The mirror reactions involving proton and neutron transfers are in accord with predictions based on the charge symmetric nature of nuclear forces. The measured absolute cross sections for the formation of analog states are equal, within the accuracy of the present experiment, consistent with isospin remaining a good quantum number in the nuclear surface regions. More precise measurements on such systems will be required to determine small isospin mixing effects.

The elastic and inelastic scattering differential cross sections display characteristic oscillations. The inelastic scattering selectively populates states of collective character in C^{12}, in agreement with previous measurements on similar heavy-ion reactions.

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De-Excitation of Highly Excited Nuclei*

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We describe a scheme of calculations useful for computer evaluation of the de-excitation of nuclei having large excitation energies and angular momenta. The scheme is based on the statistical model. Cascade emission of γ rays is taken into account. In particular, we include consideration of both dipole and quadrupole γ-ray emission, and of the crucial role played by the lowest excited state at every angular momentum in the nuclei involved. We are thus, for example, in a position to study phenomena connected with particle emission following γ-ray emission. As input in a sample calculation ($Ce^{140}+O^{16}$ at 90 MeV, lab) we use information obtained from experimental data and from the optical and shell models, but independent of the data with which comparisons are made. We find generally reasonable agreement, and no serious disagreements, with a variety of experimental data, with no parameter adjustment. The predicted energy spectra of emitted α particles and γ rays show important features not demonstrated in previous, less complete calculations. The γ spectrum displays a low average energy of less than 1.0 MeV. The number of photons emitted by quadrupole radiation is typically of the same order of magnitude as the number emitted by dipole radiation. Other results are also described.

INTRODUCTION

To interpret measurements on many nuclear reactions, it is useful to know accurately the relevant behavior predicted by the nuclear-evaporation model. Until recently, however, predictions exact and complete enough to represent adequately the consequences of this model were restricted to the small ranges of excitation energy and angular momentum within which they could be calculated. These ranges can now be greatly extended, because the use of very fast computers having large memories makes it feasible to cope with the voluminous bookkeeping such calculations entail.

Perhaps more important than the newly developed capacity to analyze old data is the prospect of learning something new about nuclei. When the calculations are carried completely through, previously unsuspected consequences of the theory are revealed, and these in turn suggest new experiments. A recent, familiar example of this synergy is the recognition[1] that the analysis of a compound-nucleus excitation function can provide an estimate of the energy of the lowest excited state at each angular momentum in the product nucleus,

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[1] J. R. Grover, Phys. Rev. 127, 2142 (1962).

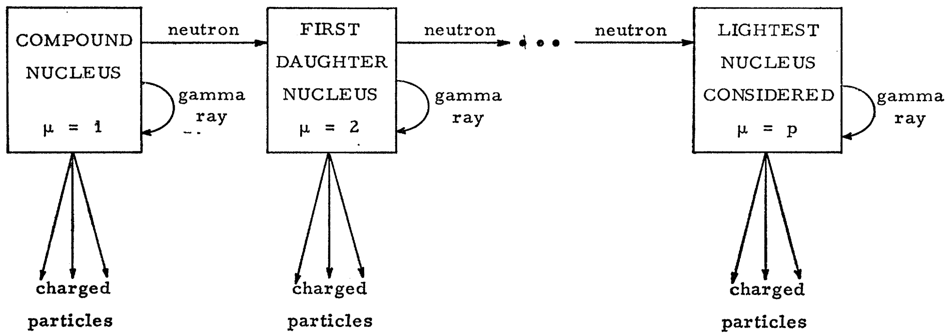

FIG. 1. Schematic diagram of calculative procedure.

sometimes to quite large angular momenta.[1–5] More examples have come to light as a result of our work, reported here and in other papers accompanying and to follow this one. That even further examples will turn up can already be foreseen (e.g., in connection with calculations of the role of fission, or of the angular distributions of emitted particles).

In this paper we describe our adaptation of the familiar nuclear-evaporation formulas to a large digital computer, and present the results of a sample calculation, compared where possible with experimental data. In two companion papers[6,7] we describe in detail some of the predicted features of the de-excitation process newly revealed by our calculation.

CALCULATIVE PROCEDURE

Over-All Scheme

The over-all[8] scheme of our calculation is diagrammed in Fig. 1. The nuclear species involved are represented by squares, and the various particles and quanta which may be emitted, by arrows. The symbol μ refers to a nuclear species (i.e., implies only a certain assignment of mass number A and atomic number Z) and not to a nucleus in a specified state. In each species μ, the nuclei are, in general, distributed with respect to both energy and angular momentum. The calculation is performed step by step as follows.

The de-excitation of the compound nucleus $\mu=1$ is treated first. Emission spectra for neutrons, γ rays and various charged particles are calculated, together with the concomitant product nucleus population distributions. It is important to remember that, in general, emission of a γ ray may leave the nucleus with enough excitation energy for particle emission. Therefore, the daughter population distributions which result from γ-ray emission are added back to the emitting population. Accordingly, in Fig. 1, the γ-ray arrows are doubled back to re-enter their own square. In the next step, the nuclear species that results from neutron emission, $\mu=2$, is treated; the various spectra and population distributions are calculated, as already described. This chain of calculations is carried down through the species that result from successive neutron emissions, $\mu=1, 2, 3, \cdots, p$, until finally a product is reached which cannot be formed with enough excitation energy to emit a neutron ($\mu=p$).

This scheme can readily be expanded to include the de-excitation of the species resulting from charged-particle emission, although we have not yet done so.

The final outputs of our calculation consist of γ-ray and particle-emission spectra for each of the nuclear species considered, and the relative probability of formation, in the ground and many excited states, of each of these species.

Equations

To describe how we take into account the cascade emission of γ rays, it proves convenient to define two kinds of population distribution in species μ, with respect to excitation energy E and angular momentum J. (1) Let $P_\mu'(EJ)$ represent the "initial" distribution of population *before* consideration of γ-ray emission. For example, this might be a distribution assumed at the outset of the calculation, or one resulting from neutron emission from nucleus $\mu-1$, etc. (2) Let $P_\mu(EJ)$ represent the "integrated" distribution of population obtained by addition to $P_\mu'(EJ)$ of all the contributions by cascade γ-ray emission from states of μ at higher energies than E.[9] Thus, P_μ is so defined that its value corresponding to the ground state, or any other discrete state, gives directly the cross section for producing

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[3] R. A. Esterlund and B. D. Pate, Nucl. Phys. **69**, 401 (1965).
[4] J. M. Alexander and G. N. Simonoff, Phys. Rev. **133**, B93 (1964).
[5] G. Kumpf and V. A. Karnaukhov, Zh. Eksperim. i Teor. Fiz. **46**, 1545 (1964) [English transl.: Soviet Phys.—JETP **19**, 1045 (1964)].
[6] J. R. Grover and J. Gilat, first following paper, Phys. Rev. **157**, 814 (1967).
[7] J. R. Grover and J. Gilat, second following paper, Phys. Rev. **157**, 823 (1967).
[8] J. R. Grover, Phys. Rev. **123**, 267 (1961). The procedure described here is a simplification and extension of the procedure described in this paper.

[9] Thus, where the total cross section σ_μ(total) for production of species μ, including nuclei which will go on to emit more particles, is given by $\sigma_\mu(\text{total})=\sum_J \int P_\mu'(EJ)dE$, it should be borne in mind that, in general, $\sum_J \int P_\mu(EJ)dE>\sigma_\mu(\text{total})$. However, as can be seen from Eqs. (9–12), $S_{\mu n}(EJ:EJ)$ is normalized in such a way that $\sum_J \int P_{\mu+1}'(EJ)dE=\sigma_{\mu+1}(\text{total})$.

species μ in that state (aside from the conversion of the continuous function to its discrete-state equivalent).

