# Measurements of Electron Thermal Diffusivity in Afterglow Plasmas*

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(Received 12 January 1966; revised manuscript received 9 December 1966)

This paper describes an experimental determination of the electron thermal diffusivity $D_T$ in a room-temperature neon afterglow plasma. The measurements were made in the absence of a magnetic field and covered an electron density range from $10^9$–$10^{12}$ cm$^{-3}$. The principle of the experiment was to heat the electrons by a short microwave pulse, and then observe electron temperature transients at other locations within the plasma. The perturbations in the electron temperature were determined by taking advantage of the temperature sensitivity of the electro-acoustic Tonks-Dattner resonances. For electron-ion collision frequencies $\nu_{ei}$ much larger than the electron-neutral collision frequency $\nu_{en}$, the experimental data for $D_T$ are inversely proportional to the electron density and agree closely with the transport theory of Spitzer and Härm. In the limit of the weakly ionized plasma, i.e., $\nu_{ei} \ll \nu_{en}$, the electron thermal diffusivity is independent of electron density and agrees within 25% with Shkarofsky's theory.

## I. INTRODUCTION

THE transport of particles and energy in a plasma can, in principle, be found from Boltzmann's transport equations. In spite of the complexities involved, there exist numerous theoretical works[1–5] on the transport coefficients, especially in the limit of the fully ionized plasma, in both the presence and the absence of a magnetic field. The transport coefficient which is most accessible for experimental investigations and has therefore been given most attention, is the *electrical* conductivity, which can be determined from microwave measurements.[6] Another transport coefficient which has been dealt with extensively in theory, but very little in experiments, is the *thermal* conductivity $\mathcal{K}$. In general, the rate of thermal energy that can flow across a temperature gradient $\nabla T$ is given by

$$Q = -\mathcal{K}\nabla T. \tag{1}$$

The thermal conductivity of a plasma depends on both the degree of ionization and on the temperature of the plasma components. We have studied a related quantity called the thermal diffusivity which, for electrons, is defined as the ratio of the thermal conductivity to the specific heat of the electron gas, i.e.,

$$D_T = \mathcal{K}/(\tfrac{3}{2}kn_e), \tag{2}$$

where $k$ is Boltzmann's constant and $n_e$ is the electron number density.

In order to simplify the problem we restrict our investigation to afterglow plasmas with all components in initial thermal equilibrium at a temperature $T_0$. Experimentally, the electrons were heated by a short pulse of microwave radiation. The thermal diffusivity was determined by measuring the resulting temperature increases with Tonks-Dattner resonance.[7] (Because of the large mass of the ions, their temperature increase from direct absorption of microwaves can be neglected.)

Other experimental studies of heat transfer in plasmas include the excellent work by Sekiguchi and Herndon.[8] They measured $D_T$ in effectively fully ionized neon and helium plasmas and found good agreement with the theory of Spitzer and Härm.[9] Their temperature measurement was based on the effect of afterglow quenching which had been used previously by Goldstein and Sekiguchi.[10] Measurements of the electron thermal conductivity in a magnetic field were made by Rostas, Bhattacharya, and Cahn,[11] who also applied the afterglow-quenching technique. In their experiment the magnetic field unavoidably disturbed the uniformity of the plasma. Nevertheless, Rostas *et al.* were able to obtain data that agreed qualitatively with the theoretical predictions of Landshoff.[12]

Since Sekiguchi and Herndon[8] have already measured the thermal conductivity of the fully ionized plasma, we decided to devote special attention to the thermal properties of the weakly ionized plasma. Our experimental approach and heat-transfer analysis is only valid in this limit, at which we have found good agreement with the theoretical prediction of Shkarofsky.[13] At higher densities we have, nevertheless, obtained data that agree closely with Spitzer's theory.[9]

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* This work was supported in part by the U. S. Air Force Cambridge Research Laboratories, Office of Aerospace Research, under Contract No. AF19(628)-4183.

