Strip Approximation and the Shape of the Diffraction Peak

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It is pointed out that the sharpness of the diffraction peak is incompatible with the usual justification of the strip approximation to the Mandelstam representation. Discrepancies between the Regge residue functions needed to fit experiment and those obtained from dynamical calculations using the new form of the strip approximation are discussed. It is shown that if the correct shape of the diffraction peak is to be reproduced there are special requirements as to the form of the force—that it oscillate in sign within the strip—which are not satisfied by the Born approximation to that force. We indicate how the correct expression for the force may rectify this, and try to discover what difference having the right sort of residues would make in the slope of the trajectories in π-π scattering, by imposing the experimental shape on the residues artificially. The trajectory slopes are found to be improved, but the drastic nature of the change produced by imposing the experimental form on the output residues makes a bootstrap impossible if only the Born approximation to the force is used.

I. INTRODUCTION

In a series of recent papers a method has been proposed for calculating scattering amplitudes in the spirit of the bootstrap philosophy,[1–4] and its consequences have been determined for π-π scattering by machine computations.[5–7] The agreement between theory and experiment, though encouraging in many qualitative features,[7,8] is very disappointing in numerical detail. This has been the most comprehensive such calculation to date, and it is now clear that some new physics must be injected into the theory if we are to have any hope of obtaining solutions closer to nature.

One such piece of physics which is currently being investigated is the possibility of calculating more of the double spectral functions by using the Mandelstam iteration procedure.[7,9–11] However, this procedure is applicable only when elastic unitarity can be expected to hold at least approximately, that is, in the strip regions. All the calculations so far envisaged rely on the assumption that the strips entirely dominate the scattering amplitude. The justification[1,3] for this assumption has always been the observed concentration of the scattering into low-energy resonances, or, at higher energies, into narrow peaks along the forward and backward directions. It is now clear, however, that the rate at which the scattering amplitude falls off with momentum transfer away from the diffraction peak is much greater than would be expected from a simple bunching of the important singularities within the strip region.

As we discuss in the next section, some form of cancellation between the contribution of different parts of the double spectral function is required to give the observed behavior. This may mean either that the inner parts of the double spectral function, which are not included in the strips, and which we can not calculate, are important, or, more optimistically, that there are suitable oscillations within the strip region. This latter view has been the assumption underlying attempts to fit high-energy cross sections with Regge poles, in which the Regge residue functions have the required rapid decrease with momentum transfer.[12–14]

Such a decrease of the residue functions has not been evidenced by our previous calculations[6–7] and (see Sec. III) is unlikely to result from the sort of approximate forces which we have been using. In fact, we find that an oscillating-force function is required. We discuss the way in which such a force function might actually result from the combination of attractive and repulsive potentials which we anticipate. Unfortunately a more exact calculation of such forces is difficult (though work is in progress, using the Mandelstam iteration), and, in order to see what sort of behavior might be expected, we propose (Sec. IV) a method of imposing the required behavior on the output, which is analogous to the way in which the required threshold behavior is obtained in the solution of N/D equations, despite the approximate nature of the force.

This imposition is found to alter the output Regge functions. It worsens the inconsistency between the magnitudes (but not of course the shapes which are compelled to be similar) of the input and output residue functions of the attempted bootstrap, which is hardly surprising in view of the mutilation of the force by our imposition, but it also improves the shapes of the trajectories. In the final section we conclude that if the strip approximation is ever to give satisfactory results

[1] G. F. Chew, Phys. Rev. **129**, 2363 (1963).
[2] G. F. Chew, Phys. Rev. **130**, 1264 (1963).
[3] G. F. Chew and C. E. Jones, Phys. Rev. **135**, B208 (1964).
[4] C. E. Jones, Phys. Rev. **135**, B214 (1964).
[5] D. C. Teplitz and V. L. Teplitz, Phys. Rev. **137**, B142 (1965).
[6] P. D. B. Collins and V. L. Teplitz, Phys. Rev. **140**, B663 (1965).
[7] P. D. B. Collins, Phys. Rev. **142**, 1163 (1966).
[8] G. F. Chew and V. L. Teplitz, Phys. Rev. **136** B1154 (1964).
[9] B. H. Bransden *et al.*, Nuovo Cimento **30**, 207 (1963).
[10] N. F. Bali, G. F. Chew, and S.-Y. Chu, Phys. Rev. **150**, 1352 (1966).
[11] N. F. Bali, Phys. Rev. **150**, 1358 (1966).
[12] W. Rarita and V. L. Teplitz, Phys. Rev. Letters **12**, 206 (1964).
[13] R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).
[14] T. O. Binford and B. R. Desai, Phys. Rev. **138**, B1167 (1965).

there are conditions which the full force (resulting from iterating the potential) will have to satisfy, and which are very different from the force taken in the first Born approximation, which has so far been used in bootstrap calculations.

II. THE STRIP APPROXIMATION AND THE FORWARD PEAK

The new form of the strip approximation[3] was put forward as a suitable method of performing an approximate bootstrap calculation consistent with the principles of maximal analyticity of the first and second kinds; that is, satisfying the Mandelstam representation, and with all the poles being Regge poles. It involves parametrizing strips of the double spectral functions adjacent to the physical region so as to ensure the occurrence of the low-energy resonance poles in each given channel, and Regge asymptotic behavior in the crossed channels.