To follow the neutron-emission cascade from nucleus μ to nucleus $\mu+1$, we first calculate $P_{\mu+1}'$ from P_μ, through[10]

$$P_{\mu+1}'(EJ)=\sum_{\bar{J}}\int P_\mu(\bar{E}\bar{J})S_{\mu n}(\bar{E}\bar{J}:EJ)d\bar{E}. \quad (1)$$

Following this, $P_{\mu+1}$ is calculated from $P_{\mu+1}'$:

$$P_{\mu+1}(EJ)=P_{\mu+1}'(EJ)+\gamma \text{ contribution}, \quad (2)$$

and we are ready for the next neutron emission. Here, $S_{\mu n}$ is the normalized relative probability that neutron emission takes place between nucleus μ, at energy and spin $\bar{E}\bar{J}$, and nucleus $\mu+1$ at EJ. Expressions for $S_{\mu n}$, and for the corresponding quantities for emission of γ rays $S_{\mu\gamma}$, and in general for any kind of particles $S_{\mu i}$, are given below [Eqs. (9)–(13)].

The "γ contribution" could, in principle, be evaluated through the following series of equations [examples of cascade γ-ray calculations in the spirit of Eqs. (3) and (4) may be found in Refs. 11–13]:

$$\begin{aligned}
P_{\mu+1}^{(2)}(EJ)&=P_{\mu+1}'(EJ)+\sum_{\bar{J}}\int P_{\mu+1}'(\bar{E}\bar{J})S_{\mu+1\gamma}(\bar{E}\bar{J}:EJ)d\bar{E}\\
&\vdots\\
P_{\mu+1}^{(n+1)}(EJ)&=P_{\mu+1}^{(n)}(EJ)+\sum_{\bar{J}}\int [P_{\mu+1}^{(n)}(\bar{E}\bar{J})-P_{\mu+1}^{(n-1)}(\bar{E}\bar{J})]S_{\mu+1\gamma}(\bar{E}\bar{J}:EJ)d\bar{E},\\
&\vdots
\end{aligned} \quad (3)$$

where

$$P_{\mu+1}^{(n)}(EJ)\to P_{\mu+1}(EJ) \quad \text{as} \quad n\to\infty. \quad (4)$$

However, for the problems we wish to consider, this series would often converge very slowly. We therefore used the following approximation, instead of Eqs. (3) and (4) (we drop the subscript $\mu+1$ for clarity):

$$P(EJ)dE\approx P'(EJ)dE+\sum_n \mathcal{P}_n(E^0,\Delta:EJ)dE. \quad (5)$$

We divide the energy coordinate into small intervals 2Δ, such that, beginning with the maximum possible energy E^0, the intervals are E^0 to $E^0-\Delta$, $E^0-\Delta$ to $E^0-3\Delta$, $\cdots$, $E^0-[2n-1]\Delta$ to $E^0-[2n+1]\Delta$, $\cdots$, $E+\Delta$ to $E-\Delta$. The contribution coming from γ-ray emission of nuclei in the interval $E^0-(2n-1)\Delta$ to $E^0-(2n+1)\Delta$ is given by $\mathcal{P}_n(E^0,\Delta:EJ)$, for $n\geqslant 1$. The contribution from nuclei in the interval E^0 to $E^0-\Delta$ is given by $\mathcal{P}_0(E^0,\Delta:EJ)$.

Now the $\mathcal{P}_n$'s can be calculated as follows:

$$\begin{aligned}
\mathcal{P}_0(E^0,\Delta:EJ)&=\sum_{\bar{J}}\int_{E^0-\Delta}^{E^0}P'(\bar{E}\bar{J})S_\gamma(\bar{E}\bar{J}:EJ)d\bar{E} && \text{for} \quad E\leqslant E^0-\Delta,\\
\mathcal{P}_1(E^0,\Delta:EJ)&=\sum_{\bar{J}}\int_{E^0-3\Delta}^{E^0-\Delta}[P'(\bar{E}\bar{J})+\mathcal{P}_0(E^0,\Delta:\bar{E}\bar{J})]S_\gamma(\bar{E}\bar{J}:EJ)d\bar{E} && \text{for} \quad E\leqslant E^0-3\Delta,\\
&\vdots\\
\mathcal{P}_n(E^0,\Delta:EJ)&=\sum_{\bar{J}}\int_{E^0-(2n+1)\Delta}^{E^0-(2n-1)\Delta}[P'(\bar{E}\bar{J})+\sum_{k=0}^{n-1}\mathcal{P}_k(E^0,\Delta:\bar{E}\bar{J})]S_\gamma(\bar{E}\bar{J}:EJ)d\bar{E} && \text{for} \quad E\leqslant E^0-(2n+1)\Delta,\ n>1.
\end{aligned} \quad (6)$$

The accuracy of Eq. (5) depends on Δ in such a way that there is an optimum value of Δ for any given problem. For population of excited states at energies well above the lowest excited state at every angular momentum (yrast levels[14]), where the level densities are high, the use of smaller values of Δ leads to increased accuracy. However, for transitions to excited states near the yrast levels, or to the yrast levels themselves, where the level spacings become of order 0.2 to 0.5 MeV or even larger, the use of too small values of Δ would cause the calculation of many spurious low-energy γ-ray transitions in regions where no levels exist. We have made studies to investigate the adequacy of Eq. (5), and to decide how to choose an optimum value of Δ. At energies well above the yrast levels, Eq. (5) is usually reasonably accurate if $2\Delta\leqslant 0.5$ MeV. This relatively coarse mesh suffices because the γ-ray spectrum has an uncomplicated shape, with its maximum occurring at energies several times larger than 0.5 MeV. For excitation energies near or at the yrast levels, the value of 2Δ should be comparable to the average spacing

[10] To avoid cumbersome expressions, we omit the limits on integrals and sums whenever they are easily obtained by application of the appropriate conservation laws (of energy, angular momentum, etc.).

[11] D. Sperber, Phys. Rev. **142**, 578 (1966).

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[13] V. M. Strutinski, L. V. Groshev, and M. K. Akimova, Nucl. Phys. **16**, 657 (1960).

[14] By "yrast" level of a given nucleus, at a given angular momentum, we mean the level with least energy at that angular momentum. See Ref. 15 for the etymology of this word.

between the lowest and second lowest excited state at every angular momentum. For Dy^{152} this spacing is calculated[15] to be about 0.2 MeV. In the sample calculation reported here, we use $2\Delta=0.5$ MeV, in order to limit the time spent in computation. Our results should therefore be reasonably trustworthy for γ-ray energies larger than about 1.5 MeV, but only representative for small energies; this is discussed in more detail in Ref. 6.

We interrupt our discussion, briefly, to mention that, since our calculation is carried out on an energy grid of mesh 2Δ, we do not perform our calculations directly with values of P, P', and $\mathcal{P}$ per se, but with quantities like

$$\int_{E-\Delta}^{E+\Delta} P(EJ)dE,$$

etc. This leads to some "technological" problems; such a problem is to devise a fast but reasonably accurate conversion of calculated spectra into integrated segments consistent with the form given above. Other problems of computer-program design that we encountered are mentioned further on, where the approximations involved affect the accuracy of our calculation. An exhaustive discussion of such technical details, however, is not appropriate here and is not given.

Since the calculation is carried out on an energy grid, the great convenience of Eq. (5) becomes apparent. If the calculation of the de-excitation of nucleus μ is begun in the highest energy interval ($n=0$) of a distribution $P_\mu'(EJ)$, we see that as soon as this energy interval has been completely processed for all J's (i.e., as soon as the spectrum for each type of decay has been computed, normalized, and entered into the proper bins as partial contributions to the appropriate product population distributions), the next lower energy interval ($n=1$) already contains values of P instead of P'. Likewise, as processing of each energy interval is completed, that interval just below it has been completely corrected to P and is ready for processing, etc. Thus, the modification of the distribution P_μ' to P_μ and the calculation of the product distribution $P_{\mu+1}'$ are carried out simultaneously. The reason this device works can be seen from Eqs. (6). In the equation of $\mathcal{P}_n$, we see that as each successive energy interval is processed, another term is added in the summation over k on the right side, beginning with $k=0$ for the topmost energy interval, $k=1$ for the first energy interval below the topmost, etc., and continuing to the $(n-1)$th energy interval, at which point the term in square brackets is itself completely calculated. Returning to Eq. (5), we see that the evaluation of all $\mathcal{P}$'s up to and including $\mathcal{P}_n$ completes the calculation of P, as the indicated summation has, of course, been performed concurrently with the $\mathcal{P}$ evaluations.

