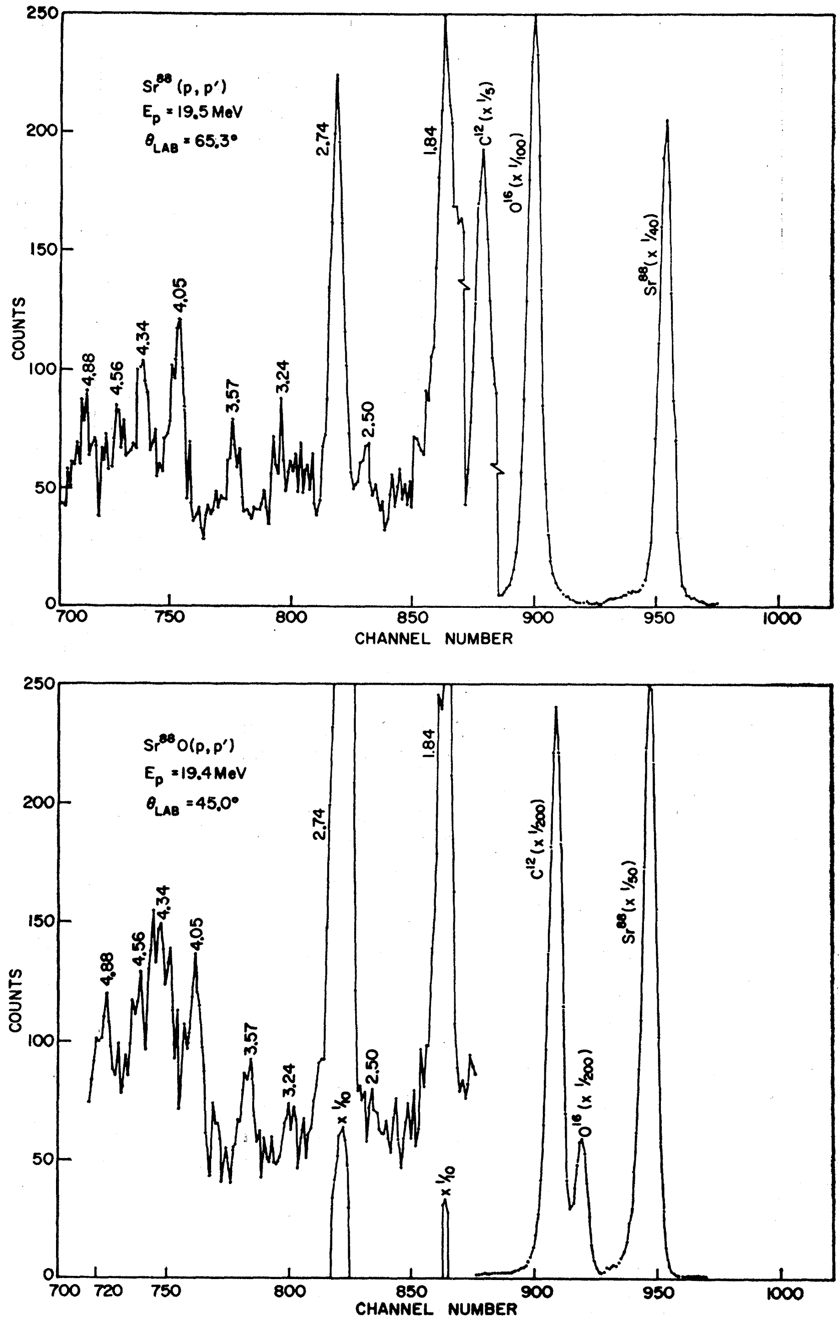


FIG. 1. Pulse-height spectra for 19.5-MeV protons incident on a natural strontium metallic foil and 19.4-MeV protons incident on $Sr^{88}O_2$.

tified with another strontium isotope, probably 86 or 87.

The 1.84-, 2.74-, and 3.24-MeV states of Sr^{88} were used as a calibration to obtain the Q values for the higher excited states. Energy losses were considered in the calculation of the Q values.

For the scattering from Y^{89} a 3-mm thick lithium-drifted silicon solid-state detector, which was cooled to the temperature of dry ice, was placed 8.35 in. from the target. The solid angle was determined by a collimator 0.189 in. in diam placed directly in front of the detector. Measurements were made from 15–110° in 5° intervals and from 110–140° in 10° intervals with an incident proton energy of 18.9 MeV. The over-all energy resolution was 80 keV. A pulse-height spectrum at an angle of 40° is shown in Fig. 2.

Proton scattering from the well-known states of Fe^{54} was used as a calibration to obtain Q values. The uncertainty in the Q values is ± 10 keV due to the uncertainty in locating the peak position within a half channel.

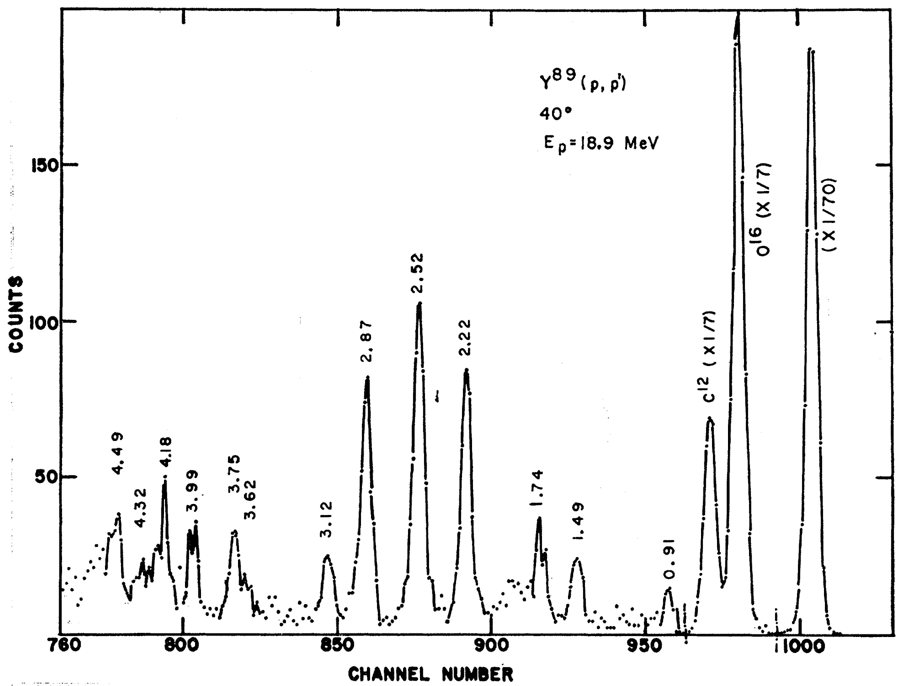


FIG. 2. Pulse-height spectrum for 18.9-MeV protons incident on Y^{89}.

Energy losses were not considered since this was a small effect compared to the error involved in locating the peak position.

An error of $\pm 5\%$ is associated with the absolute normalization of the cross sections. This is due to error in the target thickness, solid angle measurement, and beam current integrator error.

Error bars are not shown for the elastic distributions. The maximum statistical uncertainty for Sr^{88} was 1.5% and for Y^{89} was 3%.

B. The $Sr^{88}(He^3,d)$ Experiment

The target was obtained by evaporating natural strontium metal onto a 60 μg/cm^2 carbon foil, which was mounted in the vacuum cell.[18] Particles were identified and their energies measured with a two-detector system similar to that described elsewhere.[22] The front detector was a surface-barrier silicon detector approximately 1000-μ thick and the rear detector was a Li-drifted silicon detector 4700-μ thick. A solid angle of 4.6×10^{-4} sr was subtended. The incident He^3 energy was approximately 37.7 MeV.[22]

Spectra were obtained with particle discrimination and without particle discrimination. This permitted obtaining the He^3 elastic relative cross sections. At this laboratory the absolute cross sections of He^3 scattered from Y^{89} at 43.7 MeV have been obtained and analyzed to obtain optical-model parameters.[22] Since the optical-model parameters do not vary greatly with energy, these parameters were used to obtain the theoretical elastic He^3 cross sections at 37.7 MeV. The normalization of the observed-to-calculated cross sections indicated the target thickness to be 0.15 mg/cm^2.

A deuteron spectrum is shown in Fig. 3. The Q values were obtained using the Y^{89} state at 0.91 MeV as a calibration. The error in the absolute normalization of the deuteron cross sections was estimated to be 30%.