Thus the strip A in Fig. 1 is parametrized by[15]

$$\rho(s,t)=\Delta_t[(\pi/2)\Gamma(t)P_{\alpha(t)}(-1-s/2q_t^2)]\theta(s-s_1), \quad (2.1)$$

where

$$\Gamma(t)=[2\alpha(t)+1]\gamma(t)(-q_t^2)^{\alpha(t)}, \quad (2.2)$$

$\alpha(t)$ being the trajectory function, and $\gamma(t)$ the reduced residue function. This gives a contribution to the amplitude

$$R^{s_1}(s,t)=\tfrac{1}{2}\Gamma(t)\left\{-\frac{\pi}{\sin\pi\alpha(t)}P_{\alpha(t)}(1+s/2q_t^2)-\int_{-4q_t^2}^{s_1}\frac{P_{\alpha(t)}(-1-s/2q_t^2)}{(s'-s)}ds'\right\}. \quad (2.3)$$

This parametrization manifestly contains the t channel poles for $\alpha=$ integer, and has the behavior

$$\rho(s,t)\underset{s\to\infty}{\longrightarrow}s^{\alpha(t)}. \quad (2.4)$$

Since it is observed that the scattering amplitude is only large in the low-energy resonance region and, at high energies, within a narrow peak in the forward or backward directions, it was hoped that, if the parametrization of the strips were fixed appropriately, the scattering amplitude would be correctly represented wherever, in the physical region, it is large.

The set of Regge parameters which was found to bootstrap[7] bore some resemblance to that found experimentally, but was far from being in satisfactory agreement. The trajectory functions were not too dissimilar from those obtained in fitting the experimental data by Phillips and Rarita,[13] but the form of the residue functions was quite different, and we did not find the sharp falling away of the amplitude with momentum transfer

[15] The notation used is the same as that of Ref. 7 and earlier papers on this subject.

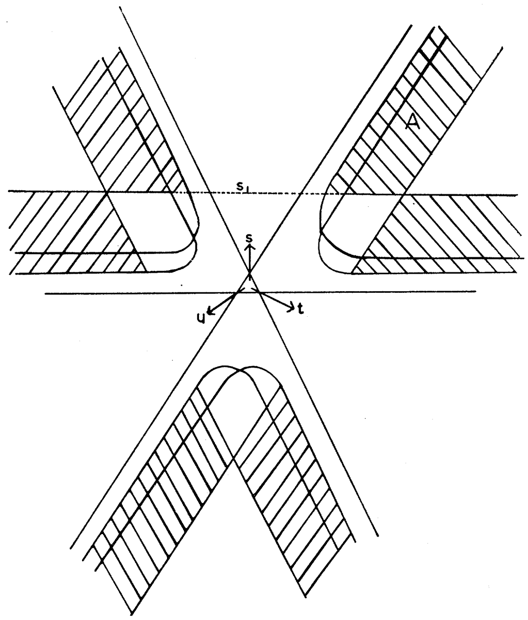

FIG. 1. The Mandelstam diagram showing the strips (shaded).

which is observed. This rapid decrease appears to be a general feature not only of elastic scattering,[16,17] but also of the production processes.[18]

For both π-p and p-p elastic scattering experiments,[16,17] the differential cross section can be fitted by the form

$$(d\sigma/dt)=Ae^{at}, \quad (2.5)$$

where $-t$ is the square of the momentum transfer, while a is roughly constant for π-p scattering, but is a function of the energy for p-p scattering, which exhibits the well-known shrinkage. This behavior holds for momentum transfers up to about 1 GeV/c, within which space the amplitude has fallen about 3 decades, but for larger momentum transfers the falloff is less rapid in both cases. p-p scattering has been measured[19] for momentum transfers squared of 2 to 25 (GeV/c)^2 at beam momenta from 1 to 30 GeV/c and is found to obey the relation[20]

$$(d\sigma/dt)=Be^{-bp_\perp}, \quad (2.6)$$

where $p_\perp\equiv p\sin\theta$ is the transverse momentum transfer, and b is constant independent of the energy. In terms of the Mandelstam variables

$$p^2\sin^2\theta=-t[1+t/(s-4m^2)], \quad (2.7)$$

and for high energies ($s\gg t$) this is indistinguishable from

[16] K. J. Foley *et al.*, Phys. Rev. Letters, **15**, 45 (1965).
[17] D. Harting *et al.*, Nuovo Cimento, **38**, 60 (1965).
[18] E. W. Anderson *et al.*, Phys. Rev. Letters, **16**, 855 (1966).
[19] G. Cocconi *et al.*, Phys. Rev. **138**, B165 (1965).
[20] J. Orear, Phys. Rev. Letters **12**, 113 (1964); Phys. Letters **13**, 190 (1964).

the form

$$(d\sigma/dt)=Be^{-b(-t)^{1/2}}. \tag{2.8}$$

π-p scattering has been measured at comparable energies only for momentum transfers squared up to 4 $(\mathrm{GeV}/c)^2$, and there seems to be some uncertainty in the results, Orear *et al.*[21] finding that $(d\sigma/dt)(\pi p)$ falls well below $(d\sigma/dt)(pp)$ for a beam momentum of 12 GeV/c, while Harting *et al.*[17] find very similar values for p-p and π-p at beam momenta 12.4 and 18.4 GeV/c This type of decrease of the amplitude outside the diffraction peak is[22] the fastest permitted by the analytic properties of the scattering amplitudes and the constancy, or near constancy, of the total cross sections at high energies, and is certainly much faster than is expected from the strip approximation.

The most straightforward interpretation of the strip approximation would be that since all the important singularities are supposed to be contained in the strips (shaded regions of Fig. 1) we have

$$A(s,t)=\frac{1}{\pi}\int_4^\infty \frac{D_t(s,t')}{(t'-t)}dt'$$

$$\approx\frac{1}{\pi}\int_4^{s_1} \frac{D_t(s,t')}{(t'-t)}dt' \text{ (for points in the high-energy } s \text{ channel forward peak)}, \tag{2.9}$$

giving

$$A(s,t)\underset{t\to-\infty}{\longrightarrow}\frac{f(s)}{t}. \tag{2.10}$$

This must be true if $D_t(s,t)$ has the same sign for $4<t<s$, but, if $D_t(s,t)$ oscillates in t, cancellations can occur which increase the rate of falloff.[23] Thus if D_t has N zeros at t_i $(i=1,N)$ such that $D_t(s,t)\prod_i(t_i-t)$ is positive, then

$$A(s,t)\prod_i(t_i-t)=\frac{1}{\pi}\int_4^{s_1}\frac{D_t(s,t')}{(t'-t)}\prod_i(t_i-t')dt'$$

$$\underset{t\to-\infty}{\longrightarrow} f(s)/t,$$

so that

$$A(s,t)\underset{t\to-\infty}{\longrightarrow} f(s)/t^{N+1}. \tag{2.11}$$

Clearly an exponential decrease requires an infinite member of oscillations, but there is no reason to suppose that the exponential behavior will continue out to indefinitely large values of momentum transfer, and probably a few oscillations will suffice. For instance Serber has suggested[24] that

$$(d\sigma/dt)\propto 1/t^5 \tag{2.12}$$

would fit the p-p experiments over a wide range of momenta, which implies $N\approx 2$ only. There will have to be a great extension of the data before the true beahvior for large momentum transfers at high energy can be identified, but at least we can already be confident that the simple $1/t$ form is incorrect.