[15] J. R. Grover, third following paper, Phys. Rev. **157**, 832 (1967).

The energy spectrum of particle i emitted from nucleus μ is

$$\mathcal{S}_{\mu i}(\epsilon)d\epsilon=\left[\sum_{\bar{J}}\int P_\mu(\bar{E}\bar{J})\sum_J S_{\mu i}(\bar{E}\bar{J}:EJ)d\bar{E}\right]d\epsilon, \quad (7)$$

where

$$\epsilon=\bar{E}-E-B_{\mu i} \quad (8)$$

is the kinetic energy of particle i and $B_{\mu i}$ is the binding energy of particle i to nucleus μ. For the subscript i we use $i=n$ for neutrons, $\gamma1$ for dipole γ rays, $\gamma2$ for quadrupole γ rays, γ for γ rays of unrestricted multipolarity, p for protons, α for alpha particles, d for deuterons, t for tritons, and h for He^3 nuclei.

The normalized relative probability $S_{\mu i}(\bar{E}\bar{J}:EJ)$ that emission of particle i takes place between nucleus μ at energy $\bar{E}$ and angular momentum $\bar{J}$, and the appropriate daughter nucleus at EJ, has already been introduced in Eqs. (1), (3), and (6) for the special cases of neutron and γ-ray emission. These functions are defined so that

$$\sum_i \sum_J \int S_{\mu i}(\bar{E}\bar{J}:EJ)dE=1, \quad (9)$$

which means, of course, that they are related to their corresponding average emission rates $R_{\mu i}(\bar{E}\bar{J}:EJ)$ through

$$S_{\mu i}(\bar{E}\bar{J}:EJ)dE \approx R_{\mu i}(\bar{E}\bar{J}:EJ)\Big/\sum_j\sum_J\int R_{\mu j}(\bar{E}\bar{J}:EJ)dE. \quad (10)$$

We estimate $R_{\mu i}(\bar{E}\bar{J}:EJ)$ using the following well-known expressions. For particle emission,[16–21]

$$R_{\mu i}(\bar{E}\bar{J}:EJ)dE = h^{-1}[\omega(EJ)/\omega_\mu(\bar{E}\bar{J})]T_{\mu i}(\bar{E}\bar{J}:EJ)dE. \quad (11)$$

For γ-ray emission,[19,22,23]

$$R_{\mu\gamma}=\sum_L R_{\mu\gamma L},$$

where

$$R_{\mu\gamma L}(\bar{E}\bar{J}:EJ)d\epsilon \approx \xi_{\mu L}h^{-1}[\omega_\mu(\bar{E}\bar{J})]^{-1}\epsilon^{2L+1}\omega_\mu(EJ)d\epsilon. \quad (12)$$

In the above, $\omega_\mu(EJ)$ is the level[24] density (where the

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[17] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).
[18] R. G. Thomas, Phys. Rev. **97**, 224 (1955). See especially Appendix B of this paper.
[19] R. Vandenbosch and J. R. Huizenga, Phys. Rev. **120**, 1313 (1960).
[20] T. Ericson, Phil. Mag. Suppl. **9**, 425 (1960).
[21] T. D. Thomas, Nucl. Phys. **53**, 558 (1964).
[22] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).
[23] S. A. Moszkowski, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), Vol. 2, p. 863.
[24] For what is meant by a nuclear "level" in this paper, we adhere to the custom used by J. M. B. Lang and K. J. LeCouteur (Ref. 44).

magnetic substates are considered to belong to the same level, and are not counted separately) of the emitting nucleus μ, and $\omega(EJ)$ is the level density of the particle-emission product. The symbol $T_{\mu i}(\bar{E}\bar{J}:EJ)$ represents the sum

$$T_{\mu i}(\bar{E}\bar{J}:EJ)=\sum_{S=|J-s|}^{J+s}\sum_{l=|\bar{J}-S|}^{\bar{J}+S}T_l(\epsilon) \qquad (13)$$

of transmission coefficients $T_l(\epsilon)$ corresponding to particle i incident with energy ϵ on the residual nucleus having angular momentum J and excitation energy E, to form nucleus μ with energy $\bar{E}$ and angular momentum $\bar{J}$; the index l designates the lth partial wave, and s the intrinsic spin of the emitted particle. For dipole γ-ray emission, $L=1$; for quadrupole, $L=2$, etc. The $\xi_{\mu L}$ values are constants of proportionality that adjust $R_{\mu\gamma L}$ to experimental values, as described in the following section.

Approximations

In using Eq. (12) we ignore the limitations on the γ-ray emission rates described by the "sum rules."[22] As explained in one of the companion papers,[6] the competition between neutron and γ-ray emission is most important where dipole γ rays can be emitted with approximately 10 MeV at most, and it is precisely this region in which Eq. (12) is normalized to experimental values, although for small angular momenta. Thus, for the conclusions to which we restrict ourselves in this report, Eq. (12) appears to be a satisfactory approximation; however, there are other important facets of the compound-nucleus problem (e.g., capture reactions) that demand a more accurate treatment. An appropriate correction to Eq. (12) would be to multiply the right side by a factor reflecting the relevant inverse cross section, i.e., the cross section for photons incident on the nucleus at EJ, to form the nucleus at $\bar{E}\bar{J}$. Perhaps the most obvious improvement this would make for Eq. (12) is that the revised equation would then take account of the effects of the photonuclear giant resonances. We would mainly need the dipole photonuclear inverse cross sections for excited states. The few available data for excited states, which are almost exclusively for the lowest-lying excited states of light nuclei, reveal that the photonuclear dipole giant resonance is similar to that for the ground state.[25] However, the indiscriminate use of ground-state inverse cross sections at all excitation energies seems to us to be unjustified at present. The angular-momentum dependence of the correction to Eq. (12) is probably less serious than the energy dependence. At one extreme, we can assume that all γ-ray emissions obey the single-particle model,[22] in which all the angular momentum in the nucleus comes from a single nucleon, which changes from one orbital to another during the transition. At the other extreme, we can assume a model in which most of the angular momentum resides in a passive core, with only a small contribution from the nucleon which changes orbitals during the transition. For dipole γ-ray emission in the single-particle model, we evaluated the appropriate angular-momentum coupling factor[23] for several cases, and found that transitions for which $|\bar{J}-J|=1$ are several times faster than those for which $|\bar{J}-J|=0$, if all other factors are the same, but that transitions for which $\bar{J}-J=1$ proceed with speeds comparable to those for which $J-\bar{J}=1$. In the core-plus-single-particle model, the correction to Eq. (12) is proportional to[26] $2J+1$, which is to say, for all but the smallest values of J, transitions for which $\Delta J=-1$, 0, $+1$ proceed with comparable speeds. Thus the essential difference between the two models, for dipole emission, has mainly to do with the relative strength of the $\Delta J=0$ transition. The proper effective correction probably lies somewhere between the extremes, and is most likely energy-dependent. At either extreme, however, the γ-ray de-excitation should display closely similar properties, insofar as we confine our attention to the competition between particle and γ-ray emission, and in this context the approximation we have made seems sensible. A more subtle effect not taken into account in Eq. (12) is a possible systematic variation of the average matrix element as a function of energy and angular momentum in the vicinity of the yrast levels (see Ref. 15).

We do not take account of parity; this can lead to serious errors near reaction thresholds,[27] and therefore limits the applicability of our program to energies 1 MeV or so above the thresholds of any reactions considered.

Since we use the "channel-spin" coupling scheme,[22] we neglect the coupling of the spin of the emitted particles with their orbital angular momentum. In fact, we simply neglect all dependence of the T_l values on E and J, because there is no unequivocal experimental information bearing on what these dependences ought to be. The spin-orbit coupling scheme leads to more complicated equations, and, in our case, should give results very little different from those we obtain with the simpler formulas.[28]

The right-hand side of Eq. (10) and the integrands of Eqs. (1), (3), (6), and (7) are quantities which should be computed separately for each nuclear level involved, then averaged over an appropriate energy interval, each contribution being weighted in proportion to the population of its level. Instead, they are approximated by quantities calculated from several factors, each of which is effectively averaged separately over the involved levels, with implicitly assumed population weighting factors that may be inappropriate for some problems.

