n-d Total Cross Section, Three-Nucleon Problem, and the Dineutron*

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The *n-d* total cross section was measured at 241 different energies to a precision of 5.7–1.0% and an energy resolution of 2.0–5.1% for $2.25 \leq E_n \leq 15.0$ MeV. The results support the correctness of present three-body calculations, and exhibit no evidence for a Wigner cusp near the predicted threshold for dineutron production. In the absence of such a cusp, an experiment of this type furnishes very little information regarding the existence of the dineutron, or of violations of charge independence.

I. INTRODUCTION

RECENTLY, Aaron, Amado, and Yam[1] made exact three-body calculations, using separable two-body interactions, on the *n-d* system for neutron energies of 0–14 MeV. The properties of the *n-d* system were successfully derived in terms of the properties of the *n-p* system. The *n-p* singlet and triplet scattering lengths, the *n-p* singlet effective range, the deuteron binding energy, and a factor relating the fraction of the time that the deuteron is not in an *S*-wave triplet state were used as the parameters of the two-body interactions. The calculated parameters of the *n-d* system are given in Table I, which exhibits the good agreement with experiment. In addition, the agreement between the calculated differential cross sections and experimental values was encouraging.

In another recent calculation, Phillips[2] considered the three-body system using the multiple-scattering theory of Faddeev and charge independence. The approximations of the theory included the use of a nonlocal separable potential to describe the nucleon-nucleon interaction over the energy range 0–22 MeV, and a phenomenological three-body force to represent the effects on the *n-d* system of the nucleon-nucleon tensor and short-range interactions. The singlet scattering length and effective range were taken to be averages of the *n-n* and *n-p* scattering lengths and effective ranges. The triplet scattering length, the deuteron binding energy, and a factor describing the two-nucleon tensor and short-range interactions were also used to describe the exact two-body interactions. The calculated *n-d* scattering lengths and triton binding energy are included in Table I, which shows good agreement with experiment. The agreement between the calculated differential scattering cross sections and experimental values was also favorable. However, all of these calculations showed poor agreement with the experimental *n-d* total cross sections over the entire energy range of 1 to 15 MeV. The experimental results over this range are dominated by the widely quoted results of Seagrave and Henkel.[3] Needless to say, the above theories should correctly predict the total cross section, which is the most unambiguous of the neutron cross sections.

TABLE I. Summary of theoretical and experimental results for the *n-d* system. The experimental results are those from Ref. 26. The scattering lengths for the doublet and quartet spin states are designated by a_d and a_q, while B.E.(t) is the binding energy of the triton.

	Phillips	Aaron *et al.*	Experimental	Percentage difference between theory and experiment: Phillips	Aaron *et al.*
a_d	0.7 F	0.7 F	0.7 ± 0.3 F	⋯	⋯
a_q	6.28 F	6.20 F	6.38 ± 0.06 F	1.6	2.8
B.E.(t)	9.1 MeV	8.53 MeV	8.49 MeV	7.2	0.5

II. THE DINEUTRON

The anomalous behavior of scattering and reaction cross sections at the thresholds of competing reactions was first investigated by Wigner.[4] There have been a number of papers on the subject,[5–15] since the pioneering work of Wigner, which have advanced the description of the phenomenon.

It has been suggested[16,17] that threshold effects such as a cusp or rounded step might appear in the *n-d* elastic-scattering and total cross section at the $H^2(n,n_2)H^1$ threshold if the dineutron (n_2) exists in a bound state. Since the exclusion principle prevents two neutrons from occupying a $^3S_1$ state, the forces acting in the S state of the dineutron cannot be compared with the

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[1] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. **140**, B1291 (1965).
[2] A. C. Phillips, Phys. Rev. **142**, 984 (1966).
[3] J. D. Seagrave and R. L. Henkel, Phys. Rev. **98**, 666 (1955).
[4] E. P. Wigner, Phys. Rev. **73**, 1002 (1948).
[5] R. H. Capps and W. G. Holladay, Phys. Rev. **99**, 931 (1955).
[6] G. Breit, Phys. Rev. **107**, 1612 (1957).
[7] A. I. Baz', Zh. Eksperim. i Teor. Fiz. **33**, 923 (1957) [English transl.: Soviet Phys.—JETP **6**, 709 (1958)].
[8] R. G. Newton, Ann. Phys. (N.Y.) **4**, 29 (1958).
[9] R. G. Newton, Phys. Rev. **114**, 1611 (1959).
[10] R. K. Adair, Phys. Rev. **111**, 632 (1958).
[11] M. E. Ennis and A. Hemmendinger, Phys. Rev. **95**, 772 (1954).
[12] P. R. Malmberg, Phys. Rev. **101**, 114 (1956).
[13] H. W. Newson, R. M. Williamson, K. W. Jones, J. H. Gibbons, and H. Marshak, Phys. Rev. **108**, 1294 (1957).
[14] W. E. Meyerhof, Phys. Rev. **128**, 2312 (1962).
[15] W. E. Meyerhof, Phys. Rev. **129**, 692 (1963).
[16] A. I. Baz' and Ya. A. Smorodinski, *Comptes Rendus du Congrès International de Physique Nucléaire; Interactions Nucléaires aux Basses Energies et Structure des Noyaux, Paris, July, 1958* (Dunod Cie., Paris, 1959), p. 579.
[17] L. Fonda, Nuovo Cimento Suppl. **20**, 116 (1961).