III. REGGE-POLE PARAMETERS AND THE SHAPE OF THE FORWARD PEAK

The bootstrap solution which we obtained in Ref. 7 has the following form for the ρ and P trajectory functions.

$$\begin{aligned}\alpha_\rho(t)&=0.55+0.27/(1-t/70),\\ \gamma_\rho(t)&=125\alpha_\rho'(t)(\bar{t}_\rho-t)Q_{\alpha_\rho}(t)[2.55)/(q_{\bar{t}_\rho}{}^2)^{\alpha_\rho(t)+1},\\ \alpha_P(t)&=0.625+0.375/(1-t/110),\\ \gamma_P(t)&=230\alpha'P(t)(\bar{t}P-t)Q_{\alpha P(t)}(2.22)/(q_{\bar{t}_\rho}{}^2)^{\alpha_\rho(t)+1},\end{aligned} \tag{3.1}$$

with $\bar{t}_\rho=40$ and $\bar{t}_P=50$. The reason for this form of parametrization of the residue function is given in Ref. 6.

The contribution of a Regge pole to the scattering amplitude is[15]

$$A^I(s,t)=\beta(I,I')\pi[2\alpha(t)+1]\bar{\gamma}(t)(q_t^2/q_{\bar{t}}^2)^{\alpha(t)}$$

$$\times P_{\alpha(t)}(1+s/2q_t^2)\left(\frac{1+(-1)^{I'}e^{-i\pi\alpha}}{2\sin\pi\alpha}\right), \tag{3.2}$$

where

$$\bar{\gamma}(t)=\gamma(t)(q_{\bar{t}}^2)^{\alpha(t)}. \tag{3.3}$$

Since the Pomeranchon passes through $\alpha=1$ at $t=0$, its contribution to $A(s,0)$ is wholly imaginary and we have

$$\operatorname{Im}\{A^I(s,0)\}\underset{s\to\infty}{\longrightarrow}\beta(I,0)\frac{\pi}{2}[2\alpha(0)+1]\bar{\gamma}(0)\left(\frac{s}{q_{\bar{t}}^2}\right)^{\alpha(0)}, \tag{3.4}$$

and with

$$\frac{d\sigma^I}{dt}=\frac{4\pi}{sq_s{}^2}|A^I(s,t)|^2, \tag{3.5}$$

we find that the inverse width of the π-π diffraction peak is

$$[\Delta t]^{-1}\equiv\left(\frac{d^2\sigma}{dt^2}\bigg/\frac{d\sigma}{dt}\right)^I_{t=0}=2\frac{d}{dt}[\operatorname{Im}A^I(s,t)]_{t=0}/\operatorname{Im}A^I(s,0)$$

$$=2\left[\frac{2\alpha'(0)}{2\alpha(0)+1}+\bar{\gamma}'(0)/\bar{\gamma}(0)+\alpha'(0)\ln(s/2q_{\bar{t}}^{-2})\right]. \tag{3.6}$$

With the parameters 3.1 we obtain an inverse width at $s=40$ GeV^2

$$[\Delta t]^{-1}=2\ \mathrm{GeV}^{-2}$$

[21] J. Orear *et al.*, Phys. Rev. Letters **15**, 309 (1965).
[22] A. Martin, Nuovo Cimento **37**, 671 (1965).
[23] Y. S. Jin and A. Martin Phys. Rev. **135**, 1369 (1964).
[24] R. Serber, Phys. Rev. Letters **10**, 357 (1963).

compared with the "experimental" result (see Sec. V) of 6.4 GeV^{-2}.

This disagreement stems partly from the small $\alpha_P'(0)$ which we have obtained ($\alpha_P'(0)=0.2$ GeV^{-2} compared with 0.34 in Ref. 13) but also from the failure of $\bar{\gamma}(t)$ to fall fast enough.

Away from the diffraction peak the situation gets worse, because whereas our residue behaves like $1/t$ for $t\ll 0$, Phillips and Rarita[13] require an exponentially decreasing function. At the very least, from Eq. (2.12), we require a $\gamma(t)$ which falls like $1/(t)^{2.5}$ as used in Ref. 14. For the region beyond $-t=1$ GeV/c, $\alpha(t)$ is almost constant, so the whole of the t dependence of the differential cross section

$$\left(\frac{d\sigma}{dt}\right)^I \underset{s\to\infty}{\longrightarrow} \frac{16\pi}{s^2}\left|\beta(I,I')\pi[2\alpha(t)+1]\bar{\gamma}(t) \times\left(\frac{s}{q_t^2}\right)^{\alpha(t)}\left(\frac{1+(-1)^{I'}e^{-i\pi\alpha(t)}}{2\sin\pi\alpha(t)}\right)\right|^2, \tag{3.7}$$

will have to come from the behavior of $\bar{\gamma}(t)$, and we expect $[\bar{\gamma}(t)]^2\propto\exp[-b(t)^{1/2}]$ if the strip approximation is to give the right sort of behavior.

