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Solar Neutrinos from the ^{3}He$(p,e^+\nu)^4$He Reaction

CARL WERNTZ*† AND J. G. BRENNAN*
The Catholic University of America, Washington, D. C.
and
U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland
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An upper limit to the cross section for the reaction ^{3}He$(p,e^+\nu)^4$He is calculated by comparing its matrix element with the matrix element of the ^{3}He$(n,\gamma)^4$He reaction, for which an experimental upper limit of 100 μb has been obtained. Expressing the cross section in the form $\sigma_\beta=(S/E_p)\exp(-2\pi\eta)$, where E_p is the center-of-momentum proton energy and $\eta=2\times3\ \mathrm{M}e^2/4\hbar^2k_p$ with k_p the relative proton wave number, we get a value for the constant S of $S\cong3.7\times10^{-20}$ keV b, as against the older value of 6.3×10^{-18} keV b. This result indicates that the ^{3}He$(p,e^+\nu)^4$He reaction is a negligible source of solar neutrinos compared to the ^{8}B decay.

I. INTRODUCTION

IT has been assumed until recently that the most important source of measurable solar neutrinos is the decay[1] of ^{8}B. Since the production rate of this nucleus within the sun depends strongly[1,2] on the internal solar temperature, a measurement of the neutrino flux, it has been hoped, would tell us what this temperature is. However, Kuzmin[3] pointed out that the reaction

$$^3\mathrm{He}+p\rightarrow{}^4\mathrm{He}+e^++\nu\,, \tag{1}$$

unimportant for energy production, must be considered as a second important source of high-energy neutrinos. Using the rate for this reaction calculated by Salpeter,[4] he estimated that 20 to 50% of the neutrinos in the high-energy end of the flux spectrum on earth could come from the ^{3}He$(p,e^+\nu)^4$He reaction. Since the temperature dependence for the rate of this reaction is opposite to that for the production of ^{8}B, Kuzmin suggested that interpreting the measured neutrino flux in terms of an internal solar temperature may be difficult.

It is the purpose of this paper to present a new estimate of the cross section for the reaction of Eq. (1). Our cross section turns out to be about two orders of magnitude *smaller* than that of Salpeter, and consequently the ^{3}He$(p,e^+\nu)^4$He reaction is probably an insignificant source of neutrinos.

The older estimate was based on the calculation of the overlap of an s-wave proton continuum wave function and a $1s$ neutron function in ^{4}He, in analogy with the early $p(p,e^+\nu)d$ calculations.[5] As a matter of fact, if one approximates the final ^{4}He state by a $(1s)^4$ configuration and the initial ^{3}He$+p$ state by a $(1s)^3(2s)$ configuration (the $2s$ state is a continuum state) and completely antisymmetrizes in space, spin, and isospin the capture rate induced by the usual allowed β-decay operator is identically zero.

The neutrino-producing reaction must involve a $1^+\rightarrow0^+$ nuclear transition, since there is a $\Delta T=0$ selection rule for Fermi $0^+\rightarrow0^+$ transitions. Then the axial-vector operator responsible for the transition is

$$\mathbf{M}_\beta=\tfrac{1}{2}\sqrt{2}G_A\sum_i\boldsymbol{\sigma}_i\tau_i^{(-)}\exp i\mathbf{q}\cdot\mathbf{r}_i+\mathrm{H.c.}, \tag{2}$$

where $\boldsymbol{\sigma}_i$, $\boldsymbol{\tau}_i$, and $\boldsymbol{\gamma}_i$ are, respectively, the spin, isospin, and position of the ith nucleon and $\mathbf{q}$ is the momentum transfer to the leptons. In the allowed approximation, $\mathbf{q}\cdot\mathbf{r}_i\cong0$, this operator is independent of the spatial coordinates and cannot couple initial and final states whose spatial parts belong to different representations of the symmetry group S_4. As we show below, the $(1s)^4$ ground state has [4] space symmetry and the $(1s)^3(2s)$ ^{4}Li state [31] symmetry.