forces binding the $^3S_1$ state of the deuteron, but must be compared with those acting in the $^1S_0$ state of the n-p system. The latter state is unbound; however, it is so close to being bound that a modest increase in the n-n interaction over the corresponding n-p interaction might lead to a bound state of the dineutron.

Alzetta, Ghirardi, and Rimini[18] have evaluated the magnitude and shape of the threshold effect in n-d scattering, assuming the dineutron to exist. The processes considered were $n+d\rightarrow n+d$ and $n+d\rightarrow p+n_2$. The threshold effect in the n-d total scattering cross section is due to the S wave of the $p+n_2$ channel, which can only be a pure $^2S_{1/2}$ state.

Alzetta *et al.* used a formulation in which the n, d, p, and dineutron interact through a separable interaction. The total Hamiltonian is given by

$$H=H_0+H_1,$$
$$H_0=\sum_{i=1}^{2}\int d^3p\left[\frac{p^2}{2\mu_i}+E_i\right]|i\mathbf{p}\rangle\langle i\mathbf{p}|, \tag{1}$$
$$H_1=-\sum_{i,j=1}^{2}\int\int d^3pd^3p'\ |i\mathbf{p}\rangle f_i(p)f_j(p')\langle j\mathbf{p}'|,$$

where the summation indices 1 and 2 indicate the $n+d$ and $p+n_2$ channels, respectively, μ_i is the reduced mass in the ith channel, E_i is the binding energy corresponding to the ith threshold, and $|i\mathbf{p}\rangle$ is the state of the ith nucleon pair with relative momentum $\mathbf{p}$. The Hulthen-type form factors $f_i(p)$ were chosen to be

$$f_i(p)=\alpha_i/(p^2+\beta_i^2), \tag{2}$$

where α_i and β_i are the adjustable parameters of the model. Qualitatively, the β_i's are the inverse ranges of the separable interactions.

The S-wave doublet n-d scattering cross sections were calculated over the energy region 0–2 MeV using the phase shifts of Adair, Okazaki, and Walt.[19] The α_i were evaluated in terms of the β_i and the binding energies of the deuteron, triton, and dineutron, using a doublet n-d scattering length of 8.26 F. The evaluation of β_1 follows from the size of the deuteron, while β_2 is adjustable. The model predicts a rounded step with a width of 200–300 keV and a height of 15–23% of the total scattering cross section for dineutron binding energies of 50, 150, and 300 keV.

The possible low-energy reactions resulting from the neutron bombardment of deuterium are

1. $n+d\rightarrow n+d$,
2. $n+d\rightarrow t+9.3865$ MeV (lab),
3. $n+d\rightarrow n+n+p-3.3389$ MeV (lab),
4. $n+d\rightarrow p+n_2-3.3389$ MeV (lab)$+1.5008$ B.E.(n_2),

where the mass tables of Mattauch, Thiele, and Wapstra[20] were used to calculate the reaction energies. Up to the three-body threshold (3.3389 MeV in the laboratory system), the contributions to the total cross section originate from reactions 1 and 2; that is,

$$\sigma_{\mathrm{T}}=\sigma_{\mathrm{el}}+\sigma_{\mathrm{c}}=\tfrac{1}{3}\sigma_{\mathrm{d}}+\tfrac{2}{3}\sigma_{\mathrm{q}}+\sigma_{\mathrm{c}}, \tag{3}$$

where σ_{T}, σ_{el}, and σ_{c} are the total, total-elastic-scattering, and capture cross sections, respectively. The elastic-scattering cross sections corresponding to the doublet and quartet spin states are designated by σ_{d} and σ_{q}, respectively. Since the capture cross section is so small (0.5 ± 0.1 mb at thermal energies, 29.4 μb at 14.4 MeV), it may be neglected and

$$\sigma_{\mathrm{T}}\approx\sigma_{\mathrm{el}}=\tfrac{1}{3}\sigma_{\mathrm{d}}+\tfrac{2}{3}\sigma_{\mathrm{q}}. \tag{4}$$

