

Measurements of the Velocity of Sound in He³ and He⁴ Gas at Low Temperatures with Implications for the Temperature Scale*

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Measurements of the velocity of sound in He³ and He⁴ gas have been made at constant temperature as a function of pressure in the temperature range 1.2 to 3.8°K. The measurements, made using a fixed-frequency, variable-length resonant technique, have an uncertainty of approximately 0.07%. The results have been analyzed in terms of an expansion of the velocity of sound in powers of the pressure to determine (i) the absolute temperature of the experimental system and (ii) the second virial coefficient of the gases in this temperature range. The calculations indicate that the T_{58} He⁴ temperature scale may be 3 to 6 mdeg K lower than the absolute temperature scale. The results of the second-virial-coefficient calculations agree well with the best previous determinations taken from gas isotherm measurements.

I. INTRODUCTION

AN experiment has been performed to measure the velocity of sound in He³ and He⁴ gas as a function of pressure at low temperatures. The results and the interpretation of these results extend the knowledge of the thermodynamic properties of these gases in the temperature range of 1.2 to 3.8°K.

The sound velocities were measured as a function of pressure along an isotherm. Walstra¹ has exhibited a relation giving the square of the velocity of sound as a polynomial expansion in the pressure,

$$c^2 = A_0 + A_1 p + A_2 p^2 + \dots \quad (1)$$

The constant term in the expansion is simply the square of the velocity of sound in an ideal gas, $\gamma_0 RT/M$, where γ_0 is the ratio of the specific heats of an ideal gas, R the gas constant, T the absolute temperature, and M the atomic weight of the gas. Thus knowledge of this term permits one to calculate the absolute temperature of the experimental system. The coefficient of the linear term is a differential relationship involving $B(T)$, the second virial coefficient of the gas, and T . Consequently the second virial coefficient can be calculated once the linear term has been determined for various temperatures.

Cataland and Plumb² have recently reported results of acoustic measurements in He⁴ gas in the temperature range 2 to 5°K. They have analyzed their results using Eq. (1) to determine the absolute temperature of their experimental system. A comparison between the temperature so determined and the T_{58} He⁴ vapor pressure scale³ indicates that the T_{58} temperature scale may be lower than the absolute thermodynamic scale by as

much as 12 ± 2 mdeg K at 5°K and 6 ± 2 mdeg K at 2.3°K. The present work, using an entirely different type of acoustical apparatus and using both He³ and He⁴ as the thermodynamic fluid, lends support to their findings in the temperature range 2.2 to 3.2°K and extends the comparison to 1.2°K.

II. EXPERIMENTAL TECHNIQUE

Cryogenics. The experimental apparatus was contained in a double Dewar system of conventional design. Temperature control was effected using two different methods depending on the temperature regime.

Above the lambda temperature T_λ , regulation was achieved using a manostat in the pumping line and stirring the liquid by dissipating power in a resistor located at the bottom of the bath. The manostat, which is related generically to one described by Walker,⁴ allowed control of the temperature to within ± 1 mdeg K of a given reference temperature over extended periods of time.

Temperature stability when $T < T_\lambda$ was attained using an electronic thermoregulator.

The bath temperatures were measured using different techniques above and below the lambda temperature. Above T_λ , a He⁴ vapor-pressure thermometer which had a $\frac{1}{8}$ -in. i.d. vacuum-jacketed tube connecting the thermometry bulb and room temperature was used. The vapor-pressure bulb was located in the bath immediately adjacent to the sound cavity. Its vertical position in the bath was midway between the top and bottom extremities of the copper tube which formed the resonant cavity.

Pressures were measured using a mercury manometer constructed of 11 mm bore diam glass tubing. The vacuum side was pumped continuously with a mechanical pump which was protected by a liquid-nitrogen cold trap. A thermocouple gauge between the manometer and cold trap gave a continuous measure of the reference pressure. The heights of the mercury columns were determined using a Wild cathetometer.

⁴ E. J. Walker, *Rev. Sci. Instr.* **30**, 834 (1959).

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¹ W. K. Walstra, *Physica* **13**, 643 (1947).

² G. Cataland and H. Plumb, *J. Res. Natl. Bur. Std.* **69A**, 531 (1965).

³ F. G. Brickwedde, H. Van Dijk, M. Durieux, J. R. Clement, and J. K. Logan, *J. Res. Natl. Bur. Std.* **64A**, 1 (1960).

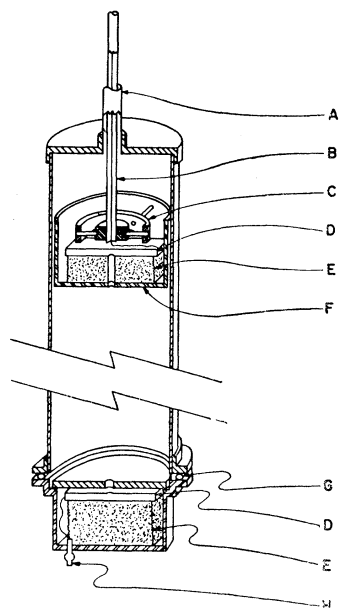


FIG. 1. Section view of the acoustic resonator (not to scale). (A) $\frac{3}{8}$ -in.-o.d. stainless steel support tube (0.008 in. wall) (B) $\frac{1}{8}$ -in.-o.d. stainless steel support tube for movable reflector (0.006 in. wall). (C) Gimbals assembly. (D) Knolls No. 1340 microphone. (E) Micro-lite Fiberglass insulation. (F) Brass reflector. (G) Indium O-ring seal. (H) Insulating lead-through.

This could be read directly to 0.1 mm; readings to 0.01 mm could be estimated. The uncertainty in any particular measurement was approximately 0.03 mm. Corrections were allowed for the value of g at Minneapolis and for the variation of the density of the mercury with temperature.³ Corrections were also applied for the height of the meniscus observed⁵ although with the rather large bore tubing such corrections are small.

Recently Sydoriak and Sherman⁶ have pointed out possible systematic errors resulting from the use of vapor-pressure thermometers in determining temperatures below T_λ . They indicate that He II film reflux causes both a pressure drop between the points where the film vaporizes and where it recondenses, and a temperature drop between the helium in the bath and that in the bulb due to heat flux through the walls of the bulb from the recondensing helium. Consequently the temperature of the bath below T_λ was measured by observing the pressure of the surface of the bath. An oil manometer filled with Octoil S was employed in these determinations. The manometer was constructed of 6 mm bore diam glass tubing. The oil was outgassed at room temperature by extended pumping at approximately 5μ Hg. The manometer remained under vacuum when not in use. The density of the oil was determined by comparing the height of the oil column with a mercury column when measuring a common pressure. A linear variation of the density of the oil with temperature was observed.