The behavior $\alpha(t)\to$ constant as $t\to-\infty$ is inescapable, as it follows directly from the form of the N/D equations.[3] The end point of a trajectory is reached when there develops a pole in l of $N_l(s)$ from the solution of the integral equation

$$N_l(s)=B_l^v(s)+\int_4^{s_1}\frac{B_l^v(s')-B_l^v(s)}{(s'-s)}\rho_l(s')N_l(s')ds', \tag{3.8}$$

where we have assumed elastic unitarity with

$$\rho_l(s)=\left(\frac{s-4}{s}\right)^{1/2}\left(\frac{s-4}{4}\right)^l. \tag{3.9}$$

Such poles are generated from the poles in $B_l^v(s)$ at $l=-1,-2\cdots$, but the exact values of l for which such poles occur is determined by the solution of the Fredholm equation.

As Jones has shown,[4] as $s\to-\infty$ we get

$$\alpha(s)\to\alpha(\infty)+\text{const}/s+\text{terms of order } 1/s^2, \tag{3.10}$$

so it is clear that the rapid decrease of $(d\sigma/dt)$ can not be due to the behavior of $\alpha(t)$.

We can determine the behavior of $\gamma(s)$ as follows:

$$D_l(s)=1-\frac{1}{\pi}\int_4^{s_1}\frac{\rho_l(s')N_l(s')}{s'-s}ds', \tag{3.11}$$

and $\alpha(s)$ is the function such that $D_{\alpha(s_R)}(s_R)=0$. The residue is obtained from[6]

$$\frac{\gamma(s_R)}{\alpha'(s_R)}=\frac{N_\alpha(s_R)}{D_\alpha'(s_R)} =\frac{\int_4^{s_1}ds'\rho_\alpha(s')N_\alpha(s')B_\alpha^v(s')/(s'-s_R)}{\int_4^{s_1}ds'\rho_\alpha(s')N_\alpha(s')/(s'-s_R)^2}. \tag{3.12}$$

If N and B^v are positive in the strip region ($4\to s_1$), as is the case for a ρ exchange potential in the Born approximation, then we see that

$$\gamma(s)\underset{s\to\infty}{\longrightarrow}\alpha'(s)s,$$

which from Eq. 3.10 gives $\gamma(s)\to 1/s$. Thus the type of asymptotic behavior which we found in Ref. 7, for the residue function, was almost inevitable. The only way of obtaining a residue function which has the required properties is for $B_l^v(s)$ to oscillate in sign in such a way as to give a cancellation of the type described in the previous section. N, of course, is determined by B^v, and the solution of Eq. 3.8 does not usually result in oscillations of N unless they are present in B^v.

It is quite possible that such oscillations do in fact occur, since we now know that the Pomeranchon gives rise to an important repulsive potential,[25] which, when combined with the attractive ρ potential (and taking the force B^v to be just the potential, i.e., the first Born approximation to the force), results in $B_l^v(s)$ having regions of different sign. Unfortunately, as was discussed at some length in Ref. 7, it is not satisfactory to include such combinations of attraction and repulsion in the first Born approximation. But it seems possible that a combination of the attractive ρ potential and the repulsive P potential may, when the complete Born series for B_l^v is used, produce the sort of oscillations which we require.

We also found in Ref. 7 that the presence of the P repulsion may imply a considerable inelasticity in the upper strip region, so the assumption of elastic unitarity may also need modification. A possible way of doing this would be to use Froissart's method[26] in which we take

$$R_l(s)\equiv[\sigma_l^{\text{tot}}(s)/\sigma_l^{\text{el}}(s)]=[\rho(s)]^{-1}[\text{Im}A_l(s)/|A_l(s)|^2]$$

to be the ratio of the total to the elastic partial-wave cross section. The only alteration required is to replace $\rho_l(s)$ by $R_l(s)\rho_l(s)$ wherever it appears in Eqs. 3.8, 11, 12. Since clearly $R_l(s)\geq 1$ by definition, it can not oscillate in sign, and will thus not produce the required

[25] G. F. Chew, Phys. Rev. **140**, B1427 (1965).
[26] M. Froissart, Nuovo Cimento **22**, 191 (1961).

behavior in Eq. 3.12. The only way of obtaining the proper form for $\bar{\gamma}(s)$ is for $B_l^v(s)$ to have suitable oscillations.

IV. MODIFICATION OF THE N/D EQUATIONS

Since it is clear that the required behavior of γ is not going to ensue from the approximations which we have used in the past, it seems worth exploring the possibility of imposing the required behavior. Of course, the more one imposes the desired behavior the less convincing is the uniqueness of the bootstrap solution, but in our present ignorance there can be no objection to being less ambitious, and insisting on the right sort of solution by adding extra conditions to the bootstrap equations, provided we are guided by experiment.

We have seen that there are very special requirements on $B_l^v(s)$ which are not satisfied by the first Born approximation. This is, of course, also true as regards the threshold behavior, which one does not expect will be produced correctly with only approximate forces,[27] and so one usually considers the "reduced" amplitude

$$B_l(s)=A_l(s)/q_s^{2l}, \tag{4.1}$$

and solves the N/D equations for $B_l(s)$. $A_l(s)$ is thus constrained to have the behavior $A_l(s)\propto q_s^{2l}$ at threshold.

We propose taking

$$\bar{B}_l(s)=A_l(s)/q_s^{2l}\phi_l(s), \tag{4.2}$$

where $\phi_l(s)$ is the function by which we need to multiply our previous solutions in order to obtain the correct form for γ. Then the elastic unitary condition within the strip,

$$\mathrm{Im}A_l(s)=[(s-4)/s]^{1/2}|A_l(s)|^2, \tag{4.3}$$

becomes

$$\mathrm{Im}\bar{B}_l(s)=\rho_l(s)\phi_l(s)|\bar{B}_l(s)|^2. \tag{4.4}$$

We set

$$B_l(s)=N_l(s)/D_l(s), \tag{4.5}$$

and define

$$\bar{B}_l^v(s)\equiv B_l^v(s)/\phi_l(s), \tag{4.6}$$

the "reduced" force function.

