(iii) $g_1(x)=1/p(x)$, where $p(x)$ satisfies the following conditions:

(a) $p(x)>0$ for all $x\geqslant a(>0)$,
(b) $p(x)$ is *concave* in $[a,\infty)$,
(c) $p(x)\to\infty$ as $x\to\infty$.

Then

$$z^\alpha f(z)\to A\pi(\cot\pi\alpha+i)$$

as $|z|\to\infty$ in any direction, the convergence being uniform with respect to θ for $0\leqslant\theta\leqslant 2\pi$. Here

$$z=re^{i\theta},\quad (r>0,\ 0\leqslant\theta\leqslant 2\pi)$$

$$z^\alpha=r^\alpha e^{i\alpha\theta},$$

$$f(z)=\int_{\to 0}^{\to\infty}\frac{g(t)dt}{t-z}\quad\text{for}\quad 0<\theta<2\pi,$$

$$f(x\pm i0)=P\int_{\to 0}^{\to\infty}\frac{g(t)dt}{t-x}\pm i\pi g(x)\quad\text{for}\quad\theta=\begin{pmatrix}0\\2\pi\end{pmatrix}.$$

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Neutral Semileptonic Decays of K Mesons*

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The question of the existence of neutral leptonic currents coupled to the neutral strangeness-changing current is discussed in the light of recent experimental limits on $K_2^0\to\mu^+\mu^-$ and $K_2^0\to e^+e^-$ decay rates.

I. INTRODUCTION

NEW experimental results on K_2^0 decays into lepton pairs have been reported recently.[1] At present, the total branching ratio corresponding to the mode $K_2^0\to\mu^+\mu^-$ is[2]

$$\frac{\Gamma(K_2^0\to\mu^+\mu^-)}{\Gamma(K_2^0\to\text{all modes})}<2.5\times10^{-6};\tag{1}$$

and for $K_2^0\to e^+e^-$,[3]

$$\frac{\Gamma(K_2^0\to e^+e^-)}{\Gamma(K_2^0\to\text{all modes})}<5\times10^{-5}.\tag{2}$$

Such decay modes are expected to occur from electromagnetic induction of neutral leptonic currents. However, the question arises whether or not they could also appear as a consequence of the existence of direct weak couplings[4] between neutral leptonic currents and neutral strangeness changing current. We should like to discuss here some implications of the new experimental upper limits mentioned above, upon the possible existence of such weak couplings.

By analogy to the usual semileptonic weak Hamiltonian, one expects neutral strangeness-changing semileptonic decays to be described by an effective Hamiltonian of the type

$$H(\text{neutral})=\frac{G}{\sqrt{2}}\sum_l g_l(J_3{}^2)^\mu\bar{l}i\gamma_\mu(1+i\gamma_5)l+\text{H.c.},\tag{3}$$

$$l=\nu_e,\ e^-,\ \nu_\mu,\ \mu^-.$$

Here, we have assumed that neutral leptonic currents have V-A structure, like the charged currents, and that $(J_3{}^2)^\mu$ is the $\Delta S=1$, $\Delta Q=0$ component of the usual octet of hadronic currents, consisting of a vector part plus an axial-vector part: $(J_3{}^2)^\mu=(V_3{}^2)^\mu+(A_3{}^2)^\mu$. The constant G is the Fermi coupling constant: $G=1.02\times10^{-5}/m_p{}^2$; and g_l are dimensionless unknown parameters (in principle different for each lepton pair) which depress the intensity of the neutral decay rates with respect to the corresponding charged modes. We assume that the parameters g_l are real or pure imaginary, and we shall discuss the physical implications accordingly. We shall also comment on some implications of the upper limits given above upon the predictions of a

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[1] See N. Cabibbo, in *Proceedings of the Thirteenth Internatjonal Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, California, 1967).

[2] W. Vernon *et al.* (communication at the Berkeley Conference). Other recent experiments on this branching ratio give the following results: $[\Gamma(K_2^0\to\mu^+\mu^-)/\Gamma(K_2^0\to\text{all modes})]<8\times10^{-6}$ [M. Bott-Bodenhausen, X. de Bouard, D. G. Cassel, D. Dekkers, R. Felst, R. Mermod, I. Savin, P. Scharff, M. Vivargent, T. R. Willits, and K. Winter, Phys. Letters 23, 277 (1966)]; $\Gamma(K_2^0\to\mu^+\mu^-/\Gamma(K_2^0\to\text{all modes})<5\times10^{-5}$ [A. Abashian *et al.*, University of Illinois Report (unpublished)]. All these figures are 90% confidence limits.