The He⁴ content of the He³ sample used in the velocity measurements was analyzed with a mass spectrometer

after most He³ measurements were completed and was found to be 16 parts He⁴ in 10 000 parts He³. This agreed well with the original analysis provided by Mound Laboratory, the supplier of the He³.

Acoustics. The most common technique employed for the measurement of the velocity of sound is ultrasonic interferometry; a description of a typical acoustic interferometer is given by Stewart and Stewart.⁷ However, early measurements of the velocity of sound in He⁴ gas at low temperatures by Van Itterbeek^{8,9} indicated that ultrasonic techniques were unsuitable at low pressures because of large attenuation. Van Itterbeek and DeLaet⁹ obviated this difficulty by using a low-frequency, fixed path-length resonant cavity. This technique, in turn, requires one to know the effective length of the cavity which is difficult to determine. Because of this inherent source of systematic error a variable length, fixed frequency design (Fig. 1) was chosen for these measurements.

The apparatus consists of a low-frequency transducer¹⁰ loosely coupled to a gas column contained in a cylindrical copper tube. The detector, an identical transducer, is located in the movable reflector. The length of the copper tube is 23.2 cm; its inner diameter and wall thickness are 3.56 cm and 0.10 cm, respectively. The transducers are electrodynamic type moving coil hearing aid transducers. At liquid-helium temperatures their useful frequency response covers a range of 1000 to 3500 cps with an impedance of approximately 1000 Ω . Frequencies used in the experiment were in the range 1900 to 2600 cps with most measurements made near 2200 cps. Thus the resonant cavity was operated in its plane wave mode at all times. Under these conditions the distance between successive pressure maxima is $\lambda/2$, where λ is the wavelength of the sound propagating in the tube.

Electronics. The transmitting transducer was driven by a Hewlett-Packard model 200AB audio oscillator; its frequency was monitored continuously using a Hewlett-Packard model 5245 L frequency counter. The output transducer fed a General Radio 1232-A amplifier-null detector. The amplifier could be operated either tuned or with a flat frequency response. The former feature is useful in eliminating the possibility that harmonics of the fundamental frequency are present in the signal received at the detector. Harmonics, if above the cutoff frequency for the propagation of plane waves in the tube, would travel at a velocity other than the plane wave velocity we seek to measure.

Technique of Measurement. The measurements were made in the following manner. With the bath temperature regulated and the pressure in the apparatus fixed, a maximum in the receiver signal was located by ad-

⁷ J. L. and E. S. Stewart, *J. Acoust. Soc. Am.* **24**, 22 (1952).

⁸ A. Van Itterbeek and G. Forrez, *Physica* **20**, 767 (1954).

⁹ A. Van Itterbeek and W. DeLaet, *Physica* **24**, 59 (1958).

¹⁰ Knolls No. 1340 supplied by Minnesota Hearing Aid and Optical Center, Minneapolis, Minnesota.

⁵ W. G. Brombacher, D. P. Johnson, and J. L. Cross, *Natl. Bur. Std. (U.S.) Monograph* **8**.

⁶ S. G. Sydoriak and R. H. Sherman, *J. Res. Natl. Bur. Std.* **68A**, 547 (1964).

TABLE I. Sound velocities in He⁴.

Temperature (°K)	Pressure (atm)	Measured velocity (m/sec)	Corrected velocity (m/sec)	Experimental uncertainty in corrected velocity (%)	Calculated velocity (m/sec)	Temperature (°K)	Pressure (atm)	Measured velocity (m/sec)	Corrected velocity (m/sec)	Experimental uncertainty in corrected velocity (%)	Calculated velocity (m/sec)
3.816	0.3427	108.30	108.34	0.07	108.35	3.182	0.0154	106.96	107.13	0.09	107.14
	0.2722	109.80	109.84	0.07	109.83		0	107.49			
	0.1990	111.27	111.32	0.11	111.30		0.2032	99.31	99.35	0.10	99.36
	0.1309	112.53	112.59	0.07	112.60		0.1394	101.22	101.27	0.05	101.26
	0.0917	113.22	113.30	0.08	113.33		0.0945	102.44	102.50	0.05	102.51
	0.0860	113.37	113.45	0.07	113.43		0.0418	103.80	103.90	0.05	103.91
	0.0641	113.72	113.83	0.07	113.83		0.0328	104.03	104.14	0.05	104.13
	0.0390	114.13	114.26	0.07	114.27		0.0236	104.25	104.37	0.06	104.37
	0.0195	114.44	114.62	0.07	114.61		0	104.95			
	0				114.95		2.978	0.1662	96.41	96.45	0.07
3.595	0.2429	106.35	106.39	0.11	106.39	0.1332	97.51	97.56	0.09	97.54	
	0.2122	107.04	107.08	0.09	107.09	0.0926	98.77	98.82	0.06	98.82	
	0.1755	107.88	107.93	0.11	107.92	0.0538	99.92	99.99	0.06	100.00	
	0.1494	108.45	108.50	0.07	108.49	0.0346	100.48	100.57	0.05	100.56	
	0.1198	109.06	109.12	0.08	109.13	0.0132	101.02	101.17	0.10	101.18	
	0.0984	109.51	109.57	0.07	109.58	0	101.55				
	0.0734	110.01	110.08	0.13	110.10	2.671	0.1180	91.84	91.88	0.07	91.89
	0.0523	110.42	110.51	0.15	110.53	0.0934	92.86	92.91	0.05	92.89	
	0.0386	110.70	110.81	0.08	110.80	0.0732	93.59	93.64	0.07	93.67	
	0.0280	110.89	111.03	0.08	111.02	0.0572	94.18	94.24	0.06	94.25	
3.379	0.0163	111.05	111.24	0.08	111.25	0.0440	94.64	94.70	0.08	94.72	
	0				111.57	0.0336	95.04	95.12	0.08	95.07	
	0.3736	98.20	98.23	0.24	98.24	0.0261	95.24	95.33	0.09	95.32	
	0.1946	103.45	103.50	0.07	103.49	0.0204	95.43	95.53	0.21	95.51	
	0.1040	105.73	105.79	0.05	105.79	0.0176	95.49	95.60	0.22	95.60	
	0.0679	106.57	106.64	0.06	106.65	0.0143	95.49	95.62	0.18	95.71	
	0.0380	107.23	107.34	0.06	107.35	0	96.16				
	0.0248	107.50	107.63	0.06	107.62	2.130	0.0413	83.67	83.72	0.04	83.73
	0.0191	107.59	107.74	0.06	107.75	0.0381	83.85	83.90	0.04	83.90	
	0.0154	107.66	107.82	0.11	107.83	0.0350	84.03	84.08	0.04	84.08	
3.337	0				108.16	0.0322	84.20	84.25	0.06	84.24	
	0.1312	104.33	104.39	0.06	104.39	0.0298	84.31	84.37	0.05	84.37	
	0.1158	104.70	104.76	0.07	104.77	0.0258	84.51	84.57	0.06	84.59	
	0.0909	105.30	105.37	0.07	105.37	0.0234	84.66	84.72	0.06	84.71	
	0.0677	105.83	105.91	0.05	105.92	0.0220	84.72	84.78	0.08	84.79	
	0.0480	106.31	106.41	0.05	106.39	0.0196	84.85	84.93	0.08	84.92	
	0.0333	106.58	106.69	0.09	106.73	0.0158	85.03	85.11	0.11	85.11	
	0.0262	106.76	106.89	0.07	106.89	0.0124	85.17	85.26	0.15	85.28	
	0.0200	106.89	107.04	0.08	107.03	0	85.87				

