Special Permutation Invariants of Four SU_3 Octets*

TINO AHRENS
Georgia Institute of Technology, Atlanta, Georgia
(Received 5 December 1966)

A simple proof is given that a biquadratic form of octets exists which is a scalar under SU_3, an eigenvector of the permutation operator p_{13} (and p_{24}), and whose constituent quadratics transform like octet components. The form is given and so is another, subject only to the last two conditions.

I. INTRODUCTION

IN view of the widely successful adoption of formalisms involving internal symmetries analogously to symmetries in space-time, it seems reasonable to extend this analogy into the realm of permutation symmetries. A step one might take in this direction consists of formulating a scalar interaction of four SU_3 octets so as to render it invariant under the permutation of the first and third and of the second and fourth octets, quite in analogy with the well-known permutation-invariant scalar interactions of four Dirac spinors.[1,2] In view of the present predilection for octets, it would then be gratifying if at least one such scalar existed, which is formed purely from quadratics that also transform like octet components. More in the language of physics: One might hope for a permutation-invariant scalar interaction formed only by the coupling of currents belonging to octets. An inspection of a diagonalizing matrix which Frahm has constructed, together with the reordering matrix for all biquadratic scalars formed from SU_3 octets, shows that such an invariant can be formed.[3] Specifically, the invariant in question is

$$8_\epsilon\cdot 8_0'+8_0\cdot 8_\epsilon', \tag{1}$$

where the numbers refer to the SU_3 multiplet structure of the currents and the subscripts to the circumstance as to whether the particular multiplet resulted as an "even" or as an "odd octet" from the reduction of the direct product of two octets; the prime indicates that octets of different currents may interact with one another.

That there indeed is a scalar interaction of pure octet currents exhibiting permutation invariance and that this interaction is of the form indicated by expression (1) can also be shown straightforwardly by direct use of the commutation properties and traces of the fundamental representation of SU_3. Apart from its appeal of simplicity, this provides a check on the pertinent result of Frahm's all-inclusive but necessarily complex procedure. Furthermore, additional pure octet current interactions may show up which could be of physical interest, although they will not be scalars. Also, the method employed here should be applicable in similar proofs, whenever regular representations are involved.[4]

II. PROOF

The general biquadratic scalar of octets in SU_3 space, with the quadratics belonging to octets as well, may be written in the form

$$a(f)_m(f)_m'+b(f)_m(d)_m'+c(d)_m(f)_m'+d(d)_m(d)_m', \quad (f)_m\equiv\bar{\Psi}_k f_{klm}\Psi_l, \quad (f)_m'\equiv\Psi'_k f_{klm}\Psi_l', \tag{2}$$

with sums to be extended over all indices appearing twice. The Ψ and Ψ' may be interpreted as field operators, while a, b, c, and d are arbitrary, in general complex constants. The f_{klm} and d_{klm} are related to the pertinent SU_3 Clebsch-Gordan coefficients as, e.g., the matrix elements of the three-dimensional rotation generators in Cartesian coordinates to the Clebsch-Gordan coefficients combining two triplets to form another triplet.[5] The latter analogy is, of course, not one-to-one, since SU_3 has two such sets of Clebsch-Gordan coefficients, while SU_2 has only one. The bars are there to permit from the outset distinction between particle creation and destruction as far as the behavior in space-time is concerned.

The question is: Under what conditions on the constants a, b, c, and d, if under any, will expression (2) be invariant up to sign under the permutation p_{13} (or p_{24}); i.e., which are the nontrivial solutions of the following equation:

$$af_{ksm}f_{rlm}+bf_{ksm}d_{rlm}+cd_{ksm}f_{rlm}+dd_{ksm}d_{rlm} =\pm(af_{klm}f_{rsm}+bf_{klm}d_{rsm}+cd_{klm}f_{rsm}+dd_{klm}d_{rsm}). \tag{3}$$

Now we invoke the previously mentioned properties of the fundamental representation,[5]

$$[\lambda_k,\lambda_l]=2if_{klm}\lambda_m, \tag{4a}$$

$$\{\lambda_k,\lambda_l\}=2d_{klm}\lambda_m+\tfrac{4}{3}\delta_{klm}, \tag{4b}$$

$$\mathrm{Tr}(\lambda_k\lambda_1)=2\delta_{kl}. \tag{4c}$$

* Work supported by the National Science Foundation.

[1] C. P. Frahm, Ann. Phys. (N. Y.) **40**, 159 (1966).

[2] An analogous step involving SU_2 has been taken previously by T. Ahrens [Progr. Theoret. Phys. (Kyoto) **34**, 867 (1965); Nuovo Cimento **43**, 1158 (1966)].

[3] Actually, Frahm's diagonalizing matrix does not exhibit this invariant directly, but yields it as a linear combination of its fifth and sixth rows. See Ref. 1, Table II.

[4] In the case of SU_2 and its (3-dimensional) regular representation such a unique permutation invariant does not exist.