[3] M. Bott-Bodenhausen *et al.*, see Ref. 2. See also A. Abashian *et al.*, Ref. 2. The latter group finds $(\Gamma(K_2^0\to e^+e^-)/\Gamma(K_2^0\to\text{all modes}))<5\times10^{-5}$ (90% confidence limit).

[4] Perhaps mediated by neutral intermediate vector boson(s).

theory of neutral leptonic currents which has recently been proposed.[5]

II. IMPLICATIONS WITHIN THE FRAMEWORK OF A CURRENT-CURRENT INTERACTION

Let us first discuss the consequences of a Hamiltonian of the type H(neutral) defined in Eq. (3). We shall distinguish the case where g_l is real from the one where g_l is pure imaginary.

A. The Parameters g_l are Real

Then, H(neutral) allows $K_2^0 \to \bar{l}l$ and forbids $K_1^0 \to \bar{l}l$. This follows from the CP-conserving property of H(neutral), and from the fact that the point-like coupling V-A forbids the lepton pair to be in a $^3P_0$ state. The decay rate $\Gamma(K_2^0 \to l^+l^-)$ predicted by H(neutral) is

$$\Gamma(K_2^0 \to l^+l^-) = (G^2/2\pi) g_l^2 f^2 M^2 m_l^2 (M^2 - 4m_l^2)^{1/2}, \quad (4)$$

where M is the mass of K^0, m_l the lepton mass, and f a dimensionless coupling constant appearing in the matrix element $\langle 0 | (A_2^3)^\mu | K^0 \rangle = fMp^\mu$, where $p^2 = M^2$. From charge independence, $\langle 0 | A_2^3 | K^0 \rangle = \langle 0 | A_1^3 | K^+ \rangle$; therefore the value of f can be estimated from the experimental decay rate $\Gamma(K^+ \to \mu^+\nu_\mu)$. We get

$$f \sin\theta = 7.08 \times 10^{-2}, \quad (5)$$

where θ is the Cabibbo angle,[6] which appears in the definition of the charged hadronic current. From Eqs. (4) and (5), and the experimental upper limits given in Eqs. (1) and (2) we calculate the corresponding upper limits for g_μ and g_e. We obtain

$$|g_\mu| < 4.62 \times 10^{-4} \sin\theta (= 1.22 \times 10^{-4}) \quad (6a)$$

and

$$|g_e| < 0.41 \sin\theta (= 0.11). \quad (6b)$$

Provided that a Hamiltonian of the type H(neutral) exists at all, these limits tell us that the *coupling of neutral lepton currents to the strangeness-changing current as compared to the corresponding coupling of charged lepton currents is depressed, at least by a factor* $\sim 5 \times 10^{-4}$ *for* μ *pairs and one order of magnitude for* e *pairs.*

The experimental upper limit quoted in Eq. (1) should also be compared to the theoretical estimates expected from electromagnetic induction of μ pairs. A recent calculation by Sehgal[7] gives the result

$$\frac{\Gamma(K_2^0 \to \mu^+\mu^-)}{\Gamma(K_2^0 \to \text{all modes})} \sim 4 \times 10^{-9};$$

i.e., three orders of magnitude below present experimental limits. In spite of the smallness of g_μ [see Eq. (6a)] we see that, in the $K_2^0 \to \mu^+\mu^-$ search, there is still a large interval where neutral leptonic currents could be detected unambiguously.

Next, let us consider $K^+ \to \pi^+ l^+ l^-$ decays. Here, the relevant hadronic vertex is

$$\langle \pi^+(p') | V^\mu(0)_2^3 | K^+(p) \rangle = (p+p')^\mu g_+(t) + (p-p')^\mu g_-(t), \quad (7)$$

where $t = (p-p')^2$, p and p' are the energy-momenta of K^+ and π^+, with $p^2 = M^2$ and $p'^2 = m^2$, $g_+(t)$ and $g_-(t)$ unknown form factors. Using H(neutral), defined in Eq. (3), we get, for the decay rate

$$\Gamma(K^+ \to \pi^+ l^+ l^-) = \frac{G^2}{(2\pi)^3} g_l^2 \frac{1}{(2M)^3} \times \int_{4m_l^2}^{(M-m)^2} dt [m_l^2 |A(t)|^2 + \tfrac{1}{3}(t - m_l^2) |F(t)|^2] \times \{t(t-4m_l^2)[(M-m)^2 - t][(M+m)^2 - t]\}^{1/2}, \quad (8)$$

with

$$A(t) = [(M^2 - m^2)/t] g_+(t) + g_-(t),$$

and

$$F(t) = -t^{-1}\{[(M+m)^2 - t][(M-m)^2 - t]\}^{1/2} g_+(t).$$

At the limit of exact $SU(3)$ invariance and $t=0$, the matrix element $\langle \pi^+ | V_2^3 | K^+ \rangle$ is proportional to $\langle \pi^0 | V_1^3 | K^+ \rangle$, which is known (in modulus) from the $K^+ \to \pi^0 e^+ \nu_e$ decay rate. Thus, we get

$$g_+(0) = \sqrt{2} f_+(0), \quad (9)$$

where $f_+(0)$ is the usual form factor[8] appearing in K_{e3}^+, taken at zero momentum transfer.