justing the position of the reflector. Since it was difficult to determine the exact position of the maximum, those positions on either side of the peak where the signal was 1 dB and 3 dB down from the maximum were measured. The mean of these values was used as the position of the maximum. The positions of the reflector were determined by measuring the distance to a fixed point on the tube supporting the reflector using a depth micrometer mounted on the Dewar head. The reflector was then moved to the next maximum and its position was similarly noted. The depth micrometer could be read to 0.01 mm. The scale was judged to be linear within 0.01 mm from the results of a series of comparison measurements using different portions of the scale of a Wild cathetometer as a standard.

The pressure of the gas in the resonant cavity was measured with a mercury manometer using the Wild cathetometer several times during each velocity measurement. The gas in the cell communicates with the room-temperature gas manifold system through the annular space between the support tube and tube sup-

porting the movable reflector (*A* and *B* in Fig. 1). The same corrections as those applied to the vapor-pressure measurements were applied to these observations.

All measurements made during a given run were made along a single isotherm. Each velocity measurement required approximately 30 min. The time between changing the pressure in the sound cavity and beginning the next velocity measurement was also approximately 30 min. After changing pressures the resonant system was adjusted to a position where the output signal was 3 dB down from a maximum. In this condition the output signal changes rapidly with a slight variation of the position of the reflector or of the sound velocity. The subsequent velocity measurement was not begun until the signal from the detector was stable. Calculation indicates a variation in the cell temperature of one millidegree was observable using this technique.

Calculation of the Velocity of Sound. A knowledge of the wavelength of the sound and the frequency of the wave permits one to calculate the velocity of sound immediately. However, several corrections must be

TABLE II. Sound velocities in He³.

Temperature (°K)	Pressure (atm)	Measured velocity (m/sec)	Corrected velocity (m/sec)	Experimental uncertainty in corrected velocity (%)	Calculated velocity (m/sec)	Temperature (°K)	Pressure (atm)	Measured velocity (m/sec)	Corrected velocity (m/sec)	Experimental uncertainty in corrected velocity (%)	Calculated velocity (m/sec)
3.598	0.1530	125.91	125.99	0.10	125.99	1.816	0.0831	94.69	94.75	0.05	94.75
	0.1164	126.51	126.61	0.07	126.61		0.0723	95.17	95.23	0.04	95.25
	0.0648	127.35	127.48	0.07	127.47		0.0518	96.11	96.18	0.08	96.15
	0.0317	127.83	128.01	0.07	128.02		0.0371	96.70	96.80	0.10	96.77
	0.0228	127.91	128.12	0.07	128.17		0.0293	97.02	97.13	0.10	97.08
	0.0152	128.08	128.34	0.07	128.29		0.0218	97.21	97.33	0.08	97.37
3.337	0				128.54	0.0133	97.55	97.71	0.21	97.70	
	0.1510	120.84	120.91	0.06	120.90	0				98.19	
	0.1256	121.30	121.38	0.05	121.40	0.0780	86.97	87.02	0.06	87.03	
	0.0984	121.84	121.93	0.04	121.92	0.0674	87.60	87.65	0.07	87.66	
	0.0785	122.18	122.29	0.08	122.30	0.0594	88.08	88.14	0.07	88.14	
	0.0581	122.57	122.70	0.05	122.69	0.0504	88.61	88.67	0.07	88.66	
	0.0442	122.81	122.95	0.06	122.96	0.0404	89.16	89.24	0.10	89.23	
	0.0315	123.02	123.19	0.06	123.20	0.0325	89.58	89.66	0.09	89.66	
	0.0220	123.17	123.37	0.08	123.37	0.0252	89.95	90.04	0.10	90.05	
	0.0144	123.26	123.52	0.12	123.52	0.0184	90.31	90.42	0.13	90.41	
	0.00969	123.34	123.65	0.21	123.61	0.0116	90.60	90.74	0.18	90.75	
	0					0				91.33	
	0.1284	114.07	114.14	0.05	114.14	0.0646	78.68	78.71	0.04	78.71	
2.988	0.1258	114.13	114.20	0.06	114.20	0.0562	79.38	79.42	0.05	79.40	
	0.1033	114.65	114.73	0.05	114.74	0.0474	80.04	80.08	0.06	80.11	
	0.0615	115.63	115.74	0.04	115.73	0.0396	80.65	80.71	0.04	80.72	
	0.0372	116.16	116.29	0.07	116.30	0.0319	81.25	81.31	0.06	81.30	
	0.0243	116.42	116.59	0.06	116.59	0.0258	81.70	81.78	0.07	81.75	
	0.0178	116.54	116.74	0.09	116.74	0.0222	81.94	82.02	0.05	82.01	
	0				117.14	0.0174	82.25	82.34	0.08	82.35	
	0.1210	107.53	107.59	0.05	107.58	0.0104	82.69	82.80	0.11	82.84	
2.682	0.1132	107.74	107.81	0.07	107.82	0				83.55	
	0.0948	108.29	108.36	0.05	108.37	1.232	0.0272	72.47	72.52	0.04	72.51
	0.0768	108.81	108.89	0.04	108.89	0.0253	72.65	72.70	0.05	72.73	
	0.0582	109.33	109.43	0.08	109.42	0.0226	72.98	73.04	0.06	73.03	
	0.0456	109.66	109.77	0.06	109.77	0.0202	73.24	73.30	0.07	73.29	
	0.0329	109.98	110.11	0.08	110.11	0.0178	73.48	73.54	0.06	73.55	
	0.0252	110.17	110.32	0.06	110.32	0.0152	73.75	73.83	0.08	73.82	
	0.0186	110.29	110.46	0.10	110.49	0.0122	74.05	74.13	0.10	74.12	
	0.0128	110.46	110.68	0.15	110.65	0.0101	74.23	74.32	0.09	74.32	
	0				110.98	0.00678	74.49	74.59	0.17	74.63	
	0.0876	94.50	94.55	0.04	94.54	0				75.22	

