where quantities with the subscript (0) are calculated for $(N-1)/2$ pairs and no unpaired particles in the major shell. The subscript γ refers to that orbital for which $e_{(0)\alpha}$ is smallest. Now, using values of Δ_0 and λ_0 formally calculated for $N/2$ "pairs," it is easy to show that

$$(\sum_\alpha 2\tilde{\epsilon}_{0\alpha}V_{0\alpha}{}^2-\Delta_0{}^2G^{-1})-(\sum_\alpha 2\tilde{\epsilon}_{(0)\alpha}V_{(0)\alpha}{}^2-\Delta_{(0)}{}^2G^{-1})\cong\lambda_{(0)}. \quad \text{(A8)}$$

Setting $E_0{}^0=\sum_\alpha 2\tilde{\epsilon}_{0\alpha}V_{0\alpha}{}^2-\Delta_0{}^2G^{-1}$, one obtains

$$E_\beta-E_0{}^0=\sum_\alpha{}^0 e_{0\alpha}-\tfrac{1}{4}G\Delta_0{}^2(\sum_\alpha{}^0 e_{0\alpha}{}^{-1})^2. \quad \text{(A9)}$$

Equation (A9) is the same formula as Eq. (A6), except that Δ_0, λ_0, etc. are numerically solved for, using N odd [e.g., in Eqs. (14) and (15)]. Referring back to Eq. (11), and making the identification $E_{\beta w}{}^0$ for $E_0{}^0$ when N is odd, and $E_{\beta w}{}^0$ for E_0 when N is even, one sees the convenience of the above-described formulas (A6), (A9) for rapid calculations.

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Isobaric-Spin Mixing in Be^8 States*

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The $Li^7(He^3,d)Be^8$ reaction has been studied using a magnetic spectrometer in order to obtain precise values for the widths of the isobaric-spin-mixed states at 16.6 and 16.9 MeV in Be^8. The results are $\Gamma(16.6)=113\pm3$ keV and $\Gamma(16.9)=77\pm3$ keV. These values specify the isobaric-spin amplitudes of the wave functions for these states and show that, while the mixing is not complete, it is quite substantial. This mixing gives rise to interference effects that are manifest in such reactions as $B^{10}(d,\alpha)Be^8$. It is pointed out that attempts to infer a cross-section ratio $\sigma(16.6)/\sigma(16.9)$ from such a reaction by taking the areas under the peaks in an experimental energy spectrum can be seriously in error if the interference effects are neglected. The excitation energies for the states were found to be 16.627 ± 0.005 MeV and 16.901 ± 0.005 MeV.

I. INTRODUCTION

The unusual properties of the $J^\pi=2^+$ Be^8 levels at 16.6 and 16.9 MeV have prompted several recent experiments and calculations designed to improve our understanding of these interesting states. The observation of the direct capture of protons by Li^7 to form the 16.6-MeV state, but not the 16.9-MeV level,[1,2] and the fact that the $Li^7(d,n)Be^8$ reaction strongly populates the lower state at threshold[3,4] and at higher bombarding energies,[5,6] whereas the upper state is only weakly excited, led to the proposal[2,7] that the 16.6-MeV state has primarily the configuration Li^7+p. The additional assumption[2] that the 16.9-MeV state has the configuration Be^7+n is supported by measurements of the yield ratios in the $Be^9(He^3,\alpha)Be^8$ and $Be^9(p,d)Be^8$ reactions.[8,9] Because Li^7 and Be^7 are well described by the configurations $\alpha+t$ and $\alpha+He^3$, respectively,[10,11] the configurations of the Be^8 states can be further expanded, with the results shown in Table I.

On the basis of Coulomb-energy calculations, one expects to find the first $T=1$ level of Be^8 at an excitation energy between 16.5 and 17 MeV. The apparently successful single-particle description of the only two levels in this energy range precludes the identification of either of these states as the expected $T=1$ level because a state with a wave function of the form $|Li^7+p\rangle$ or $|Be^7+n\rangle$ cannot be an eigenstate of total isobaric spin. One is therefore led, on the basis of the single-

Table I. Simple single-particle model of Be^8 states.

Energy (MeV)	J^π	Configuration	L-S partition
16.63	2^+	$Li^7+p=\alpha+t+p$	[431]
16.90	2^+	$Be^7+n=\alpha+He^3+n$	[431]

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particle description of these states, to the interesting conclusion that the $T=1$ strength is divided approximately equally between the states.[2] That is, each state consists of approximately equal amplitudes of $T=0$ and $T=1$:

$$|16.6\rangle=\alpha|T=0\rangle+\beta|T=1\rangle\,, \tag{1}$$

$$|16.9\rangle=\beta|T=0\rangle-\alpha|T=1\rangle\,, \tag{2}$$

where $|\alpha|\cong|\beta|$, and where $\alpha^2+\beta^2=1$. Expressing these wave functions as the two possible orthogonal combinations of two isobaric-spin basis states is justified by the fact that there is no other 2^+ level in the vicinity to perturb the wave functions.