Letting N have the cuts of the force, and D the unitary cut from 4 to s_1, we obtain the equations.

$$N_l(s)=\bar{B}_l^v(s)+\frac{1}{\pi}\int_4^{s_1}\frac{\bar{B}_l^v(s')-\bar{B}_l^v(s)}{(s'-s)}\times\rho_l(s')\phi_l(s')N_l(s')ds', \tag{4.7}$$

$$D_l(s)=1-\frac{1}{\pi}\int_4^{s_1}\frac{\rho_l(s')\phi_l(s')N_l(s')}{(s'-s)}ds'. \tag{4.8}$$

[27] M. Bander and G. L. Shaw, Ann. Phys. (N.Y.) **31**, 506 (1965).

Then

$$\frac{\gamma(s_R)}{\alpha'(s_R)}=\phi_\alpha(s')\frac{N_\alpha(s_R)}{D_\alpha'(s_R)}=\phi_\alpha(s_R)\times\frac{\int_4^{s_1}\rho_\alpha(s')\phi_\alpha(s')N_\alpha(s')\bar{B}_\alpha(s')/(s'-s_R)ds'}{\int_4^{s_1}\rho_\alpha(s')\phi_\alpha(s')N_\alpha(s')/(s'-s_R)^2ds'}. \tag{4.9}$$

Then assuming that (s) is positive within the strip we have

$$\gamma(s)\underset{s\to-\infty}{\longrightarrow}\alpha'(s)\phi_\alpha(s)s\longrightarrow\frac{1}{s}\phi_\alpha(s). \tag{4.10}$$

The introduction of ϕ in this way allows us to impose any desired behavior on γ. The only important restriction is that ϕ must be free of singularities within the strip, whereas it would be more natural for a function which is to undergo an exponential decrease within $t=-1$ $(\mathrm{GeV}/c)^2$ to have important singularities at about $t=+1$ $(\mathrm{GeV}/c)^2$. This is unlikely to be true, however, because we can not have $\mathrm{Im}\gamma$ large in the resonance region, which means at least to beyond the f_0 mass. We expect that $\mathrm{Im}\gamma$ will be important only in the upper part of the strip[1], and so it is only here that making $\gamma(s)\propto\phi(s)$ may be something of a mistreatment. The exponental behavior can be expected to hold in the resonance region, but not in the upper part of the strip. Varying the width of the strip enables us to determine how serious this problem is.

In the next section we shall use the experimental data to determine a suitable form for ϕ.

V. THE EXPERIMENTAL SITUATION

Unfortunately, there is very little direct evidence about π-π scattering, and our knowledge of such quantities as the total cross section at high energies, and the shape of the diffraction peak, has to be deduced from other processes. Since, we assume, the high-energy behavior is controlled by the highest-lying trajectories, we can obtain the desired information from the factorization theorem[28]

$$\bar{\gamma}_{i\pi\pi}(t)=[\gamma_{i\pi N}(t)]^2/\gamma_{iNN}(t), \tag{5.1}$$

for any trajectory i.

According to Phillips and Rarita,[13] we can represent

$$\bar{\gamma}_{P\pi N}(t)=C_0\alpha(t)e^{C_1t} \tag{5.2}$$

with $C_1\approx2.5$ GeV^{-2}, while Rarita dnd Teplitz[12] find

$$\bar{\gamma}_{PNN}(t)=\alpha(t)b(0)e^{at} \tag{5.3}$$

[28] M. Gell-Mann, Phys. Rev. Letters **8**, 263 (1963); V. Gribov and I. Pomeranchuk, *ibid.* **8**, 343 (1962).

with $a=3.65$ GeV^{-2}. The factor α was included by these authors to ensure that the residue should vanish where the P trajectory cuts $\alpha=0$, thus avoiding the problem of ghost poles. In both cases the parametrization

$$\alpha_P(t)=-1+4/[2-\alpha_P{}^1(0)t] \tag{5.4}$$

was used, with $\alpha_P'(0)=0.34$ GeV^{-2}.

Then

$$\tilde{\gamma}_{P\pi\pi}(t)=C\alpha(t)e^{(2C_1-a)t}=C\alpha(t)e^{1.35t},$$

$$\tilde{\gamma}_{P\pi\pi}'(0)/\tilde{\gamma}_{P\pi\pi}(0)=\alpha_P'(0)/\alpha(0)+1.35=1.68\text{ GeV}^{-2}. \tag{5.5}$$

Substituting this in Eq. 3.6 gives, with $2q_{\bar{t}}{}^2=2$ GeV^2,

$$[\Delta t]_{\pi\pi}{}^{-1}=6.4\text{ GeV}^{-2},$$

while

$$[\Delta t]_{pp}{}^{-1}=10.8\text{ GeV}^{-2},$$

and

$$[\Delta t]_{\pi p}{}^{-1}=8.5\text{ GeV}^{-2},\text{ all at }s=40\text{ GeV}^2.$$

It should be remembered that in the π-p case there will be a correction of about 10% from the effect of the P' trajectory which accounts for the lack of shrinkage at these energies. The corresponding contribution to p-p is very small, because the P' and ω give contributions which more or less cancel each other. We will ignore these corrections. The above numbers are in reasonable agreement with more recent data. Taking $(d\sigma/dt)\propto e^{bt}$, Foley *et al.*[16] find $b_{pp}=8.68\pm0.79$ at 19.84 GeV/c, and $b_{\pi p}=8.98\pm0.65$ at 19.75 GeV/c, while Harting *et al.*[17] find $b_{pp}=8.58\pm0.24$ at 18.4 GeV/c and $b_{\pi p}=7.53\pm0.21$. Straightforward factorization gives

$$b_{\pi\pi}=9.28\pm2.09\text{ GeV}^{-2}\quad\text{(Foley)}$$
$$=6.48\pm0.66\text{ GeV}^{-2}\quad\text{(Harting)}.$$

This latter result is in good agreement with the earlier Regge parameters, but the two together give some idea of the reliance to be placed on the numbers.

Since our P trajectory will not usually cut $\alpha=0$, we choose

$$\tilde{\gamma}_{P\pi\pi}\propto e^{1.68t}, \tag{5.6}$$

or, measuring t in π mass units, $\tilde{\gamma}_{P\pi\pi}\propto e^{0.0336t}$, which means choosing $\phi_l(s)=e^{0.0336s}$. This gives a form for ϕ which is independent of l. Since the trajectory exists only over a small range of l there does not seem to be much point in including l dependence.