applied to the measurements. The first arises from the design of the instrument. When the movable reflector traverses a half-wavelength interval the tube fixing its position expands or contracts since it moves relative to the temperature gradient between the resonant cavity and room temperature. Corrections for this expansion were applied using data of Beenakker and Swenson¹¹ for stainless steel. Application of the correction assumes that this temperature gradient remains approximately constant during the course of a velocity measurement (i.e., for times of the order of 30 min). The correction amounts to approximately 30 parts in 10 000.

Other corrections are required because of the interaction between the gas and the wall and ends of the tube. This interaction leads to a resistance term in the wave equation and consequently to a complex propagation constant. The real part determines the sound velocity of the waves in the tube, c_t . This is related to the velocity of sound in free space c by the expression

$$c_t = c(1 - \alpha), \quad (2)$$

¹¹ J. J. M. Beenakker and C. A. Swenson, *Rev. Sci. Instr.* **26**, 1204 (1955).

where

$$\alpha = [\eta^{1/2} + (\gamma - 1)(\kappa/c_p)^{1/2}][2r(\pi\nu\rho)^{1/2}]^{-1} + [(\gamma - 1)(\kappa/c_p)^{1/2}][l(\pi\nu\rho)^{1/2}]^{-1}. \quad (3)$$

Here r is the radius of the tube, η the coefficient of viscosity, ν the frequency, γ the ratio of specific heats of the real gas, κ the coefficient of thermal conductivity, c_p the specific heat at constant pressure, ρ the density of the gas, and l the length of the tube. The first term on the right of Eq. (3) was calculated for infinite tubes by Stokes and Kirchhoff.¹² The second term, calculated by Thiesen,¹³ takes into account the interaction of the wave with the ends of the tube. The corrections depend on the pressure of the gas and thus vary for different measurements from between 2 to 30 parts in 10 000.

III. RESULTS OF VELOCITY MEASUREMENTS

The values of the sound velocities measured in He⁴ are given in Table I; those for He³ in Table II. The value

¹² See, for example, Baron Rayleigh, *The Theory of Sound* (Dover Publications, Inc., New York, 1945), Sec. 347.
¹³ M. Thiesen, *Ann. Physik* **24**, 401 (1907).

TABLE III. Coefficients in the pressure expansion of the square of the velocity of sound for He⁴.

Temperature (°K)	A ₀ m ² sec ⁻²	A ₁ m ² sec ⁻² atm ⁻¹	A ₂ m ² sec ⁻² atm ⁻²
3.816	13 213±4	-3931± 55	-1080± 150
3.595	12 448±2	-4357± 43	-1200± 180
3.379	11 699±1	-4637± 29	-2260± 110
3.337	11 553±5	-4850±160	-1200±1100
3.182	11 015±3	-5133± 83	-2390± 410
2.978	10 312±4	-5680± 96	-2330± 500
2.671	9246±9	-5910±290	-7500±2100
2.130	7373±7	-7860±490	-23 300±8200

of the correction term for each measurement given by Eq. (3) can be deduced by comparing the column "Measured velocity," which gives the uncorrected velocity values, with the "Correct velocity" column. The velocity values which are calculated from a least-squares fit of the velocity squared as a polynomial expansion in the pressure, Eq. (1), are given in the column headed "Calculated velocity."

The experimental uncertainties listed in Tables I and II are calculations of the standard deviation of each of the measurements. The recognized factors which contribute to the uncertainties listed are:

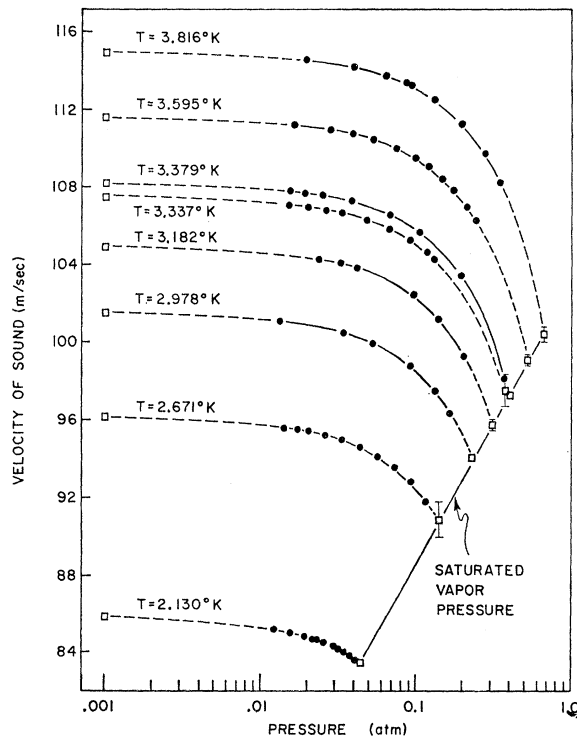


FIG. 2. Measured sound velocities in He⁴. The solid circles are the experimental points; the open squares are values calculated from the least-squares fit of the experimental data. The solid line is the least-squares fit of the data. This line is shown dashed when extrapolated beyond the range of the experimental data.