[25] N. W. Tanner, G. C. Thomas, and E. D. Earle, Nucl. Phys. **52**, 29 (1964).

[26] D. Sperber (private communication).

[27] J. R. Grover (to be published); and private communication to J. Delorme.

[28] D. G. Sarantites and B. D. Pate, Nucl. Phys. **A93**, 545 (1967).

This approximation is probably of little importance in the cases we consider in the next section, although presently available data do not allow us to disregard it completely. Discussions and some examples of corrections for this approximation are given in Refs. 29 to 33. It may be possible that the elucidation of some puzzling effects[34] seen in compound-nucleus reaction data involve correlations between the reduced widths for formation and decay of the intermediate nuclei, and will thus require an adequate treatment of the averaging problem. Even more striking examples of such data are those measured by Miller and his co-workers.[35–37]

It may be important to take into account the effect of isotopic-spin selection rules, in calculations of nuclear evaporation.[38] We illustrate the operation of such selection rules with an example from the reaction system chosen for our sample calculation. States with $T=21/2$ in Dy^{151} exist only above 12–13 MeV,[39–41] and with $T=11$ in Dy^{152}, only above ≈ 15 MeV. We see that the latter would indeed be populated in $Ce^{140}(O^{16},4n)$ reactions (the maximum energy of Dy^{152} in our sample calculation is 25.5 MeV). Since the binding energy of a neutron to Dy^{152} is about 10 MeV,[41,42] the emission of neutrons from the $T=11$ states of Dy^{152} having energies less than 22–23 MeV would be isotopic-spin-"forbidden," because they have available to them only states of Dy^{151} with $T=19/2$. Now, proton emission to Tb^{151}, all states of which have $T\geqslant 21/2$, is not similarly forbidden, so that under the above circumstances proton emission from Dy^{152} could have a relative competitive advantage. The data bearing on this possible effect are very scant,[38] so that at present it is not clear whether isotopic-spin selection should be included. With these considerations in mind, however, the previously mentioned observations of Miller and his co-workers seem especially provocative.[35–37]

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[30] L. Dresner, in Proceedings of the International Conference on the Neutron Interactions with Nucleus, Columbia University, 1957 [Atomic Energy Commission Report No. TID-7547 (unpublished)], p. 71.
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[32] G. R. Satchler, Phys. Letters **7**, 55 (1963).
[33] P. Axel, Phys. Rev. **126**, 671 (1962).
[34] B. L. Cohen, Phys. Rev. **92**, 1245 (1953); B. L. Cohen and E. Newman, *ibid.* **99**, 718 (1955). In the analysis of activation cross sections for many reactions induced by 21.5- and 11.5-MeV protons, and 14.5-MeV neutrons, these workers find that proton emission appears to compete more effectively with neutron emission when protons are the incident particles. Also, D. Bodansky [Ann. Rev. Nucl. Sci. **12**, 79 (1962)] noticed, in a comparison of neutron- and α-induced reactions, that neutron emission seems favored over proton emission when neutrons are the incident particle.
[35] J. M. Miller (private communication) points out that when the same compound nucleus is made by different target-projectile combinations, there appears to be a tendency for favored emission from the compound nuclei of the same kind of particle as that incident. This effect applies to the whole spectrum, and not just to compound-elastic scattering. The puzzle pointed out by Cohen (Ref. 34), on the basis of an analysis now known to be inadequate, thus seems confirmed.
[36] K. L. Chen and J. M. Miller, Phys. Rev. **134**, B1269 (1964). We refer here to a comparison between the $Sc^{45}(\alpha,\alpha' n)Sc^{44}$ and $Ti^{47}(d,\alpha n)Sc^{44}$ reactions. Unfortunately there are possibly sizable contributions from direct reactions in this case, but the observed effect seems to us to be too large to be accounted for entirely by direct reactions.
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[39] J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. **129**, 2718 (1963).
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[42] P. A. Seeger, Nucl. Phys. **25**, 1 (1961).

INPUT DATA

The sample calculation described in the following section is for O^{16} incident on Ce^{140} at 90 MeV (lab). The nucleus $\mu=1$ is therefore Dy^{156}, and since 90 MeV is just above the threshold for $Ce^{140}(O^{16},7n)Dy^{149}$, we need input data appropriate for particle emission from dysprosium isotopes $A=156$ to 150.

We believe that the approximations we use to facilitate the calculations are reasonably accurate, so that comparisons of our results with experimental values can serve as a test of the adequacy of our input data (for known compound-nucleus reactions). Thus, we choose input data that are justified by criteria other than the experiments to which we compare our calculations. Where possible, we use direct experimental results rather than interpolated values, and interpolated data rather than extrapolated ones. Where there are only theoretical predictions for input data, we select those we are most interested in testing.

Level Densities

For energies far above the yrast levels[15] we use[20,43–48]

$$\omega(EJ)=\mathfrak{N}(2J+1)(E+\delta)^{-2} \times\exp[-J(J+1)/(2\sigma^2)+2(a[E+\delta])^{1/2}], \quad (14)$$

where $\sigma^2=\hbar^{-2}\mathcal{J}[(E+\delta)a^{-1}]^{1/2}$, the moment of inertia $\mathcal{J}$ being that of a rigid sphere: $\mathcal{J}=0.4MR^2$ with M the mass of the nucleus and $R=(1.2\times10^{-13})A^{1/3}$ cm. The sign sense of the condensation energy δ is taken so that $\delta=0$ for even-even nuclei, $\delta=\delta_e>0$ for odd-mass nuclei, and $\delta=2\delta_e$ for odd-odd nuclei. An average value of $\delta_e=0.9$ MeV, calculated from Cameron's table,[48] is used for all the nuclei involved. The level-density parameter a, interpolated from experimental data as interpreted by Lang,[46] is taken to be $a=A/7.24$ MeV^{-1}. Through parameter $\mathfrak{N}$ the calculated level density is normalized

[43] H. A. Bethe, Rev. Mod. Phys. **9**, 84 (1937).
[44] J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954).
[45] T. D. Newton, Can. J. Phys. **34**, 804 (1956).
[46] D. W. Lang, Nucl. Phys. **26**, 434 (1961).
[47] D. W. Lang, Nucl. Phys. **42**, 353 (1963).
[48] A. G. W. Cameron and R. M. Elkin (unpublished); A. G. W. Cameron, Can. J. Phys. **36**, 1040 (1958).

to an "experimental"[49] level density $\omega_\mu{}^0(J^0,E^0)$ at a particular spin J^0 and energy E^0 in one of the involved nuclei μ, i.e.,

$$\mathfrak{N}=\omega_\mu{}^0(E^0J^0)(E^0+\delta)^2(2J^0+1)^{-1} \times\exp[J^0(J^0+1)/\sigma^2-2(a(E^0+\delta))^{1/2}]. \quad (15)$$

In our example, we estimate that, in Dy^{152} at an excitation energy of $E^0=8.35$ MeV and a spin of $J^0=0$, the level density is $\omega_5{}^0(8.35,0)=8000$ levels/MeV.[49] The corresponding value of $\mathfrak{N}$ is then used for all the nuclei involved in our example. In the vicinity of the yrast levels, the statistical arguments which give rise to Eq. (14) are no longer valid. By definition, there are no levels below the yrast level for any J, i.e., $\omega(E,J)=0$ for $E<E_J$. At $E\geqslant E_J$, we assume that the energy dependence of the level density can be adequately represented by the quadratic form

$$\ln\omega(EJ)=a_0+a_1E+a_2E^2. \quad (16)$$

The coefficients a_0, a_1, and a_2 are fixed by the following three requirements:

(1) At[50] $E=E_J$, $\omega(E_JJ)=1$ MeV^{-1}, or

$$a_0+a_1E_J+a_2E_J{}^2=0. \quad (17)$$

(2) The rate of increase of the level density at E_J i.e., $[\partial \ln\omega(EJ)/\partial E]_{E=E_J}$, is made to be equal to that given by the input value of the "yrast temperature" T_J (see Refs. 15, 51), or

$$a_1E_J+2a_2E_J=T_J{}^{-1}. \quad (17')$$

(3) The quadratic form of Eq. (16) is made to join smoothly with the level density as given by Eq. (14) at an energy E' chosen so that

$$\left(\frac{\partial \ln\omega(EJ)}{\partial E}\bigg|_{E=E'}\right) \text{Eq. (16)} =\left(\frac{\partial \ln\omega(EJ)}{\partial E}\bigg|_{E=E'}\right) \text{Eq. (14)}, \quad (18)$$

$$\omega(E'J) \text{ Eq. (16)}=\omega(E'J) \text{ Eq. (14)}.$$

In our sample calculation, we need to know the yrast-level energies E_J for angular momenta up to about $J=50$. This is far beyond the range of the present experimental knowledge of these levels, and we rely on theoretical estimates. Here we use yrast-level energy values calculated from the nuclear shell model, but taking into account the effect of pairing forces; this work is described in a companion paper.[15] In particular, yrast-level energies similar to those used in this calculation are shown in Fig. 2 of that paper (the values used here were calculated by a less accurate method than is described in Ref. 15). The same calculation also provides values for the "yrast temperatures" T_J. It has proved practical[15] to evaluate T_J in an interval between E_J and about $E_J+0.5$ MeV.