We have already noted that for momentum transfers greater than about 1 (GeV/c)^2 the differential cross section flattens off, and (as already noted) according to Orear,[20] for p-p scattering, can be represented by

$$(d\sigma/dt)=Be^{-bs\mathrm{in}\theta}, \tag{5.7}$$

where $b=6.58$ (GeV/c)$^{-1}$. This is observed to hold for $p\sin\theta=1$ to 4 GeV/c and $E_{\mathrm{lab}}=10-32$ GeV. We prefer to use

$$(d\sigma/dt)=Be^{-b(-t)^{1/2}}. \tag{5.8}$$

for these high energies. Unfortunately there is much less data on high-momentum-transfer π-p scattering. At 8 and 12 GeV/c incident pion momentum, Orear *et al.*[21] find the π-p differential cross section to be more that a decade lower than p-p at $t=-3$ (GeV/c)^2 and conclude that π-p scattering is very different from p-p. On the other hand, at a higher energy (18.4 GeV/c) Harting *et al.*[17] find π-p and p-p very similar for t out to -3.5 (GeV/c)^2. With this uncertainty it seems reasonable to take b to be the same for both p-p and π-p, and so by factorization for π-π. Thus at large t we want

$$\tilde{\gamma}_{P\pi\pi}\propto e^{-(b/2)(-t)^{1/2}}, \tag{5.9}$$

with $b=5.58$ (GeV/c)$^{-1}$. A form which combines the behaviors of Eqs. (5.6) and (5.9) is[29]

$$\tilde{\gamma}_{P\pi\pi}=C\exp\{-2(R/A)[(1-tA^2)^{1/2}-1]\} \tag{5.10}$$
$$\approx e^{tAR},\quad t\ll1/A^2$$
$$\approx e^{-2R(-t)^{1/2}},\quad t\gg1/A^2$$

giving $R=1.645$ (GeV/c)$^{-1}$ and $A=1.02$ (GeV/c)$^{-1}$. The change over from the one behavior to the other occurs at $t\approx-1$ (GeV/c)^2, as it should. In π mass units this becomes

$$\tilde{\gamma}_{P\pi\pi}=C\exp\{-3.24[(1-t/50)^{1/2}-1]\}. \tag{5.11}$$

This expression, though it represents the desired behavior, is unpleasant because of the (not unexpected) square-root singularity at $t=50$, which is well within the strip. However, it is worth comparing the results or using Eq. 5.11 to calculate the potential as against Eq. 5.6, even if it is not a behavior that can readily be imposed on the output.

We have found the form of $\phi_l(s)$ appropriate to the Pomeranchuk trajectory which controls the diffraction peak, but the chief force in π-π scattering is the ρ trajectory exchange, and we need to know the behavior of its residue. Unfortunately, its contribution to N-N scattering, where it will account for the differences between p-p and n-p, is very small,[30] and so has not been determined. For π-N scattering the ρ trajectory is important in the charge exchange process $\pi^-p\rightarrow\pi^0n$. This has been fitted by several authors,[13,31–34] but they obtain rather various results, because it seems to be necessary to have a large spin-flip amplitude B as well as the nonflip amplitude A which concerns us. Also the so-called cross-over effect[35] can be explained either by the residues changing sign, or by A and B having

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[29] E. M. Henley and I. J. Muzinich, Phys. Rev. **136**, B1783 (1964).

[30] F. Hadjioannou, R. J. N. Phillips, and W. Rarita, Phys. Rev. Letters **9**, 183 (1962).

[31] R. K. Logan, Phys. Rev. Letters **14**, 414 (1965).

[32] M. Barmawi, Phys. Rev. **142**, 1088 (1966).

[33] G. Hohler *et al.*, Phys. Letters, **20**, 79 (1966).

[34] B. R. Desai, Phys. Rev. **142**, 1255 (1966).

[35] The name cross-over effect has been given [Ref. 13] to the fact that $(d\sigma/dt)_{\pi^-p}>(d\sigma/dt)_{\pi^+p}$ for t near 0, but $(d\sigma/dt)_{\pi^-p}<(d\sigma/dt)_{\pi^+p}$ for $t<-0.05$ (GeV/c)^2.

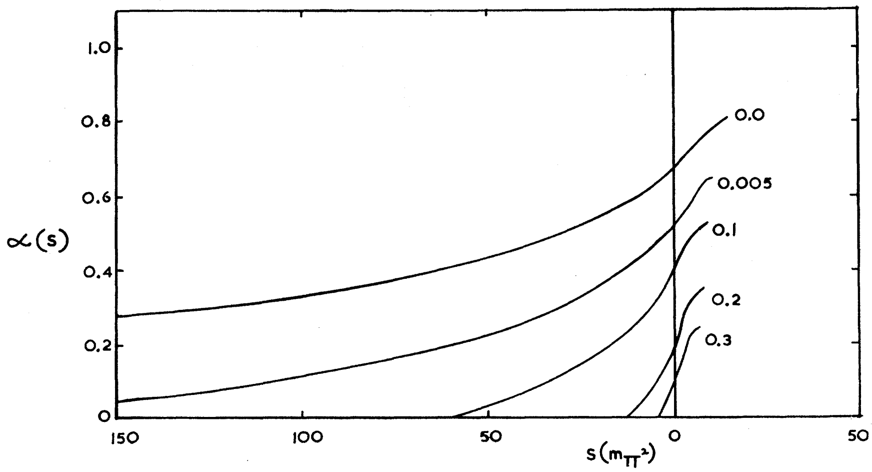

FIG. 2. The Regge trajectories with the force of Eq. (6.1) $g=2.3$, $\phi(s)=e^{bs}$ with the values of shown and $S_1=150$.

opposite signs. Rarita and Phillips[13] find several possible solutions for $\gamma_{\rho\pi N}$ depending upon which of these possibilities is adopted, but all with a rapid exponential decrease, more rapid than for $\gamma_{P\pi N}$. On the other hand, Desai's[34] fit requires only a slowly varying nonflip residue while Hohler *et al.*[33] were able to use constant residues.