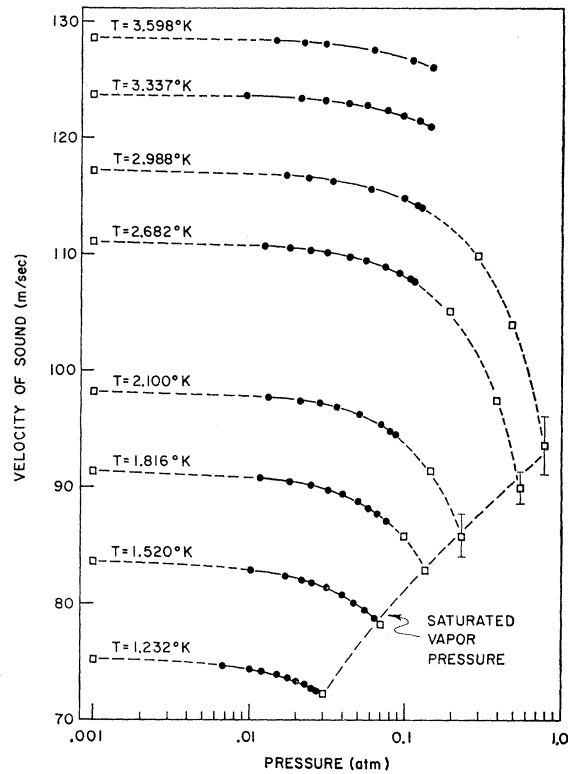


FIG. 3. Measured sound velocities in He³. The solid circles are the experimental points; the open squares are values calculated from the least-squares fit of the experimental data. The solid line is the least-squares fit of the data. This line is shown dashed when extrapolated beyond the range of the experimental data.

(a) The random error in the measurement of the wavelength. This is of the order of 0.05 to 0.10% and thus is much larger than the uncertainty in the frequency which is of the order of 0.01%. The uncertainty in the measured value of the velocity is thus essentially that of the wavelength determination.

(b) The corrections which must be applied to velocities measured in a tube to convert to free-space velocities. These corrections are estimated to be uncertain by approximately 5%, and thus contribute an uncertainty of 0.01% to measurements at low pressures where they are most significant.

TABLE IV. Coefficients in the pressure expansion of the square of the velocity of sound for He³.

Temperature	A ₀	A ₁	A ₂
3.598	16 524± 6	-4245± 81	
3.337	15 323± 4	-4630± 110	-300± 600
2.988	13 722± 4	-5260± 110	-1180± 720
2.682	12 316± 4	-5710± 120	-3490± 800
2.100	9642±15	-7100± 590	-10 700± 5100
1.816	8341± 4	-8870± 180	-12 700± 1800
1.520	6981± 8	-11 200± 400	-15 600± 4900
1.232	5658±11	-12 500±1200	-81 000±30 000

(c) An uncertainty in the value of the expansion coefficient of stainless steel of approximately 3%. This contributes an uncertainty of 0.01% to all measurements.

(d) Variations in the pressure in the resonant cavity observed during the course of an experimental run. These arise from two sources. The refrigerant bath level changes slowly in time causing the average temperature of the gas manifold to increase. Since the volume is constant the pressure also increases. Secondly, as the position of the movable reflector is changed, the volume of the experimental cell varies implying a variation also in the pressure. This variation in pressure is converted into a variation in velocity using Eq. (1). The subsequent uncertainty in velocity amounts to 0.01 to 0.03%.

The corrected velocities of Tables I and II are displayed as a function of pressure in Figs. 2 and 3.

Values of the expansion coefficients A_0 , A_1 , and A_2 of Eq. (1) which were obtained from a least-squares fit of the corrected velocity data are given in Tables III and IV. The uncertainties listed are the standard deviations of each of the A_i 's. These are a function of the root-mean-square deviations of the experimental velocities from the fitted polynomial. All values of the temperature given in Tables I-IV are temperatures calculated from the expression

$$A_0 = c_0^2 = \gamma_0 RT/M, \quad (4)$$

where c_0 is the velocity of sound in the limit of zero pressure. An extensive discussion of this point follows in Sec. IV.

The degree of the polynomial used to represent the velocity data was chosen by examining the variance associated with polynomials of successive degrees. If we let X_i denote the independent variable, Y_i the dependent variable, W_i the weight associated with Y_i , and $P^*(X)$ the least-squares polynomial of degree α , the variance Ω_α of $P_\alpha(X)$ is defined by

$$\Omega_\alpha = \frac{1}{n - (\alpha + 1)} \sum_{i=1}^n W_i [P_\alpha(X_i) - Y_i]^2, \quad (5)$$

where n is the number of data points. The polynomial of degree α was chosen when $\Omega_\alpha < \Omega_{\alpha+1}$.¹⁴ In every case except the 3.598°K isotherm for He³ a polynomial of degree two was chosen using this criterion.

IV. ACOUSTIC THERMOMETRY

We have noted above the use of sound velocity measurements to determine absolute temperatures. Experimentally this straightforward idea has had limited success. The first attempt to apply the technique to

low-temperature problems is that of Van Itterbeek.¹⁵ Sound velocities were observed using an acoustic interferometer operating at ultrasonic frequencies. Extrapolation of the velocity measurements to zero pressure gave values of the temperature which varied between 32 and 181 mdeg K larger than the bath temperature. This was followed by the work of Van Itterbeek¹⁶ and Van Itterbeek and DeLaet⁹ who used the low-frequency resonant cavity referred to above. Using this technique, the discrepancies ΔT between the temperature determined by sound velocity measurements (the acoustic temperature) and the bath temperature were reduced but still varied between +14 and +28 mdeg K. Miss DeLaet, continuing the above work, later reported discrepancies of +12 to +21 mdeg K between 2.8 and 4.2°K.¹⁷ Finally, Cataland and Plumb reported the construction of an acoustic interferometer for measuring temperatures in the range between 2 and 20°K. In a series of papers^{18-20,2} they report: a measurement¹⁸ at 4.2°K in which the temperature discrepancy is +12 mdeg K; measurements¹⁹ at 2.0 and 2.2°K yielding discrepancies of +3 mdeg K with reproducibilities of ± 2 mdeg K; a preliminary result²⁰ giving a discrepancy of +6 mdeg K at 4.2°K; and an approximately linear variation² of ΔT from 2.2 to 5°K which changes from $+6 \pm 2$ mdeg K at 2.3 K to $+12 \pm 2$ mdeg K at 5°K.

The reason for the differences between the acoustic temperature and bath temperature reported by the various groups is not completely clear; Van Itterbeek¹⁶ attributes it partially to the heat transported via support members into the experimental cavity. It is significant to note that the acoustic temperatures observed in all the reports cited above were higher than the corresponding bath temperatures.