III. OPTICAL-MODEL ANALYSIS

The elastic scattering distributions are described in terms of an optical-model potential of the form

$$U(r)=-V(e^x+1)^{-1}+4iW_D(d/dx')(e^{x'}+1) -iW(e^{x'}+1)^{-1}+(h/m_\pi c)^2V_s\mathbf{L}\cdot\boldsymbol{\sigma}r^{-1} \times(d/dr)(e^x+1)^{-1}+V_c(r_c),$$

where

$$x=(r-r_0A^{1/3})/a,\quad x'=(r-r_0'A^{1/3})/a',$$

and V_c is the Coulomb potential of a uniformly charged sphere of radius $r_cA^{1/3}$ F.

A. The (p,p') Experiment

The optical-model analyses of Sr^{88} and Y^{89} were carried out using the automatic search routine of Perey[21] and the results are shown in Table I. There were 29 data points for Sr^{88} and 23 for Y^{89}. For the search, an error of 5% was associated with the experimental points. For the first search, the standard geometrical parameters of Perey[21] were used and only V and W_D were varied. The resulting χ^2 was quite large for both nuclei. In the second search, the imaginary diffuseness and the spin-orbit potential were allowed to vary in addtion to the real and imaginary potential, and χ^2 was greatly reduced. The results of the second search were

[22] E. F. Gibson, B. W. Ridley, J. J. Kraushaar, M. E. Rickey, and R. H. Bassel, Phys. Rev. 155, 1194 (1967).

TABLE I. Optical-model parameters. The italicized parameters were left fixed during the search. σ_R is the reaction cross section.

	Pot.	Energy (MeV)	V (MeV)	r_0 (F)	a (F)	W_D (MeV)	W (MeV)	r_0' (F)	a' (F)	V_s (MeV)	r_c (F)	σ_R (mb)	χ^2
$Sr^{88}+p$	1	19.5	49.3	*1.25*	*0.65*	*14.7*	0.0	*1.25*	*0.47*	*7.5*	*1.25*	1183	318.
	2		46.4	*1.25*	*0.65*	10.5	0.0	*1.25*	0.65	4.2	*1.25*	1318	32.7
	3		56.1	1.15	*0.65*	8.5	0.0	*1.25*	0.78	5.5	*1.25*	1369	4.6
	4		47.1	*1.25*	0.70	16.9	0.0	*1.25*	0.52	0.35	*1.25*	1306	26.4
	5		57.1	1.14	0.71	9.4	0.0	*1.25*	0.74	6.0	*1.25*	1385	3.2
	6		51.0	*1.20*	*0.70*	10.9	0.0	*1.25*	*0.65*	4.7	*1.25*	1329	13.1
$Y^{89}+p$	1′	18.9	50.5	*1.25*	*0.65*	*13.7*	0.0	*1.25*	*0.47*	*7.5*	*1.25*	1151	108.
	2′		49.6	*1.25*	*0.65*	13.2	0.0	*1.25*	0.53	3.4	*1.25*	1214	18.6
	3′		47.0	1.29	*0.65*	16.9	0.0	*1.25*	0.45	3.9	*1.25*	1212	12.1
	4′		49.3	*1.25*	0.71	12.8	0.0	*1.25*	0.52	5.2	*1.25*	1256	9.9
	5′		54.0	1.17	0.79	8.7	0.0	*1.25*	0.71	6.4	*1.25*	1374	6.2
	6′		52.6	*1.20*	*0.70*	9.8	0.0	*1.25*	*0.65*	5.7	*1.25*	1285	12.5
$Sr^{88}+He^3$		37.7	180.5	1.13	0.74	0.0	15.8	1.58	0.79	0.0	1.4		
$Y^{89}+d$		40.	117.6	1.0	0.87	13.9	0.0	1.27	0.82	0.0	1.4		

then used as intial parameters for additional searches in which an increasing number of parameters were allowed to vary.

Similar analyses of elastic proton scattering data[14,15] from Zr^{90}, Zr^{92}, and Zr^{94} at nearly the same incident energy show that the distributions can be reproduced quite well with an average potential having $r_0=1.2$ F, $a=0.70$ F, $r_0'=1.25$ F, and $a'=0.65$ F and allowing the potential well depths to vary. This was also done in the sixth search for Sr^{88} and Y^{89}. The comparison with experimental data is shown in Fig. 4. The resulting parameters, potential 6 and 6′, were used in the distorted-wave analysis of the inelastic angular distributions. In a later section, the effect of using the various optical-model parameters given in Table I in the distorted-wave theory will be discussed.

B. The $Sr^{88}(He^3,d)$ Experiments

The elastic scattering distributions of He^3 particles and deuterons were not obtained in this experiment. The He^3 elastic scattering parameters were obtained from the analysis of the elastic scattering of 43.7-MeV He^3 particles from Y^{89}.[22] The deuteron parameters were obtained from the analysis of the deuteron elastic scattering study of Becker *et al.*[23]

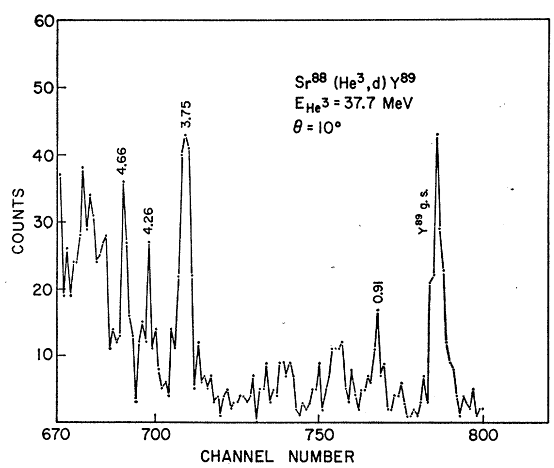


FIG. 3. Deuteron spectrum obtained from the $Sr^{88}(He^3,d)Y^{89}$ reaction.

IV. SPECTROSCOPIC INFORMATION

A. The (p,p') Experiment

The experimental inelastic angular distributions and comparison with the distorted-wave theory[24] using the real collective-model form factor are shown in Figs. 5 and 6. Optical-model potentials 6 and 6′ were used in the calculations and effects of Coulomb excitation were

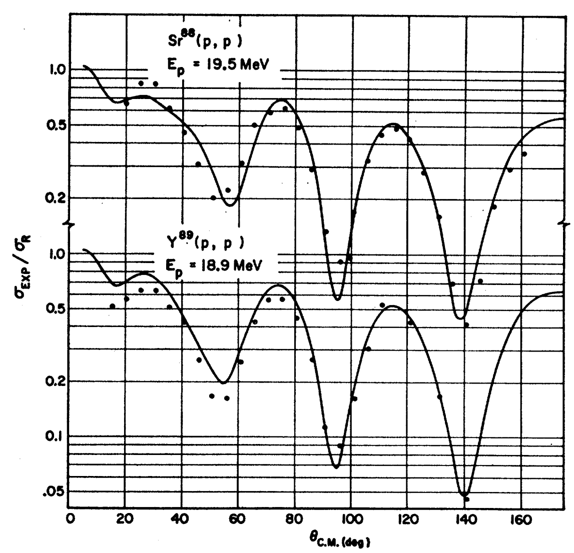
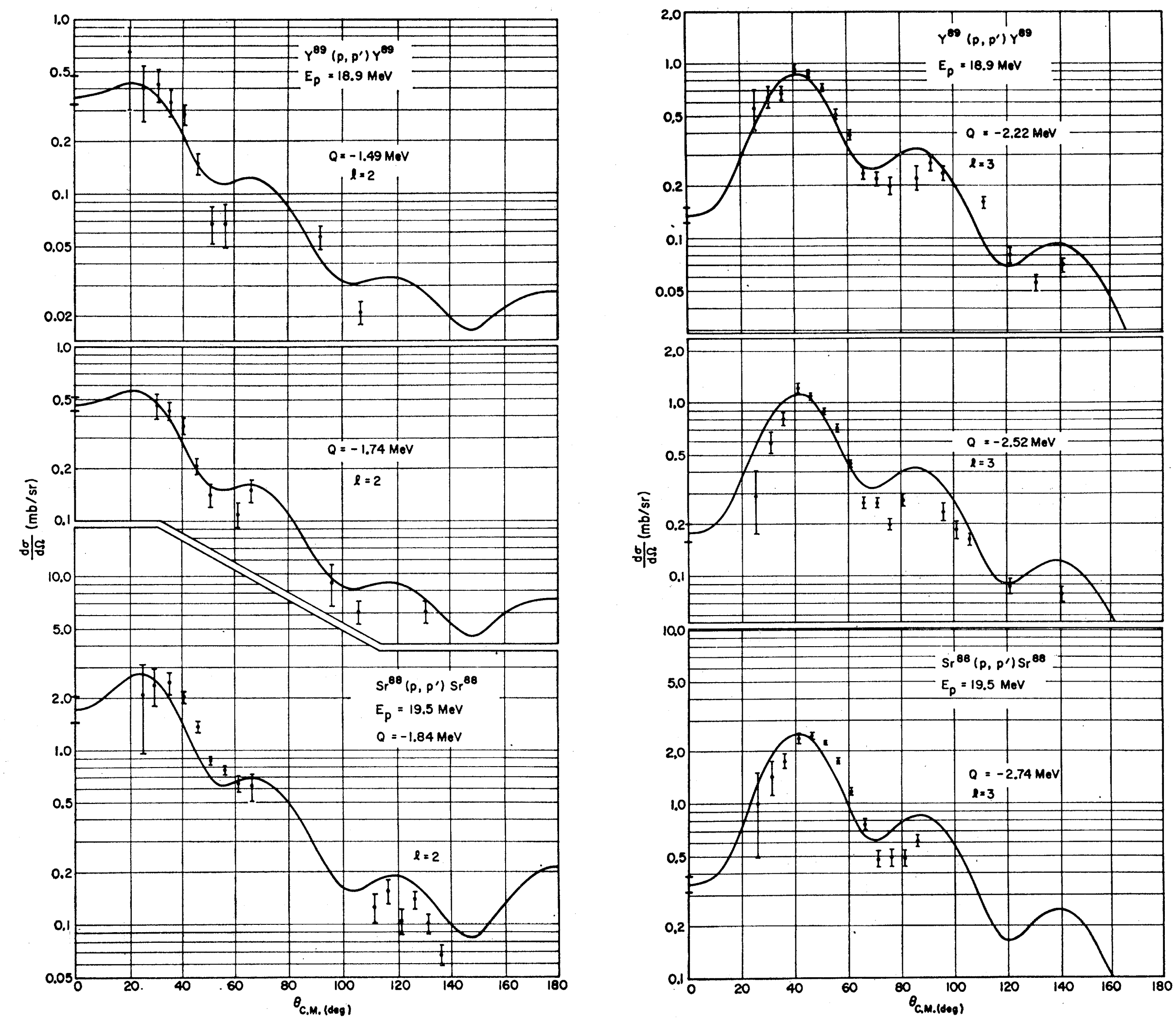