An estimate of $g_-(t)$ can be obtained using the *zero-energy theorem* developed by Callan and Treiman.[9] The version of this theorem which interests us here, states the following relations:

$$\langle 0 | [V^\mu(0)_2^3, F_{(-)}^5(0)] | K^+(p) \rangle = \frac{f_\pi}{\sqrt{2}} \lim_{p' \to 0} \langle \pi^+(p') | V^\mu(0)_2^3 | K^+(p) \rangle,$$

and

$$[V^\mu(0)_2^3, F_{(-)}^5(0)] = A^\mu(0)_1^3.$$

These relations follow from the algebra of hadronic currents proposed by Gell-Mann[10] and the assumption of partially conserved axial-vector current[11]; f_π is the

[5] M. L. Good, L. Michel, and E. de Rafael, Phys. Rev. **151**, 1195 (1966).

[6] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963). Throughout this work we shall assume that $\theta_V = \theta_A = 0.26$.

[7] L. M. Sehgal, Nuovo Cimento **45**, 785 (1966).

[8] In general: $\langle \pi^0(p') | V^\mu(0)_1^3 | K^+(p) \rangle = (p+p')^\mu f_+(t) + (p-p')^\mu \times f_-(t)$.

[9] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966). See also V. S. Mathur, S. Okubo, and L. K. Pandit, *ibid.* **16**, 371 (1966).

[10] M. Gell-Mann, Phys. Rev. **125**, 1064 (1962); and Physics **1**, 63 (1964).

[11] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); S. L. Adler, Phys. Rev. **137**, B1022 (1965).

π coupling constant in $\pi \to \mu+\nu$ decay and $F_{(-)}{}^5$ the charge associated with the axial current $A_1{}^2$:

$$F_{(-)}{}^5(0)=\int d^3x\, A(x)_1{}^2.$$

Using these relations and the expression given in Eq. (7), we get

$$\sin\theta[g_+(M^2)+g_-(M^2)]=\sqrt{2}f_K/f_\pi\,, \qquad (10)$$

where f_K is the K coupling constant in $K\to\mu+\nu$ decay, i.e., $\sin\theta\langle 0|(A^\mu)_1{}^3|K^+(p)\rangle=f_K p^\mu$.

To estimate the decay rate defined in Eq. (8) we have used the information contained in Eqs. (9) and (10) in the following way: (i) We have assumed that the g_+ form factor is dominated by the K^* pole at 891 MeV, i.e.,

$$g_+(t)=g_+(0)(1+t/M_{K^*}{}^2)\,.$$

(ii) We have assumed that the ratio $\xi=g_-(t)/g_+(t)$ is a constant.[12] Then, the integration in Eq. (8) yields to the results

$$\Gamma(K^+\to\pi^+\mu^+\mu^-)=5.53g_\mu{}^2\,10^7(\mathrm{sec}^{-1}) \qquad (11\mathrm{a})$$

and

$$\Gamma(K^+\to\pi^+e^+e^-)=1.17g_e{}^2\,10^8(\mathrm{sec}^{-1})\,. \qquad (11\mathrm{b})$$

Using the upper limits obtained in Eqs. (6a) and (6b), and the K^+ mean life, we get the branching ratios

$$\frac{\Gamma(K^+\to\pi^+\mu^+\mu^-)}{\Gamma(K^+\to\text{all modes})}<1.01\times10^{-8} \qquad (12\mathrm{a})$$

and

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to\text{all modes})}<1.55\times10^{-2}. \qquad (12\mathrm{b})$$

These numbers are to be compared with the corresponding experimental upper limits[13]

$$\frac{\Gamma(K^+\to\pi^+\mu^+\mu^-)}{\Gamma(K^+\to\text{all modes})}<3\times10^{-6} \qquad (13\mathrm{a})$$

and[14]

$$\frac{\Gamma(K^+\to\pi^+e^+e^-)}{\Gamma(K^+\to\text{all modes})}<1.1\times10^{-6}. \qquad (13\mathrm{b})$$