Because of the wide range of temperature discrepancies in the reports noted above, an attempt was made in the present experiment to determine the acoustic temperatures of the various measurements for comparison with the temperature of the liquid helium bath.

Acoustic-Thermometry Results. Table V gives the results of the acoustic-temperature measurements obtained in this work. The uncertainties listed for the vapor pressures represent upper limits on the variations

¹⁵ A. Van Itterbeek, *J. Acoust. Soc. Am.* **29**, 584 (1957).

¹⁶ A. Van Itterbeek, *Proceedings of the Fifth International Conference of Low Temperature Physics and Chemistry*, edited by J. R. Dillinger (University of Wisconsin Press, Madison, Wisconsin, 1958), p. 206.

¹⁷ J. DeLaet, *Verhand. Vlaam. Acad. Wet.* **66**, 3 (1960) cited in Ref. 18.

¹⁸ G. Cataland, M. Edlow, and H. H. Plumb, *Temperature, Its Measurement and Control in Science and Industry*, edited by Charles M. Herzfeld (Reinhold Publishing Corporation, New York, 1962) Vol. III, Part 1, p. 129.

¹⁹ G. Cataland and H. H. Plumb, *Proceedings of the Eighth International Conference of Low Temperature Physics*, edited by R. O. Davis (Butterworths Scientific Publications, Ltd., London, 1963), p. 439.

²⁰ H. H. Plumb and G. Cataland, *J. Res. Natl. Bur. Std.* **69A** 375 (1965).

¹⁴ L. G. Parratt, *Probability and Experimental Errors in Science* (John Wiley & Sons, Inc., New York, 1961), p. 134.

of the vapor pressure during a set of velocity measurements along an isotherm. Larger variations than those encompassed by the uncertainties occasionally occurred; however, corrections for variations in temperature were applied to the measured velocities.

The acoustic measurements directly determine the temperature of the gas contained in the experimental apparatus. Since the bath temperatures do not, we choose to label all velocities measured in the experiment by their respective acoustic temperatures.

There are some velocity measurements reported in Sec. III for temperatures not given in Table V. These were made prior to the observations given in Table V. A systematic error in manometry was present in the earlier set of measurements which affected the absolute value of the bath temperature during velocity measurements along an isotherm but had only a second-order effect on relative changes. Each set of measurements constitute velocity measurements along an (unknown) isotherm; they each yield an acoustic temperature when extrapolated to zero pressure. Hence the observed acoustic temperature is used for these results.

Discussion of Possible Errors. The temperature discrepancies reported above indicate that either the gas inside the cavity is warmer than the bath or that systematic errors are present in the observations of ΔT .

If the gas inside the chamber were warmer than the bath a thermal flux would exist through the gas and copper walls of the apparatus into the bath which would be approximately proportional to the thermal conductivity of the gas. Consequently a change in the thermal conductivity should change the steady state temperature difference. However while a difference of approximately a factor of two exists between the measured thermal conductivities of He³ and He⁴ in the temperature range in question,²¹ the observed temperature discrepancies are approximately equal.

A temperature difference between the gas and the coolant bath would also require a steady flux of heat from room temperature. The major source of the incoming thermal power is the stainless steel tube attached to the movable reflector. To decrease the flux from this source a length of braided copper cable shield was soldered between the end of the tube and the inside of the resonant cavity. Where previously the heat entering the cavity along the tube only be dissipated through the gas, a thermal shunt was not provided through the braided copper. However a subsequent measurement of sound velocities showed no change in ΔT .

We thus conclude that the temperatures of the gas within the experimental chamber and of the coolant bath are substantially equal and we must look to systematic errors as the source of the observed temperature discrepancies. Possible systematic errors would

TABLE V. Acoustic thermometry results.

Gas	T (°K) Acoustic	Vapor pressure (mm Hg 0°C)	T_{58} (°K)
He ³	1.232±0.002	0.7490±0.0040	1.226±0.001
He ³	1.520±0.002	3.892 ±0.013	1.516±0.001
He ³	1.816±0.001	13.085±0.020	1.814±0.001
He ³	2.100±0.003	31.34 ±0.03	2.099±0.001
He ³	2.682±0.001	107.90 ±0.15	2.678±0.001
He ³	3.337±0.001	287.50 ±0.20	3.329±0.001
He ⁴	2.130±0.002	33.68 ±0.10	2.126±0.002
He ⁴	2.671±0.003	106.01 ±0.15	2.668±0.001
He ⁴	3.337±0.001	287.90 ±0.20	3.330±0.001

be found in (a) the velocity measurements, (b) the manometry, or (c) the temperature scale used for determining the bath temperature.

A systematic error in velocity could result in a constant value of the ratio $\Delta c/c$ (or equivalently $\Delta T/2T$). Referring to Table V we see this is not the case, the temperature discrepancy ΔT decreasing with increasing T below T_λ and increasing with increasing T above T_λ .

We note also that the discrepancies for a given temperature are essentially equal for both He³ and He⁴ while their velocities are significantly different. This would eliminate the possibility of a systematic error for only particular ranges of velocities.

It appears more likely that an experimental error, if it exists, occurs in the manometry. The prevailing trend of ΔT as a function of temperature changes in the vicinity of the lambda temperature. It is perhaps significant then that the method employed for measuring the bath temperature was changed at this point.

Germane to this question of the measurement of bath temperatures is the problem of stirring the coolant bath when its temperature is above T_λ . This was effected by dissipating power in a resistor located at the bottom of the bath. To test the adequacy of the power dissipated a trial was made in which this power level was changed from its normal value of 30 mW to amounts varying between 120 and 375 mW. At the largest values the bath is boiling vigorously. No change in the observed value of ΔT was seen nor was there any change in the normal scatter of measured velocities relative to previous measurements.

Another consideration adds to the belief that the power dissipated in the resistor promotes adequate stirring of the coolant bath. Figure 4, which shows the results of the acoustic thermometry, indicates that the value of ΔT is apparently continuous across T_λ . Below T_λ the dissipation of power is not required since the extremely large effective thermal conductivity of He II permits one to assume an equilibrium situation. Hence the evidence indicates that the same assumption above T_λ is not implausible.

A final likely source of systematic error is a discrepancy between the true thermodynamic temperature

²¹ K. Fokkens, W. Vermeer, K. W. Taconis, and R. de Bruyn Ouboter, *Physica* **30**, 2153 (1964).