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The following section of this paper describes how $D_T$ can be found from a simplified energy-balance equation. The basic characteristics of the apparatus are described in Sec. III and the use of Tonks-Dattner resonances as a diagnostic tool for the determination of electron temperature perturbations is described in Sec. IV. The results are presented and discussed in Sec. V.

## II. ENERGY TRANSFER PROCESSES

### A. General Considerations

Characteristics for the cylindrical afterglow plasma are first, that the density is uniform in the axial direction, i.e., $\partial n_e/\partial z=0$, and second, that the density changes very little on the time scale of typical energy-transfer processes. Therefore, in this model the electron density can be represented by the function $n_e(r)$.

Let us define the energy density of the electron gas as

$$E_e=\tfrac{3}{2}kn_e(r)T_e(z,r,t)\,, \tag{3}$$

where $T_e$ is the electron temperature. The change in energy density per second can, according to Goldstein and Sekiguchi[10] be expressed as

$$\frac{\partial E_e}{\partial t}=\left[\frac{dE}{dt}\right]_{\text{source}}-\tfrac{3}{2}k\frac{n_e}{\tau}[T_e-T_0]+\nabla\cdot(\mathcal{K}\nabla T_e)\,. \tag{4}$$

The first term on the right-hand side of Eq. (4) describes the energy increase due to the heating source. The second term represents the rate of energy lost by electrons colliding with ions and neutrals, where $\tau$ is the relaxation time for the local collisional energy transfer. For binary collisions we can write

$$\tau^{-1}=\tau_{en}^{-1}+\tau_{ei}^{-1}\,, \tag{5}$$

where the subscripts *en* and *ei* refer to electron-neutral and electron-ion collisions, respectively. Following standard terminology, we shall call the plasma *weakly* ionized whenever $\tau_{en}\ll\tau_{ei}$, and *fully* ionized whenever $\tau_{ei}\ll\tau_{en}$. At this point it is interesting to note [see Eq. (4)] that in an effectively fully ionized plasma the electron energy lost in collisions with ions is proportional to $n_e^2$, since $\tau\approx\tau_{ei}\propto 1/n_e$. On the other hand, for a weakly ionized plasma the rate of energy losses to the neutral atoms will only be proportional to $n_e$ because the relaxation time $\tau_{en}$ is independent of electron density.

The last term in Eq. (4) describes the energy lost by heat conduction; our objective has been to determine the thermal conductivity $\mathcal{K}$ or its related quantity, the thermal diffusivity $D_T$, as defined in Eq. (2). Since the coefficients in Eq. (4) depend strongly on the nature of the collisional processes, we start out by seeking separate solutions for the limits of the fully and weakly ionized plasmas.

### B. Fully Ionized Plasma

According to Spitzer and Härm[9] the thermal conductivity of a fully ionized plasma is density-independent, so that $\nabla\mathcal{K}=0$. Consequently, Eq. (4) becomes

$$\frac{\partial T_e}{\partial t}=\left[\frac{dT_e}{dt}\right]_{\text{source}}-\frac{1}{\tau_{ei}}[T_e-T_0]+\frac{\mathcal{K}}{\tfrac{3}{2}kn_e}\nabla^2T_e\,. \tag{6}$$

Let us now define the increase in the electron temperature as

$$\theta(z,r,t)=T_e(z,r,t)-T_0 \tag{7}$$

and rewrite Eq. (6), without the source term, as

$$\frac{\partial\theta(z,r,t)}{\partial t}=-an_e(r)\theta(z,r,t)+\frac{b}{n_e(r)}\nabla^2\theta(z,r,t)\,, \tag{8}$$

where $1/\tau_{ei}=an_e$ and $b=2\mathcal{K}/(3k)$. We have assumed here that $\theta\ll T_0$, so that $\tau_{ei}$ and $\mathcal{K}$ can be considered to be independent of temperature. With the purpose of separating the radial and axial temperature dependence, we try a solution of the form