Given this uncertainty, it seems best to rely on a dynamical argument and note that the forces producing the ρ and P trajectories in π-π scattering are very similar. The difference is only in the actual strength of the exchanged ρ as given by the crossing matrix, and we found them to have a very similar structure in Ref. 7. Thus it is hard to see how the residue for ρ and P can differ very greatly, and since the P residue is now fairly well determined it seems reasonable to adopt exactly the same behavior for the ρ.

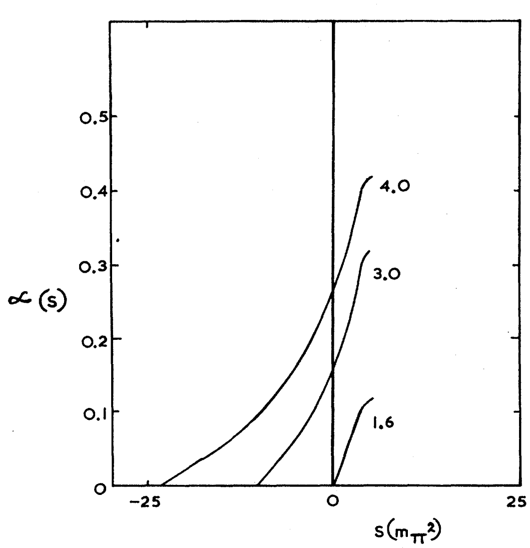

FIG. 3. The Regge trajectories with the force of Eq. (6.1) $\phi(s)=e^{0.0336s}$ the values of g (the exchange width) shown, and $s_1=150$.

VI. DISCUSSION OF THE RESULTS

The effect of imposing the required behavior on the residue function is twofold.

Firstly, it alters the form of the potential calculated from the cross-channel strips. In fact, from Eq. 3.7 of Ref. 7 one can see that putting a strong t dependence into $\gamma(t)$ will strengthen the force for the higher partial waves (in the S channel) relative to the lower partial waves. We can thus expect some steepening of the output trajectories. The over-all potential is weakened, however, so we can expect that the force required to generate a reasonable output trajectory will require output parameters even further removed from the experimental values than was the case in Ref. 7.

The more important change is the imposition of the required behavior on the output via Eqs. 4.7, 4.8. The alteration of the form of the N/D equations produces a considerable alteration in the form of the output trajectories. This can conveniently be examined using the force from the exchange of an elementary (fixed spin) ρ particle in first Born approximation,

$$B_l^v(s)=3gm_\rho[1+2s/(m_\rho^2-4)]Q_l(1+m_\rho^2/2q_t^2)\,, \quad (6.1)$$

where g is the width of the ρ in units of $m\pi$ (≈ 0.8

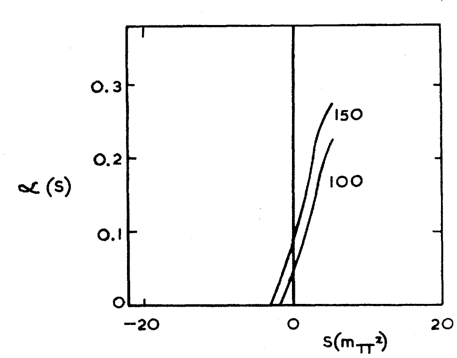

FIG. 4. The Regge trajectories with the force of Eq. (6.1) $\phi(s)=e^{0.0336s}$, $g=2.3$, and the values of s_1 shown.

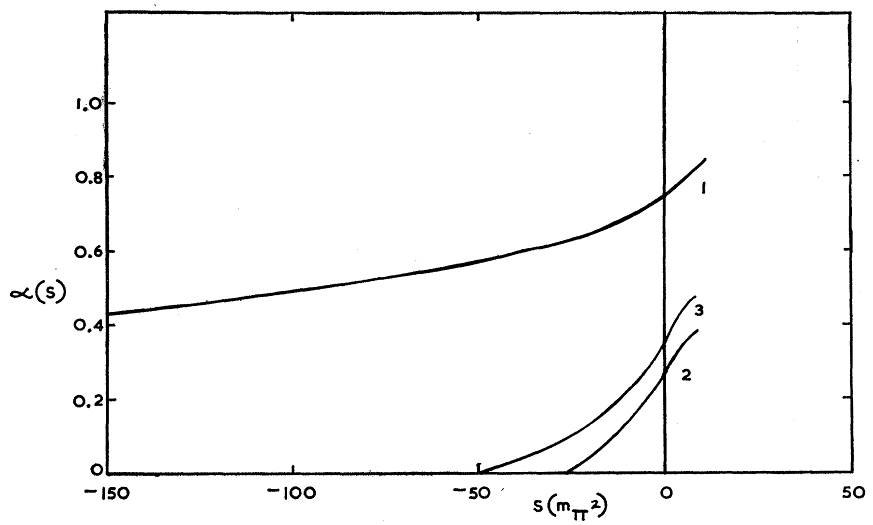

FIG. 5. The ρ trajectory using: (1) input from Eq. (3.1) ρ alone; (2) input from Eq. (3.1) with residue multiplied by $e^{0.0336t}$; (3) input from Eq. (3.1) multiplied by Eq. (5.11). There is no output trajectory when behavior Eq. (5.6) is imposed.

experimentally). In Fig. 2 we show the output trajectory resulting from various choices of $\phi(s)$. It will be seen that increasing b has the effect of lowering and steepening the trajectories. This is as we would wish, as it brings our trajectories closer to those of Ref. 13. In Fig. 3 we plot the trajectories for our experimentally determined $\phi(s)$ with various exchanged widths. It can be seen that there is no hope of obtaining anything like the physical trajectories using the correct ρ width, and the first Born approximation.