Intermediate-coupling calculations have been performed by Barker[12] and these have produced results which are close to those of the simple single-particle model described above. Barker points out that the measured widths of the 16.6- and 16.9-MeV states can be used to obtain the magnitudes of α and β in the wave functions of Eqs. (1) and (2). This can be seen in the following way. The 16.6- and 16.9-MeV levels are bound with respect to all types of particle emission except α particles. Since the electromagnetic decay widths are a few eV at most, the natural widths of the levels are essentially the α-particle widths, $\Gamma=\Gamma_\alpha$. Now, the reduced width for α-particle emission for, say, the 16.6-MeV state is the probability for finding an α particle at the nuclear surface in the outgoing channel (only $l_\alpha=2$ is allowed) and is given by the square of the surface integral

$$\theta_\alpha^2(16.6)=N_c\left|\int_{R_c}\langle 16.6|\varphi_c\rangle d\sigma\right|^2, \tag{3}$$

where $|\varphi_c\rangle$ is the channel wave function and R_c is the channel radius; N_c is a normalizing factor which we do not need to evaluate for the present purpose. Similarly, the reduced width for the 16.9-MeV level is

$$\theta_\alpha^2(16.9)=N_{c'}\left|\int_{R_{c'}}\langle 16.9|\varphi_{c'}\rangle d\sigma\right|^2, \tag{4}$$

where c' now designates the new channel. Next, we substitute from Eqs. (1) and (2); because $|T=1\rangle$ is orthogonal to both $|\varphi_c\rangle$ and $|\varphi_{c'}\rangle$, which are $\alpha+\alpha$ and therefore pure $T=0$ channels,

$$\theta_\alpha^2(16.6)=\alpha^2N_c\left|\int_{R_c}\langle T=0|\varphi_c\rangle d\sigma\right|^2, \tag{5}$$

$$\theta_\alpha^2(16.9)=\beta^2N_{c'}\left|\int_{R_{c'}}\langle T=0|\varphi_{c'}\rangle d\sigma\right|^2. \tag{6}$$

Since the 16.6- and 16.9-MeV levels are so close in energy and so similar in properties, we argue that $|\varphi_c\rangle$ and $|\varphi_{c'}\rangle$ at R_c and $R_{c'}$, respectively, are essentially identical and that $N_c=N_{c'}$. Thus,

$$\theta_\alpha^2(16.6)/\theta_\alpha^2(16.9)=\alpha^2/\beta^2. \tag{7}$$

The reduced widths θ_α^2 are related to the α-particle widths Γ_α by penetration factors ($\theta_\alpha^2=\Gamma_\alpha/P$) and P varies by only about 1% between the two levels. Therefore, we have, finally,

$$\Gamma_\alpha(16.6)/\Gamma_\alpha(16.9)=\alpha^2/\beta^2. \tag{8}$$

This is a useful result because it allows the evaluation of the wave-function coefficients by the relatively simple measurement of the natural widths of the states. Furthermore, it is an immediate consequence of the description of the states by Eqs. (1) and (2) that the intensity ratios for the formation of the states by reactions which proceed via either $T=0$ only or $T=1$ only are given by

$$T=0 \text{ only:}\quad \text{intensity ratio } 16.6/16.9=\alpha^2/\beta^2 \tag{9}$$

$$T=1 \text{ only:}\quad \text{intensity ratio } 16.6/16.9=\beta^2/\alpha^2. \tag{10}$$

Examples of $T=0$ reactions are $B^{10}(d,\alpha)Be^8$ and $Li^6(Li^6,\alpha)Be^8$. Cases for $T=1$ are more difficult to imagine, but one such would be $C^{12}(d,Li^6)Be^8$, where the Li^6 ion is emitted in its 3.56-MeV, $T=1$ state. [The observation of Li^{6*} (3.56-MeV) ions from the $B^{11}(He^3,Li^6)Be^8$ reaction has recently been reported.[13]] Also, the dipole excitation of C^{12} by photons should predominantly produce $T=1$ states, so that the $C^{12}(\gamma,\alpha)Be^8$ reaction should be governed by Eq. (10).