If the dineutron exists, the effect in the total cross section is

$$\sigma_{\mathrm{T}}(\text{effect})\approx\tfrac{1}{3}[^2\sigma_0(\text{effect})+{}^2\sigma_1]+\tfrac{2}{3}[^4\sigma_0+{}^4\sigma_1]+\sigma_{\mathrm{r}}, \tag{5}$$

where $^2\sigma_0$(effect), $^2\sigma_1$, $^4\sigma_0$, and $^4\sigma_1$ are the S- and P-wave scattering cross sections for the doublet and quartet spin states, respectively, and σ_{r} is the cross section for the reaction $n+d\rightarrow n_2+p$.

Since a bound state of the dineutron is prohibited by the assumption of charge independence of nuclear forces (neglecting the weak moment-moment interaction), the observation of a threshold effect in the total cross section would be of extreme importance to the nuclear-force problem.

III. EXPERIMENTAL PROCEDURE

The precision, accuracy, and resolution of our measurements[21] of the structured neutron total cross sections of Be, C, N, O, Na, Mg, Si, P, S, and Ca have convinced us that we could improve on the existing measurements of the n-d total cross section and detect a threshold effect of the magnitude predicted by Alzetta *et al.* if such an effect exists.

Our pulsed-beam, time-of-flight technique for measuring $\sigma_{\mathrm{T}}(E)$ by way of a transmission experiment has been described in full detail[21]; therefore, only a summary will be given here. Deuterium ions were accelerated in a 2-MV Van de Graaff accelerator which was equipped with a post-acceleration beam-pulsing system to produce 1.5-nsec bursts of ions on a thick target of natural Li. The bombardment of natural Li with 2-MeV deuterons produces a continuous spectrum of neutrons spanning the energy range 1.8–15.0 MeV. The continuum of neutron energies which results from the highly exoergic reaction is due to the great width of the first two excited states in Be^8, the presence of continua from such three-body processes as $Li^7(d,n)2\alpha$, and the use of a thick target.

[18] R. Alzetta, G. C. Ghirardi, and A. Rimini, Phys. Rev. **131**, 1740 (1963).

[19] R. K. Adair, A. Okazaki, and M. Walt, Phys. Rev. **89**, 1165 (1953).

[20] J. H. E. Mattauch, W. Thiele, and A. H. Wapstra, Nucl. Phys. **67**, 1 (1965).

[21] D. G. Foster, Jr., and D. W. Glasgow, Nucl. Instr. Methods **36**, 1 (1965).

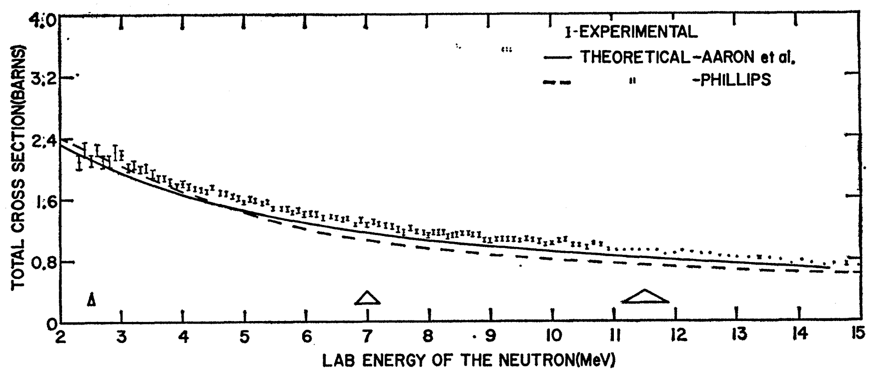

FIG. 1. Experimental and theoretical n-d total cross sections. The error limits are the standard deviations. The error limits are the size of the points over the energy range 11–14.5 MeV. The triangles have the same width at half-maximum as the observed resolution function.

Strict precautionary measures were taken to suppress the background produced by the externally pulsed deuterium-ion beam. A beam-alignment and focusing system was used to produce a beam spot less than 3 mm in diameter on the target and to confine the ions to the axis of the beam tube. The ions were allowed to strike only the target and certain strategically placed tantalum slits. In addition, the detector was encased in lead and mounted in a shielded room whose concrete walls and ceiling were 1.2-m thick. The target assembly was viewed by the detector through a collimator which pierced a shield of water 1.2-m thick. Pulse-shape discrimination was used in the detecting system to reduce further the effects of spurious radiation.