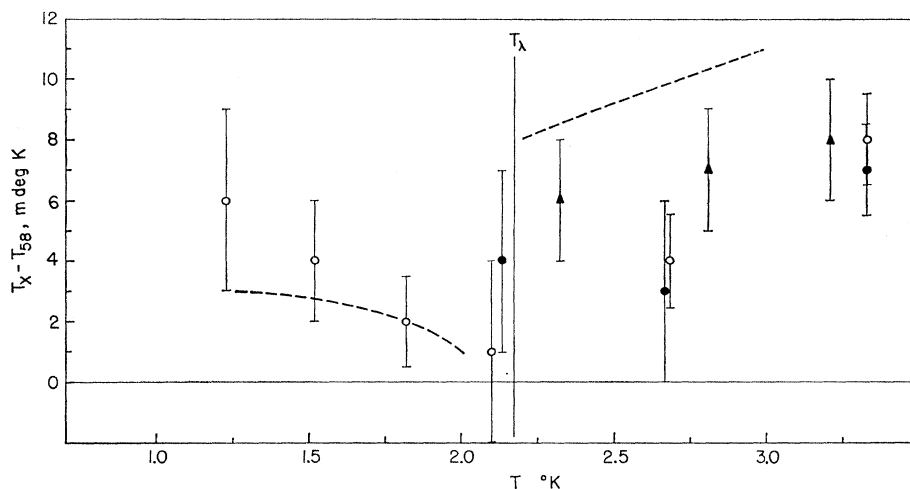


Fig. 4. A summary of the temperature discrepancies cited by various groups. The open circles are values given in this report for He³; the closed circles for He⁴. The dashed lines are values deduced from latent heat measurements (see Ref. 22). The solid triangles are values obtained in He⁴ using the NBS acoustic thermometer (see Ref. 2).

and the T_{58} He⁴ vapor-pressure scale. Such a possibility is more tenable when evidence given by Roberts, Sherman, and Sydoriak²² is cited. In discussing the merits of the T_{62} He³ vapor-pressure scale, these authors consider such a possibility by pointing out:

(i) Thermodynamic calculations of the latent heat of vaporization of He³ and He⁴, when compared with the measured values, indicate the T_{58} scale may be low by amounts varying between 3 mdeg K at 1°K to 1 mdeg K at 2°K. Above T_λ the deviation ranges from 8 mdeg K at 2.2°K to 11 mdeg K at 3.0°K.

(ii) Preliminary acoustic thermometry results indicate a ΔT of $+3 \pm 2$ mdeg K at 2.0 and 2.2°K. This must not be appended with the results of Cataland and Plumb² indicating a linear deviation of 6 mdeg K at 2.3°K to 8 mdeg K at 3.2°K.

These results together with those of the present work are also shown in Fig. 4. Taken together they offer further support to the thesis that a systematic error is present in the T_{58} temperature scale.

V. SECOND VIRIAL COEFFICIENT

A least-squares fit of the velocity measurements yields the coefficient of the linear term, $A_1(T)$, in Eq. (1). This can be expressed¹ in terms of the second virial coefficient as

$$A_1(T) = \frac{\gamma_0}{M} \left[2B(T) + \frac{4}{3} T \frac{dB}{dT} + \frac{4}{15} T^2 \frac{d^2B}{dT^2} \right]. \quad (6)$$

Previous measurements of $B(T)$ in this temperature range^{1,23,24} indicate that $B(T)$ can adequately be rep-

resented by the functional form

$$B(T) = \alpha + \beta/T. \quad (7)$$

This in turn means that $A_1(T)$ is given by the relation

$$A_1(T) = \frac{\gamma_0}{M} \left[2\alpha + \frac{6\beta}{5T} \right]. \quad (8)$$

Thus by plotting A_1 from the velocity of sound measurements as a function of T^{-1} we can determine the constants α and β and therefore the second virial coefficient.

A least-squares fit of the values yields the results

He⁴: 2.130 to 4.219°K

$$B(T) = (22 \pm 4) - (433 \pm 22)/T \quad \text{cm}^3/\text{mole}, \quad (9)$$

He³: 1.232 to 3.598°K

$$B(T) = (5.0 \pm 1.9) - (256.2 \pm 7.7)/T \quad \text{cm}^3/\text{mole}. \quad (10)$$

The temperature range for the He⁴ results has been extended to 4.219°K by using two results obtained previously using a similar apparatus.²⁵ The values of A_1 from these measurements are:

$$\begin{aligned} 4.003^\circ\text{K} \quad A_1 &= -3620 \pm 100 \quad \text{m}^2/\text{sec}^2\text{-atm}, \\ 4.219^\circ\text{K} \quad A_1 &= -3287 \pm 42 \quad \text{m}^2/\text{sec}^2\text{-atm}. \end{aligned}$$

The best previous measurements of the second virial coefficients are those of Keller based on pV -isotherm results.^{23,24} His data has been reanalyzed by Roberts, Sherman, and Sydoriak²² who employed a technique of least-squares curve fitting which makes allowances for an uncertainty in both the dependent and independent variable. The isotherm data give the value of B for each temperature at which an isotherm is measured.

²² T. R. Roberts, R. H. Sherman, and S. G. Sydoriak, J. Res. Natl. Bur. Std. **68A**, 567 (1964).

²³ W. E. Keller, Phys. Rev. **97**, 1 (1955); **100**, 1790 (1955).

²⁴ W. E. Keller, Phys. Rev. **98**, 1571 (1955).

²⁵ D. T. Grimsrud, Master of Science thesis, University of Minnesota, 1963 (unpublished).