Let us examine this point in more detail, considering first the ^{4}He ground state. Cohen[6] has analyzed the symmetry properties of the ^{4}He ground state. While binding-energy calculations using all the different components of the wave functions have not been carried out for the 4-body case as they have been for 3-nucleons, one can use the results of the 3-body work[7] to make an estimate of the composition of the ^{4}He ground state. One would expect that the composition of the ground state is [4] S state, $\sim92\%$; [22] S state, $\sim0.5\%$; [22] D state, $\sim7\%$; and [31] P state $\sim1\%$. In this listing we have given, in order, the spatial symmetry of each state, its orbital angular momentum, and its percentage admixture. If one uses the S_4 tables of Jahn[8] to find the component states of a $T=1$, $J^\pi=1^+$ 4-body state, one finds that the states of greatest spatial symmetry

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TABLE I. *s*-wave SU_4 multiplet. The singlet and triplet neutron scattering lengths are denoted by ${}^1a_n$ and ${}^3a_n$ and the proton hard-sphere radii by ${}^1r_p$ and ${}^3r_p$. σ_{np} is the triplet low-energy cross section for ${}^3\text{He}(n,p){}^3\text{H}$. Under SU_4 invariance, ${}^1a_n={}^3a_n={}^1r_p={}^3r_p$ and ${}^3\sigma_{np}=0$.

System	T, J^π	Number of states	Experimental quantity
${}^3\text{H}+n$	$1, 0^+$	1	${}^1a_n$
	$1, 1^+$	3	${}^3a_n$
${}^3\text{H}+p$	$1, 0^+$	1	
${}^3\text{He}+n$	$0, 1^+$	3	${}^3\sigma_{np}$
	$1, 1^+$	3	
${}^3\text{He}+p$	$1, 0^+$	1	${}^1r_p$
	$1, 1^+$	3	${}^3r_p$

for each orbital angular momentum are [31] *S* state, [31] *P* state, [31] *D* state, and [211] *F* state. One would expect the [31] *S* state to be the most important since it can be made up of the product of $(1s)^3$ and a fourth *S*-state proton function. However, the $p+{}^3\text{He}$ system in this state cannot make a transition to the ${}^4\text{He}$ [4] *S* state because of the change in spatial symmetry.

An alternative but equivalent way of looking at the problem is to assume the SU_4 invariance of the Wigner supermultiplet theory. The three 1^+ ${}^3\text{He}+p$ states $(m=\pm 1, 0)$ are members of a {15} representation, while the ground state of ${}^4\text{He}$ is a scalar {1}. The operators $\sigma^j\tau^t$ of the allowed axial-vector operator are generators of the group SU_4 and cannot couple the two different representations. This is the same point which Foldy and Walecka[9] have made in connection with μ^- capture in doubly closed-shell nuclei. In their case, the momentum transfer is large enough that the dipole term in the expansion of $\exp i\mathbf{q}\cdot\mathbf{r}$ is large and μ^- capture in the 1^- state populates the negative-parity {15} states which are generalized giant dipole states.

The fact that the low-energy even-parity continuum states in the $n+t$, $p+t$, $n+{}^3\text{He}$ states[10] make up a {15} SU_4 representation is strongly supported by experimental evidence. Table I shows how the 15 states are distributed among the physical 4-nucleon states. We also give the experimental quantities which are pertinent to our discussion.

Tombrello[11] has shown that the singlet and triplet $p+{}^3\text{He}$ *s*-wave phase shifts are essentially hard sphere, with the hard-sphere radii given by ${}^1r_p=+3.05$ F, ${}^3r_p=+3.15$ F. Under SU_4 invariance one would have ${}^1r_p={}^3r_p$. One can obtain the singlet and triplet $n+t$ scattering lengths by combining the coherent scattering length $A_{\text{coh}}=\frac{3}{4}\,{}^3a_n+\frac{1}{4}\,{}^1a_n$ and the thermal cross section $\sigma=\pi(3\,{}^3a_n^2+{}^1a_n^2)$.

The coherent scattering length has been recently measured[12] to be 3.52 ± 0.23 F and the thermal cross section[13] lies between 1.3 and 1.4 b. One obtains complex roots unless A_{coh} is taken near its lower limit and σ near to 1.4 b. The experimental values are not inconsistent with

$${}^1a_n={}^3a_n=3.29\text{ F}.$$

Less direct information is available with regard to the ${}^3\text{He}+n$ and $t+p$ systems. Under exact SU_4 invariance the 1^+ $T=0$ and $T=1$ *s*-wave states are also described by the same scattering length and $n+{}^3\text{He}$ scattering in the 1^+ state should be completely elastic. (With equal scattering amplitudes in the two isospin-states, the relative mixture of isospin states in the incoming wave which gives pure ${}^3\text{He}+n$ also obtains in the outgoing wave.) Passell and Schirmer[14] have shown that at thermal neutron energies the reaction ${}^3\text{He}(n,p)t$ goes, within experimental limits, entirely through the 0^+ state with $\sigma(0^+)/\sigma(\text{total})=1.01\pm0.03$.