Estimates of ξ_L

Through the parameters ξ_L, the average γ-ray emission rates are normalized to the "experimental" rates $\hbar^{-1}\langle\Gamma_{\mu\gamma L}{}^0(E^0J^0)\rangle$ already known or readily estimated for nucleus μ, at a suitable reference energy E^0 and spin J^0. Thus

$$\xi_{\mu L}{}^{-1}=b_{\mu L}{}^{-1}(J^0E^0)\textstyle\sum_J \displaystyle\int \epsilon^{2L+1}\omega_\mu(E^0-\epsilon J)d\epsilon, \quad (19)$$

where

$$b_{\mu L}(J^0E^0)=\langle\Gamma_{\mu\gamma L}{}^0(E^0J^0)\rangle/D_\mu{}^0(E^0J^0) \quad (20)$$

and $D_\mu{}^0=(\omega_\mu{}^0)^{-1}$ is the average level spacing.

For dipole γ-ray emission, a reasonably trustworthy estimate for $b_{\mu 1}(E^0J^0)$ can be obtained from the slow-neutron-capture data,[49] assuming that the cited widths for γ radiation are due chiefly to dipole γ-ray emission. Thus, for Dy^{152} at $E^0=8.35$ MeV and $J^0=0$ we estimate[49] $b_{51}(8.35,0)\approx 4.4\times 10^{-4}$.

For quadrupole emission, which is seen in the next section to play a surprisingly important role, little suitable experimental information seems to be available. The single-particle–model estimates suggest[23] that the mean γ-ray emission rate is about 10^2 to 10^3 times faster for dipole than for quadrupole radiation. On the other hand, compilations of experimentally known reduced γ-ray emission rates[52,53] show that electric-quadrupole emission rates tend to be 10–100 times faster than the single-particle estimates, while electric-dipole emission rates tend to be retarded by like factors. Wilkinson[52] and others[54] have pointed out, however, that the very

[49] Slow-neutron data seem to be the richest source of experimental level densities and γ-ray emission widths, but unfortunately one seldom finds data for exactly the nucleus of interest. Values interpolated and extrapolated from data for neighboring nuclei are often the best one can obtain. The cited values are estimated from data for a Sm^{147} target, tabulated in the compilation by D. J. Hughes, B. A. Magurno, and M. K. Brussel, Brookhaven National Laboratory Report No. BNL-325 (U. S. Government Printing and Publishing Office, Washington, D. C., 1960), 2nd ed.

[50] In general, the effective "level density" at E_J would not be exactly 1 MeV^{-1}, of course. However, for practical purposes, the uncertainty in our knowledge of the level density may be combined with the uncertainty in the value of E_J. Then the involved inaccuracy of E_J coming from the inaccuracy of the level density is of the order of a few hundred keV, smaller than the mesh of the energy grid on which the calculations are performed, and comparable to or smaller than the estimated inaccuracy of the calculated yrast-level energies (Ref. 15).

[51] A few typical values of T_J that we used in the sample calculation are $J=1$, $T_1=0.63$ MeV; 4, 0.59; 9, 0.56; 11, 0.32; 15, 0.60; 20, 0.46; 25, 0.50; 30, 0.45; 35, 0.61; 40, 0.83; 45, 0.86. We now know (see Ref. 15) that these estimates are probably somewhat high.

[52] D. H. Wilkinson, in *Nuclear Spectroscopy: Part B*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York and London, 1960), p. 852

[53] J. Lindskog, T. Sundström, and P. Sparrman, *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1965), Vol. 2, p. 1599.

[54] D. J. Hughes and R. L. Zimmerman, *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. 1, p. 356.

circumstances that lead to the ready measurement and identification of electric-quadrupole transitions and transition rates tend to weight the accumulated data in favor of the fastest ones. In addition, since our calculation disregards parity, it does not distinguish between electric and magnetic multipoles; the magnetic-quadrupole transitions do not seem to be enhanced. For our sample calculation, we have chosen, quite arbitrarily, to use one-tenth of the theoretical single-particle (dipole)/(quadrupole) ratio of 10^{+4} for reduced transition rates, which gives $b_{52}(8.35,0)\approx 10^{-6}$.

Transmission Coefficients

With the ready availability of such codes as ABACUS-2,[55] optical-model transmission coefficients are easily obtained for many nucleus-particle combinations. One needs only a reasonable set of input parameters, namely, a set known to fit exactly the excited nucleus-particle combinations involved in the calculations.

The paucity of suitable information about the interactions of excited nuclei with nucleons and other particles enforces the use of the corresponding ground-state values (in the form of optical-model parameters known to fit scattering data). Support for this approximation is provided by some data gathered by Miller and co-workers[56] which indicate that the ratio of cross sections for interaction of protons and α particles with excited Co^{59} nuclei is not dependent on the excitation energy.

For the calculations described here, we used transmission coefficients suitable for (1) neutrons incident on Dy^{152}, (2) protons incident on Tb^{152}, and (3) α particles incident on Gd^{148}, and left their values unchanged for all the nuclei Dy^{156} to Dy^{150}. A more elaborate treatment does not seem appropriate at this time. The following optical-model parameters were used (expressed, in each case, in the notation of the cited authors): (1) for neutrons[57] of energy less than 7 MeV (c.m.), $V_{CR}=50.0$ MeV, $V_{CI}=7.0$ MeV, $V_{SR}=9.5$ MeV, $a=0.65\times10^{-13}$ cm, $b=0.98\times10^{-13}$ cm, $r_0=1.25\times10^{-13}$ cm; for neutrons of energy $\geqslant 7$ MeV, $V_{CR}=45.5$ MeV, $V_{CI}=9.5$ MeV, $V_{SR}=8.6$ MeV, a, b, r_0 remaining the same; (2) for protons,[58] $V_{CR}=55.0$ MeV, $V_{CI}=11$ MeV, $V_{SR}=5$ MeV, $a=0.65\times10^{-13}$ cm, $b=1.2\times10^{-13}$ cm, $r_0=1.25\times10^{-13}$ cm; (3) for α particles,[59] $V=-50$ MeV, $W=-18.6$ MeV, $d=0.576\times10^{-13}$ cm, $r_c=6.189\times10^{-13}$ cm, $r_0=7.959\times10^{-13}$ cm, where we take $V_c=(2Ze^2)/r$ for $r>r_c$ and $V_c=(2Ze^2)/r_c$ for $r<r_c$. To average the dependence of the neutron- and proton-transmission coefficients on spin-orbit coupling, we used $T_l=(2l+1)^{-1}\times(lT_{l,l-\frac{1}{2}}+[l+1]T_{l,l+\frac{1}{2}})$, in obvious notation.