According to the recent survey by Lauritsen and Ajzenberg-Selove,[14] the mean values for the widths were given as

$$\Gamma_\alpha(16.6)=97\pm11 \text{ keV}\,, \tag{11}$$

$$\Gamma_\alpha(16.9)=83\pm10 \text{ keV}\,, \tag{12}$$

so that

$$\alpha^2/\beta^2=1.17\pm0.19. \tag{13}$$

The most recent published measurements of $T=0$ reactions are those of Browne and Erskine[15] for the $B^{10}(d,\alpha)Be^8$ reaction at $E_d=7.5$ MeV and of Kibler[16] for the $Li^6(Li^6,\alpha)Be^8$ reaction at $E_{Li^6}=4.3$–5.5 MeV. In each case, intensity ratios were extracted from the energy spectra measured at various angles by integrating the yields under the peaks corresponding to the 16.6- and 16.9-MeV levels. The results, based on integrated angular distributions, were given in terms of "cross-section ratios":

$$B^{10}(d,\alpha)Be^8:\quad \sigma(16.6)/\sigma(16.9)=1.15\pm0.05\,, \tag{14}$$

$$Li^6(Li^6,\alpha)Be^8:\quad \sigma(16.6)/\sigma(16.9)=1.20\pm0.05. \tag{15}$$

[12] F. C. Barker, Nucl. Phys. **83**, 418 (1966).

[13] F. C. Young, P. D. Forsyth, and J. B. Marion, Nucl. Phys. **A91**, 209 (1967).

[14] T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).

[15] C. P. Browne and J. R. Erskine, Phys. Rev. **143**, 683 (1966); and (private communication).

[16] K. G. Kibler, Phys. Rev. **152**, 932 (1966).

These values are in good agreement with the intensity ratio predicted from α^2/β^2 based on the natural widths. Although qualitative agreement is to be expected, the detailed agreement is almost certainly fortuitous because the analyses of the experimental widths and cross-section ratios cited above were carried out without ragard for the interference effects that exist between the states[12] (see Sec. IV). Because of these interference effects, it is not meaningful to integrate the areas under peaks in an energy spectrum and to arrive at a cross-section ratio. Rather, one must analyze the complete energy spectrum taking into account the interference and extract an *intensity ratio.* The present experiment was undertaken to provide a precise set of widths for the 16.6- and 16.9-MeV states so that a rigorous test of the predicted intensity ratio could be made.

II. EXPERIMENTAL PROCEDURE

In order to make a precision measurement of the widths of the Be^8 states, it is necessary to choose a reaction in which the states in question are clearly defined relative to the background arising from three-body decay processes and from the tails of other levels. It is also necessary that the peaks be well removed in energy from contaminant peaks. A final requirement is that the 17.64-MeV state of Be^8 be reasonably prominent in the spectrum because observation of this narrow level ($\Gamma=11$ keV) provides a convenient means of assessing the energy resolution of the measuring apparatus. This latter requirement immediately rules out the use of any $T=0$ reactions, such as $B^{10}(d,\alpha)Be^8$ or $Li^6(Li^6,\alpha)Be^8$, since these reactions will not appreciably populate the dominantly $T=1$ 17.64-MeV state. It was felt that a time-of-flight measurement using the $Li^7(d,n)Be^8$ reaction would not provide the desired accuracy, and the $Li^6(He^3,p)Be^8$ reaction suffers from background problems. The $Li^7(He^3,d)Be^8$ reaction, on the other hand, was found by Cocke and Barnes[17] to yield clean peaks for all three states and, at certain angles of observation, to be free of contaminant peaks in the regions of interest. It was therefore decided to use the $Li^7(He^3,d)Be^8$ reaction for a precision measurement of the widths of the 16.6- and 16.9-MeV states.

The angular distribution of the $Li^7(He^3,d)$ reaction, as measured by Cocke and Barnes[17] at a bombarding energy of 11 MeV, shows a pronounced forward maximum for the 16.6-MeV level, as is expected for a proton transfer (stripping) reaction leading to a state characterized as Li^7+p. The group leading to the 16.9-MeV state, however, exhibits a more-or-less isotropic angular distribution (since stripping should contribute only a small amount for this final state which is mainly of the configuration Be^7+n). For observation angles greater than about 40°, the peak heights in the energy spectra for the two states are approximately the same. (The *yield,* however, favors the 16.6-MeV state because of its greater width.) Since an accurate comparison of the widths (and it is the width *ratio* that is more important here than the individual values) is easiest to make when the yields are approximately equal, the angle 40° was chosen for the measurement. Furthermore, at this angle, the peaks from the $O^{16}(He^3,d)F^{17}$ reaction (oxygen is the only serious contaminant) do not interfere with the analysis of the peaks due to the Be^8 states.