The detector consisted of type NE–213 liquid organic scintillator contained in a glass cell, which was coupled to an Amperex 58–AVP photomultiplier. The electronics consisted of a fast start-stop system coupled to a circulating-line vernier chronotron, a memory and control system, a beam-current monitor system, and a system that provided a check on the stability of the entire experiment.

The sample was a 2.5 cm$\times$2.5 cm$\times$10 cm parallelepiped of deuterated polyethylene enriched to 94.7 at.% in deuterium. The length of the sample was such as to give about 17% transmission. The effect of carbon and hydrogen impurities in the sample was corrected for by using a blank in the open-beam measurement which duplicated the areal density of the impurities. The effect of neutrons not coming directly from the source was evaluated by placing a 30-cm long iron shadow bar between the source and the detector. The sample, blank, and shadow bar were supported by a lightweight steel frame. The frame was suspended from a remotely operated assembly by stranded stainless-steel wires and was positioned vertically and horizontally with a reproducibility of ± 0.2 mm. The sample was positioned 86 cm from the source so that it barely obscured the source from the detector, which was located 6.16 m from the source. This configuration of source-sample and source-detector separations minimizes inscattering from the sample. The inscattering correction, which was calculated to be much less than 0.1%, was ignored in the analysis of the data.

The data were taken in cycles consisting of blank, sample, and background, with a counting-time ratio of 5:20:2, respectively. The various runs were normalized to the same number of source neutrons by measuring the charge collected on the Li target only during the time that the vernier chronotron was not analyzing and storing the consequences of an event. The form of the resolution function and the relativistic energy-scale calibration were established in ancillary experiments which utilized the spectrum of neutrons from the $Be^9(d,n)B^{10}$ reaction, the spectrum of neutrons and associated γ rays from the $Li(d,n)$ reaction, and the differential channel-width spectrum. The total cross sections were determined in two individual experiments separated by a number of months in order to observe the reproducibility of the data.

The raw data were reduced to final form by a succession of computer programs, written in FORTRAN IV for a Univac 1107 computer. In the course of the first step, which processes data taken under identical experimental conditions, the program corrects for small changes in the zero point of the time-of-flight scale by a method which introduces small correlations among neighboring points. Thus, there are off-diagonal elements in the covariance matrix which describes the statistical uncertainties in the cross sections. The square root of an element on the main diagonal of the matrix is identical with the standard deviation. Later members of the set of programs take full account of the covariance in combining the results of runs taken under different conditions, correcting them for impurities and other sources of error, and comparing the results with those obtained at other laboratories. The total cross section was determined at 241 different energies to a precision of 5.7–1.0% and an energy resolution of 2.0–5.1% for $2.25 \leq E_n \leq 15.0$ MeV.

Our results are shown in Fig. 1. For the sake of clarity, not all of the data points are plotted at lower energies. The error limits are the standard deviations; however, the presence of covariance should be borne in mind. Typical resolutions are shown by the full width of the triangles at half-maximum. The theoretical curves are discussed in Sec. IV.

Table II summarizes comparisons of our work with

TABLE II. Comparison of results with previous work.

Year	Reference	Number of points	Energy range (MeV)	Difference (%)
1946	a	8	2.6 -6.0	−0.8
1951	b	2	4.5 -5.5	+2.1
1953	c	2	2.60–2.96	−1.4
1955	d	12	2.50–8.13	+5.6
1958	e	2	7.17–8.77	+1.2
1964	f	31	2.79–3.57	+0.4

[a] R. G. Nuckolls, C. L. Bailey, W. E. Bennett, T. Bergstralh, H. T. Richards, and J. H. Williams, Phys. Rev. **70**, 805 (1946).
[b] E. Wantuch, Phys. Rev. **84**, 169 (1951).
[c] See Ref. 19.
[d] See Ref. 3.
[e] A. Bratenahl, J. M. Peterson, and J. P. Stoering, Phys. Rev. **110**, 927 (1958). Comparison is made only with the two points of lowest energy from this reference. The deuterium sample used in our work was the same as that used by these authors.
[f] See Ref. 22.

earlier results obtained at other laboratories. This summary should be viewed with caution, since it conceals some much larger discrepancies at particular energies. Nonetheless, the table shows good agreement with all previous work except the widely quoted results of Seagrave and Henkel,[3] which averaged nearly 6% higher than our own values in the same energy range. In particular, the agreement with Willard *et al.*[22] is excellent.

IV. COMPARISON WITH THREE-BODY CALCULATIONS

The total cross sections calculated by Aaron, Amada, and Yam[1] and by Phillips[2] are both shown in Fig. 1, together with our measured values. In both cases, the agreement with experiment is much better than originally reported by those authors, because they relied primarily on the data of Seagrave and Henkel, which Table II suggests are systematically about 6% too high.

