PHYSICAL REVIEW VOLUME 157, NUMBER 3 15 MAY 1967

Hall Effect of *n*-Type GaAs in High Electric Fields

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(Received 15 December 1966)

Pulsed-resistivity and Hall-effect measurements have been performed at room temperature on *n*-type GaAs samples, as a function of electric field, up to the threshold for microwave oscillations. High sensitivity was obtained by using a null detection method. The changes in resistivity and Hall mobility are $\sim 15\%$ at threshold. For intermediate electric fields the dependence is quadratic in field. The Hall-constant exhibits a nonmonotonic over-all increase of less than 2%, tentatively explained by a variation in the scattering factor. The results are then consistent with a distribution function not significantly different from a displaced Maxwellian, in the range of field studied.

I. INTRODUCTION

It has been known for some time[1] that samples of *n*-GaAs exhibit microwave oscillations for applied electric fields exceeding a few thousand volts cm^{-1}. The pressure experiments of Hutson *et al.*[2] and the $\text{GaAs}_{1-x}\text{P}_x$ alloy experiments of Allen *et al.*[3] support the idea of a negative-conductance mechanism, in which electrons are transferred from the low-mass central [000] valley to higher-lying large-mass ⟨100⟩ valleys of the conduction band of GaAs.[4,5] It has been shown[6] that threshold fields of the right order of magnitude can be calculated from the transferred-electron model; a recent calculation of the high-field electron distribution function by Vassel and Conwell,[7] in which the assumption of a displaced Maxwellian is not made, indicates in addition a substantial transfer of electrons below the experimentally observed threshold fields. However there has been no experimental determination of the relative population in the [000] and the ⟨100⟩ valleys in terms of the applied electric field.

In this paper, we report[8] measurements of the resistivity and Hall effect of *n*-type GaAs samples up to an electric field of about 2.2 kV cm^{-1} at 300°K. It is found that the Hall constant exhibits very small variations ($<2\%$) as a function of field, which we ascribe to changes in the scattering factor of electrons in the lower minimum of the conduction band. The change in the number of these electrons is apparently negligible over the range of measurement. This interpretation leads to a fair agreement with what may be deduced from calculations assuming a displaced Maxwellian distribution function.[6] Reasons for this agreement are given.

In Sec. II we describe the experimental techniques; the results are presented in Sec. III and discussed in Sec. IV.

II. EXPERIMENTAL TECHNIQUES

The specimens were all taken from the same *n*-type GaAs single crystal, with a resistivity $\rho \sim 0.2\ \Omega$ cm and a low-field Hall mobility $\mu^0 \sim 7\times10^3$ $\text{cm}^2\ \text{V}^{-1}\ \text{sec}^{-1}$. Dumbbell-shaped samples with Hall sideprobes were cut with an ultrasonic drill (see Fig. 1) and evaporated and alloyed Ohmic contacts were provided; the distance between the end contacts was 2 mm. The sample holder was designed so as to minimize stray inductances.

Applied voltage pulses of 10 nsec duration were used, generated by discharging a 50-Ω delay line through a mercury relay. As the departures from linearity in *n*-GaAs are small below threshold,[1] a bridge method was necessary to measure the changes in resistivity with good accuracy (Fig. 2). The voltage pulse was applied to a symmetrical circuit including the sample and a set of coaxially mounted disk resistors having a resistance equal, within 0.5%, to the low-field resistance of the specimen ($\sim 30\ \Omega$); each arm was in series with a coaxial attenuator connected to one of the inputs of a pulse transformer. The difference pulse (proportional to the unbalance signal of the bridge) appeared at the secondary of the transformer and was fed through appropriate attenuators to input A of a type 661-4S2-5T1A Tektronix sampling oscilloscope system. The voltage across the specimen was measured with two

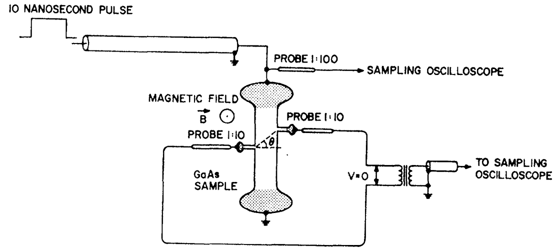

FIG. 1. Circuit used for resistivity measurements.