FIG 4. Comparison of the elastic angular distributions of Sr^{88} and Y^{89} with predictions of the optical model potentials 6 and 6′. σ_R is the cross section for Rutherford scattering.

[23] L. C. Becker, J. C. Hiebert, and E. Newman, Bull. Am. Phys. Soc. **11**, 44 (1966).

[24] R. H. Bassel, G. R. Satchler, R. M. Drisko, and E. Rost, Phys. Rev. **128**, 2693 (1962).



(a) (b)

FIG. 5. Comparison of inelastic angular distributions with predictions of the distorted-wave theory using a real collective-model form factor. The horizontal marks on the ordinate indicate the limits within which the theoretical curve can be shifted vertically and still give a reasonably good fit to the data as judged visually.

not included. The distorted-wave code JULIE[25] was used for the calculations.

In Fig. 5 the Sr^{88} and Y^{89} states relevant to the discussion of core excitation have been grouped together. This discussion will be given in the next section. In Fig. 6(a) the forward peaking of the angular distribution of the 4.05-MeV state of Sr^{88} indicates $l=2$, while the rest of the angular distribution is characteristic of $l=4$. In Fig. 6(b) the angular distribution for the Y^{89} state at 0.91 MeV could also be fit with an angular momentum transfer of $l=4$. Only the $l=5$ curve is shown since that state corresponds to the transition of the $2p_{1/2}$ to the $1g_{9/2}$ orbit, which requires a change in parity and thus $l=5$. The results of the $Sr^{88}(He^3,d)$ experiment also show that the 0.91-MeV state corresponds to the $2p_{1/2}$ to $1g_{9/2}$ transition.

In the study of 19.4-MeV proton scattering from Zr^{92} and Zr^{94} the distorted-wave theoretical curves resulting from the use of the real and complex collective-model form factors were compared.[18] It was found that both gave the same characteristic shape for a given angular momentum transfer. However, the values of the distortion parameter β obtained using the real form factor were about 10% larger for the 2^+ state and 30% larger for the 3^- state than when the complex form factor was used. In Table II the values of the distortion parameter β using the real collective-model form factor are tabulated for several states of Sr^{88} and Y^{89}.

The values of β can also be compared with the results of Coulomb-excitation experiments and lifetime meas-

[25] R. H. Bassel, G. R. Satchler, and R. M. Drisko, U. S. Atomic Energy Commission Report No. ORNL-3240, 1962 (unpublished).

TABLE II. Tabulation of β values.[a]

A	Q (MeV)	l	β	$B(El)/B_{sp}(El)$
89	1.49	2	0.072	
89	1.74	2	0.067	
88	1.84	2	0.13	9.6
89	2.22	3	0.16	
89	2.52	3	0.16	
88	2.74	3	0.20	24.2

[a] The listed values of β, as outlined in the text, were derived from distorted-wave calculations using only a real interaction. It is estimated that β_2 is 10% higher and β_3 30% higher than would be obtained using a complex interaction.

TABLE III. Comparison of $\sigma_{pp'}$ values using various optical-model potentials.

A	Potential	$\sigma_{pp'}$ (mb)
88	1	19.6
	2	15.9
	3	11.4
	5	12.7
	6	15.0
89	1′	23.0
	2′	17.8
	3′	22.6
	5′	26.4
	6′	19.6

urements. For the 2^+ state of Sr^{88} at 1.84 MeV the Coulomb-excitation experiment[26] gives a value of the reduced transition probability $B(E2)_{exc}$ equal to $(0.20 \pm 0.09)e^2 \times 10^{-48}$ cm^4. Mean-lifetime measurements yield[27,28] $(1.55 \pm 0.40) \times 10^{-13}$ sec and 1.4×10^{-13} sec. Our value of β (0.13) corresponds to a $B(E2)_{exc}$ of $0.11e^2 \times 10^{-48}$ cm^4 and a mean lifetime of 1.8×10^{-13} sec, in good agreement with the Coulomb excitation and lifetime experiments. A value of 9.6 (Table II) was obtained for $B(E2)/B_{sp}(E2)$, while Alster *et al.*[7] obtained a value of 5.0.

A mean-lifetime measurement, extracted from inelastic electron scattering data, is also available for the 3^- state of Sr^{88} at 2.74 MeV,[28] and has a value of 1.08×10^{-10} sec for the decay to the ground state. Our β value of 0.2 corresponds to a mean lifetime 1.34×10^{-10} sec. If the complex collective-model form factor were used, it would probably result in a smaller value of β for the 3^- state and, since the lifetime is inversely proportional to β^2, a larger value for the lifetime. The amount of error incurred in extracting the lifetime from the inelastic scattering measurements is not certain, so that one can only say that the results of both experiments are compatible.

Since the value of the distortion parameter β is compared with results of other types of experiment, it was thought interesting to study the dependence of β on the particular choice of the optical-model parameters. For this study the state of Sr^{88} at 1.84 MeV and the state of Y^{89} at 1.75 MeV were chosen. Since all of the $l=2$ theoretical curves are very similar in shape, the relative magnitudes of the predicted cross sections can be obtained by comparison of the total reaction cross section $\sigma_{pp'}$, using the various optical-model parameters. $\sigma_{pp'}$ is defined by

$$\sigma_{pp'} = 2\pi \int \sigma_l(\theta) \sin\theta d\theta ,$$

and $\sigma_l(\theta)$ is obtained from the distorted-wave calculations. This comparison is given in Table III. It is interesting to note that the magnitude of the theoretical cross section predicted using the Sr^{88} optical-model potential 6 is 75% of that predicted using the Y^{89} optical-model poetntial 6′. Therefore, it seems reasonable that one should associate an error of $\pm 25\%$ with β^2 because of optical-model uncertainties.

The energy-level diagrams and comparison with other experiments are shown in Figs. 7 and 8. The errors associated with the Sr^{88} Q values are 3.57 ± 0.01, 4.05 ± 0.02, 4.34 ± 0.03, 4.56 ± 0.03, and 4.88 ± 0.03 MeV. As stated previously, the error in the Y^{89} Q values is ± 10 keV.

The spins of the 2^+ state at 1.84 MeV and the 3^- state at 2.74 MeV of Sr^{88} have been determined from angular correlation studies.[29] Hamburger[30] has obtained Q values for states in Sr^{88} and Y^{89} by means of inelastic deuteron scattering, but has not made any spin and parity determinations.