$$\theta(z,r,t)=CZ(z,t)R(r,t)\,, \tag{9}$$

where $C$ is a constant. The *axial* part of the solution can be found when the coefficients in Eq. (8) are constant. This limits the validity to a plasma of uniform density, which, for practical reasons, we will set equal to the average density $\langle n_e\rangle$. If the electrons are now instantaneously heated by a short microwave pulse in a narrow region around $z=0$ at the time $t=0$ (effectively a $\delta$ impulse) we find

$$Z(z,t)\propto\{\exp[-z^2/(4\langle D_T\rangle t)-(t/\langle\tau_{ei}\rangle)]\}/\sqrt{t}\,, \tag{10}$$

where

$$\langle D_T\rangle=\mathcal{K}/(\tfrac{3}{2}k\langle n_e\rangle)$$

and

$$1/\langle\tau_{ei}\rangle=a\langle n_e\rangle\,. \tag{11}$$

The plasma density in an afterglow is fairly uniform close to the axis, so we would expect Eq. (10) to be quite accurate in the bulk of the plasma.

An approximate picture of the *radial* temperature distribution can easily be obtained in the steady-state situation where $dR/dt=0$. Near the origin we then find

$$R(r)\propto J_0[r/(\langle D_T\rangle\langle\tau_{ei}\rangle)^{1/2}]\,. \tag{12}$$

Since $\nu_{en}$ is always much larger than $\nu_{ei}$ close to the walls of the discharge tube, there is no sense in seeking a solution of Eq. (8) in this region. Instead, we direct our attention to the weakly ionized plasma, where a solution is both physically meaningful and useful in predicting how much energy is lost out of the plasma volume.

### C. Weakly Ionized Plasma

With the substitution of Eq. (9) the energy balance equation now becomes

$$\frac{\partial\theta(z,r,t)}{\partial t}=-\frac{\theta(z,r,t)}{\tau_{en}}+\frac{D_T}{n_e(r)}\left[\frac{\partial n_e(r)}{\partial r}\right]\cdot\left[\frac{\partial\theta(z,r,t)}{\partial r}\right]+D_T\nabla^2\theta(z,r,t). \quad (13)$$

The other terms from $(\nabla\mathcal{K})\cdot(\nabla T_e)$ in Eq. (4) equal zero because $\partial n_e/\partial z=0$ and $(\partial n_e/\partial r)\cdot(\partial\theta/\partial z)=0$.

Because of the low electron density close to the tube wall, there exists in this region a thermal barrier[8,10] where the amount of energy transferred by conduction is very small. Associated with the thermal barrier there is a large relaxation time for heat transfer in the radial direction. Consequently, the time dependence of the function $R(r,t)$ is so weak that it can be considered to be a function of $r$ only. The variables can then be separated by setting

$$D_T\frac{1}{R}\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\frac{D_T}{n_e(r)}\frac{1}{R}\left[\frac{dn_e(r)}{dr}\right]\left[\frac{dR}{dr}\right]=C_1 \quad (14a)$$

and

$$D_T\frac{1}{Z}\frac{d^2Z}{dz^2}-\frac{1}{Z}\frac{dZ}{dt}=C_2, \quad (14b)$$

where

$$C_1+C_2=\tau_{en}^{-1}.$$

With the $\delta$-impulse heating source we obtain the axial solution

$$Z(z,t)\propto\{\exp[-z^2/(4D_Tt)-t/\tau_{\rm eff}]\}/\sqrt{t}, \quad (15)$$

where $\tau_{\rm eff}$ is an effective relaxation time which includes unknown radial temperature gradients. However, $\tau_{\rm eff}$ can be eliminated by making measurements of the electron temperature at different positions $z$ along the tube.