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P6035 probes connected to the primary of another pulse transformer, the secondary being connected through attenuators to input B of the oscilloscope. All connecting cables had sufficient length to avoid interference of reflected pulses with the primary one. By sampling at a fixed time after the beginning of the pulse, dc voltages proportional, respectively, to the unbalance signal of the bridge and to the field across the sample were obtained at the outputs of the oscilloscope and recorded with a 7030 AM Moseley X-Y pen recorder. Several curves were taken for each GaAs sample, changing the sampling time, in order to eliminate the effect of any small oscillations on the flat top of the pulses.

For accurate measurement of the Hall effect a novel null-detection technique was used. Instead of inferring the Hall angle by measuring the ratio of longitudinal and transverse electric fields, the angle was held fixed by varying the magnetic field. The achievement of the desired angle was ensured by setting to zero the voltage between two side contacts which were offset at the required angle; the sampling oscilloscope was thus used only as a null detector. Accurate measurements of the magnetic field at the null setting then allowed small changes in the Hall constant to be detected. The tangent of the Hall angle θ is given by $\tan\theta = \mu B$, where μ is the Hall mobility and B is the applied magnetic field. In order to keep θ constant when μ varies by the small amount $\Delta\mu$, a corresponding change ΔB is required, given by

$$(\Delta B/B^0) = -(\Delta\mu/\mu^0), \tag{1}$$

where the index zero labels the quantities taken at low applied electric field. In the experiment, the sample side arms were offset by $65\,\mu$ and the width of the specimen was about 0.5 mm; therefore, under null conditions, $\tan\theta$ had a constant value of about 0.13. The magnetic field was furnished by a regulated 4-in. Varian magnet, and measured with a type-240 Bell incremental gaussmeter. As the accuracy was poor in the Ohmic region, B^0 was deduced by taking advantage of the measured quadratic dependence of μ on electric field and extrapolating to zero (Fig. 3). A check was made by using pulses in the microsecond range and a 545 Tektronix oscilloscope equipped with a type-D differential preamplifier: The two values for B^0 agreed within 2%.

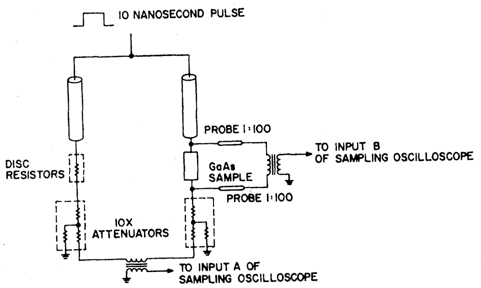


FIG. 2. Null-detection technique used for measurements of the Hall angle.

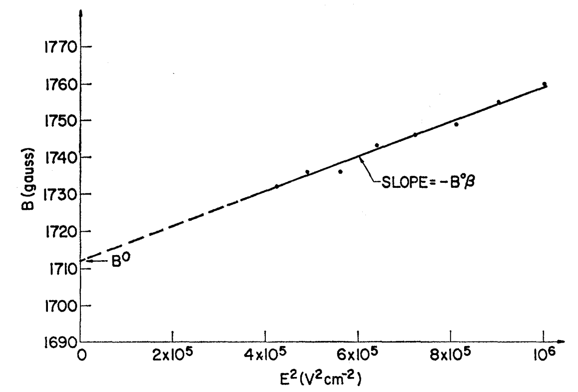


FIG. 3. Magnetic field at zero Hall signal in terms of the square of the electric field. This illustrates the procedure used to determine B^0 and β.

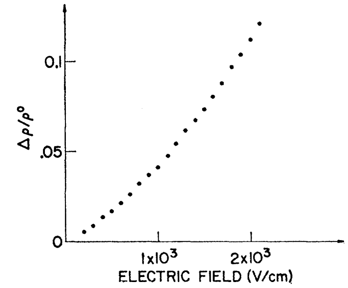
III. RESULTS

The resistivity of n-GaAs increases with applied electric field E (Fig. 4). At fields up to about 800 V/cm it follows a quadratic law $\rho = \rho^0(1+\alpha E^2)$; we found a value $\alpha = 3.7 \pm 0.2 \times 10^{-8}$ V^{-2} cm^2. As it has been observed that in short samples a large cathode drop[9] affects the current-voltage characteristics, enlarged end contacts were used to eliminate the cathode drop. It was verified that the results were consistently independent of sample length; in particular the same results were obtained by measuring the voltage across the whole sample, or between a sidearm and either end.