This experiment verifies that the assignment of the state at 3.24 MeV of Sr^{88} is 2^+. A 2^+ assignment is also made for the 3.57-MeV state of Sr^{88} and the state at 4.05 MeV is either 2^+ or 4^+. States at 4.56 and 4.88 MeV were identified, but angular distributions were not obtained.

The state of Y^{89} at 0.915 MeV is known to be $\frac{9}{2}^+$.[13] From a study of the $Y^{89}(n,n'\gamma)$ reaction, Shafroth *et al.*[8] have determined that the state at 1.51 MeV is $\frac{3}{2}^-$ and the state at 1.75 MeV is $\frac{5}{2}^-$, and have given probable assignments of $\frac{5}{2}^+$ to the state at 2.22 MeV and either $\frac{7}{2}^+$ or $\frac{9}{2}^+$ to the state at 2.52 MeV. Tentative assignments were also made for three other states. The $Y^{89}(n,n'\gamma)$ reaction has also been studied by Buchanan *et al.*[31] and tentative assignments made. In their work a state at 3.49 MeV was found and given a tentative assignment of $\frac{5}{2}^-$. A state at this energy had been indicated from a study of inelastic neutron scattering.[32] From β-decay studies[33] of Zr^{89}, it was suggested that

[26] D. G. Alkhazov, D. S. Andreev, V. D. Vasilev, Yr. P. Gangrskii, I. Kh. Lemberg, and Yu. I. Udralov, Bull. Acad. Sci. USSR, Phys. Ser. **27**, 1263 (1963).

[27] S. Ofer and A. Schwarzschild, Phys. Rev. Letters **3**, 384 (1959).

[28] Richard H. Helm, Phys. Rev. **104**, 1466 (1956).

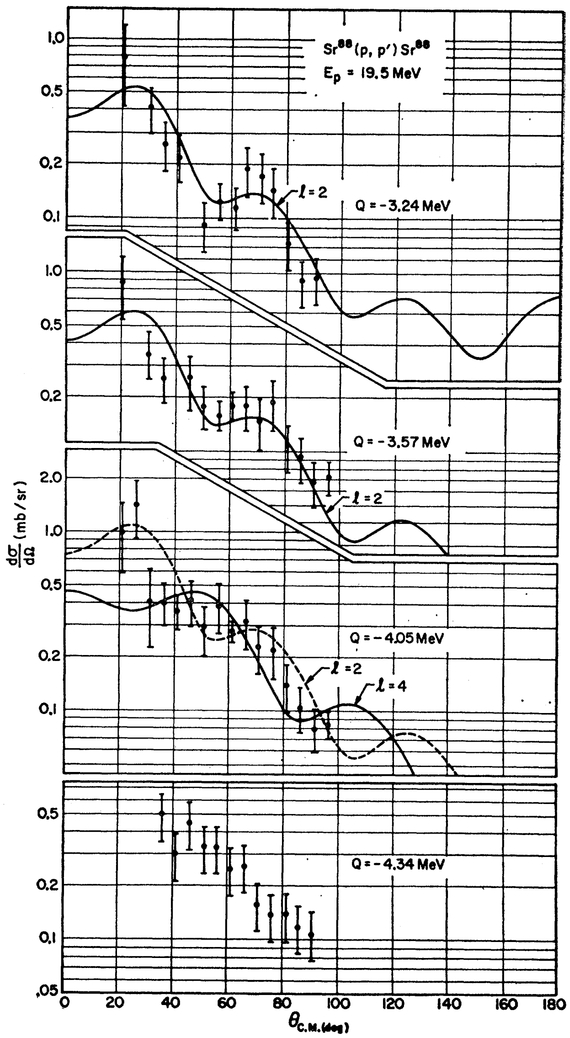
[29] S. Shastry and R. Bhattacharyya, Nucl. Phys. **55**, 397 (1964).

[30] E. W. Hamburger, Nucl. Phys. **39**, 139 (1962).

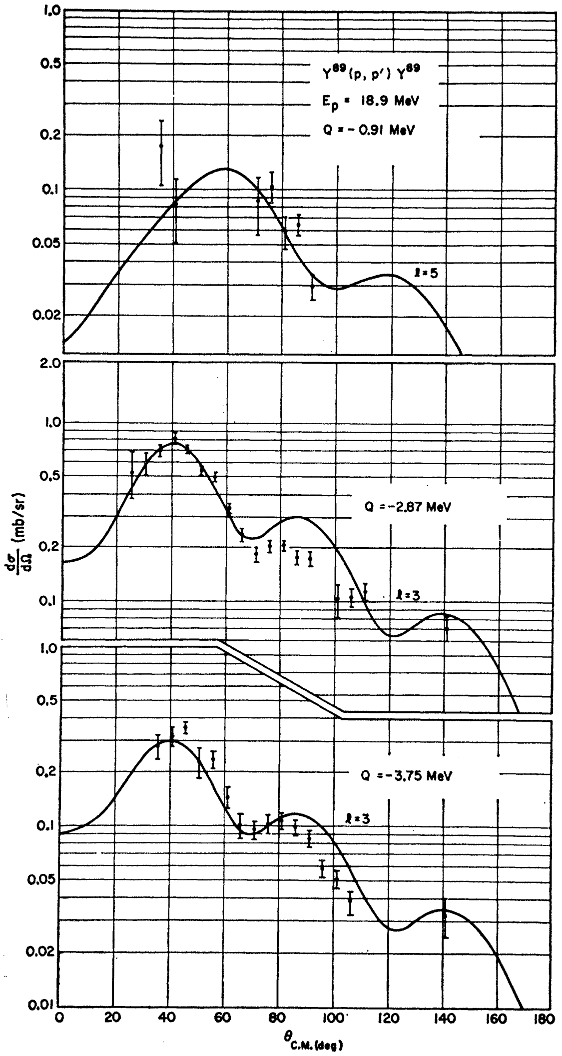
[31] Patricia S. Buchanan, Suresh C. Mathur, and W. E. Tucker, Bull. Am. Phys. Soc. **10**, 1117 (1965); P. S. Buchanan, S. C. Mathur, W. E. Tucker, I. L. Morgan, and E. L. Hudspeth, Phys. Rev. (to be published).

[32] I. L. Morgan and J. T. Prud'homme, Bull. Am. Phys. Soc. **4**, 103 (1959).

[33] D. M. VanPatter and S. M. Shafroth, Nucl. Phys. **50**, 113 (1964).



(a)



(b)

the state at 2.61 MeV is $\frac{9}{2}^+$. Alster *et al.*[7] have also identified states at 2.84 MeV with $l=3$, 3.1 MeV with either $l=2$, or 4, 3.70 MeV with $l=3$, 3.98 MeV with $l=2$, and 4.17 MeV with $l=2$.

A proton scattering experiment on Y^{89} at 14.7 MeV was performed by Awaya.[34] Angular distributions corresponding to thirteen excited states were obtained, but no angular momenta transfers were assigned. The Q values obtained are in very good agreement.