### D. Determination of $D_T$

On the basis of the preceding discussion it is possible to simplify the problem by studying heat transfer in one dimension only. From the heat-transfer analysis we know that the temperature increase due to a $\delta$-impulse heating source is

$$\Delta T_e(z,t)\propto\{\exp[-z^2/(4\langle D_T\rangle t)-(t/\tau_{\rm eff})]\}/\sqrt{t}, \quad (16)$$

where $\Delta T_e(z,t)=T_e(z,t)-T_0$. We notice that at constant times after the application of the heating pulse the temperature increase goes as $\exp(-z^2)$. In a later section we shall present measurements demonstrating this relation.

In principle, the thermal diffusivity could be found by fitting Eq. (16) to the observed electron temperature transient if $\tau_{\rm eff}$ were known. However, the need to know $\tau_{\rm eff}$ can be eliminated in a simple way by measuring electron temperature transients in the two positions $z_1$ and $z_2$ at the same time $t_r$ after the heating pulse. By dividing these temperature transients we find

$$\langle D_T\rangle=(z_2^2-z_1^2)/\{4t_r\ln[\Delta T_e(z_1,t_r)/\Delta T_e(z_2,t_r)]\}. \quad (17)$$

For the determination of the temperature perturbations $\Delta T_e(z_1,t_r)$ and $\Delta T_e(z_2,t_r)$ we have taken advantage of the temperature sensitivity of the Tonks-Dattner resonances. The principle of this particular electron "thermometer" is discussed after the description of the experimental arrangement.

## III. APPARATUS

The afterglow following the pulsed electrical breakdown of a gas constitutes a stable plasma with well-known properties; the axial uniformity of the density and the slow rate of decay have already been discussed. The cylindrical discharge tube in which the plasma was produced is shown in Fig. 1. The tube was pumped out to pressures below $10^{-7}$ Torr before it was filled with spectroscopically pure neon. The discharge was ignited by a high-voltage pulse of 20–30-$\mu$sec duration with a repetition rate of about 200 cps.

The group of microwave instruments shown to the right in Fig. 1 is for the excitation and detection of the Tonks-Dattner resonances. The resonances were excited by strip lines approximately 5 mm wide, and were used for measurement of the electron density and the changes in the electron temperature due to the heating. Two strip lines were used in order to monitor the axial uniformity of the plasma. Independently, we have complemented the strip-line density measurements with conventional microwave-cavity techniques.

The heating of the electrons was made with a short (1–2$\mu$sec) pulse in the waveguide. This pulse was applied at a certain time in the afterglow, i.e., at a certain known electron density. The length of the heated plasma region was 2 cm. This length is much smaller than the distance between the waveguide and the strip line, allowing us to use the approximation of a spatial $\delta$ impulse for the heating source. One problem with the waveguide heating source is that it was operated at a frequency of about 1.4 GHz, which corresponds to a cutoff plasma density of about $2.4\times10^{10}$ cm$^{-3}$. For densities below this value the electrons will be uniformly heated, but for the highest peak densities we have tried to investigate, which were of the order of $10^{12}$ cm$^{-3}$, the electrons in the inner region of the plasma will only be negligibly heated. However, as heat diffuses down the tube the radial temperature distribution tends to become more uniform, so that estimates of the thermal diffusivity can still be obtained from Eq. (17).

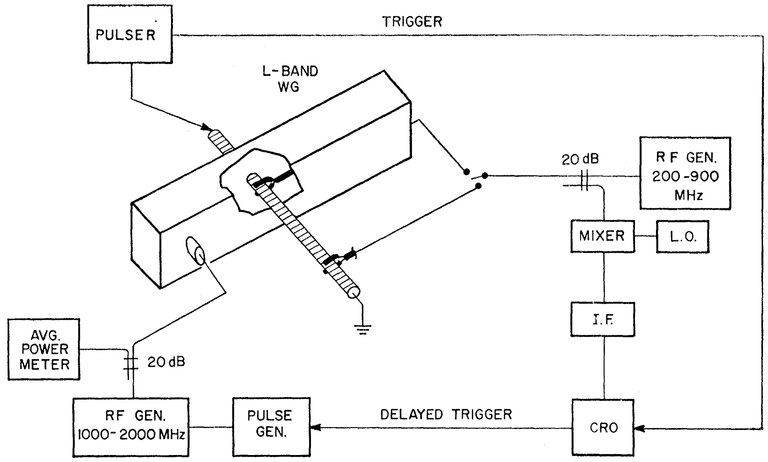


FIG. 1. Schematic diagram of the experimental arrangement. The hot cathode discharge tube measured 50 cm in length and 3 cm in diameter.