The Hall mobility decreases with increasing electric field, the change reaching about 15% at 2.2 kV cm^{-1} (Fig. 5). The behavior is quadratic for low and intermediate fields, according to $\mu = \mu^0(1+\beta E^2)$, with $\beta = -(3.0 \pm 0.2) \times 10^{-8}$ V^{-2} cm^2 (see Fig. 3). For fields larger than 1 kV cm^{-1}, the dependence becomes linear.

The Hall coefficient $R_H = \rho\mu$ can be calculated from the measurements (Fig. 6). It passes through a maximum around 900 V cm^{-1}, and becomes constant near

FIG. 4. Relative change in resistivity versus applied electric field.



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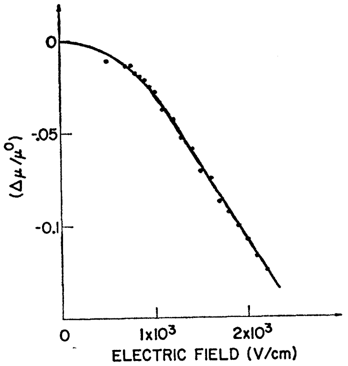

FIG. 5. Relative change in Hall mobility versus applied electric field.

threshold, in the limit of experimental error. Note that the maximum relative variation is smaller than 2%.

IV. DISCUSSION

The Hall coefficient for two-band electron conduction is given by

$$R_H=\frac{r_1n_1\mu_{c1}^2+r_2n_2\mu_{c2}^2}{e(n_1\mu_{c1}+n_2\mu_{c2})^2}, \tag{2}$$

when n_1 is the number of carriers in the low-mass central [000] valley 1, μ_{c1} is their conductivity mobility, and r_1 is a factor of the order of unity depending on the scattering mechanisms and on the shape of the distribution function in valley 1; n_2, μ_{c2}, and r_2 are the corresponding quantities for electrons in the higher-lying large-mass ⟨100⟩ valleys 2. At room temperature in low fields n_1 is much larger than n_2, as the subsidiary minima lie 0.36 eV higher in energy[10]; furthermore μ_{c2} is much smaller than μ_{c1} owing to the large mass ratio in valleys 1 and 2. The mobility μ_{c2} has been estimated[11,12] to be about 150 cm² V⁻¹ sec⁻¹. Therefore, if we consider only small changes ΔR_H in the Hall constant about the thermal equilibrium value R_H^0, we may write

$$(\Delta R_H/R_H^0)\approx(\Delta r_1/r_1^0)-(\Delta n_1/n_1^0). \tag{3}$$

The number of electrons in valley 1 is expected to decrease monotonically with the applied electric field. The nonmonotonic variation of R_H which we observe (Fig. 6) therefore indicates that the changes in R_H are probably due to changes in the scattering factor. This result is also supported by the fact that R_H becomes constant at high fields; the scattering factor varies according to the changes in shape of the distribution function, which should become constant when the hot-electron region is reached. Furthermore, the small over-all increase of r_1 means that the distribution function is only slightly non-Maxwellian at high fields. This is in qualitative agreement with the calculations of Vassel and Conwell.[7]

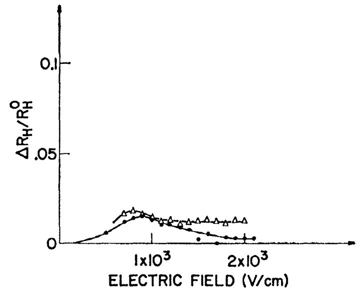

FIG. 6. Relative change in Hall constant versus applied electric field. The triangles and circles represent data for two different samples.

On the basis of our interpretation, the results (Fig. 6) show that $\Delta n_1/n_1^0$ is at most 1% at 2.0 kV cm⁻¹. However, in these samples the threshold for oscillations was reached at an average field of 2.3 kV cm⁻¹. It has been shown by detailed measurements[13] on other samples that the maximum local field at threshold is 3.6–3.8 kV cm⁻¹, but that the average field may be reduced considerably below this value by slight inhomogeneities in the sample. These inhomogeneities become effective in distorting the field distribution only in the immediate neighborhood of the threshold. Thus we believe that, in the present samples, the local and average fields are nearly equal up to about 2 kV cm⁻¹, but probably differ progressively as the threshold is reached. As a result, the present measurements do not give a reliable value for Δn_1 at threshold, but should be valid at lower fields. The change in the number of carriers in valley 1 is related to their mean energy, and the experimental data thus show that the mean energy of the electrons in the lower minimum of the conduction band is only slightly larger than the thermal equilibrium value; a crude comparison with thermal changes in the Hall constant[14] yields an equivalent "electron temperature" smaller than 500°K over the range of electric field values used.