The theoretical calculations of Phillips, which explicitly assume charge independence, agree best with our data in the vicinity of the three-body threshold. (See Fig. 2.) The calculations of Aaron *et al.*, which do not explicitly assume charge independence, agree best with our data over the energy range 4.8 to 15.0 MeV.

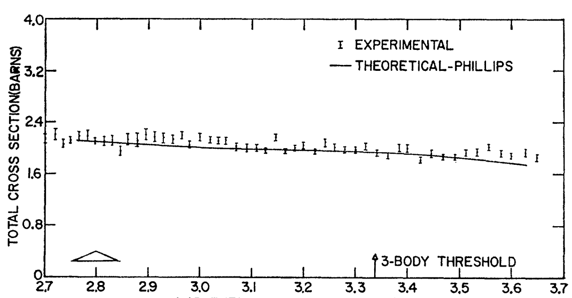

FIG. 2. Experimental and theoretical *n-d* total cross sections in the vicinity of the three-body threshold. The error limits are the standard deviations. The triangle has the same width at half-maximum as the observed resolution function.

[22] H. B. Willard, J. K. Bair, and C. M. Jones, Phys. Letters **9**, 339 (1964).

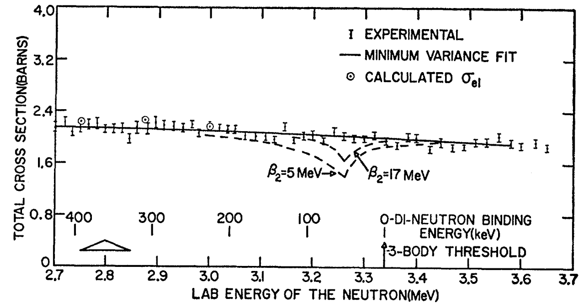

FIG. 3. The *n-d* total cross section in the vicinity of the three-body threshold. The predicted cusps are shown as dashed curves for a dineutron binding energy of 50 keV and inverse range β_2 of the $p-n_2$ interaction. The elastic-scattering cross sections were calculated using the phase shifts of Adair *et al.* (Ref. 19). The error limits and resolutions are the same as in Fig. 2.

This better agreement at higher energies probably reflects the degree to which higher-momentum components were considered in the two-body interaction. Both theories are in reasonable agreement with our experimental results at low energies, as they should be, since they both give the correct scattering lengths as $E \rightarrow 0$. It is apparent that the over-all agreement between the experimental and theoretical results supports the correctness of using separable two-body interactions in the quantum-mechanical three-body problem.

V. THE EXISTENCE OF THE DINEUTRON

The calculations of Alzetta, Ghirardi, and Rimini[18] show that, under a specific assumption regarding the scattering lengths which we shall discuss below, the neutron total cross section of deuterium provides a critical test for the existence of the dineutron, since the threshold for production of the dineutron would be marked by a prominent Wigner cusp. Willard, Bair, and Jones[22] have explored the threshold region experimentally and failed to find such a cusp, and Thornton *et al.*[23] have remarked that this *essentially disproves* the existence of the dineutron.

It is now clear that the specific assumption concerning the scattering lengths is incorrect, and therefore that the absence of a prominent cusp was to be expected, regardless of whether or not the dineutron exists. If a cusp had been observed, however, it would indeed have been conclusive proof of the existence of the dineutron, and would have constituted a major contradiction to the current theoretical picture of the three-nucleon system.

For the sake of completeness, let us first give the discussion of our results which parallels that of Willard, Bair, and Jones. The lower limit of zero for the dineutron binding energy corresponds to the three-body threshold at 3.3389 MeV in the laboratory system. The upper

[23] S. T. Thornton, J. K. Bair, C. M. Jones, and H. B. Willard, Phys. Rev. Letters **17**, 701 (1966).

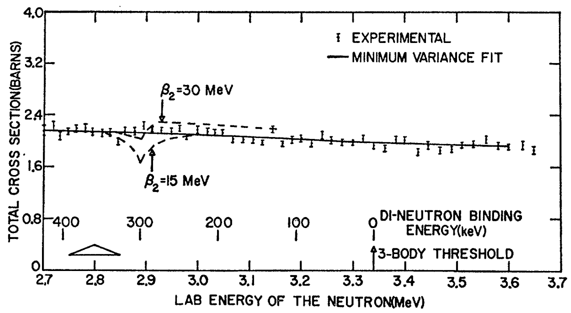

FIG. 4. The n-d total cross section in the vicinity of the three-body threshold. The predicted cusps are shown as dashed curves for a dineutron binding energy of 300 keV and inverse range β_2 of the $p-n_2$ interaction. The error limits and resolution are the same as in Fig. 2.