The computational accuracy of ABACUS-2 as used with the IBM 7094 computer is such that transmission coefficients smaller than about 5×10^{-7} are subject to errors of the order of or greater than $\pm10\%$, the absolute (i.e., round-off) error being of the order of $\pm5\times10^{-8}$. Although this accuracy is usually sufficient for calculation of the emission of neutrons and protons, even for circumstances in which γ-ray emission competes effectively, we found in our sample calculation that much smaller transmission coefficients of reasonable (i.e., within a factor three, say) accuracy are required for a description of α-particle emission that is at least qualitatively complete and quantitatively typical. To estimate approximate values for proton- and neutron-transmission coefficients less than 5×10^{-7}, we extrapolated the optical-model values (using only values $>5\times10^{-7}$) provided by ABACUS-2, assuming that $\log T_l$ varies linearly with $\log\epsilon$. For the α-particle transmission coefficients, the linear relationship assumed above is not accurate enough. Here, we obtained the estimate $T_0\approx P_0\approx3\times10^{-30}$ for an α-particle energy of 3.3 MeV (c.m.) interpolated from Rasmussen's calculated penetrabilities,[60,61] P_l. For other angular momenta we used the rough approximation suggested by Rasmussen,[61,62] $P_l/P_0\approx\exp[-2.027l(l+1)Z^{-1/2}A^{-1/6}]$, where Z and A are the atomic and mass numbers of the product of α-particle emission. Then, assuming a quadratic relationship between $\log T_l$ and $\log\epsilon$ such that the calculated values for $T_l>5\times10^{-7}$ are joined smoothly to the estimated point at $\epsilon=3.3$ MeV, we obtained interpolated values of T_l for α particles.[63] For all particles, we extended the extrapolated transmission coefficients down to, but not below 10^{-18}.

The distribution in population of compound nuclei, with respect to angular momentum, was calculated using the Woods-Saxon form for both real and imaginary potentials, with[64] $V=41.8$ MeV, $W=16.4$ MeV, $R=1.26(A_1^{1/3}+A_2^{1/3})\times10^{-13}$ cm, $a=0.49\times10^{-13}$ cm.

In our sample calculation, we neglected emission of charged particles other than protons and α particles.

Other Input Data

The binding energies of neutrons, protons, and α particles to the nuclei Dy^{150} through Dy^{156}, and of O^{16} to Dy^{156}, were taken from the tables of Seeger.[42]

[55] E. Auerbach very graciously donated to us a copy of his ABACUS-2 program.

[56] J. M. Miller (private communication).

[57] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

[58] F. Bjorklund, G. Campbell, and S. Fernbach, Helv. Phys. Acta, Suppl. **VI**, 432 (1961).

[59] J. R. Huizenga and G. Igo, Nucl. Phys. **29**, 462 (1962).

[60] J. O. Rasmussen, Phys. Rev. **113**, 1593 (1959).

[61] J. O. Rasmussen, Phys. Rev. **115**, 1675 (1959).

[62] I. Perlman and J. O. Rasmussen, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 149.

[63] Estimates by hand calculation show that the above approximation for P_l/P_0 becomes progressively too small as l increases beyond $l=6$. However, at $l=10$ to 12, $T_l\approx10^{-14}$, the extrapolated transmission coefficients are still accurate to about a factor 3.

[64] E. H. Auerbach and C. E. Porter, in *Proceedings of the Third Conference on Reactions between Complex Nuclei*, edited by A. Ghiorso, R. M. Diamond, and H. E. Conzett (University of California Press, Berkeley, 1963), p. 19.

DESCRIPTION OF GROSS RESULTS AND COMPARISON WITH EXPERIMENT

Although our calculation is for a specific example, chosen because there are data with which direct comparisons can be made, many of its features should be generally typical of reactions between medium-heavy nuclei and incident heavy ions. We therefore compare our calculated results with data for which we believe our calculation has general relevance.

In Fig. 2 are plotted spectra for γ rays, neutrons, protons, and α's, summed over emissions from the nuclei μ. Although these spectra do not include emissions from the excited nuclei resulting from charged-particle emission, the shapes and relative magnitudes of the particle spectra of Fig. 2 should be reasonably good approximations to what the total calculated spectra would be like. The results plotted in Fig. 2 show that charged-particle emission is calculated to be relatively unimportant, compared with neutron emission. The γ-ray spectra are incomplete, not only for the reason already mentioned, but because we did not follow the γ-ray de-excitation completely to the ground state. In most cases, we had to cut off the γ cascade at excitation energies of 2 to 5 MeV. This shortcoming is described more completely in Ref. 6.

At this point, we split our description of gross results into two main topics: emission of neutrals, i.e., γ rays and neutrons; emission of charged particles.

Gamma Rays and Neutrons

Available data with which we may compare our results include average energies, spectra, cross sections, and total average decay energies; we consider these in the order mentioned.

For γ rays and neutrons, the average energies are $\lesssim$0.96 and 2.1 MeV, respectively, in line with what the

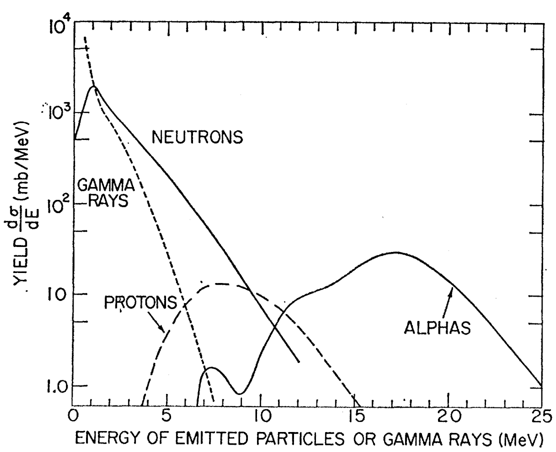

FIG. 2. Calculated summed spectra, including contributions from Dy^{150}, Dy^{151}, Dy^{152}, Dy^{153}, Dy^{154}, Dy^{155}, and Dy^{156}.

meager experimental data lead us to expect for heavy ions incident on medium-heavy nuclei. For example, at incident energies near 110 MeV the systems $Te+C^{12}$ and $Ho+C^{12}$ display average γ-ray energies[65] of 1.1 and 1.2 MeV, respectively, and for the systems $Ta+O^{16}$ and $W+O^{16}$ at incident energies near 85 and 100 MeV, respective average energies[66] of about 0.8 and 1.0 MeV were measured. No gross average neutron-energy measurements seem to be available. However, from the angular distributions measured for the recoiling product nuclei, Alexander and Simonoff[4,67] estimate that the average energy of the first neutron emitted is about 3.6±0.4 MeV for our example system. Effectively, they measure the average total energy carried away by neutrons; then, knowing the number of neutrons emitted, and assuming that the average neutron energy at each step is proportional to the square root of the average excitation energy of the emitting nucleus, they calculate the above value. Our calculation gives 2.8 MeV for this quantity, and is in agreement with the result of Thomas's calculation[68] (4.7 MeV) when the differences in assumed level-density parameter (he used $a=A/12$) and initial excitation energy of Dy^{156} (he used 87 MeV, we use 58.5 MeV) are taken into account. Our average calculated γ-ray energy is an upper limit because of the incompleteness of the calculated γ-ray cascade.

Above about 2 MeV, in Fig. 2, the γ-ray and neutron spectra display roughly exponential behavior, so it is convenient to define "temperatures" T_i as the negative reciprocals of the logarithmic derivatives of the spectra with respect to energy, i.e., $[\partial \ln(\text{spectrum})/\partial E] = -T^{-1}$. We calculate γ-ray temperatures[69] in the range $T_\gamma=0.65$ MeV (near γ-ray energies $E_\gamma\approx 7$ MeV) to 1.0 MeV (near $E_\gamma\approx 3$ MeV), and neutron temperatures in the range $T_n=1.5$ MeV (near neutron energies $E_n\approx 8$ MeV) to 1.8 MeV (near $E_n\approx 4$ MeV). The measured γ-ray temperatures range from about 0.5 MeV (near $E_\gamma\approx 2$ MeV) in the above-mentioned systems[66] $Ta+O^{16}$, and $W+O^{16}$, to 0.8 MeV (near $E_\gamma\approx 3$ MeV) in the system[65] $Ho+C^{12}$ and 1.4 MeV (near $E_\gamma\approx 2.5$ MeV), in the system[65] $Te+C^{12}$. For neutrons, temperatures of about 2.8 and 2.0 MeV (both near $E_n\approx 5$–6 MeV), respectively, have been measured in the system $Au+O^{16}$ at 160 MeV[70] and at 0–142 and 0–164 MeV

[65] J. F. Mollenauer, Phys. Rev. **127**, 867 (1962). The average γ-ray energies reported in this paper may possibly be a little too large. See Ref. 66.
[66] Yu. Ts. Oganesyan, Yu. V. Lobanov, B. N. Markov, and G. N. Flerov, Zh. Eksperim. i Teor. Fiz. **44**, 1171 (1963) [English transl.: Soviet Phys.—JETP **17**, 791 (1963)].
[67] G. N. Simonoff and J. M. Alexander, Phys. Rev. **133**, B104 (1964).
[68] T. D. Thomas, Nucl. Phys. **53**, 577 (1964).
[69] The incompleteness of our γ-ray spectra should not seriously affect the γ-ray "temperature" above 2 MeV.
[70] P. R. Broek, Phys. Rev. **124**, 233 (1961).