Neither of these conclusions is much affected by the choice of the strip width, s_1, as Fig. 4 shows, though if s_1 is increased too far we tend to get "ghost" trajectories (i.e., having residues of the wrong sign) indicating that our approximation scheme has broken down.

With results of this sort, there is clearly no point in looking for self-consistent trajectories, since, though presumably attainable with sufficient effort, they would be quite unlike the physical trajectories. But to show the effect of using the altered form of γ in the input we compare, in Figs. 5, 6, and 7, the trajectories obtained from input Eq. (3.1) with those obtained when this input is multiplied by our two forms of $\phi(t)$, and when the same form is imposed on the output. Multiplying the input by $\phi(t)$ produces slightly lower but steeper trajectories, but imposing the required behavior results in much greater steepening and lowering. It is not too surprising that a force greatly in excess of the first Born approximation is needed, as our whole approach has been based on recognizing the inadequacy of this approximation, and in any case we have severely multilated it by our imposition of the exponential output behavior. Whether the full Born series will give adequate strengths remains to be seen.

VII. SUMMARY AND CONCLUSIONS

We have seen that the new strip approximation as it stands can not produce output residue functions which

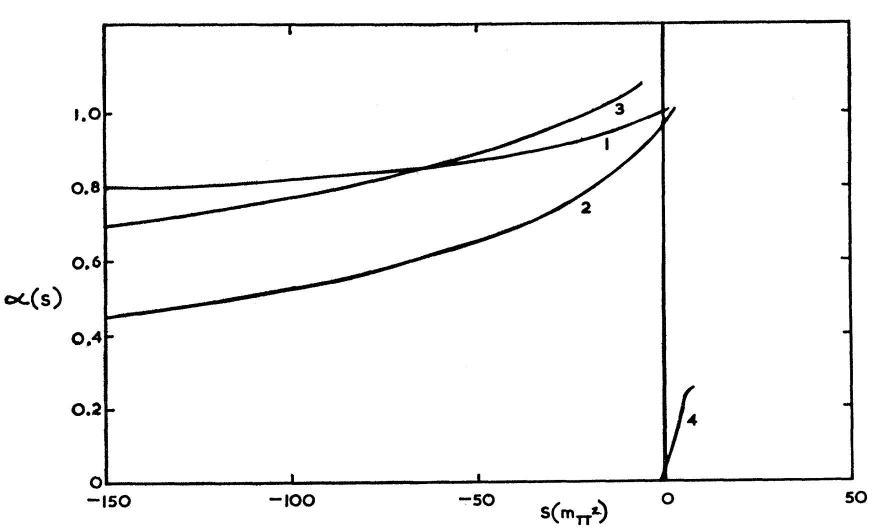

FIG. 6. The P trajectory using: (1) input from Eq. (3.1) $\rho+P$; (2) input from Eq. (3.1) $\rho+P$ with residues multiplied by $e^{0.0336t}$; (3) input Eq. (3.1) residues multiplied by Eq. (5.11); (4) same as (2) with behavior Eq. (5.6) imposed.

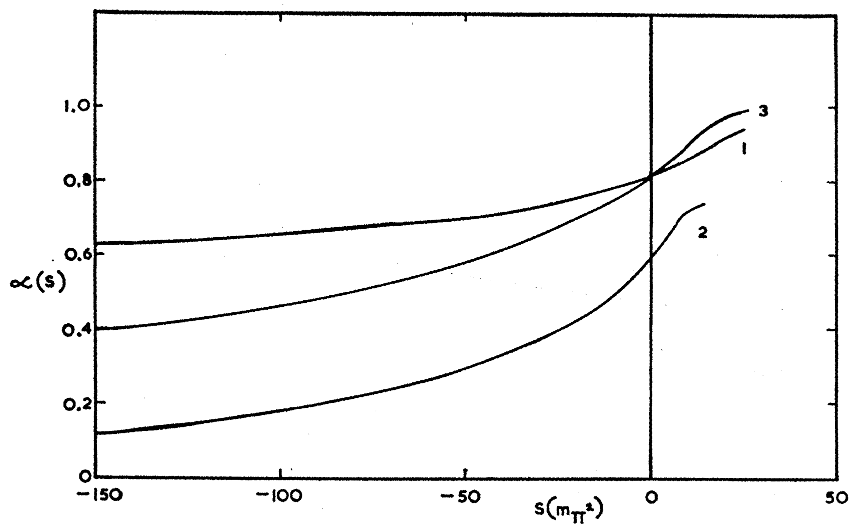

FIG. 7. The ρ trajectory using cases (1), (2), (3) of Fig. 6. There is no output trajectory when behavior Eq. (5.6) is imposed.

are like those needed to fit the experimental data. In fact, if we believe in the Regge fits we are forced to believe in oscillations of the double spectral function in the upper part of the strip. But simply to give such a behavior to the input residue functions does not mean that the output residue will behave in the same way. In fact, we have shown that they will never behave in the same way unless the force function $B_l^v(s)$ oscillates in sign within the strip. Such a behavior of $B_l^v(s)$ might be obtained if it were calculated from the complete Born series rather than just the first Born approximation which has been used hitherto. Since it is difficult (though possible using the Mandelstam iteration) to calculate the complete Born series, we have tried to discover what the correct sort of solution would look like by imposing the desired behavior on the output residue functions. The forms of the resulting trajectory functions are found to be much improved, and more like those of Ref. 13.

We conclude that there is still some hope that the new form of the strip approximation will succeed, provided that the force is obtained by iterating the potential a sufficient number of times, but there are fairly stringent requirements, which have not been suspected previously, that the force function should oscillate in sign. If these conditions can not be satisfied the strip approximation will have to be discarded, because it will mean that there are important contributions to the dynamics from regions of the double spectral functions which we have little or no idea how to calculate. It would also mean that the current success in fitting Regge poles to the experimental data would be somewhat accidental. Indeed these fits really are more convincing in the forward direction than in their momentum-transfer dependence.[36]

ACKNOWLEDGMENT

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[36] E. Leader, Rev. Mod. Phys. 38, 476 (1966).