Targets were prepared *in situ* by evaporating lithium metal (enriched to 99.99% Li^7) onto thin (1000 Å) nickel foils. Target thickness was measured by observing elastically scattered He^3 ions, first from the nickel backing before evaporation and then after evaporation, using a 61-cm double-focusing magnetic spectrometer. The displacement of the scattering edge directly measures the target thickness. For the study of the $Li^7(He^3,d)Be^8$ reaction, bombardment was made *through* the backing foil, with the lithium target surface oriented normal to the direction of the outgoing particles (40°). Deuterons were analyzed by the magnetic spectrometer and detected with an array of 16 solid-state counters located on the focal plane. The pulse-height spectrum in each counter was recorded in a 1024-channel analyzer operating in the two-dimensional mode. In this way the pulses due to deuterons were clearly resolved from pulses due to other types of particles. In repeated runs over the Be^8 peaks, no systematic decrease in counting rate was observed, thus indicating the stability of the lithium metal targets.

The bombarding energy was 11.00 MeV. Later, it was determined from the momentum of the deuterons leaving Be^8 in the 17.638-MeV state that the beam energy at the midpoint of the lithium target was 10.972 MeV.

III. RESULTS AND DATA ANALYSIS

The momentum spectrum of the region of excitation energy including the 16.6-, 16.9-, and 17.64-MeV states, obtained by the procedures outlined above, was converted into an energy spectrum (counts per unit energy interval versus excitation energy) by using the appropriate kinematical expression. At most of the energy values two or three points were measured, and the averages are shown in Fig. 1.

The 17.64-MeV peak was analyzed first so that the effects of geometry and target conditions manifest in this peak could be assessed and then used in the analysis of the 16.6- and 16.9-MeV peaks. In addition to the natural width of the level, the experimental width of the 17.64-MeV peak includes the effects of target thickness, finite angular aperture of the magnetic spectrometer, resolution of the spectrometer, energy spread in the incident beam, and straggle in the energy of the beam due to passage through the nickel target backing. The target thickness was measured in the scattering experiment described above, and the spectrometer aperture

[17] C. L. Cocke and C. A. Barnes (private communication).

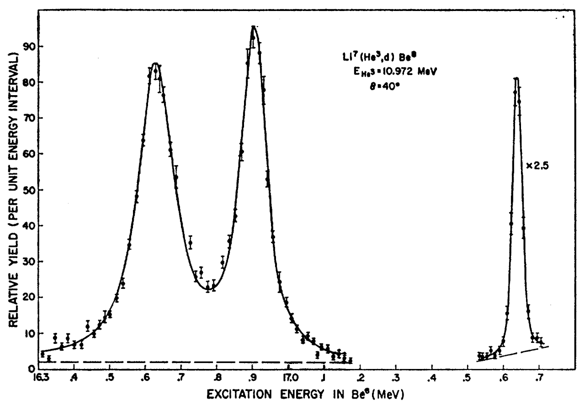

FIG. 1. Spectrum of deuterons from the $Li^7(He^3,d)Be^8$ reaction at $\theta=40°$ for a bombarding energy of 10.972 MeV (at the mid-point of the lithium target). The points are the experimental data (measured as a momentum spectrum) converted to an energy spectrum. The solid curves are the best fits to the data with uncoupled Breit-Wigner curves using the parameters of Table III and a resolution function characterized by the entries in Table II. The dashed lines represent the background assumed for the analysis.

was determined from the settings of the entrance slits. Also the spectrometer resolution function (including the effects of the sizes of the various detectors in the array and an estimate of the effect of the finite beam-spot size) was taken from previous measurements in this laboratory.[18] The effects of beam energy spread and straggle were not known *a priori*. The combination of beam energy spread and straggle can be considered as a single effect which can be determined by fitting the shape of the 17.64-MeV peak. The various contributions to the width of the 17.64-MeV peak, including the assumed shapes and full widths at half-maximum (FWHM), are listed in Table II. These four effects, together with the natural width of 11 keV, were folded together and a fit to the 17.64-MeV peak was made with the FWHM of the beam-spread and straggle contribution as a parameter. The curve that was obtained at minimum χ^2 is shown in Fig. 1, and yielded a FWHM value of 18.2 keV for the effect of beam spread and straggle. This value is close to that estimated from the geometry of the beam and the known energy loss in the target backing. This procedure also gave a value of 10.972 MeV for the energy of the beam at the center of the lithium target, assuming an excitation energy of 17.638 MeV for the narrow state.

TABLE II. Contributions to the energy resolution.