limit on the dineutron binding energy[24] may be established by observing that the allowed and favored β decay

$$_2\mathrm{He}^6 \rightarrow {}_3\mathrm{Li}^6 + e^- + \bar{\nu} + 3.51\ \mathrm{MeV}, \quad T_{1/2} = 0.81\ \mathrm{sec},$$

proceeds according to Gamow-Teller selection rules and not by the mechanism

$$_2\mathrm{He}^6 \rightarrow {}_2\mathrm{He}^4 + n_2 - 0.9635\ \mathrm{MeV}.$$

This upper limit on the binding energy corresponds to a neutron energy of 1.8932 MeV in the laboratory system. The indirect measurement of the n-n interaction by Phillips *et al.*[25] indicates that a dineutron binding energy >50 keV is $<0.1\%$ probable; consequently, the predicted threshold effect should be in the vicinity of 3.26 MeV.

The portion of our results which lies between 2.70 and 3.65 MeV is plotted in Figs. 3 and 4. This interval covers the region from 300 keV above the three-body threshold down to a dineutron binding energy of more than 400 keV. The error limits shown are the standard deviations, but the presence of covariance should be borne in mind. Typical resolutions (FWHM) were approximately 50 keV. The dashed curves show the cusp expected under various assumptions, and will be discussed below.

Although it is clear from Figs. 3 and 4 that the predicted cusp is absent, the data are not absolutely free from anomalies. We have studied the internal consistency of our results by attempting to fit a smooth minimum-variance curve to the cross sections; in particular, the form chosen was a second-degree polynomial. Since our method of data reduction introduces correlations among neighboring points, there is a tendency for chance errors to simulate weak fine structure. The method of fitting is a generalization of the usual least-squares procedure, and a few details are given in the Appendix.

A fit to 49 points between 2.69 and 3.58 MeV gave a value of χ^2 equal to 52.5, compared to the expected value $n-3=46\pm9.6$. Although this is quite satisfactory from a statistical point of view, the region between 2.90 and 3.05 MeV is systematically above the fitted line, and the region from 3.05 to 3.20 MeV is systematically below the fitted line except for an isolated point at 3.15 MeV. An independent fit to the critical region from 3.10 to 3.40, indeed, gave a χ^2 of 21.0, compared to the expected value of $n-3=13$. However, there is a 7% probability of getting a larger value of χ^2 by chance, and the resulting fit yields a cross section at 3.15 MeV which is only 1% less than the fit to the entire region from 2.69 to 3.58 MeV. We conclude, therefore, that no significant cusp is present.

The calculated curves in Figs. 3 and 4 were computed by the method of Alzetta *et al.*,[18] which we have outlined in Sec. II above. Clearly, for the expected small binding energy of the dineutron the cusp is extremely prominent, although its prominence diminishes rapidly with increasing binding energy. The crucial assumption in this calculation is that the set of phase shifts proposed by Adair, Okazaki, and Walt[19] correctly describe the interaction. The scattering measurements of Adair, Okazaki, and Walt did not actually determine a unique set of phase shifts, so they resolved the ambiguity by using the zero-energy scattering lengths. Unfortunately, the low-energy scattering measurements[26] give two alternative sets of scattering lengths, which are listed in Table III. Set 1 is the set originally favored and was the set adopted by Adair *et al.* in determining their phase shifts, and thus subsequently by Alzetta *et al.* and then by Willard *et al.* This set is characterized by a large doublet scattering length and a small quartet scattering length, and it is the large doublet length which yields the large cusp in Figs. 3 and 4 if the dineutron exists. The downward cusp is in agreement with the study by Fonda.[17]

However, Alfimenkov *et al.*[27] have now shown conclusively, by their measurements of the transmission of polarized neutrons through a polarized deuterium target, that it is set 2 which is correct, so that the quartet scattering length is large and the doublet scattering length is small. This conclusion agrees with the calculations of Aaron *et al.* and of Phillips which we have discussed in Secs. I and IV. If we repeat the calcu-

TABLE III. Alternative sets of S-wave scattering lengths of the neutron-deuteron system.