(thick targets)[71]; see, however, Refs. 72 and 68. Also, a temperature of about 1.6_5 MeV has been measured in the system $Sn+Ar^{40}$ at 0–170 MeV (thick target).[73] The available data thus suggest that the γ-ray temperatures are indeed lower than the neutron temperatures. The experimental observation of this temperature difference between γ's and neutrons constitutes an important confirmation of the model we are using, as we explain in one of the companion papers.[6] Unfortunately, the γ-ray and neutron spectra have not both been measured for the same system in any case of which we are aware, while our interpretations of some of the above data are not completely unambiguous.

A comparison between the results of our calculation and some quantities interpolated from the experimental data of Alexander and Simonoff[4,67] is given in Table I.

The cross sections in column 4 are calculated assuming that there is no contribution from direct reactions. As a first rough correction, we assume that 0.7 of all interactions proceed through the compound nucleus,[74] while most of the direct reactions lead to products other than Dy isotopes. The corrected values are given in column 5. Intuitively, we expect the direct reactions to reduce the assumed distribution of compound nuclei of high spins relative to low spins, but there seem to be too few experimental data bearing on this point to justify expensive attempts at obtaining a better correction at this time. The experimental cross section for Dy^{151} is smaller than the calculated values in either columns 4 or 5, but is reasonably close; i.e., it is within range of reasonable adjustments of the input data. For example, the smoothed experimental cross section for Dy^{151} goes through 313 mb at 92 MeV, instead of 90 MeV, so that a plausible downward adjustment in a (from 21 to 16 MeV^{-1}) or a small upward adjustment in E_J (of order 1 MeV at $J\approx 15$ to 35), or even smaller changes of both, if both are varied together, will suffice to bring agreement.

Cross sections for the production of γ rays in heavy-ion reactions have been reported, but our calculations are not yet complete enough to allow a full comparison, since we were not able to follow the γ-ray cascades all the way to the ground state in all the involved nuclei. The integrated spectrum from Fig. 2 is 6.7 b, from which we calculate that, on the average, >7.5 γ rays are emitted per compound nucleus formed. A rough correction for the missing γ rays brings this figure to $\gtrsim 11$. In the experimental examples already cited, the average numbers of γ rays per compound nucleus are measured

TABLE I. Comparison of the experimental measurements for the system $Ce^{140}+O^{16}$ (90 MeV, lab) with the corresponding calculated quantities.[a,b]

Product	Reaction	Available energy (MeV)	Cross section[c] (mb)	Cross section[d] (mb)	Average total neutron energy (MeV)	Average total γ energy (MeV)
Dy^{153}	$(O^{16},3n)$	33.0	**7**	**5**	**13.7**	**19.3**
Dy^{152}	$(O^{16},4n)$	25.5	**441**	**309**	**10.4**	**15.1**
					10.0	*15.5*
Dy^{151}	$(O^{16},5n)$	15.7	**447**	**313**	**8.5**	**7.2**
				210±80	*10.2*	*5.5*
Dy^{150}	$(O^{16},6n)$	7.8	**4**	**3**	**5.4**	**2.4**
				<18 (2.5)		

a The values printed in boldface type are calculated; those in italics are the experimental data of Alexander and Simonoff (Refs. 4 and 67).
b For self-consistency, the "experimental" total γ-energy values cited for Dy^{151}, and for Dy^{152} (for which there are no measurements) are read from the smooth curves in Fig. 5 of Ref. 67. The directly measured value for Dy^{151} is 4.4 MeV at an available energy of 15.4 MeV.
c Assuming that all reactions lead through a Dy^{156} compound nucleus.
d Assuming that only 70% of the reactions proceed via a Dy^{156} compound nucleus.

to be 11 and 14, respectively, for the systems $Te+C^{12}$ and $Ho+C^{12}$, and 7 to 9 for the systems $Ta+O^{16}$ and $W+O^{16}$.

We calculate that 17% of the γ rays are emitted as quadrupole radiation, although the ratio of dipole- to quadrupole-emission widths in the input was "normal," i.e., $\Gamma_{\gamma 1}/\Gamma_{\gamma 2}=440$. This is in line with Mollenauer's evidence[65] that a large proportion of the γ-ray emission from systems involving incident 45-MeV helium ions is quadrupole radiation. Our results are not, strictly speaking, in agreement with his observations, but within the uncertainties of his experimental error, considering the difficulties in the interpretation of his data, and bearing in mind that our calculations are for a different system, we believe that "enhanced" quadrupole emission is a significant prediction of the model (see Ref. 6).

The total average energy carried away by γ rays can, at present, be calculated only approximately by our program, and these values are given in Table I, column 7. We believe these numbers are accurate to within about 0.3 MeV, and thus are useful for comparisons. We intend to compute more accurate values in the future. The experimental data of Simonoff and Alexander,[67] also quoted in Table I, are gratifyingly close to the calculated figures, and can evidently be brought into closer agreement by small adjustments of a and/or E_J which are in the same direction as the adjustments which would bring agreement with the cross sections. Indeed, the data of Simonoff and Alexander reach $T_\gamma=7.2$ MeV in the production of Dy^{151} at 92 MeV, according to Fig. 5 of their paper. The analogous counter experiments yield results that seem somewhat equivocal, and do not allow a satisfying comparison to be made. Of course, these data are for systems other than our example, and include the emissions from the excited products of charged-particle emission. In the systems[65] $Te+C^{12}$ and $Ho+C^{12}$ at ≈ 110 MeV, the total excitation energy carried away by γ rays is measured to be 12.2 and 17.0 MeV, respectively, and

71 W. G. Simon, University of California Radiation Laboratory Report No. UCRL-11088 (unpublished).
72 Fission is an important competing reaction in the $Au^{197}+O^{16}$ and similar systems (see Refs. 68 and 74), so the particle spectra reported here may well have important contributions from excited fission products, i.e., from nuclei with $A\approx 100$.
73 H. Kumpf, L. Kumpf, and S. Shuang-hui, Yadern. Fiz. 1, 264 (1965) [English transl.: Soviet J. Nucl. Phys. 1, 186 (1965)].
74 T. Sikkeland, Phys. Rev. 135, B669 (1964).

from the systems[66] Ta+O^{16} and W+O^{16} at ≈85 and 100 MeV, it is 6 to 8 MeV.

Charged Particles

Experimental spectra for protons and α's emitted from systems somewhat similar to our example have been published by the Yale group.[75,76] In all cases, these spectra have their maxima near the Coulomb barrier, and at energies well above the Coulomb barrier they display temperatures about the same as the neutron temperatures from the same systems.[70] These features are also true of our calculated spectra.