Keller²⁴ and Roberts, Sherman, and Sydoriak²² have indicated that a proper interpolation formula for Keller's values of B has the functional form given by Eq. (7). However, no uncertainties are listed for the coefficients they have calculated. Therefore, in order to calculate the dependence of B upon T and obtain the uncertainties in the coefficients of this expression we have made a least-squares fit of the reanalyzed data from Ref. 22 and have obtained the following temperature dependence for Keller's values of B :

$$\text{He}^4: 2.147 \text{ to } 3.954^\circ\text{K}$$

$$B(T) = (24 \pm 4) - (426 \pm 10)/T \text{ cm}^3/\text{mole}, \quad (11)$$

$$\text{He}^3: 1.510 \text{ to } 3.779^\circ\text{K}$$

$$B(T) = (4.2 \pm 3.1) - (268.7 \pm 7.8)/T \text{ cm}^3/\text{mole}. \quad (12)$$

Figures 5 (He³) and 6 (He⁴) display Eqs. (9)–(12). Keller's individual values of the second virial coefficient are also shown.^{23,24} The horizontal shading on each figure bounded by the lines BB denotes the 95% con-

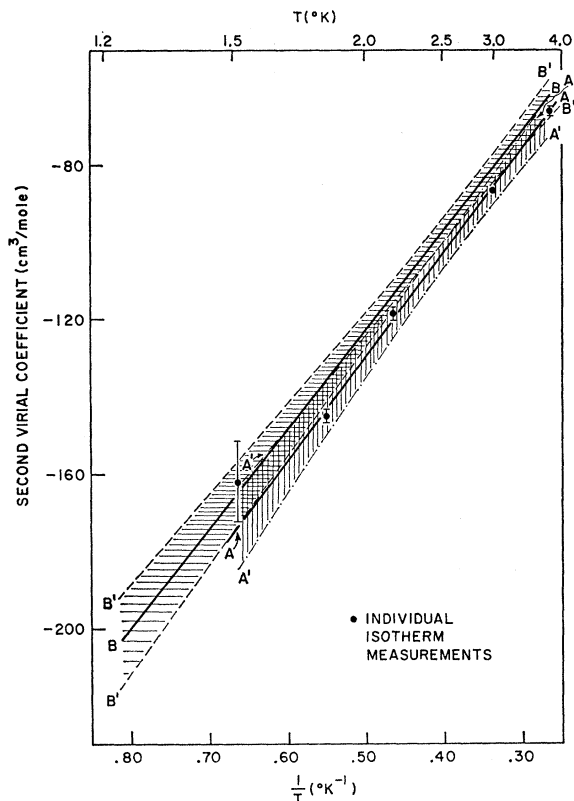


FIG. 5. Second virial coefficient results for He³. The horizontal shading bounded by the lines BB denotes the 95% confidence interval about the solid line BB which gives the second virial coefficient calculated from the sound velocity measurements presented in this report. The vertical shading bounded by the lines AA defines the 95% confidence band referred to the line AA . Line AA gives the second virial coefficient obtained from pV -isotherm measurements (see Ref. 24). The individual isotherm measurements of B from Ref. 24 are shown as solid circles.

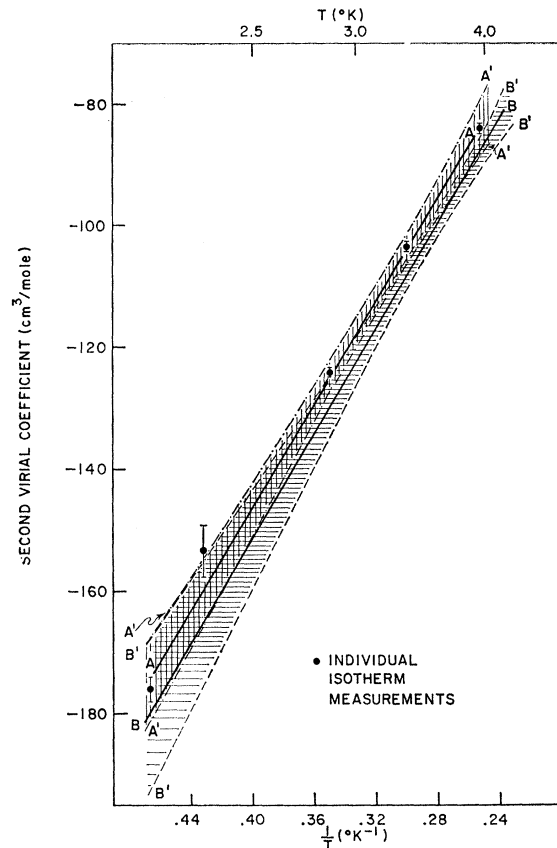


FIG. 6. Second virial coefficient results for He⁴. The horizontal shading bounded by the lines BB denotes the 95% confidence interval about the solid line BB which gives the second virial coefficient calculated from the sound velocity measurements presented in this report. The vertical shading bounded by the lines AA defines the 95% confidence band referred to the line AA . Line AA gives the second virial coefficient obtained from pV -isotherm measurements (see Ref. 23). The individual isotherm measurements of B from Ref. 23 are shown as solid circles.

fidence interval (giving the interval which contains the true statistical value of $B(T)$ with a probability of 0.95) about the line BB calculated from sound velocity measurements. The vertical shading bounded by the lines AA defines the 95% confidence band referred to the line AA calculated from isotherm measurements. The confidence bands were obtained using an expression given by Mandel²⁶ appropriately modified to account for the use of weighted data in the curve fitting procedures.

VI. CONCLUSIONS AND SUMMARY

The results of this experiment are conveniently placed in three major categories.

Isothermal measurements of absolute sound velocities as a function of pressure have been reported for He³ in the temperature range 1.2 to 3.6°K and for He⁴

²⁶ John Mandel, *The Statistical Analysis of Experimental Data* (Interscience Publishers, Inc., New York, 1964), p. 288.

between 2.1 and 3.8°K. The estimated uncertainties of the results are approximately 0.07%.

The absolute temperature of the experimental system has been calculated from the velocity measurements. This value has been compared with the value of the temperature of the coolant bath measured on the T_{58} He⁴ vapor-pressure scale. A discrepancy has been found between the two values. The source of the temperature difference, though not clearly understood, may well arise from an error in the T_{58} temperature scale.

Calculations of the second virial coefficient are reported which agree within experimental uncertainty with the best previous measurements obtained using different techniques. The results extend the knowledge

of the second virial coefficient of He³ to 1.2°K and indicate the suitability of sound velocity measurements for the determination of virial coefficients.

VII. ACKNOWLEDGMENTS

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Erratum

Ground State of Liquid Helium—Boson Solutions for Mass 3 and 4, WALTER E. MASSEY [Phys. Rev. 151, 153 (1966)]. (a) In Fig. 15 $J^{(l)} = J_{\text{ext}}/d$. (b) In Table X, columns 2, 4, and 5; rows 2 and 3 the numbers should be changed. The following are the listed numbers with the correct number in parenthesis: 502.86 (505.40), 71.97 (74.53), 14.31 (14.74) and 529.99 (530.21), 82.23 (82.46), 15.52 (15.55). Also, J_{ext} , J_a , and J_b are in units of Å.