## IV. PLASMA DIAGNOSTIC BY MEANS OF TONKS-DATTNER RESONANCES

### A. Electron Density

It has been suggested by Parker *et al.*[14] that electroacoustic resonances excited in low-density cylindrical plasmas might be useful as a diagnostic tool. In another communication[15] we have applied Tonks-Dattner resonances for the detection of sound waves in afterglow plasmas. Here we use them for the measurement of afterglow plasma densities and especially for the temperature perturbations that are needed in order to determine the thermal diffusivity from Eq. (17).

In Fig. 1 the dipolar electric field between the arms of the strip line will excite standing electroacoustic waves close to the tube wall as the plasma frequency $\omega_p$ passes certain discrete values. Neglecting sheath effects, Schmitt *et al.*[16] have assumed a Bessel function for the radial-density profile[17] and by means of the WKB method obtained the following condition for resonance:

$$(i-1/4)\pi=\frac{2a\omega^3}{3\sqrt{3}v_T\alpha J_1(\alpha)\omega_{p0}^2}\left[1-\frac{J_2(\alpha)}{5J_1^2(\alpha)}\frac{\omega^2}{\omega_{p0}^2}\right]. \quad (18)$$

Here, $i=1, 2, 3, \cdots$, $a$ is the tube radius, $\omega$ is the exciting frequency, $\omega_{p0}$ is the on-axis plasma frequency, $v_T=(kT_e/m)^{1/2}$, the $J$'s are Bessel functions, and $\alpha=2.405$. To test the validity of the WKB method Schmitt[18] also obtained a solution in terms of Airy functions; the two methods agree within 5% at the second-order ($i=2$) resonance. For convenience, we have used the simple analytic form represented by Eq. (18).

The accuracy of Eq. (18) is limited by the assumption of a Bessel-function electron-density profile which is linear at the wall. Furthermore, its derivation is strictly valid only when $v_Tk(r)/\omega \ll 1$, which does not hold at the wall. [The term $k(r)$ is the local wave number for the electroacoustic wave.]

The microwave power reflected from the plasma exhibits the resonances, seen as characteristic minima in Fig. 2. Referring to Eq. (18), $i=1$ corresponds to the leftmost resonance. Figure 3 is a plot of this equation showing how the resonance densities on the tube axis depend on the exciting frequency $f$ and resonance number $i$. The frequency range used in this experiment covered a density region from approximately $10^9$ to $10^{12}$ electrons per $cm^3$.

Deviations from the Bessel function density profile, which are very likely to occur at the higher plasma density for which recombination losses[19] and higher diffusion modes[20] might prevail, will result in inaccurate density determinations. For densities below $10^{11}$ $cm^{-3}$

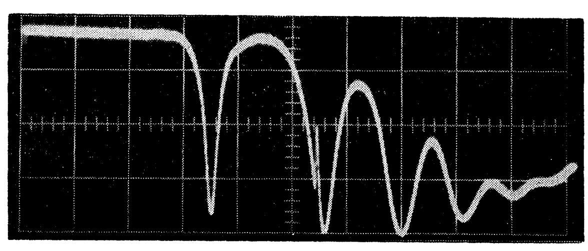

FIG. 2. Tonks-Dattner resonances in a 2-Torr neon afterglow. The resonances were excited by a frequency of 500 MHz. The time scale was 0.5 msec/cm. The little spike just before the second resonance was caused by a heating pulse in the waveguide. A detailed picture of this heating effect is shown in Fig. 4.