Set	Doublet scattering length (F)	Quartet scattering length (F)
1	8.26 ± 0.12	$2.6\ \pm0.2$
2	$0.7\ \pm0.3$	6.38 ± 0.06

[24] N. Feather, Nature **162**, 213 (1948).
[25] R. H. Phillips and K. M. Crowe, Phys. Rev. **96**, 484 (1954).
[26] D. Hurst and N. Alcock, Can. J. Phys. **29**, 36 (1951).
[27] V. P. Alfimenkov, V. I. Lushchikov, V. G. Nikolenko, Yu. V. Taran, and F. L. Shapiro, in Proceedings of the International Conference on Nuclear Physics, Gatlinburg, Tennessee, 1966, Abstract 9.7, p. 85 (unpublished).

lation of Alzetta *et al.* using set 2 instead of set 1, the Wigner cusp is found to be *undetectably small*, regardless of whether or not the dineutron exists. Consequently, our observations *do not disprove* the existence of the dineutron.

Similarly, other recent efforts to disprove the existence of the dineutron have somewhat overstated the case. For example, Baz' *et al.*[28] conclude that the amplitude of the Wigner cusp in the total cross section would be of the order of 0.001%, thus placing it altogether beyond practical detection. However, their assumption of a production cross section of the order of 10 μb is based on a misinterpretation of the work of Katase *et al.*,[29] who failed to find evidence for dineutron production via the reaction

$$d+\mathrm{H}^3 \rightarrow n_2+\mathrm{He}^3-2.989\ \mathrm{MeV}+\mathrm{B.E.}(n_2).$$

We have pointed out above that the binding energy of the dineutron is almost certainly less than 50 keV. Thus, the threshold for this reaction is greater than 2.9 MeV, whereas Katase *et al.* attempted to detect it at a bombarding energy of only 0.2 MeV.

Schiffer and Vandenbosch[30] attempted to detect dineutrons produced during fission, using the proposed reaction $\mathrm{Al}^{27}(n_2,p)\mathrm{Mg}^{28}$ as a detector, and set an upper limit of 10^{-9} dineutrons/fission. Baz' *et al.* have used this limit to set an upper limit of 1 to 10 μb for the production cross section for dineutrons in the interaction of light nuclei. The limit set by Schiffer and Vandenbosch assumes a cross section for the $\mathrm{Al}^{27}(n_2,p)\mathrm{Mg}^{28}$ reaction comparable to the capture cross section for the neutron. Because of the weak binding and correspondingly large separation of the two neutrons, however, it is much more probable that the dineutron will be stripped of one of its neutrons, instead of being captured altogether, so that the production of Mg^{28} is highly improbable. From a crude geometrical model, we estimate that the capture cross section might be only 1% of that assumed by Schiffer and Vandenbosch, with a corresponding decrease in the sensitivity of their detector.

The limit deduced by Baz' *et al.* from the work of Schiffer and Vandenbosch assumes that the interaction cross section of the dineutron with light nuclei is comparable to that observed in fission. This argument also is open to question. The yield of light particles in ternary fission is largely statistical in nature, and is governed mainly by the energetics[31] rather than by the interaction cross sections of the emerging light particles. If the formation cross section for the dineutron is small, this will be balanced by a correspondingly small probability for breakup before the dineutron escapes from the region of fission.

The very large interaction cross section implied by the calculation of Alzetta *et al.*, of course, is a consequence of incorrectly assuming the large-doublet set of scattering lengths. We suggest, however, that the upper limit of Baz' *et al.* may have to be raised to 100 or 1000 μb, and that the nonexistence of the dineutron is not yet conclusively proved. However, it now seems fair to say that with present experimental techniques very little can be said concerning the existence of the dineutron in any experiment involving threshold effects.

VI. CONCLUSIONS

We have measured the n-d total cross section spanning the energy range 2.25–15.0 MeV. The agreement between our experimental cross sections and the results of Aaron, Amado, and Yam, and of Phillips is additional evidence that the use of separable two-body interactions is a good approximation to the three-body problem. A search of the n-d total cross section near the energy threshold for dineutron production provided no evidence for a Wigner cusp. In view of very recent evidence that resolved the ambiguity in the n-d scattering lengths, and in view of the small magnitude of the cross section for dineutron production, we conclude that very little can be said concerning the existence of a dineutron, or violation of charge independence, in an experiment of this type.

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APPENDIX: MINIMUM-VARIANCE FIT TO COVARIANT DATA

Following the discussion of Hamilton,[32] we seek a minimum-variance fit to the n-component data vector $\boldsymbol{\sigma}$, corresponding to the energy vector whose components are the n points E_i at which the measurements are made. We attempt a fit of the form

$$\boldsymbol{\sigma}_0=\mathbf{D}\mathbf{c}, \tag{A1}$$

where the coefficient vector $\mathbf{c}$ has three components,

[28] A. I. Baz', V. I. Gol'danskii, and Ya. B. Zel'dovich, Usp. Fiz. Nauk SSR **85**, 445 (1965) [English transl.: Soviet Phys.—Usp. **8**, 177 (1965)].