Origin	Assumed shape	FWHM, 17.64 (keV)	FWHM, 16.6–16.9 (keV)	Uncertainty (keV)
Target thickness	Square	6.1	8.8	3
Angular spread of magnet aperture	Square	23.8	27.0	3
Spectrometer resolution	Triangle	6.4	9.4	1
Beam spread and straggle	Gaussian	Determined by χ^2 fit	18.2	2

[18] D. E. Groce, Ph.D. thesis, California Institute of Technology, 1963 (unpublished).

The magnitudes of the various contributions to the over-all resolution at the 17.64-MeV peak were then converted to values appropriate for the average energy of the 16.6- and 16.9-MeV peaks. (Since the values change only slowly with excitation energy, no correction was made for the energy change between the 16.6–16.9-MeV doublet.) These values and their uncertainties are also listed in Table II.

Using the resolution function determined by the above procedure, the 16.6- and 16.9-MeV peaks were fitted with Breit-Wigner curves in order to determine their widths. Two separate (i.e., uncoupled) Breit-Wigner curves were used; the justification for the neglect of interference between the two levels in the special case of the reaction used is given in Sec. IV. Figure 2 shows the results of the fitting procedure. Minimum values of χ^2 were obtained for widths of 113 and 77 keV for the 16.6- and 16.9-MeV states, respectively. The energy spectrum computed with these values is shown in Fig. 1. The agreement between the computed curve and the experimental points, even in the valley between the peaks (where interference between the levels or with the background, if present, would be evident), is exceptionally good. Combining the uncertainties in the various contributions to the resolution function with a reasonable estimate of the reliablity of the χ^2 fitting procedure results in the assignment of ± 3 keV as the uncertainty in each of the width measurements:

$$\Gamma(16.6)=113\pm 3\text{ keV}, \tag{16}$$

$$\Gamma(16.9)=77\ \pm 3\text{ keV}. \tag{17}$$

An additional spectrum measured at 15° confirmed the result for $\Gamma(16.9)$ obtained at 40°, but the 16.6-MeV peak was coincident with a peak from the $O^{16}(He^3,d)F^{17}$ reaction at the forward angle, so that no value for $\Gamma(16.6)$ could be extracted from the data.

The ratio of these widths gives the ratio of the squares of the wave-function coefficients:

$$\Gamma(16.6)/\Gamma(16.9)=1.47\pm 0.07=\alpha^2/\beta^2. \tag{18}$$

Using the normalization condition $\alpha^2+\beta^2=1$, we find $\alpha=0.772$ and $\beta=0.636$, so that

$$|16.6\rangle=0.772|T=0\rangle+0.636|T=1\rangle, \tag{19}$$

$$|16.9\rangle=0.636|T=0\rangle-0.772|T=1\rangle, \tag{20}$$

where the relative sign of α and β has been chosen in accordance with the discussion in Sec. IV.

The value of $\Gamma(16.6)$ determined in this experiment is somewhat larger than that obtained in previous measurements, whereas $\Gamma(16.6)$ is somewhat smaller. Therefore the intensity ratio for $T=0$ reactions, $I(16.6)/I(16.9)=\Gamma(16.6)/\Gamma(16.9)=1.47\pm 0.07$, is significantly larger than the previous value [Eq. (13)] of 1.17. The reason for this apparent discrepancy is probably the neglect of interference effects in the previous analyses and this will be discussed in Sec. V.

The peak-fitting procedure yielded not only values of the widths, but also values for the two excitation energies (based on $E_x=17.638$ MeV for the narrow state); the results are given in Table III. The energy difference between the states (274 keV) has a somewhat smaller uncertainty (± 3 keV) than the uncertainties on the absolute excitation energies because the latter uncertainties are correlated.

IV. INTERFERENCE BETWEEN THE Be^8 LEVELS

Because of the peculiar nature of the 16.63- and 16.90-MeV states, in that they consist of the two possible orthogonal combinations of the two isobaric-spin basis states, interference between these states should be observable in certain situations. There are three important cases to consider:

(*i*) *Only T=0 channel open.* This is the case, e.g., of the $B^{10}(d,\alpha)Be^8$ reaction. The $T=0$ parts of the states act coherently, so that the doubly differential cross section (i.e., the energy spectrum) should have the form

$$\frac{d^2\sigma}{d\Omega dE}=\left|N(\theta)\left\{\frac{A}{E-E_1+i\Gamma_1/2}+\frac{B}{E-E_2+i\Gamma_2/2}\right\}\right|^2, \quad (21)$$

where E is the excitation energy in Be^8 and where $E_1=16.63$ MeV, $E_2=16.90$ MeV, $\Gamma_1=113$ keV, and $\Gamma_2=77$ keV. The $T=0$ amplitude leading to the 16.63-MeV state is A, and B is the same quantity for the 16.90-MeV state; the ratio A/B is equal to α/β. The factor $N(\theta)$ contains the only dependence on the angle of emission θ. If $A>0$ and $B>0$, then in the region between the states, *destructive* interference should result since in this region $E-E_1>0$ and $E-E_2<0$. Such destructive interference has in fact been observed,[19] thereby justifying our choice $\alpha>0$ and $\beta>0$ in Eqs. (19) and (20). Another consequence of Eq. (21) is that the angular distributions for the two states should be identical. Of course, in a proper analysis the interference should be explicitly taken into account in extracting the angular distributions, but by using the approximate procedure of simply integrating the yield in each (still well-defined) peak, Browne and Erskine[15] extracted angular distributions for the two states which had essentially the same shape.

Similar behavior of the energy spectrum, but with constructive interference between the levels, should be observed in $T=1$ reactions, e.g., $C^{12}(d,Li^6_{T=1})Be^8$.

(*ii*) *T=0+T=1 channels, pure direct reaction.* This is the case, e.g., of the $Li^7(d,n)Be^8$ and the $Li^7(He^3,d)Be^8$ reactions at sufficiently high bombarding energies to

[19] C. P. Browne, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 136.

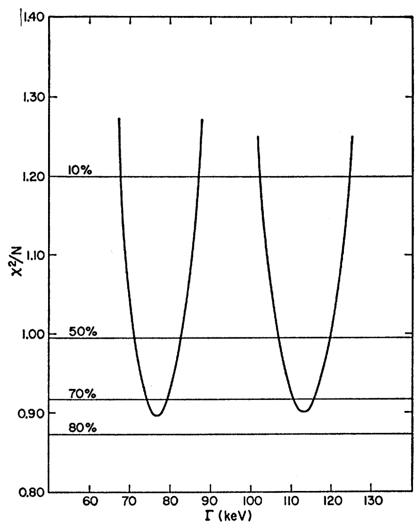

FIG. 2. χ^2 plots for determining the widths of the Be^8 states. The left-hand curve gives 77 keV for the 16.90-MeV level and the right-hand curve gives 113 keV for the 16.63-MeV level.

be pure stripping. Here, both the $T=0$ and $T=1$ contributions add coherently and the energy spectrum is of the form

$$\frac{d^2\sigma}{d\Omega dE}=\left|N_d(\theta)\left\{\frac{A}{D_1}+\frac{B}{D_2}\pm\frac{B}{D_1}\mp\frac{A}{D_2}\right\}\right|^2, \quad (22)$$

where

$$D_1=E-E_1+i\Gamma_1/2, \quad (23)$$

$$D_2=E-E_2+i\Gamma_2/2. \quad (24)$$

The signs of the last two terms depend on the isobaric-spin vector coupling coefficients for the particular case at hand. For proton transfer reactions, such as $Li^7(d,n)Be^8$ and $Li^7(He,d)Be^8$, the upper signs hold, whereas for neutron transfer reactions, such as $Be^7(d,p)Be^8$, the lower signs hold.

If the Be^8 levels were pure single-particle states, so that $A=B$, we would have

Proton transfer reactions:

$$\frac{d^2\sigma}{d\Omega dE}=\frac{2N_d^2(\theta)A^2}{(E-E_1)^2+\Gamma_1^2/4}, \quad (25)$$

Neutron transfer reactions:

$$\frac{d^2\sigma}{d\Omega dE}=\frac{2N_d^2(\theta)A^2}{(E-E_2)^2+\Gamma_2^2/4}. \quad (26)$$

TABLE III. Properties of the Be^8 $J^\pi=2^+$ doublet.

Excitation energy (MeV)	Width, c.m. (keV)
16.627±0.005	113±3
16.901±0.005	77±3
Energy difference=274±3 keV	