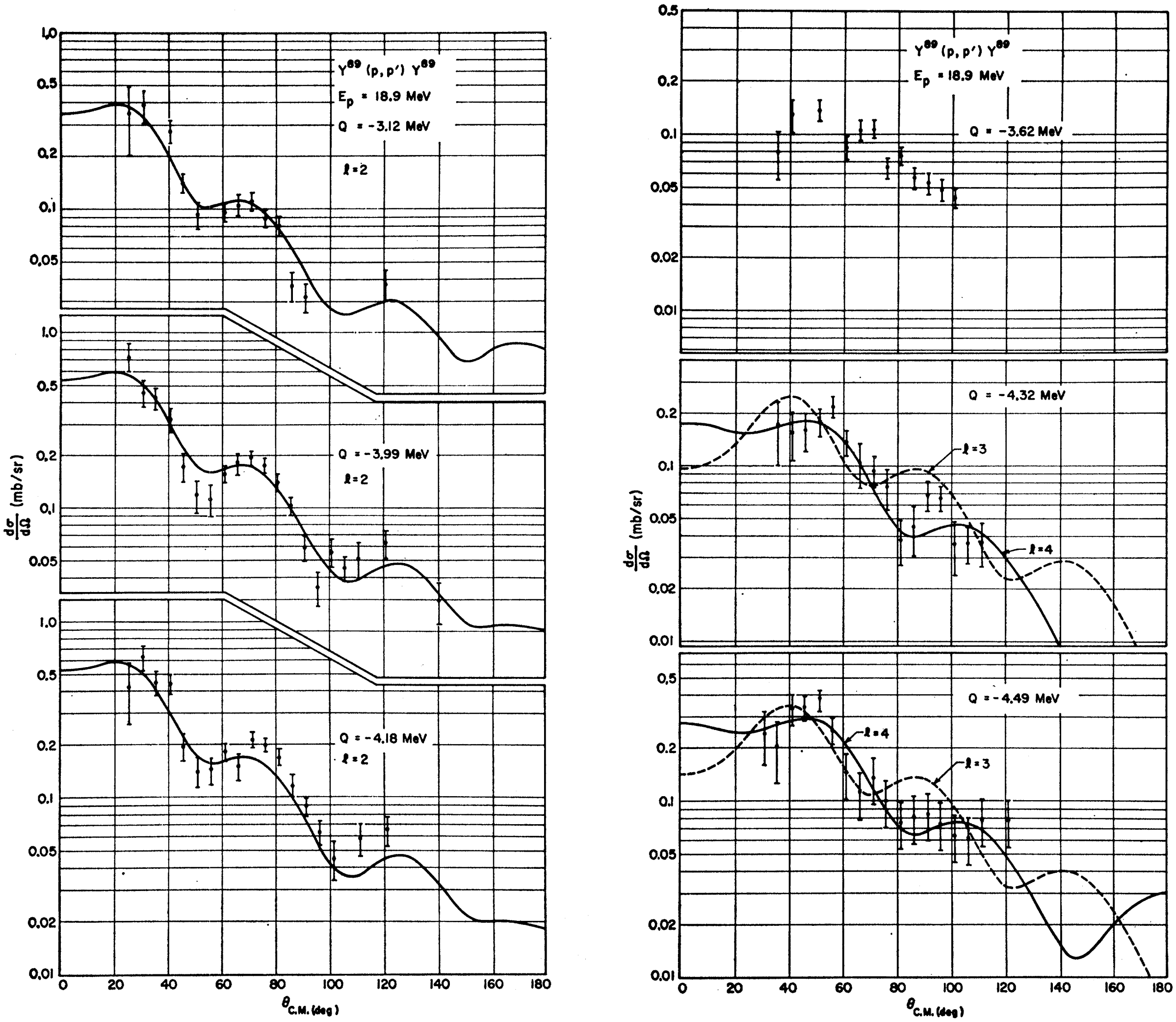
The results of proton scattering give either a $\frac{5}{2}^+$ or a $\frac{7}{2}^+$ assignment to the states at 2.22, 2.52, and 2.87 MeV. The results of Shafroth *et al.*[8] give an assignment of $\frac{7}{2}^+$ or $\frac{9}{2}^+$ to the state at 2.52 and a probable assignment of $\frac{5}{2}^+$ to the state at 2.22 MeV. Therefore the results of both experiments yield a $\frac{7}{2}^+$ assignment to the state at 2.52 MeV and a probable assignment of $\frac{5}{2}^+$ to the state at 2.22 MeV. Discussion of the $\frac{5}{2}^+$ assignment to the state at 3.75 MeV will be given in connection with the $Sr^{88}(He^3,d)$ experiment.

[34] Y. Awaya, Phys. Letters 21, 75 (1966).

B. The $Sr^{88}(He^3,d)$ Experiment

The experimental angular distributions are shown in in Fig. 9. The Q values were determined to be 3.76, 4.26, and 4.66 MeV with an error of 10 keV.

The theoretical curves were calculated using the distorted-wave code JULIE. The local single-particle wave functions for the stripped proton were computed by assuming that the proton moves in a Woods-Saxon well of diffuseness 0.7 F, radius of 1.2 F, and a spin-orbit coupling term which is 25 times the Thomas term for nucleons and also in the Coulomb potential of a uniformly charged sphere of radius 1.25 F. (Factors of $A^{1/3}$ in the description of the radius are to be understood.) The binding energies and potential well depths are given in Table IV.



(c) (d)

FIG. 6. Comparison of inelastic angular distributions with predictions of the distorted-wave theory using a real collective-model form factor.

The Y^{89} ground state corresponds to the stripping into a $2p_{1/2}$ orbit and the first excited state, into the $1g_{9/2}$ orbit. The $1g_{9/2}$ orbit closes a shell and, as expected, the stripping to the next orbit has a large excitation energy, nearly 3 MeV above the 0.91-MeV state. On the basis of the shell model the next orbit expected is the $2d_{5/2}$. In Fig. 9 the states at 3.75, 4.26, and 4.66 MeV are fitted with the distorted-wave curve corresponding to stripping into the $2d_{5/2}$ orbit. The theoretical curve for the $2d_{3/2}$ stripping would be very similar to the $2d_{5/2}$ and it is difficult to say whether or not the states at 4.26 and 4.66 MeV correspond to $2d_{5/2}$ or $2d_{3/2}$ stripping. The state at 3.75 MeV is expected to correspond to $2d_{5/2}$ stripping since that is the next orbit above the $1g_{9/2}$ orbit. The inelastic proton scattering results indicate either a $\frac{5}{2}^+$ or $\frac{7}{2}^+$ spin and parity assignment for this state. Therefore, from the results of the stripping experiment and the inelastic proton scattering, the state at 3.75 MeV is determined to be $\frac{5}{2}^+$. The state at 4.26 MeV does not correspond with any of the states found from proton scattering. The uncertainties in the Q values are quoted as ± 10 keV and this indicates that an additional state at 4.26 MeV has been located. This information is also shown in Fig. 8.

TABLE IV. Potential parameters for the Y^{89} single-particle wave functions.

Orbit	$2p_{1/2}$	$1g_{9/2}$	$2d_{5/2}$
Binding energy (MeV)	7.2	6.3	3.2
Well depth (MeV)	60.3	62.9	65.7

The relationship between the theoretical distorted-wave predictions and the experimental angular distri-

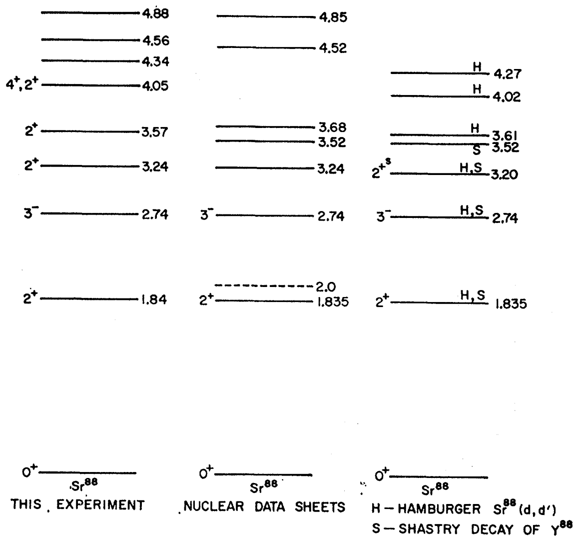


FIG. 7. Energy level diagram of Sr^{88} comparing present results with previous work.

butions is given[35] by

$$(d\sigma/d\Omega)=N[(2J_f+1)/(2J_i+1)]C^2\sum_l S_l\sigma_l(\theta)\,,$$

where J_i and J_f are the spins of the initial and final

TABLE V. Spectroscopic information from the $Sr^{88}(He^3,d)$ reaction.

Y^{89}	Configuration	$\frac{(2J_f+1)}{(2J_i+1)}C^2S$ Absolute	$\frac{(2J_f+1)}{(2J_i+1)}C^2S$ Relative	$\sum\frac{(2J_f+1)}{(2J_i+1)}C^2S$ Theoretical
g.s.	$2p_{1/2}$	3.23	2.0	2.0
0.91	$1g_{9/2}$	16.2	10.0	10.0
3.75	$2d_{5/2}$	1.79	1.10	5.58
4.26	$2d_{5/2}$ or $2d_{3/2}$	0.85	0.52	
4.66	$2d_{5/2}$ or $2d_{3/2}$	1.04	0.64	

states, respectively, l is the orbital-angular-momentum transfer, C is the isobaric-spin Clebsch-Gordan coupling coefficient, S_l is the spectroscopic factor which contains the information about the nuclear structure, and $\sigma_l(\theta)$ is the cross section obtained from the distorted-wave calculation. N is the normalization factor, which depends upon the deuteron and He^3 wave functions. If the Hulthén wave function is used for the deuteron and the Irving-Gunn wave function for the He^3, then the normalization factor N has the value of 4.4,[36] and this number was used in the calculations.