[29] A. Katase, M. Seki, T. Akiyoshi, A. Yoshimura, and M. Sonoda, J. Phys. Soc. Japan **17**, 1211 (1962).

[30] J. P. Schiffer and R. Vandenbosch, Phys. Letters **5**, 292 (1963).

[31] We are indebted to I. Halpern for this argument. The basis of the calculation is outlined in I. Halpern, *Physics and Chemistry of Fission* (International Atomic Energy Agency, Vienna, 1965), Vol. II, p. 369.

[32] W. C. Hamilton, *Statistics in Physical Science* (Ronald Press Company, New York, 1964), Chap. 4.

and the $n\times 3$ "design matrix" $\mathbf{D}$ is defined by

$$D_{ij}=E_i^{j-1}. \tag{A2}$$

If the covariance matrix of the data is $\mathbf{V}$, then we must minimize the value of

$$q=(\boldsymbol{\sigma}-\boldsymbol{\sigma}_0)'\mathbf{V}^{-1}(\boldsymbol{\sigma}-\boldsymbol{\sigma}_0), \tag{A3}$$

where the prime denotes the transpose. In the usual case of uncorrelated data, $\mathbf{V}$ is diagonal and Eq. (A3) reduces to the conventional method of least squares. A straightforward calculation furnishes the solution

$$\mathbf{c}=(\mathbf{D}'\mathbf{V}^{-1}\mathbf{D})^{-1}\mathbf{D}'\mathbf{V}^{-1}\boldsymbol{\sigma}. \tag{A4}$$

Most of the computational time for solution is consumed in inverting the covariance matrix $\mathbf{V}$. Since $\mathbf{c}$ has three components, the expectation value of q is given by

$$\langle q\rangle=(n-3)\pm(2n-6)^{1/2}, \tag{A5}$$

and the complete distribution of q is the usual χ^2 distribution for $n-3$ degrees of freedom.

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Deuteron Stripping on a Realistic Nucleus*

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A model for deuteron stripping is formally examined in which both neutron and proton can excite collective degrees of freedom of the target nucleus. A set of coupled equations is derived and truncated by defining optical-model potentials. Appropriate optical-model potentials for computing are discussed and compared with potentials for related reactions such as proton elastic scattering. An expression for the stripping amplitude is derived in the weak-coupling limit. It describes all possible excitations in the model, not just the single-particle component of the final state as current stripping theory does. The purpose of the paper is to show what is involved in obtaining a realistic description of stripping and to make the approximations as explicit as possible.

1. INTRODUCTION

THE model underlying present theories of deuteron stripping[1,2] is a quasi-three-body concept which treats the reaction as if it involved three bodies: the neutron, the proton, and the target nucleus or core. Because degrees of freedom of the core may be excited, the neutron-core and proton-core interactions are represented for positive relative energies by optical potentials. It is generally agreed that the optical potential should be given by the potential which describes elastic scattering at the appropriate energy in the two-body system.

To obtain a computable model a further simplification is made. A distorted-wave Born approximation (or an improvement using inelastic amplitudes for core excitation) is used for the stripping amplitude in which the motion of the deuteron in the initial state is treated as if it were a particle moving with the neutron-proton center-of-mass motion.

Many careful analyses of experimental data[1] using the conventional theory have been made over the past few years and we are now in a position to be able to evaluate its effectiveness as a description of the reaction. To say the least, the model is not such an effective description that it leaves only trivial questions to be answered. Some of its qualitative deficiencies are as follows:

1. It is not at all certain[1] that the optical parameters which give the best fit are related simply to two-body scattering.
2. Spectroscopic factors for many reactions are not predicted within a factor of 2 if we believe the corresponding nuclear-structure calculations.[3,4]
3. Finite stripping cross sections are observed for states where stripping is forbidden by an unexcited-core model.[4]
4. The correct j dependence[3] of stripping cross sections is not predicted.

At this stage in the development of the theory of the reaction we think it is appropriate to present a thorough formal examination of a model for stripping which includes enough degrees of freedom to give a realistic

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[1] W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961); K. R. Greider and G. L. Strobel (to be published).

[2] P. J. Iano and N. Austern, Phys. Rev. **151**, 853 (1966).

[3] L. L. Lee, Jr., in Proceedings of the International Conference on Nuclear Physics, Gatlinburg, 1966 (unpublished).

[4] M. H. Macfarlane, in Proceedings of the International Conference on Nuclear Spectroscopy with Direct Reactions, edited by F. E. Thron (Argonne National Laboratory, Argonne, Illinois, 1964), Report No. ANL 6878 (unpublished).