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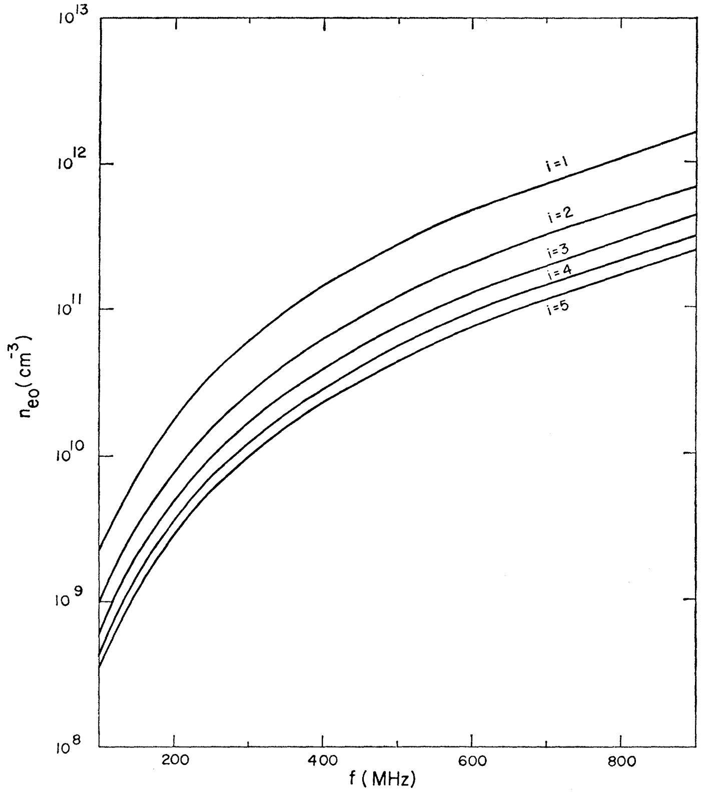


FIG. 3. Electron densities at which Tonks-Dattner resonances are excited as a function of frequency with resonance number as parameter. This calculation pertains to $T_e=300°K$ and tube radius 1.5 cm.

we have found good agreement between densities obtained from the cavity measurements and the Tonks-Dattner resonances, but for higher densities the resonance method tends to give too high results. In case of disagreement we have been using the cavity measurements; from these we found a decay time constant $\tau_d \approx 1.9$ msec in neon at a pressure of 2 Torr.

### B. Electron Temperature

From Eq. (18) the electron density at the occurrence of the $i$th Tonks-Dattner resonance is given by

$$n_{e0}(t_i)=\frac{2a\epsilon_0 m^{3/2}}{3\pi(3k)^{1/2}e^2\alpha J_1(\alpha)}\frac{\omega^3}{(i-1/4)T_0^{1/2}}, \quad (19)$$

where $t_i$ is the time in the afterglow when this resonance takes place, $T_0$ is the equilibrium temperature, $\epsilon_0$ is the vacuum permittivity, $m$ is the electron mass, and $e$ is the electronic charge. [The parameters in our experiment are such that the last term in the right-hand side parenthesis of Eq. (18) can be neglected.] Now, if the electron temperature in the present experiment is slightly increased due to a microwave pulse in the waveguide, the density at which the $i$th resonance is excited is given by

$$n_{eh}(t_i+\Delta t)=\frac{2a\epsilon_0 m^{3/2}}{3\pi(3k)^{1/2}e^2\alpha J_1(\alpha)} \times\frac{\omega^3}{(i-1/4)[T_0+\Delta T_e(t)]^{1/2}}. \quad (20)$$

It has been assumed here that the plasma density profile does not change during the weak microwave pulse and that the resonance condition [Eq. (18)] remains valid during the temperature transient. Thus, we see that the increase in the electron temperature will shift the position of the resonance to a later time by an amount $\Delta t$. By taking the difference between Eq. (19) and (20) we find

$$\Delta T_e(t)=T_0^{3/2}\frac{3\pi(3k)^{1/2}e^2(i-1/4)\alpha J_1(\alpha)}{\epsilon_0 a m^{3/2}\omega^3}n_{e0}(t_i)\frac{\Delta t}{\tau_d}. \quad (21)$$