Exactly the same argument which we have advanced in support of a suppressed ${}^3\text{He}(p,e^+\nu){}^4\text{He}$ cross section can be used to predict a very small ${}^3\text{He}(n,\gamma){}^4\text{He}$ M_1 capture rate. The spin magnetic-moment operator

$$\mathbf{M}_\gamma=\sum\left[\frac{\mu_p-\mu_n}{2}\boldsymbol{\sigma}_i\tau_i^z+\frac{\mu_p+\mu_n}{2}\boldsymbol{\sigma}_i\right], \tag{3}$$

which would usually make a large contribution in an *S* state to *S*-state transition, cannot couple the {15} part of the initial state to the principal {1} part of the ${}^4\text{He}$ ground state. In fact, the cross section is very small, an upper limit[15] of 100 μb having been placed on it. Since the rates for the β^+ and γ transitions depend on a detailed knowledge of the small components of the 4-body wave functions and an a knowledge of meson-exchange corrections to M_β and M_γ, a calculation from first principles of either rate is out of the question. What we have done is to make plausible but nonrigorous assumptions about the meson exchange terms in M_β and M_γ to get a relationship between the two operators and to use SU_4 and isospin invariance to relate the matrix elements for ${}^3\text{He}(p,e^+\nu){}^4\text{He}$ and ${}^3\text{He}(n,\gamma){}^4\text{He}$. Using the upper limit on the M_1 capture rate, we get a rough upper limit on the neutrino production cross section which turns out to be two orders of magnitude smaller than the cross section estimated by Salpeter.

II. NEUTRINO-PRODUCTION CROSS SECTIONS

We begin by writing down the matrix elements of the magnetic moment and axial-vector β decay, including

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[10] The 0^+, $T=0$ state must be excluded because it is also a scalar {1} state.

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[15] H. Gallmann, J. Kane, and R. Pixley, Bull. Am. Phys. Soc. **5**, 19 (1960).

exchange terms:

$$\mathbf{M}_\gamma=\mu_0\left\{\sum_{i=1}^{A}\left(\frac{\mu_p+\mu_n}{2}-\tfrac{1}{4}\right)\boldsymbol{\sigma}_i+\tfrac{1}{2}\mathbf{J}+\sum_{i=1}^{A}\left[\left(\frac{\mu_p-\mu_n}{2}\right)\boldsymbol{\sigma}_i+\tfrac{1}{2}\mathbf{L}_i\right]\tau_{iz}\right\}+\mathbf{M}_\gamma{}^S(\text{ex})+\mathbf{M}_\gamma{}^V(\text{exch}) \tag{4}$$

$$\mathbf{M}_\beta=\tfrac{1}{2}\sqrt{2}G_A\sum_{i=1}^{A}\boldsymbol{\sigma}_i\tau_i{}^{(-)}+\mathbf{M}_\beta(\text{exch}).$$

We have divided $\mathbf{M}_\gamma$ into isotopic-spin scalar and vector parts. The total angular-momentum operator is J, while μ_p, and μ_n are the proton and neutron magnetic moments in Bohr magnetons, $\mathbf{L}_i$ is the orbital angular-momentum operator for the ith nucleon, and $\mathbf{M}_\gamma{}^S(\text{exch})$ and $\mathbf{M}_\gamma{}^V(\text{exch})$ are the isospin scalar and vector exchange-moment operators. A reasonable assumption is that these exchange operators depend on the coordinates of pairs of nucleons. Then the isospin vector exchange operator can be expressed phenomenologically[16] as

$$\mathbf{M}_\gamma{}^V(\text{exch})=\mu_0\left(\frac{\mu_p-\mu_n}{2}\right)\sum_{i<j}\{[g_\gamma(r_{ij})P_{ij}{}^x+h_\gamma(r_{ij})]\times[(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)(\tau_i-\tau_j)^z]+j_\gamma(r_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)(\tau_i+\tau_j)^z\}+\text{tensor terms.} \tag{5}$$