Below the Coulomb barrier, the calculated α spectrum has interesting structure, the origin of which we explain in a companion paper.[7] We might hope to see these structures especially in the Yale data on gold and bismuth, and indeed the sub-barrier portions of their published α spectra[76] are not inconsistent with our calculation; in their system Au+C^{12} at 126 MeV, the spectrum at 5 MeV below the maximum is still 0.2 of the maximum value, compared to our calculated value of 0.3 of the maximum value at 5 MeV below the maximum, as shown in Fig. 3. In the absence of the special effects giving rise to the sub-barrier contributions, the spectrum should drop to about 0.05 of the maximum value at 5 MeV below the maximum. The Yale data do not quite extend to low enough energies, however, to be able to show us whether there is a low-energy maximum, which we would predict to appear at about 11 MeV in this case.[7] Unfortunately, these sub-barrier data do not quite seem to be of good enough quality to allow us to draw conclusions from a comparison with detailed calculations. In addition, there is much fission in the systems they studied,[68,72,74] and how this affects the α spectrum is yet to be learned. In another experiment,[77] the sub-barrier α spectrum arising from 167-MeV O^{16} bombardment of gold has indeed been observed at somewhat smaller energies than those reported by the Yale group. Here, the gold target was thick enough to stop the incident O^{16} ions, and the spectrum of emitted α's was measured at 0° with respect to the beam. The low-energy portion of the spectrum is thus distorted and smeared out because the outgoing α particles must burrow through gold metal of thickness ≈0 up to 112 mg/cm². In particular, 20-MeV α's born at the leading face of the target would be almost stopped. Thus, although the information we seek is present in these data, the obstacles in the way of extracting it have so far discouraged us from considering this work further.

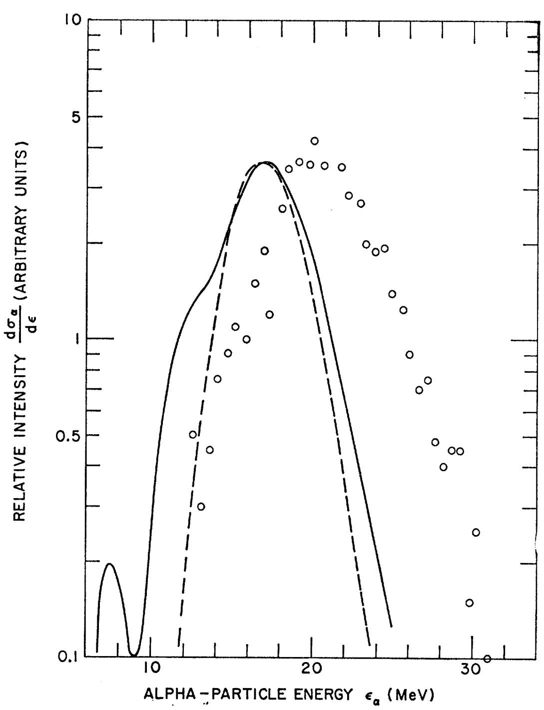

FIG. 3. Qualitative comparison of α-particle spectra calculated in this study for the reacting system Ce^{140}+O^{16} at 90 MeV (lines) with the α-particle spectrum measured for the system Au^{197}+C^{12} at 126 MeV (points) (Ref. 76). The solid line represents the summed contributions for all dysprosium isotopes $A=150$ to 156. The dashed line is the contribution from Dy^{154} only, which is formed with too much excitation energy to display the subbarrier structure caused by high angular momentum (Ref. 7).

The total calculated cross sections for the formation of protons and α particles, by emission from the nuclei μ only, are 73 and 200 mb, respectively. The emission of other charged particles, such as deuterons, tritons, and He^3 nuclei, was not included in these calculations, but their binding energies are so high that it is clear that their cross sections are much smaller than that for the protons. Thus, those compound nuclei which lead to neutron emission only, represent 0.77 of the total reaction cross section, assuming that all the reactions proceed by compound-nucleus formation. This result may be compared with the data correlation of Alexander and Simonoff[4] (see their Fig. 2) which would predict that near 58.5 MeV of excitation energy the fraction of the estimated reaction cross section[78] leading only to neutron emission is about 0.78. Remembering that possibly only about 70% of the reaction cross section may lead through a compound nucleus, and that the estimated reaction cross section is somewhat uncertain, there seems to be reasonable agreement between calculation and experiment, although within broad limits. It would be most useful here to have some cross sections, recoil ranges, and angular distributions also for production of gadolinium and terbium isotopes, to provide information about proton and α-particle emission from the excited dysprosium nuclei.

[75] W. J. Knox, A. R. Quinton, and C. E. Anderson, Phys. Rev. **120**, 2120 (1960).
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[77] D. V. Reames, Phys. Rev. **137**, B332 (1965).
[78] T. D. Thomas, Phys. Rev. **116**, 703 (1959).

Klingen and Choppin[79] published data on the systems $Te^{128}+C^{12}$ and $Te^{130}+C^{12}$, which they interpret as indicating that a very large fraction of the reaction cross section appears as products involving α-particle emission. We would be unable to explain their results with our calculation because the relevant α-particle binding energies are considerably larger in their system[41] (where the compound nuclei are Ce^{140} and Ce^{142}) than they are for $Dy^{156-150}$. Furthermore, we note that the full width at half-maximum for their excitation function for the reaction $Te^{130}(C^{12},\alpha 3n)Ba^{135m}$ is 7 MeV, while the neutron binding energy in Ba^{135} is 7.2 MeV, and in Ce^{139} is 7.5 MeV.[41] Since, in a compound-nucleus mechanism, the excitation-function peaks must be at least as wide as the relevant (usually the neutron) binding energy plus the average emission energy for the corresponding emitted particle, and since the influences of high angular momentum and of direct interactions serve to widen the peaks still more, we find it difficult to understand how such narrow peaks as Klingen and Choppin observe could come about. In any case, they cannot be due to the compound-nucleus mechanism that our computer program deals with.

In Table II, we exhibit a breakdown of the charged-particle production cross sections into contributions from the individual emitting species μ.

For protons, these results can be compared semi-quantitatively with the data of Gilat and Sisson,[80] who measured cross sections for production of the shielded nuclides Eu^{146} and Eu^{148} in the systems $La^{139}+N^{14}$, $Ba^{136}+O^{16}$, and $Ce^{140}+C^{12}$ at several bombarding energies. As can be seen from Table II, the gross proton/neutron emission ratio is roughly a constant $\approx 1.7\times 10^{-2}$ at each step of the cascade. In addition, we may assume that the population that results from proton emission will proceed to decay mainly by the emission of neutrons (and γ-rays) so that the probability of emitting two protons in any given cascade is very small. Thus, the experimental cross section for a (HI,pxn) product (where HI stands for "heavy ion") is approximately the sum of the contributions of proton emission at each step of the cascade, so that the total proton cross section should increase with increasing excitation energy. Experimentally, the peak cross section for $La^{139}(N^{14},p4n)Eu^{148}$ (at ≈ 70 MeV of excitation energy) is about 30 mb, or $\approx 2\%$ of the total reaction cross section. For $La^{139}(N^{14},p6n)Eu^{146}$ and $Ba^{136}(O^{16},p5n)Eu^{146}$ (at ≈ 94 and 87 MeV of excitation energy, respectively) the peak cross sections reach about 150 mb, or $\approx 9\%$ of the total reaction cross section.[80] From Table II, we would naively predict the summed proton cross section, which is about 6.2% of the total reaction cross section at 58.5 MeV of excitation, to rise to about 10% of the total reaction cross section at ≈ 90 MeV of excitation energy in our sample system, while the experimental data[80] indicate values near 15–20% for the systems involving $Gd^{152,153}$ compound nuclei. Bearing in mind the important differences between the systems under comparison (e.g., differences in proton binding energies,[41] in proximity to the 82-neutron closed shell,[15] etc.), this must be regarded as satisfactory agreement.

TABLE II. Partial cross sections for formation of charged particles, in the system $Ce^{140}+O^{16}$ at 90 MeV (lab).

Emitting nucleus	σ_p[a] (mb)	σ_p/σ_n	σ_α (mb)	σ_α/σ_n
Dy^{156}	18.8	1.71×10^{-2}	53.3	4.84×10^{-2}
Dy^{155}	19.0	1.85×10^{-2}	49.8	4.83×10^{-2}
Dy^{154}	15.0	1.53×10^{-2}	35.2	3.58×10^{-2}
Dy^{153}	12.8	1.37×10^{-2}	27.9	3.00×10^{-2}
Dy^{152}	7.1	1.58×10^{-2}	33.0	7.30×10^{-2}
Dy^{151}	0.3	7.02×10^{-2}	0.7	17.50×10^{-2}
Sum	73		200	

[a] σ_p, σ_n, σ_α mean cross sections for production of protons, neutrons, and alpha particles, respectively, from the emitting nucleus in column one.

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