There are two theories available with which comparison may be made. That of Vassell and Conwell[15] yields[16] a value of threshold field of 2.3 kV cm⁻¹, and a value of $\Delta n_1/n_1^0$ of 0.14 at 2 kV cm⁻¹. It is clear that this theory predicts a rate of intervalley transfer of electrons with increasing field which is much larger than the measured value. The discrepancy may arise because of the relatively low "electron temperature." In Conwell and Vassell's theory, polar optical-mode scattering is treated as elastic, which is a valid approximation only when the mean energy of the carriers is much larger than the optical phonon energy[10] of $k\times418$°K. Also, in their theory the electron drift velocity v_d is assumed to be small compared with the thermal velocity v_T; in fact, at 2 kV cm⁻¹, with an

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"electron temperature" of 500°K, v_d is 1.4×10^7 cm sec^{-1}, but v_T is only 4×10^7 cm sec^{-1}.

Assuming a displaced Maxwellian distribution function[6] Butcher and Fawcett have calculated[17] a value $(\Delta n_1/n_1^0)=0.002$ for an applied electric field of 2.25×10^3 V/cm, which would be in better agreement with our experimental data. If, as we believe, r_1 changes only by 1% or so, it means that the distribution function does not greatly differ from a Maxwellian near threshold, and this should be reasonably the case for "electron temperatures" not significantly larger than the thermal equilibrium value of 300°K. As a matter of fact Butcher and Fawcett[6] found that the temperature of the electrons in valley 1 was 450°K at 2.5×10^3 V/cm, the threshold field being 3.2×10^3 V/cm. It has to be noted that they did not make the assumption of elastic collisions, nor did they assume $v_T\gg v_d$.

For the same reasons as given for the Hall constant, the Hall mobility changes may be written

$$(\Delta\mu/\mu^0)=(\Delta r_1/r_1^0)+(\Delta\mu_{c1}/\mu_{c1}^0). \tag{4}$$

If we admit that Δn_1 is negligible the change in the conductivity mobility is directly given by the change in resistivity. The behavior of the electron mobility with electric field in polar semiconductors has been studied by Stratton,[18] who calculated the coefficient for quadratic deviations of conductivity mobility in terms of the lattice temperature, assuming a displaced Maxwellian. Because of the assumed constancy of n_1, this coefficient in our case is identical with α. Applying Stratton's result to n-GaAs at room temperature yields a value $\alpha=2\times10^{-8}$ V^{-2} cm^2, in very good qualitative agreement with experiment.

Our interpretation for the observed changes in the Hall constant gives reasonable agreement between the experimental results and the theoretical calculations based on a displaced Maxwellian distribution function. However the electron density in our samples was by far too low for electron-electron collisions to be predominant for energy and momentum exchange,[15] which is the usual situation required to ensure a displaced Maxwellian. The reason why our results appear to be in agreement with a displaced Maxwellian distribution, despite the low density, is not yet known. However, the apparent resemblance to such a distribution is not in disagreement with the instability observed at higher fields; for the latter phenomenon it is the high-energy tail of the distribution which is important. In fact, the theory of polar optical-mode scattering for a single band[18] predicts a critical electric field, above which the electrons lose their ability to interact with the lattice and experience a large increase in energy. For n-GaAs this critical field is equal to the observed threshold for oscillations.[13,19] It has to be noted that the runaway effect appears in Conwell and Vassel's calculation[15] as well, since their distribution function at high fields [Eq. (11) of their paper] does not tend to zero in the limit of high electron energies, when intervalley scattering is not considered.

ACKNOWLEDGMENTS

The authors are indebted to Dr. E. M. Conwell and Dr. W. Fawcett for communicating their theoretical numerical results. We wish to acknowledge stimulating discussions with Dr. E. M. Conwell, Dr. B. J. Elliott, and Dr. N. Braslau. We thank J. Woodall for growing the crystals, L. Foltzer and J. Rasile for technical assistance.

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