In Eq. (21) it has been assumed that $\Delta t \ll \tau_d$. This re-

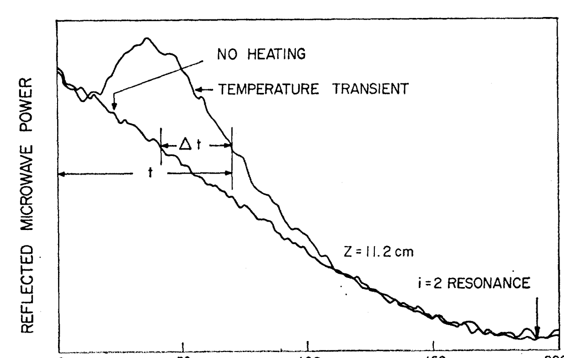


FIG. 4. Reflected microwave power (in relative units) as a function of the time $t$ elapsed after the heating pulse. The measurement was made in neon at a pressure of 2 Torr with $\langle n_e\rangle \approx 5\times 10^{10}$ $cm^{-3}$.

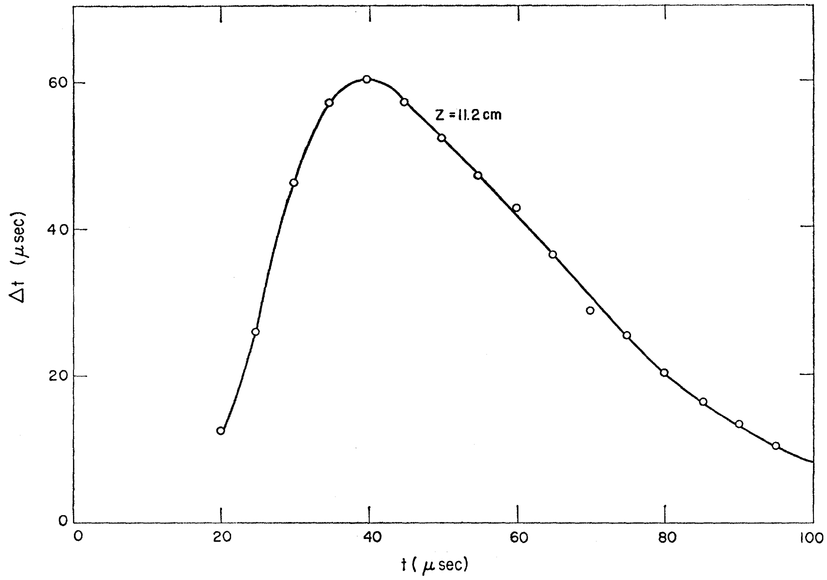


FIG. 5. Relative electron temperature transient obtained from Fig. 4. The peak temperature corresponds to approximately 20°K.

quirement, which implies that the electron density changes very little during the temperature transient, was always satisfied in our measurement. Furthermore, Eq. (21) is correct only to first order in the relative temperature increase $\Delta T_e/T_0$, which is a reasonable approximation for the low (*circa* 10-mW) heating powers in the experiment. For short heating pulses of sufficiently low power we have found, as expected, that the temperature increase is proportional to the power. On the other hand, the dissipation of larger amounts of energy, which was realized by increasing pulse duration and amplitude, resulted in a significant loss of plasma. The probable explanation for this behavior is that the ambipolar diffusion coefficient depends on the electron temperature, to a first approximation[21] as

$$D_a = D_+(1+T_e/T_0)\,,$$

where $D_+$ is the ion diffusion coefficient.

Notice that the temperature perturbations derived from Eq. (21) appear only as a ratio in Eq. (17). Consequently, any systematic errors in Eq. (21) due to inaccuracies in the dispersion relation and neglect of the sheath would tend to cancel.