$P_{ij}{}^x$ is the space-exchange operator and $g_\gamma(r)$, $h_\gamma(r)$, and $j_\gamma(r)$ are short-ranged functions of the pair separation. In the same way the weak-interaction exchange operator can be expressed[17] as

$$\mathbf{M}_\beta(\text{exch})=\tfrac{1}{2}\sqrt{2}G_A\sum_{i<j}\{[g_\beta(r_{ij})P_{ij}{}^x+h_\beta(r_{ij})]\times[(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)(\tau_i-\tau_j)^{(\pm)}]+j_\beta(r_{ij})(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)(\tau_i+\tau_j)^{(\pm)}\}+\text{tensor terms.} \tag{6}$$

We first note that the isospin scalar part of M_γ is small.[18] The coefficient $\frac{1}{2}(\mu_p+\mu_n)-\frac{1}{4}=0.19$ and $\mathbf{J}$ operating on the 0^+ ^{4}He ground state is zero. As is discussed in the Appendix, the isospin scalar exchange moment is one order of magnitude smaller than the vector. Noting also that the coefficient in front of $\boldsymbol{\sigma}_i$ in the isospin vector term, $\mu_p-\mu_n=4.70$, is larger than the coefficient of $\mathbf{L}_i$, we can approximate $\mathbf{M}_\gamma$ by the isospin vector part depending on the spins alone

$$\mathbf{M}_\gamma\cong\mu_0\left\{\sum_i\left(\frac{\mu_p-\mu_n}{2}\right)\boldsymbol{\sigma}_i\tau_{iz}\right\}+\mathbf{M}_\gamma{}^V(\text{exch}). \tag{7}$$

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It is shown in the Appendix that for the ^{3}H-^{3}He mirror pair the experimental magnetic moments and ft values imply

$$\begin{aligned}\langle+\tfrac{1}{2}|M_\gamma{}^{V(z)}(\text{ex})|+\tfrac{1}{2}\rangle&\cong-0.14(\mu_p-\mu_n)/2,\\ \langle+\tfrac{1}{2}|M_\beta{}^{(z)}(\text{ex})|-\tfrac{1}{2}\rangle&\cong0.11G_A/\sqrt{2}.\end{aligned} \tag{8}$$

Here the states are denoted by their z components of isospin, and the superscripts on the M's refer to configuration space. If the principal S state of the 3-body wave function is used to calculate the expectation values of the exchange operators of Eqs. (5) and (6), then Eq. (8) leads to

$$\langle f_r|g_\gamma(r_{12})+h_\gamma(r_{12})|f_r\rangle\cong\langle f_r|g_\beta(r_{12})+h_\beta(r_{12})|f_r\rangle, \tag{9}$$

within an accuracy consistent with our other approximations. The ket $|f_r\rangle$ represents the completely symmetric space state of the 3-nucleon ground state.

In the ^{3}He$(n,\gamma)^4$He and ^{3}He$(p,e^+\nu)^4$He reactions the major contribution from the exchange term comes from the [31] S state of the initial system and the [4] S state of the final system. Using Jahn's tables,[8] one can write

$$\begin{aligned}\Psi_i&=\tfrac{1}{3}\sqrt{3}[F_{aa}\Phi_{sa}+F_{sa}\Phi_{as}+F_{ss}\Phi_{aa}],\\ \Psi_f&=\Phi_{aa}G_{ss}.\end{aligned} \tag{10}$$

For the initial state, denoted by Ψ_i, the F's are the three space functions making up the [31] S_4 representation, while the Φ's are members of the conjugate [211] and are functions of the spin and isospin variables with total spin and isospin both unity. For the final ^{4}He ground state, denoted by Ψ_f, Φ_{aa} is a completely antisymmetric spin and isospin function, while G_{ss} is a completely symmetric space function.

The Φ's can be defined further in terms of spin and isospin functions by

$$\begin{aligned}\Phi_{sa}&=-\tfrac{1}{2}\sqrt{2}(\eta_{sa}\varphi_{ss}-\eta_{ss}\varphi_{sa}),\\ \Phi_{as}&=\tfrac{1}{2}\sqrt{2}(\eta_{as}\varphi_{ss}-\eta_{ss}\varphi_{as}),\\ \Phi_{aa}&=-\tfrac{1}{2}\sqrt{2}(\eta_{as}\varphi_{sa}-\eta_{sa}\varphi_{as}),\end{aligned} \tag{11}$$

and

$$\Phi_{aa}=\tfrac{1}{2}\sqrt{2}[\varphi_{aa}\eta_{ss}-\varphi_{ss}\eta_{aa}].$$

The φ's and η's are spin and isospin functions, respectively, whose two subscripts indicate the symmetry with respect to exchange of particles one and two and three and four, respectively. As an example, η_{ss} denotes an isospin function which is a product of a triplet state in one and two and a triplet state in three and four, with the two triplets coupled to $T=1$.