The isobaric spin formalism for stripping reactions has been developed by Macfarlane and French.[37,38] If a proton is transferred into orbit j of a target of nonzero isobaric spin $T_0(N-Z)/2$ and the corresponding j-

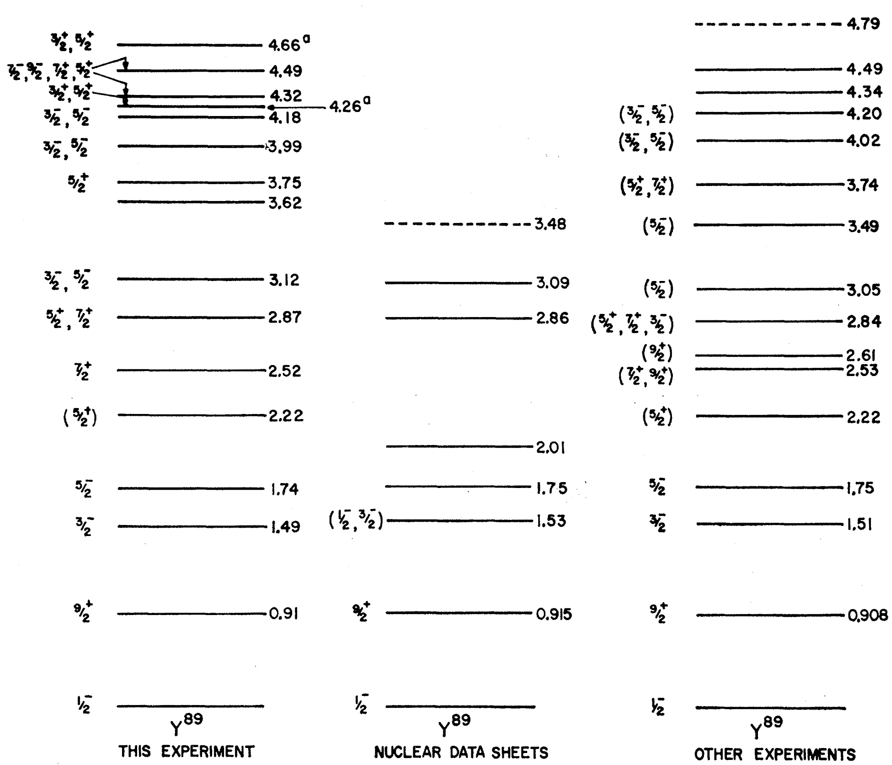


FIG. 8. Energy-level diagram of Y^{89} comparing the results of inelastic proton scattering and the $Sr^{88}(He^3,d)$ reaction with previous work. States indicated by *a* were not observed in proton scattering.

35 M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).
36 R. H. Bassel (to be published).
37 J. B. French, Nucl. Phys. **26**, 161 (1961).
38 J. B. French and M. H. Macfarlane, Nucl. Phys. **26**, 168 (1961).

TABLE VI. Comparison with predictions of the core-excitation model.

A	Q (MeV)	C.G. (MeV)	$d\sigma/d\Omega(40°)$ (mb/sr)	$\sum(d\sigma/d\Omega)$ (40°) (mb/sr)	R	R_{theoret}
89	1.49	1.64	0.25±0.05	0.55±0.06	0.83±0.19	4/6
89	1.74		0.30±0.03			
88	1.84		1.60±0.15			
90	2.18		1.0			
89	2.22	2.39	0.86±0.10	1.96±0.17	0.77±0.13	6/8
89	2.52		1.11±0.14			
88	2.74		2.55±0.25			
90	2.75		2.7			

neutron shell is not filled, then the excited states of the residual nucleus can have isobaric spin defined by $T=T_0+\frac{1}{2}$ and $\delta=T_0-\frac{1}{2}$. If the j-neutron shell is filled, only $T=T_0-\frac{1}{2}$ can occur. Summation over all final states of given isobaric spin excited by the transfer of a proton into orbit j gives

$$\sum\frac{(2J_f+1)}{(2J_i+1)}C^2S_j(T>)=\frac{(\text{neutron holes})j}{(N-Z+1)},$$

and

$$\sum\frac{(2J_f+1)}{(2J_i+1)}C^2S_j(T>)=(\text{proton holes})j-\frac{(\text{neutron holes})j}{(N-Z+1)}.$$

In Table V the quantity $[(2J_f+1)/(2J_i+1)]C^2S$ is tabulated and compared with the experimental data. The ratio of values for the $2p_{1/2}$ and $1g_{9/2}$ stripping is in very good agreement with the theoretical ratio, while the ratio for the $1g_{9/2}$ and $2d_{5/2}$ stripping is not.

V. COMPARISON WITH CORE-EXCITATION PREDICTIONS

The spin and parity assignments have been discussed in the preceding section. The assignments are such that the 1.49- and 1.74-MeV states might be described as the core excitation of the 2^+ state of Sr^{88} at 1.84 MeV. The states at 2.87 and 3.12 MeV have opposite parity and, therefore, cannot be the components of a core-excitation doublet. The state at 2.52 MeV has a $\frac{7}{2}^+$ assignment. States at 2.22 and 2.87 MeV have a possible $\frac{5}{2}^+$ assignment, so that either of these states and the state at 2.52 MeV could be the components of the core-excitation doublet corresponding to the excitation of the 2.74-MeV state of Sr^{88}.

The multiplet states in the even-odd nucleus will have angular momenta ranging from J_c+j to $|J_c-j|$ in integral steps, where J_c is the angular momentum of the excited state of the even-even core nucleus and j is the angular momentum of the single-particle orbital of the added nucleon. The center-of-gravity rule of Lawson

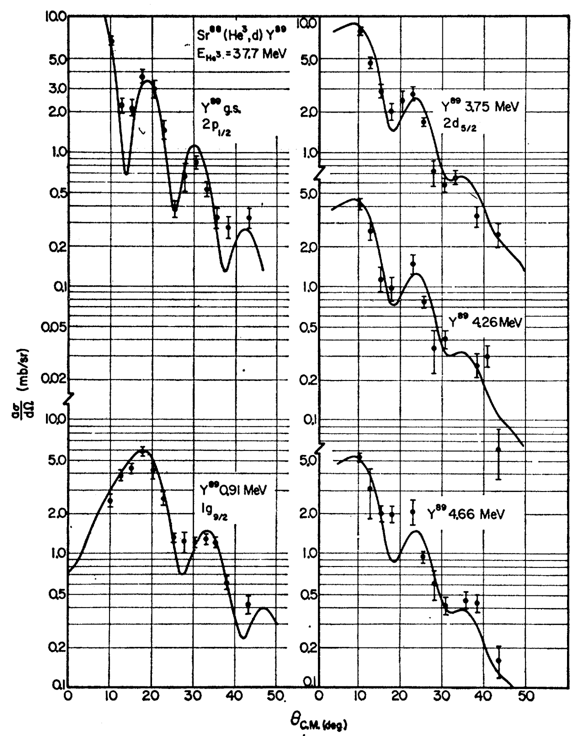


FIG. 9. Deuteron angular distributions of the $Sr^{88}(He^3,d)$ reaction and comparison with the distorted-wave theory of stripping reactions.

and Uretsky[39] states that the center of gravity of the multiplet should have the same energy as the core state, but, as discussed by de-Shalit,[40] this relationship is expected to hold only qualitatively.

The sum of the cross sections to multiplet states in the even-odd nucleus should be equal to the cross section to the parent state in the even-even nucleus. The magnitude of the cross sections to a given state in the multiplet should be proportional to its statistical weight, $(2J+1)$. Specifically, for Y^{89},

$$E_c=\frac{(2J_1+1)E_1+(2J_2+1)E_2}{(2J_c+1)(2)},$$

$$\frac{d\sigma}{d\Omega}(0\rightarrow E_c)=\sum_{i=1,2}\frac{d\sigma}{d\Omega}(0\rightarrow E_i),$$

and

$$R=\frac{(d\sigma/d\Omega)(0\rightarrow E_1)}{(d\sigma/d\Omega)(0\rightarrow E_2)}=\frac{(2J_1+1)}{(2J_2+1)},$$

where E_c and J_c are the energy and angular momentum, respectively, of the Sr^{88} core state. E_1, J_1 and E_2, J_2 are the energy and angular momentum, respectively, of the corresponding multiplet in Y^{89}.

[39] R. D. Lawson and J. L. Uretsky, Phys. Rev. **108**, 1300 (1957).
[40] A. de-Shalit, Phys. Rev. **122**, 1530 (1961).

TABLE VII. Potential parameters for calculation of single-particle wave functions.