The latter result leads to a better upper limit for g_e than the one obtained from two-body decays [see Eq. (6b)]. Indeed from Eqs. (11b) and (13b) we get

$$|g_e|<0.88\times10^{-3}, \qquad (14)$$

i.e., *the coupling of e pairs to the neutral strangeness-changing current as compared to the coupling of* $e^+\nu_e$ ($e^-\bar{\nu}_e$) *to the charged strangeness-changing current is depressed at least by three orders of magnitude.* As regards the coupling of μ pairs, it turns out that *the limit given in Eq (12a) is even lower than the theoretical estimates based on electromagnetic induction of μ pairs*, which predict[15]

$$\frac{\Gamma(K^+\to\pi^+\mu^+\mu^-)}{\Gamma(K^+\to\text{all modes})}\cong1.4\times10^{-7}.$$

We see that for a Hamiltonian of the type H(neutral) [see Eq. (3)] with g_l real, experimental upper limits on $K_2{}^0\to\mu^+\mu^-$ imply very small upper limits on $K^+\to\pi^+\mu^+\mu^-$, while experimental upper limits on $K^+\to\pi^+e^+e^-$ imply very small upper limits on $K_2{}^0\to e^+e^-$. In fact, if $K^+\to\pi^+\mu^+\mu^-$ decays were found with total branching ratio $>10^{-7}$, a Hamiltonian of the type H(neutral) with g_μ real would be ruled out by the experimental number quoted in Eq. (1).

B. The Parameters g_l are Pure Imaginary

Then, H(neutral) allows $K_1{}^0\to\bar{l}l$ and forbids $K_2{}^0\to\bar{l}l$. At present, there are no experimental results on $K_1{}^0\to\bar{l}l$ decays. However, $K^+\to\pi^+\bar{l}l$ decays are also predicted by H(neutral) with g_l pure imaginary, and from their experimental upper limits [see Eqs. (13a,b)] one can get upper limits for $|g_\mu|$ and $|g_e|$. The limit on $|g_e|$ is the one given in Eq. (14); the limit on $|g_\mu|$ can be obtained from Eqs. (11a) and (13a):

$$|g_\mu|<2.10\times10^{-3}. \qquad (15)$$

Then we can estimate the corresponding branching ratios expected for $K_1{}^0\to\mu^+\mu^-$ and $K_1{}^0\to e^+e^-$ decays. The results are

$$\frac{\Gamma(K_1{}^0\to\mu^+\mu^-)}{\Gamma(K_1{}^0\to\text{all modes})}<1.17\times10^{-6}$$

and

$$\frac{\Gamma(K_1{}^0\to e^+e^-)}{\Gamma(K_1{}^0\to\text{all modes})}<5.26\times10^{-12}.$$

These are very small numbers. However, a branching ratio of $\sim10^{-6}$ for the mode $K_1\to\mu^+\mu^-$ should be be attainable with present techniques. Of course, if $K_1{}^0\to\mu^+\mu^-$ decays are detected with a total branching ratio $>2\times10^{-6}$ this would rule out a Hamiltonian of the type H(neutral) with g_μ pure imaginary.

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[12] Note that these assumptions correspond to those usually made in the computation of $K_{\mu3}{}^+$ decay rate.

[13] U. Camerini, D. Cline, G. Gidal, G. Kalmus, and A. Kernan, Nuovo Cimento **37**, 1795 (1965) (90% confidence level).

[14] U. Camerini, D. Cline, W. F. Fry, and W. M. Powell, Phys. Rev. Letters **13**, 318 (1964) (90% confidence level).

[15] The branching ratio $[\Gamma(K^+\to\pi^+e^+e^-)/\Gamma(K^+\to\text{all modes})]$ expected by electromagnetic induction of e pairs has been estimated by Cabibbo and Ferrari to be 1.0×10^{-7}. [N. Cabibbo and E. Ferrari, Nuovo Cimento **18**, 928 (1960).] Other estimates [see M. Baker and S. L. Glashow, Nuovo Cimento **25**, 857 (1962); M. A. B. Bég, Phys. Rev. **132**, 426 (1963)] give a total branching ratio $\sim10^{-6}$. The value for $[\Gamma(K^+\to\pi^+\mu^+\mu^-)/\Gamma(K^+\to\text{all modes})]$ has been obtained by simply taking into account the phase-space ratio with respect to an average of the estimates of $\Gamma(K^+\to\pi^+e^+e^-)$.