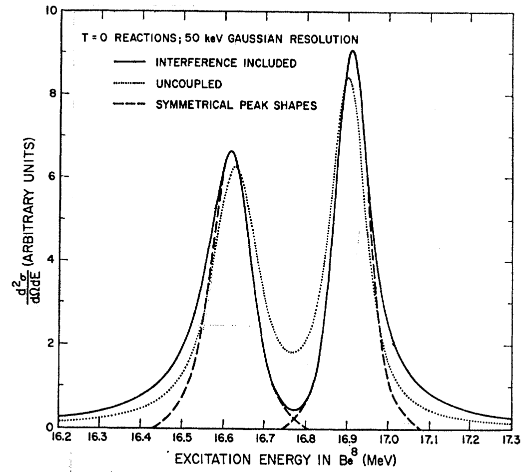

FIG. 3. Results of the "experiment" to determine the ratio of peak areas. The solid curve is that calculated using Eq. (21) with the parameters obtained from the present experiment. The dotted curve is for two *uncoupled* Breit-Wigner curves and, compared with the solid curve, shows the large effects of interference. The dashed curves were sketched symmetrically to represent one simple method for obtaining the areas under the individual peaks. The two areas are approximately equal. Notice that the separation of the peaks is 293 keV, whereas $E_2-E_1=274$ keV.

That is, only the 16.63-MeV level would be populated in the first case and only the 16.90-MeV level in the second case. Since the states are actually not purely of single-particle character, there will, in general, be a small population of the other state in each case and there will also be present an interference term. None of the various direct reactions has yet been performed at high energy with sufficient energy resolution to reveal the interference effect, although the $Li^7(d,n)Be^8$ and $Li^7(He^3,d)Be^8$ reactions[5,6,17] show the expected strong preference for only one of the two states.

Equation (22) also indicates that the angular distributions for the two levels should be identical. The only experiment performed at a sufficiently high bombarding energy to insure that the reaction takes place predominantly by a direct process was a study[9] of the $Be^9(p,d)Be^8$ reaction at 40 MeV, and it was in fact found that the angular distributions were the same. (The uncertainties in the yields to the 16.63-MeV state were large, however, because the 16.90-MeV state dominated by a factor of 20.)

(*iii*) *$T=0+T=1$ channels, direct reaction+compound nucleus formation.* This case covers the same reactions as case (ii) but at lower bombarding energies, where compound nucleus formation can also take place. If only a single level (or a few levels) in the compound nucleus is involved, this case cannot be analyzed without specific knowledge of the level properties and reaction dynamics. (In particular, one needs the phases between the direct and compound nucleus amplitudes.) In the event that a large number of compound levels is formed (as will be the case for the reactions considered here), the random-phase approximation can be used to reduce the cross-section expression to a simple one. The direct and compound nucleus amplitudes will, in general, have different magnitudes and angular dependences, so that the doubly differential cross section can be written as

$$\frac{d^2\sigma}{d\Omega dE}=\left|N_d(\theta)\left\{\frac{A}{D_1}+\frac{B}{D_2}\pm\frac{B}{D_1}\mp\frac{A}{D_2}\right\}\right.$$
$$\left.+N_c(\theta)\left\{\sum_j\frac{C_je^{i\delta_j}}{D_1}+\sum_k\frac{D_ke^{i\gamma_k}}{D_2}\pm\sum_l\frac{D_le^{i\gamma_l}}{D_1}\mp\sum_m\frac{C_me^{i\delta_m}}{D_2}\right\}\right|^2. \tag{27}$$

The direct reaction term, $N_d(\theta)\{\cdots\}$, is the same as in Eq. (22). The compound-nucleus term $N_c(\theta)\{\cdots\}$, which is coherent with the direct-reaction term, involves sums over the compound-nucleus states with amplitudes C_n and D_n, which are all assumed to be approximately equal and with phases δ_n and γ_n, which are all assumed to be random. The sums in the terms containing the random phases eliminate all of the compound-direct and compound-compound interference terms, and we have

$$\frac{d^2\sigma}{d\Omega dE}=\left|N_d(\theta)\left\{\frac{A}{D_1}+\frac{B}{D_2}\pm\frac{B}{D_1}\mp\frac{A}{D_2}\right\}\right|^2+N_c^2(\theta)$$
$$\times\left\{(C^2+D^2)\left[\frac{1}{(E-E_1)^2+\Gamma_1^2/4}+\frac{1}{(E-E_2)^2+\Gamma_2^2/4}\right]\right\}, \tag{28}$$

where, again, the direct term is the same as in case (*ii*). If $A\approx B$ and $C\approx D$, the energy spectrum for the proton-transfer case (upper signs) is, approximately,

$$\frac{d^2\sigma}{d\Omega dE}\simeq\frac{2[A^2N_d^2(\theta)+C^2N_c^2(\theta)]}{(E-E_1)^2+\Gamma_1^2/4}+\frac{2C^2N_c^2(\theta)}{(E-E_2)^2+\Gamma_2^2/4}. \tag{29}$$

This expression shows that the energy spectrum will be described by the sum of two uncoupled Breit-Wigner curves (thus justifying our use of such a form in the analysis described in Sec. III). Since $N_d^2(\theta)$ is forward-peaked, while $N_c^2(\theta)$ is symmetrical about $\theta_{c.m.}=90°$, the angular distributions will, in general, be different, and in the forward direction the 16.63-MeV level will dominate the spectrum. (For the case of the lower signs in Eq. (28), the roles of the two states are reversed.) Equation (29) correctly describes the qualitative features of the angular distributions observed for the $Li^7(d,n)Be^8$ and $Li^7(He^3,d)Be^8$ reactions[5,6,17] at bombarding energies $\lesssim 12$ MeV.