Orbit	Binding energy (MeV)	Well depth (MeV)
protons		
$2p_{1/2}$	7.44	60.3
$1g_{9/2}$	4.49	60.3
$2p_{3/2}$	9.21	60.3
$1f_{5/2}$	8.96	60.3
$1f_{7/2}$	13.8	60.3
neutrons		
$1g_{9/2}$	11.4	54.4
$2d_{5/2}$	8.4	56.1
$3s_{1/2}$	5.8	54.4

From the angular distributions of the 1.49- and 1.74-MeV states of Y^{89} shown in Fig. 5, it is immediately apparent that the sum of the cross sections to the 1.49- and 1.74-MeV states is much smaller than to the 1.84-MeV state of Sr^{88}. As can be seen from the proton spectrum in Fig. 2 and the angular distributions in Figs. 5(b) and 6(b), the states at 2.22 and 2.87 are nearly equally strong. In the following analysis the state at 2.22 MeV will be used in the discussion of the core-excitation model and the state at 2.87 MeV in discussion of the shell-model interpretation of the excited states. From Fig. 5(b) it is seen that the sum of the cross sections to the 2.22- and 2.52-MeV states of Y^{89} is about the same magnitude as that to the 2.74-MeV state of Sr^{88}. It should be noted here that if the state at 2.87 MeV were used as one component of the core-excitation doublet, the center of gravity of the 2.52- and 2.87-MeV states would be 2.67 MeV, which is very near the excitation of the Sr^{88} state at 2.74 MeV.

A quantitative comparison of the angular distributions with predictions of the core-excitation model is given in Table VI. The cross-section values at 40° are those of the theoretical curves and the errors are due to shifting the curves within the limits, as described earlier. The sum of the cross sections to the 1.49- and 1.74-MeV states of Y^{89} is approximately one third that to the 1.84-MeV of Sr^{88}, and the lower limit of the ratio R is equal to the theoretical prediction of 4/6. The sum to the 2.22- and 2.52-MeV states of Y^{89} is very nearly equal that to the 2.74-MeV state of Sr^{88} and the ratio is equal to that predicted theoretically.

One might also investigate whether the states in Y^{89} can be described in terms of coupling a hole to the core-excited states of Zr^{90}. The information obtained from 18.8-MeV proton scattering from Zr^{90} has also been included in Table VI. The 2^+ state of Zr^{90} has an excitation of 2.18 MeV, which is considerably larger than the center-of-gravity energy of 1.64 MeV and the energy of the Sr^{88} 2^+ state at 1.84 MeV. However, the cross section of the 2^+ state of Zr^{90} is 1.0 mb/sr at 40° compared with 1.6 mb/sr for Sr^{88}. The 3^- state of Zr^{90} has an excitation energy of 2.75 MeV, which is almost exactly that of the Sr^{88} 3^- state. The cross section of the 3^- state of Zr^{90} is 2.7 mb/sr at 40° compared with the Y^{89} sum of 2 mb/sr.

Alster *et al.*,[7] who have used α-particle scattering from Sr^{88} and Y^{89} to test the validity of the core-excitation model, find that the sum of the cross sections to the 1.49- and 1.74-MeV states is 25% of that to the 1.84-MeV state, and that the sum to the 2.22- and 2.52-MeV states is only 60% of that to the 2.74-MeV state.

The core-excitation model has also been studied by Gibson *et al.*[41] by means of He^3 scattering from Y^{89} and Zr^{90}. The ratios of the cross sections of the 1.49- and 1.74-MeV states and the 2.22- and 2.52-MeV states are in very good agreement with the core-excitation model. However, when comparing the sum of the Y^{89} cross sections to the corresponding state in Zr^{90}, the Y^{89} sum for the $l=2$ and $l=3$ transfers is about 65% and 73%, respectively, of that of the Zr^{90} state.

Therefore, the results of all of the experiments show that the core-excitation model is not a good description for the 1.49- and 1.74-MeV states of Y^{89}. The results of the $Zr^{90}(d,He^3)Y^{89}$ reaction[11] indicate that the 1.49-MeV state has a large $p_{3/2}$ hole contribution and the 1.74-MeV state has some $f_{5/2}$ hole contribution. The results of proton scattering show that the 2.22- and 2.52-MeV states satisfy the criteria for core excitation very well, while this degree of success is not achieved for the α-particle scattering to these states.

The values of the distortion parameter β, given in Table II show that the single-particle enhancement is much larger for the Sr^{88} 3^- state than for the 2^+ state. Therefore, one might expect that the 2^+ state could be described from a microscopic point of view (the shell model), while the 3^- was described from a macroscopic point of view (the collective model); and that this would also be reflected when attempting to explain the Y^{89} states in terms of core excitation.

VI. SHELL-MODEL ANALYSIS

The 0.91-MeV state of Y^{89} is well described by the elevation of the $2p_{1/2}$ proton to the $1g_{9/2}$ orbit. As discussed in the preceding section, the best description of the 1.49- and 1.74-MeV states of Y^{89} is in terms of single-particle transitions. Therefore, the angular distributions of Y^{89} can profitably be studied using a shell-model form factor for the interaction.

The interaction which causes the transition is the sum of two-body interactions of the incident proton with the target nucleons. From previous studies[14,15] of proton scattering on Zr^{90}, Zr^{92}, and Zr^{94}, it was found that the Yukawa potential with a range of 1 F and a depth of 205 MeV would fit both the shape and magnitude of the inelastic cross sections. This potential is also used here.

The single-particle wave functions were computed in a nonlocal Woods-Saxon potential well having a diffuseness of 0.7 F, a radius of 1.2 F, and a spin-orbit coupling

[41] E. F. Gibson, J. J. Kraushaar, B. W. Ridley, M. E. Rickey, and R. H. Bassel, Phys. Rev. **155**, 1208 (1967).

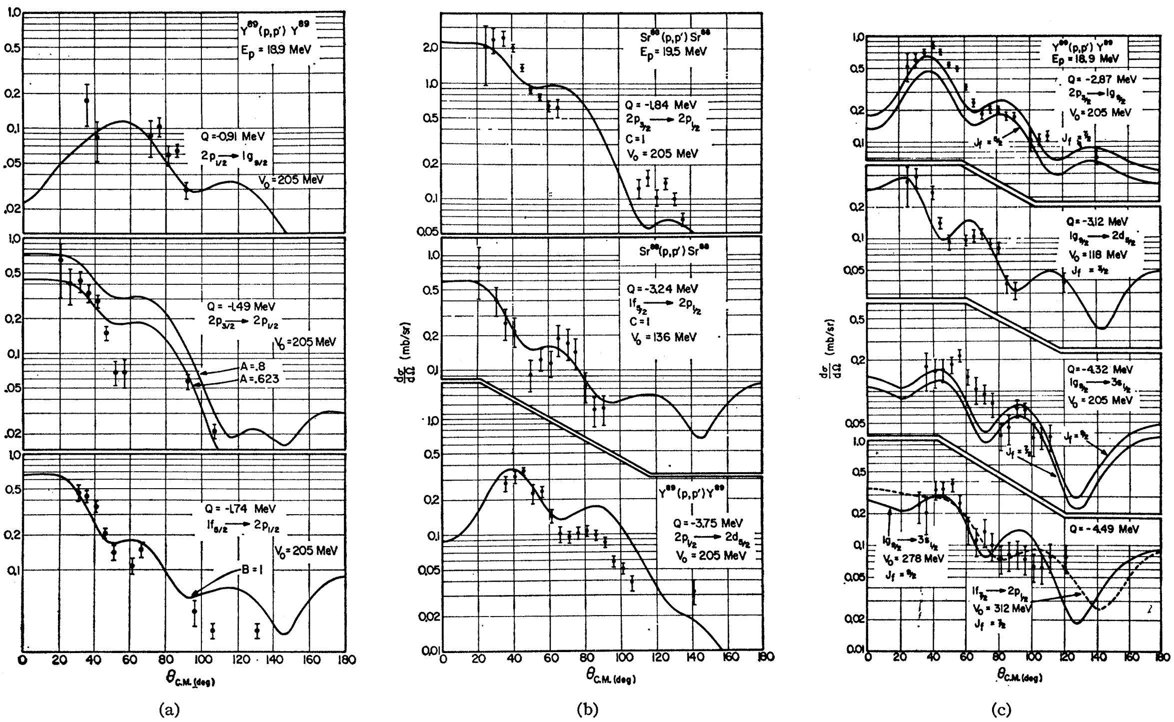


FIG. 10. Comparison of inelastic angular distributions with predictions of the distorted-wave theory using a shell-model form factor.

term which is 25 times the Thomas term for nucleons. The range of the nonlocality was 0.85 F. The effects of a uniform distribution of charge with radius 1.25 F were also included. The binding energies and potential well depths are given in Table VII. Note that the binding energies of the $2p_{3/2}$ and $1f_{5/2}$ orbits are very close, but that the $1f_{5/2}$ orbit lies above the $2p_{3/2}$ orbit for this potential. From the angular momentum of the states of Y^{89} at 1.49 and 1.74 MeV, one predicts the reverse ordering of the orbits.