III. INTERMEDIATE VECTOR BOSONS

If weak couplings are mediated by intermediate vector bosons (W's) then, the straightforward generalization of the Hamiltonian given in Eq. (3) is

$$H_W(\text{neutral})=g^2\sum_l g_l(J_3{}^2)^\mu W_{\mu\nu}\bar{l}i\gamma^\nu(1+i\gamma_5)l+\text{H.c.},$$

where $W_{\mu\nu}$ is the propagator of a neutral W with mass M_W, and $g^2/M_W{}^2=G/\sqrt{2}$. For $M_W\gtrsim 2$ BeV the decay rates predicted by a Hamiltonian of this type are roughly equal to those predicted by H(neutral) in Eq. (3). The results of the preceding section apply similarly to a Hamiltonian of the type H_W(neutral).

IV. THEORETICAL REMARKS

The new experimental results quoted in Eqs. (1) and (2) affect some of the predictions of the theory discussed in Ref. 5. In this theory, neutral hadronic currents and neutral leptonic currents appear as fundamentally uncoupled to each other. However, it was shown[5] that small perturbing effects could create links among them, originating CP-violation effects. Two possible perturbing mechanisms were suggested: one due to the mass difference between the two neutral W's which appear in the scheme[16] (β mechanism), and another due to intrinsic magnetic moment of neutral W's (α mechanism). Both models allow neutral strangeness-changing decays *either* into $\bar{\nu}_e\nu_e$ and $\mu^+\mu^-$ pairs *or* into $\bar{\nu}_\nu\nu_\nu$ and e^+e^- pairs.

The predictions arising from the β mechanism are unaffected by the new experimental limits given in Eqs. (1) and (2). In this model $K_2{}^0\to\mu^+\mu^-$ is forbidden. Here, search for $K_1{}^0\to\mu^+\mu^-$ and $K^+\to\pi^+\mu^+\mu^-$ decays is needed to decide upon this model.

The α mechanism allows both $K_2{}^0\to\mu^+\mu^-$ and $K^+\to\pi^+\mu^+\mu^-$ decays. If we call $(e\hbar/2M_Wc)\sigma$ the intrinsic magnetic moment of neutral W, then the total branching ratio of $K_2{}^0\to\mu^+\mu^-$ is $\sim\alpha^2\sigma^2$. From the experimental number quoted in Eq. (1) we see that $\sigma<0.22$.

Both the α mechanism and the β mechanism predict CP violation[17] in processes where a μ pair is emitted, due to the interference of the weak semileptonic amplitude with the weak (nonleptonic)×electromagnetic amplitude. Both mechanisms lead to CP-violating effects in the K^0-$\bar{K}^0$ system, due to the contribution of these mechanisms to an imaginary part in the off-diagonal terms of the K^0-$\bar{K}^0$ mass matrix. However, as we now know,[18] this is not enough to explain the present experimental information on the K^0-$\bar{K}^0$ system.

We should like to emphasize that these perturbing mechanisms (α mechanism and β mechanism) are by no means unique. The question of the existence of neutral leptonic currents and their implications can only be settled by systematic search for the three possible types of coupling: neutral lepton current–neutral lepton current; $\Delta S=1,\Delta Q=0$ current–neutral lepton current; $\Delta S=0$, $\Delta Q=0$ current–neutral lepton current.

Note added in proof. New experimental results have been published since the completion of this work [see M. Bott-Bodenhausen, X. de Bouard, D. G. Cassel, D. Dekkers, R. Felst, R. Mermod, I. Savin, P. Scharff, M. Vivargent, T. R. Willetts, and K. Winter, Phys. Letters **24B**, 194 (1967)]. The upper limits obtained by this group (90% confidence) are:

$$\Gamma(K_2{}^0\to\mu^+\mu^-)/\Gamma(K_2{}^0\to\text{all modes})<1.6\times10^{-6},$$
$$\Gamma(K_2{}^0\to e^+e^-)/\Gamma(K_2{}^0\to\text{all modes})<1.8\times10^{-5},$$

and

$$\Gamma(K_1{}^0\to\mu^+\mu^-)/\Gamma(K_1\to\text{all modes})<7.3\times10^{-5}.$$

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[16] In this theory, the W's are in a triplet of U_3 as first suggested by B. d'Espagnat [Phys. Letters **7**, 209 (1963)].

[17] See Ref. 5. See also L. Michel, lecture given at the Second Japanese Summer Institute in Theoretical Physics, Oisi, September 1966 (unpublished).

[18] James W. Cronin, Paul F. Kunz, W. S. Risk, and P. C. Wheeler, Phys. Rev. Letters **18**, 25 (1967); J. M. Gaillard *et al.*, *ibid.* **18**, 20 (1967).