[5] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

Because of Eq. (4c) we can expand as follows:

$$2f_{klm}f_{rsm}=f_{klm}\lambda_{mAB}\lambda_{nBA}f_{rsn}\,,\quad \text{etc.} \tag{5}$$

This is the essential step in facilitating the present proof. With the aid of Eqs. (4a)–(4c), Eqs. (3) then become

$$a[\lambda_k,\lambda_s]_{AB}[\lambda_r,\lambda_l]_{BA}-d(\{\lambda_k,\lambda_s\}_{AB}\{\lambda_r,\lambda_l\}_{BA}-16\delta_{ks}\delta_{rl}/3)+i(b[\lambda_k,\lambda_s]_{AB}\{\lambda_r,\lambda_l\}_{BA}+c\{\lambda_k,\lambda_s\}_{AB}[\lambda_r,\lambda_l]_{BA})=\pm(s\rightleftharpoons l)\,. \tag{6}$$

A nontrivial solution of the lower of Eqs. (6), valid for all k, l, r, and s, is given by $a=d=0$ and $b=-c$.[6] The corresponding interaction was exhibited above as Eq. (1). No other generally valid solutions of Eqs. (6) appear to exist. It may be of interest to note the existence of a solution to the upper equation when k, $r\neq l$, s. This solution is given by $b=c=0$ and $a=-d$. The corresponding interaction is, of course, not a scalar under SU_3. It shall be denoted by the expression

$$8_\epsilon\cdot 8_\epsilon'-8_0\cdot 8_0'\,,\quad 1,3\neq 2,4\,, \tag{7}$$

where the inequality specifies that in no term of the interaction may the components of the first and third octets carry the same index as those of the second and fourth octets.

A unitary transformation of the field operators appearing in Eq. (2) together with the corresponding similarity transformation of the f and d matrices permits an evaluation of the currents 8_ϵ and 8_0 in terms of the usual current components, like $\bar{p}n$, $\bar{\Sigma}^0\Xi^-$, etc., in the case of baryons.[7]

[6] Incorporating, e.g., the V-A interaction in space-time in Eq. (2) interchanges the permutation eigenvalues, when commuting field operators are used.

[7] The complete odd and even baryon currents are exhibited by M. Gell-Mann [California Institute of Technology Report No. CTSL-20 (1961) (unpublished)] where they are coupled to the meson octet. It seems to this author that the component $\bar{\Xi}^0\Xi^-$ should be coupled to π^+ with a plus sign relative to the other couplings.

PHYSICAL REVIEW VOLUME 157, NUMBER 5 25 MAY 1967

Unsubtracted Dispersion Relations in Weak Interactions and Unitary Symmetry. II*

K. Nishijima

Department of Physics, University of Tokyo, Tokyo, Japan†

(Received 19 December 1966)

In a previous paper a dynamical approach to weak interactions was proposed based on unsubtracted dispersion relations, and in the present paper this approach is further developed. We first show that this theory reproduces the experimental data of the nonleptonic decays of hyperons fairly well. Then, we examine the consistency conditions imposed on the second-order matrix elements in weak interactions. It is shown that the current commutation relations as well as CP violation in the $K_2\rightarrow 2\pi$ decay are inevitable consequences of the consistency conditions.

I. INTRODUCTION

IN a series of papers[1–5] a dynamical approach to weak interactions has been introduced and discussed based on unsubtracted dispersion relations. In one of these papers by Swank and the present author,[4] this approach was combined with unitary symmetry to offer a unified interpretation of the Goldberger-Treiman relation,[6] Gell-Mann–Okubo,[7] and Coleman-Glashow[8] mass formulas, and the Cabibbo theory of semileptonic interactions.[9] From the study of these problems emerged a new form of the nonleptonic decay interactions in a rather natural way. This new form is different from the current-current type that has been widely accepted in describing the nonleptonic decays of strange particles. Therefore, in order to study the consequences of this new form we applied it to the nonleptonic S-wave decay of the hyperons and found an excellent agreement with the experimental data. Furthermore, it was shown that the Lee-Sugawara sum rule[10] holds as a con-

* This work was supported by the National Science Foundation under NSF Grant No. GP 5622. Part of this work was carried out while the author was staying at the University of Tokyo as a John Simon Guggenheim fellow.

† Present address: Department of Physics, University of Illinois, Urbana, Illinois.

[1] E. R. McCliment and K. Nishijima, Phys. Rev. **128**, 1970 (1962).

[2] K. Nishijima, Phys. Rev. **133**, B1092 (1964).

[3] K. Nishijima and M. H. Saffouri, Progr. Theoret. Phys. (Kyoto) Suppl., commemoration issue, 207 (1965).

[4] K. Nishijima and L. J. Swank, Phys. Rev. **146**, 1161 (1966).

[5] K. Nishijima, Progr. Theoret. Phys. (Kyoto) Suppl. **37**, 392 (1966). This is a preliminary version of the present paper.

[6] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); **111**, 354 (1958).

[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[8] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[9] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963)

[10] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **21**, 213 (1964).