V. EFFECT OF INTERFERENCE ON THE INTENSITY RATIO

Previous measurements of the cross-section ratio $\sigma(16.63)/\sigma(16.90)$ for the $B^{10}(d,\alpha)$ and $Li^6(Li^6,\alpha)Be^8$ reactions[15,16] have yielded values of approximately 1.17 [see Eqs. (14) and (15)]. (Higher values have been obtained[15,20] in other experiments at selected angles, but the results from angular distribution measurements are felt to be more indicative of the ratios from these reactions.) This value (1.17) is significantly smaller than the intensity ratio (1.47±0.07) predicted from the width measurements presented here. As mentioned previously, the cross-section ratios were determined without taking into account the fact that there is interference between the levels. In order to obtain an estimate of the effect of interference on the cross-section ratio, the following "experiment" was performed. The energy spectrum expected on the basis of the interference formula [Eq. (21)] was computed with the energy and width values obtained from the present experiment. This curve was then folded with Gaussian functions with FWHM of 50 and 90 keV in order to simulate the effects of experimental resolution. The resulting curves were then analyzed as if no interference were present, i.e., symmetrical peak shapes were sketched to represent each peak. The ratio of the areas in each case was approximately 1.0. One example (for 50-keV resolution) is shown in Fig. 3. It is easy to see that the areas are approximately equal. (A simple procedure is to compare the product, peak height×half-width, for the two peaks. These products are equal within a few percent). The result is, however, dependent on how much "background" one assumes, since the interference curve, even after folding in the resolution function, has the appearance of two peaks resting on a smooth background. Thus, it appears that an actual intensity ratio of 1.47 can easily be misinterpreted as a much smaller value if the effects of interference are ignored. Therefore, the discrepancy between the previously measured cross-section ratios and the intensity ratio predicted on the basis of $\Gamma(16.63)=113$ keV and $\Gamma(16.90)=77$ keV is apparently removed.[21]

VI. CONCLUSIONS

The description of the Be^8 states at 16.63 and 16.90 MeV as an almost completely mixed isobaric-spin pair has so far been successful in explaining all of experimental information available regarding these states.[12,22] The present results for the widths of the states, $\Gamma(16.63)=113\pm3$ keV and $\Gamma(16.90)=77\pm3$ keV, are the most precise values currently available, and they now specify with considerable accuracy the isobaric-spin amplitudes of the wave functions for these states and also the intensity ratio to be expected for $T=0$ reactions, $I(16.63)/I(16.90)=1.47\pm0.07$. [The expected ratio for $T=1$ reactions is, of course, $I(16.90)/I(16.63)=1.47\pm0.07$]. Interference effects are expected in reactions proceeding by only one of the isobaric-spin channels, but for single-nucleon stripping reactions at low energies where compound nucleus effects are important, the interference effects almost completely disappear.

The excitation energies obtained for the Be^8 states (see Table III) are to be compared with the average values[14] of 16.628±0.005 MeV and 16.923±0.006 MeV. For the lower state, the agreement is excellent, but there is a discrepancy of 22 keV in the values for the upper state. Again it must be pointed out that interference effects are important and that they can influence the peak positions as well as the widths. The energy spacing between the levels can be increased over the actual value by 10–20 keV, depending upon the resolution of the measuring apparatus. This effect therefore probably accounts for most, if not all, of the difference in the previous spacing (295 keV) compared to that obtained here (274±3 keV).

[20] J. R. Erskine and C. P. Browne, Phys. Rev. **123**, 958 (1961).

[21] In a recent paper, C. P. Browne, W. D. Callender, and J. R. Erskine [Phys. Letters **23**, 371 (1966)] report an analysis of the α-particle spectrum from the $B^{10}(d,\alpha)Be^8$ reaction taking into account the interference effects in a manner similar to that described in Sec. IV. They quote values for the widths in agreement with those obtained here. The two analyses yield values for E_1-E_2 that differ by 3 times the combined stated probable errors; the reason for this discrepancy is not understood.

[22] G. J. Stephenson, Jr., and J. B. Marion (to be published).