The transition matrix element of $M_\beta(\text{exch})$ is easily evaluated using the above expressions for the wave functions and the definitions in Eq. (6), with the result

$$\langle\Psi_f|M_\beta{}^{(z)}(\text{exch})|\Psi_i\rangle=-\tfrac{1}{2}\sqrt{2}G_A8\langle G_{ss}|g_\beta(r_{12})+h_\beta(r_{12})|F_{ss}\rangle. \tag{12}$$

It is important to note that the same combination of functions $g_\beta(r)+h_\beta(r)$ occurs as in the 3-body case. For the exchange-moment contribution to the ^{3}He$(n,\gamma)^4$He reaction, one gets, of course,

$$\langle\Psi_f|M_\gamma^{(z)}(\text{exch})|\Psi_i\rangle=+\tfrac{1}{2}\mu_0(\mu_p-\mu_n)\times 8\langle G_{ss}|g_\gamma+h_\gamma|F_{ss}\rangle. \quad (13)$$

Because of the 3-body data which say $g_\gamma+h_\gamma\cong g_\beta+h_\beta$, one can assert that the weak-interaction matrix elements are related to the M_1 elements by a rotation in isospin space. Thus the basic relation which we use to relate the matrix element for ^{3}He$(p,e^+\nu)^4$He and the $T=1$ part of ^{3}He$(n,\gamma)^4$He is

$$\langle 0,0|\mathbf{M}_\beta|1,+1\rangle=\frac{\sqrt{2}G_A}{\mu_0(\mu_n+\mu_p)}\frac{(110|+1,-1,0)}{(110|0,0,0)}\times\langle 0,0|\mathbf{M}_\gamma|1,0\rangle. \quad (14)$$

Equation (14) has been written with the tacit assumption that the initial states are discrete states which are eigenfunctions of isotopic spin. The equation must be modified if one wants to compare the matrix elements for ^{3}He$(p,e^+\nu)^4$He and ^{3}He$(n,\gamma)^4$He. In the first place, the initial ^{3}He$+n$ function is a mixture of isotopic spin states[19]

$$\chi(n+{}^3\text{He})=\tfrac{1}{2}\sqrt{2}[\chi(T=0)+\chi(T=1)]. \quad (15)$$

Since the $T=0$ part is being neglected, a factor of the $\sqrt{2}$ must be added to the right-hand side of Eq. (14). If we invoke SU_4 invariance, the $T=1$ part represents pure elastic scattering and the radial wave function for thermal-neutron energies must take the form

$$\chi(T=1)\cong a/r-1, \quad (16)$$

where a is the common hard-sphere radius for the {15} functions. The initial $1^+(^3\text{He}+p)$ state also represents pure elastic scattering and to the same order in the relevant expansion parameter,[20] $\eta k_p r$, is given by

$$\chi(^3\text{He}+p)\cong C_0(a/r-1), \quad C_0=[2\pi\eta e^{-2\pi\eta}]^{1/2}. \quad (17)$$

Here η is the usual Coulomb parameter $2\times 3Me^2/4\hbar^2k_p$, where k_p is the relative proton wave number. We make the assumption that inside of some strong interaction radii, $\chi(T=1)$ and $\chi(^3\text{He}+p)$ have the same shape but are scaled by their values in the region just outside of that radius. Taking Eqs. (15), (16), and (17) into account and inserting the values for the Clebsch-Gordan coefficients, one finds that Eq. (14) is modified to

$$\langle^4\text{He}|\mathbf{M}_\beta^{(z)}|^3\text{He}+p\rangle=-\frac{2G_A}{\mu_0(\mu_p-\mu_n)}C_0\langle^4\text{He}|\mathbf{M}_\gamma|n+{}^3\text{He}\rangle. \quad (18)$$