In the calculation of the cross section the effect of using a nonlocal optical-model potential was also included. The range of nonlocality was 0.85 F.

The experimental cross section is related to the theoretical cross section $\sigma_l(\theta)$ by

$$(d\sigma/d\Omega) \propto \frac{(2J_f+1)}{(2J_i+1)}[V_0{}^2M_l{}^2\delta_{S0}+V_1{}^2N_{lJ}{}^2\delta_{S1}]\sigma_l(\theta).$$

The derivation of this equation has been discussed in the papers of Gray *et al.*[14] and Johnson *et al.*[42] The first term gives an angular momentum transfer l and parity change $(-)^l$, while the second term permits an additional transfer of $S=1$. J is the result of the vector addition of l and S. J_i is the spin of the target nucleus and J_f is the result of the vector addition of J_i and J. V_0 is the strength of the potential independent of spin-flip and V_1 is the strength of the spin-flip dependent potential.

For most of the transitions of interest the angular part of the matrix element M_l has been tabulated by Gray *et al.*[14] Several other matrix elements are listed in Table VIII. For the elevation of a proton from a filled shell to the partially filled $2p_{1/2}$ orbit, the matrix element was calculated from the relationship

$$M_l{}^2=(2/2j_h+1)|\langle 1\tfrac{1}{2}\|Y_l\|(lj)_h\rangle|^2,$$

and the relationships given by Johnson *et al.*[42], where j_h is the total angular momentum of the hole.

In Fig. 10(a) the angular distribution of the 0.91-MeV state is compared with shell-model predictions using a potential strength of $V_0=205$ MeV and neglecting any contribution from spin-flip. Thus, the strength of the potential found from studies of Zr^{90}, Zr^{92}, and Zr^{94} is also successful in predicting the magnitude of the angular distribution for a single-particle transition in Y^{89}. This fact can also be used to give some qualitative

TABLE VIII. Tabulation of the angular part of the matrix elements for the transition of protons from filled shells into the partially occupied $2p_{1/2}$ orbit and for the transition of the single $2p_{1/2}$ proton into the $2d_{5/2}$ orbit.

Transition	l	$M_l{}^2$
$2p_{3/2}\rightarrow 2p_{1/2}$	2	0.0796
$1f_{5/2}\rightarrow 2p_{1/2}$	2	0.0796
$1f_{7/2}\rightarrow 2p_{1/2}$	4	0.0796
$2p_{1/2}\rightarrow 2d_{5/2}$	3	0.0796

[42] M. B. Johnson, L. W. Owen, and G. R. Satchler, Phys. Rev. **142**, 749 (1966).

information about the strength of the spin-flip–dependent potential V_1. N_{55} can be calculated from the relationships derived by Johnson *et al.*[42] resulting in $N_{55}=1.095M_5$. Since the magnitude of the angular distribution is predicted quite well without inclusion of spin-flip, this indicates that $V_1 \ll V_0$. This state could also be reached with an angular momentum transfer of $l=3$ with spin-flip. If data were available at the forward angles, they might provide an upper limit on V_1.

Comparison of the 1.49- and 1.74-MeV angular distributions with shell-model predictions is also made in Fig. 10(a). For these calculations the following wave functions were assumed:

$$1.49\text{ MeV}|\tfrac{3}{2}^-\rangle = A|(p_{1/2})^2(p_{3/2})^3\rangle + (1-A^2)^{1/2}|(g_{9/2})^2(p_{3/2})^3\rangle,$$

$$1.74\text{ MeV}|\tfrac{5}{2}^-\rangle = B|(p_{1/2})^2(f_{5/2})^5\rangle + (1-B^2)^{1/2}|(g_{9/2})^2(f_{5/2})^5\rangle.$$

It was also assumed that the Y^{89} ground state corresponds to a $2p_{1/2}$ proton outside of closed shells. A value of A and B equal to 0.8 would correspond to the same $(p_{1/2})^2$ and $(g_{9/2})^2$ contribution that has been found in the Zr^{90} ground state. For the 1.49-MeV state, the predicted cross section for $A=1$ is much too large. If $A=0.8$, the agreement is better. However, a better fit corresponds to $A=0.62$. It is seen for the 1.74-MeV state that $B=1$ predicts the magnitude quite well.

In the analysis of the $Zr^{90}(d,He^3)$ reaction[11] the spectroscopic factor for the Y^{89} state at 1.51 MeV was 2.7 compared with the theoretical value of 4, and for the 1.75-MeV state, was 1.9 compared with the theoretical value of 6. The theoretical values were calculated assuming A and B to be 0.8.

It might also be interesting to investigate the 2^+ states of Sr^{88} using the shell-model analysis. Assume that the Sr^{88} wave functions have the following form:

$$\text{g.s.},\quad |0^+\rangle = |(p_{3/2})^4(f_{5/2})^6\rangle,$$

$$1.84\text{ MeV},\quad |2^+\rangle = C|p_{1/2}(p_{3/2})^3(f_{5/2})^6\rangle + (1-C^2)^{1/2}|p_{1/2}(p_{3/2})^4(f_{5/2})^5\rangle.$$

Assume also a wave function for the 3.24-MeV state which is orthogonal to the wave function of the 1.84-MeV state. The results for $C=1$ are shown in Fig. 10(b). $V_0=205$ MeV predicts the magnitude of the 1.84-MeV state fairly well, while for the 3.24-MeV state a depth of 136 MeV is needed.

An attempt was made to vary the value of C to see the effect of configuration mixing. However, it was found that contributions from configurations not considered in the preceding equations were important for the calculation of the form factor. Therefore, more realistic values of C could not be obtained from this analysis.

It was shown from the $Sr^{88}(He^3,d)$ reaction that the state at 3.75 MeV is partially due to the elevation of the $2p_{1/2}$ proton to the $2d_{5/2}$ orbit. It should be noted that neutrons can also make the same transition, but this state will be considered due only to proton excitation. The angular part of the matrix element for proton excitation is given in Table VIII. Comparison with the angular distribution is made in Fig. 10(b).

One would also expect states corresponding to proton excitation from the filled shells into the $1g_{9/2}$ orbit. In Fig. 10(c) the distorted-wave curve for the $2p_{3/2}$ to $1g_{9/2}$ transition is compared with the angular distribution of the state at 2.87 MeV. $J_f=\frac{7}{2}$ gives the best fit to the data. The distorted-wave curve for the $1f_{5/2}$ to $1g_{9/2}$ transition was also calculated, but V_0 would have to be increased by a factor of two in order to correspond with the data. It is interesting to note that the states at 2.22, 2.52, and 2.87 MeV are the most strongly excited states in the spectrum and that the strength of two of them can be accounted for by core excitation and the third by a $2p_{3/2}$ to $1g_{9/2}$ proton transition. Because of the closeness in energy of these states, the wave functions for all three states would undoubtedly be best represented by both a collective and a shell-model component.

Neutron excitation is also expected. The state at 3.12 MeV is a good candidate for the excitation of a $1g_{9/2}$ neutron to the $2d_{5/2}$ orbit, since the energy nearly corresponds to the separation of the $2d_{5/2}$ and $1g_{9/2}$ orbits found from the $Zr^{92}(p,d)$ reaction.[15] The distorted-wave curve for this transition is shown in Fig. 10(c) with $V_0=118$ MeV.

States at 4.32 and 4.49 MeV have either $l=3$ or $l=4$. An angular momentum transfer of $l=4$ can be obtained with the proton transition from the $1f_{7/2}$ orbit to the $2p_{1/2}$ orbit, and with the neutron transition from the $1g_{9/2}$ orbit to the $3s_{1/2}$ orbit. However, from the results of Table VII, one would expect an excitation energy of about 6 MeV for the transition of a $1f_{7/2}$ proton into the $2p_{1/2}$ orbit. In Fig. 10(c) the angular distribution of the 4.32-MeV state is compared with the $1g_{9/2}$ to $3s_{1/2}$ neutron transition, and the 4.49-MeV state with the $1f_{7/2}$ to $2p_{1/2}$ proton transition. The reason for this choice is that the magnitude of the $1g_{9/2}$ to $3s_{1/2}$ transition with $V_0=205$ MeV gives better agreement with the magnitude of the 4.32-MeV angular distribution.

ACKNOWLEDGMENTS

We wish to thank G. R. Satchler, P. D. Kunz, and E. Rost for helpful discussions.