The final step is to put the above relation into standard formulas for M_1 emission[21] and neutrino production.[22] The final result is

$$\frac{\sigma_\beta}{\sigma_\gamma}\cong\frac{1}{20\pi^3}\frac{E_{\max}^5}{M^2E_\gamma^3}\left(\frac{E_n}{E_p}\right)^{1/2}\frac{4G_A^2C_0^2}{\alpha(\mu_p-\mu_n)}. \quad (19)$$

Here $E_{\max}$ is the maximum positron energy, E_n is the energy of the neutron (thermal) in the ^{3}He$(n,\gamma)^4$He reaction, E_γ is the γ energy in the same reaction, E_p is the center-of-momentum proton energy in ^{3}He$(p,e^+\nu)^4$He, M is the nucleon mass (in energy units), and α is the fine-structure constant. In deriving this relation we have taken the positron to be in a plane-wave state and set its rest mass to zero.

The cross section for the capture of protons, σ_β, should have an energy dependence characteristic of that of a nonresonant low-energy capture reaction involving two charged initial nuclei; namely,

$$\sigma_\beta=(S/E_p)\exp(-2\pi\eta). \quad (20)$$

Salpeter[4] assumed such a relation and calculated a value of S. This value is not given explicitly in his paper, but if one works backwards from his tabulated mean reaction times for ^{3}He, one obtains

$$S(\text{Salpeter})=6.3\times10^{-18}\text{ keV b}. \quad (21)$$

Equation (19) can also be recast in the form of Eq. (20):

$$\sigma_\beta=\sigma_\gamma\frac{1}{10\pi^2}\frac{E_{\max}^5}{M^2E_\gamma^3}\left(\frac{3E_nM}{2}\right)^{1/2}\frac{4G_A^2}{(\mu_p-\mu_n)}\frac{e^{-2\pi\eta}}{E_p}. \quad (22)$$

Using the values[15,22] $\sigma_\gamma\leq 100\ \mu$b, $G_A=1.17\times10^{-5}$, $\mu_p-\mu_n=4.70$, $E_{\max}=18.8$ MeV, $E_\gamma=20.6$ MeV, one obtains

$$S(\text{WB})\cong 3.7\times10^{-20}\text{ keV b}. \quad (23)$$

Comparing this value of S to the older value [Eq. (20)], we see that our estimate of the rate of the ^{3}He$(p,e^+\nu)^4$He reaction is of the order of 6×10^{-3} times smaller than that of Salpeter.

III. DISCUSSION

Our result is at best an order-of-magnitude result, but it would be very surprising if the cross section for ^{3}He$(p,e^+\nu)^4$He were as large as Salpeter calculated. An earlier calculation by one of the authors,[23] using phenomenological exchange-moment operators, showed that the 100-μb cross section for ^{3}He$(n,\gamma)^4$He is at least consistent with the operators which also give the correct exchange magnetic moment in ^{3}He and ^{3}H. An important point is that the exchange-moment contribution dominates since it connects the [4] S state in ^{4}He and

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[21] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), p. 299.

[22] *Selected Topics in Nuclear Theory*, edited by F. Janouich (International Atomic Energy Agency, Vienna, 1963), p. 374.

[23] Carl Werntz, Bull. Am. Phys. Soc. **6**, 294 (1961).

the [31] S state in $p+{}^3$He. The interaction-moment operators, in general, have expectation values about 10% of the free-nucleon moment operators, the matrix elements are down by a factor of 10^{-1} and the cross sections down by about 10^{-2} for the nonresonant ^{3}He$(n,\gamma)^4$He process. It is possible that accidental cancelation between the exchange and normal nuclear-magnetic-moment operators lead to a cross section much smaller than the measured upper limit of 100 μb. In this case the cancelation of the exchange and the normal axial vector β-decay operators cannot be expected to be precisely the same, and the proportionality we have assumed between the M_1 and β processes will not hold. However, in this case one still knows that the ^{3}He$(p,e^+\nu)^4$He rate is very small (due to the cancelation) and our approximate value of $S=3.7\times10^{-20}$ keV b becomes a good upper limit.

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APPENDIX: MESON EXCHANGE EFFECTS FOR A=3

The 3-body nuclear system has been completely analyzed[24] as to the orbital angular momenta and SU_4 representations which can occur in the ground state with $T=\frac{1}{2}$, $J=\frac{1}{2}^+$. In all there are 3 S states, 3 D states, and 4 P states. Expressions have been obtained for the static magnetic moments of ^{3}He and ^{3}H in terms of the admixtures of these states as has the β-decay rate of ^{3}H into ^{3}He. If we make the assumptions that the principal [3] S state and the three D states are the most important, then expectation values of M_β and the isovector part of M_γ (omitting exchange terms) in the spin-up state can be expressed[25,17]

$$\langle +\tfrac{1}{2}|M_\gamma^{V(z)}|+\tfrac{1}{2}\rangle=\mu_0[\tfrac{1}{2}(\mu_n-\mu_p)][1-\tfrac{4}{3}|S|^2-\tfrac{2}{3}|D|^2]+\tfrac{1}{6}\mu_0[|D|^2-(20/\sqrt{21})a_6a_8], \quad \text{(A1)}$$
$$\langle +\tfrac{1}{2}|M_\beta^{(z)}|-\tfrac{1}{2}\rangle=\tfrac{1}{2}\sqrt{2}G_A[1-\tfrac{4}{3}|S|^2-\tfrac{2}{3}|D|^2].$$

We have also included the contribution of the [21] S state since its contribution, unlike that of the P states, depends only on the square of its amplitude and this quantity has been tabulated.[7] The coefficients a_6a_8 represent the product of the amplitude of two of the D states and are bounded in absolute value by $\frac{1}{2}|D|^2$. Thus, the orbital momentum contribution to the magnetic moment from the D states is within the limits

$$\tfrac{1}{6}\mu_0D^2[1\pm10/\sqrt{21}]. \quad \text{(A2)}$$

This is of the order of 35% of the spin term proportional to D^2.

Experimentally, the isovector moment is given by the sums of the moments[26] of ^{3}He and ^{3}H and has the value

$$\mu_V=\tfrac{1}{2}[\mu(^3\text{He})-\mu(^3\text{H})] = -2.533\mu_0. \quad \text{(A3)}$$

If one takes the ^{3}H wave functions to contain 7% D state and 0.5% S state, one gets from Eq. (A1)

$$\mu_V(\text{calc})=-2.240\mu_0\pm0.024\mu_0, \quad \text{(A4)}$$

so that

$$\langle +\tfrac{1}{2}|M_\gamma^{V(z)}(\text{exch})|+\tfrac{1}{2}\rangle=-0.313\mu_0\pm0.024\mu_0 \cong 0.14(\mu_n-\mu_p)/2.$$

Blin-Stoyle and Papageorgiou[17] show that from the ft values of the neutron and ^{3}H, one gets an experimental value for the Gamow-Teller matrix element of

$$\langle +\tfrac{1}{2}|M_\beta^{(z)}|-\tfrac{1}{2}\rangle=1.05G_A/\sqrt{2}\pm0.04G_A/\sqrt{2}. \quad \text{(A5)}$$

From Eq. (A1) we have

$$\langle +\tfrac{1}{2}|M_\beta^{(z)}|-\tfrac{1}{2}\rangle=0.945G_A/\sqrt{2}, \quad \text{(A6)}$$

with the result

$$\langle +\tfrac{1}{2}|M_\beta^{(z)}(\text{exch})|-\tfrac{1}{2}\rangle=0.11G_A/\sqrt{2}.$$

Sachs[25] has also given an expression for the isoscalar moment:

$$\langle +\tfrac{1}{2}|M_\gamma^{S(z)}|+\tfrac{1}{2}\rangle=\tfrac{1}{2}\mu_0(\mu_p+\mu_n)(1-2D^2)+\tfrac{1}{2}\mu_0D^2. \quad \text{(A7)}$$

Experimentally,

$$\mu_S=\tfrac{1}{2}(\mu(^3\text{He})+\mu(^3\text{H}))=+0.426\mu_0, \quad \text{(A8)}$$

whereas from Eq. (A7) we calculate

$$\langle +\tfrac{1}{2}|M_\gamma^{S(z)}|+\tfrac{1}{2}\rangle=+0.413\mu_0,$$

giving

$$\langle +\tfrac{1}{2}|M_\gamma^{S(z)}(\text{ex})|+\tfrac{1}{2}\rangle=+0.013\mu_0. \quad \text{(A9)}$$

[24] G. Derrick and J. M. Blatt, Nucl. Phys. **8**, 310 (1958).
[25] R. G. Sachs, Phys. Rev. **72**, 312 (1947).
[26] *Nuclear Spectroscopy*, edited by Fay Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B. (Note the typographical error for the ^{3}H moment which should be 2.97 . . . rather than 2.79 . . .)