In the following measurements a small part of the resonance curve around the temperature transient, which is shown as a little spike in Fig. 2, was amplified and time-expanded with a sampling oscilloscope and displayed on a recorder (Fig. 4). Following the nomenclature in Eq. (16) $t$ is defined as the time elapsed after the application of the heating pulse, not to be confused with $t_i$, which is the time of occurrence of the $i$th resonance in the afterglow. In the example shown in Fig. 4 there is a time delay of about 20 μsec before the heat arrives at the position of the strip line; the resonance curve is then displaced to the right, the amount of displacement depending on the temperature increase. When the heating effect is over the curve goes back to its original course, which shows that the time displacement has been produced by a temporary change in temperature and not by a change in density. [If the *density* had changed, this would have shown up as a shift of the whole resonance curve after the heating pulse. This shift was never observed (cf. Figs. 2 and 4).]

From Eq. (21) the shape of the temperature transients can be obtained by plotting $\Delta t$ [as defined in Eq. (20) and Fig. 4] versus $t$. Figure 5 shows the temperature transient that resulted from the recorded curve in Fig. 4. With special care, temperature changes of the order of 1% can be detected with this method. We have indicated a maximum temperature of 20°K in Fig. 4. The absolute value of the temperature change is, in fact, unimportant in our application since only the ratio of two temperature increases is needed for the determination of $D_T$ from Eq. (17).

In the preceding paragraphs it was inherently assumed that Eq. (18), which pertains only to the minimum of the resonance, can also be applied to the sides of the resonance. By using different frequencies to excite the resonance the heating pulse could be moved along the resonance curve. The temperature transients obtained in this way always agreed within the experimental uncertainty limits. This finding gives support to the application of Eq. (18) to the sides of the resonance.

## V. RESULTS AND DISCUSSION

In order to test the validity of the one-dimensional heat-transfer equation we first made some measurements of the temperature perturbation along the plasma column. If the one-dimensional approach is approximately correct, we would expect to find, at a certain time $t_r$ after the heating pulse,

$$\Delta T_e \propto \exp(-z^2/\zeta_0^2)\,,$$

[21] H. J. Oskam, Philips Res. Rep. 13, 401 (1958).

where

$$\zeta_0^2 = (4\langle D_T \rangle t_r)^{-1}.$$

The exponential relationship between $\Delta T_e$ and $z^2$ is exemplified in Fig. 6, which displays measurements made in a plasma of density $5\times10^{10}$ cm$^{-3}$. This density value falls within a region where $\nu_{ei}$ is of the same order of magnitude as $\nu_{en}$. Also for densities below and above the example presented here we have found the same relationship. From the longitudinal dependence of the temperature perturbation, the diffusivity can be found as

$$\langle D_T \rangle = \zeta_0^2/(4t_r). \tag{22}$$

The results from Eq. (22) agree within the error limits with those obtained from Eq. (17).

The results obtained from the measurement of temperature transients and the use of Eq. (17) are shown in Fig. 7. To excite the Tonks-Dattner resonances we used frequencies around 150 MHz at the lowest densities and 900 MHz at the highest densities. The diffusivity was determined at different locations along the plasma column and at 3–5 different times during the temperature transient, resulting in maximum spreads indicated by the vertical bars.

Now we want to compare our results with available theories. First we notice that the measured diffusivity apparently converges to a constant value for decreasing electron densities. This value can be found from Shkarofsky's[13] work as

$$\lim D_T = 10kT_e/3m\nu_{en}g_K, \qquad \nu_{en}/\nu_{ei} \to \infty, \tag{23}$$

where the coefficient $g_K$ is sensitive to the velocity dependence of the collision frequency. Chen[22] has found that $\nu_{en} \propto v^2$, and for this case Shkarofsky gives $g_K = 2$. The horizontal arrows at 184 and 368 $m^2$/sec in Fig. 7 indicate the calculated values for pressures of 2 and 1

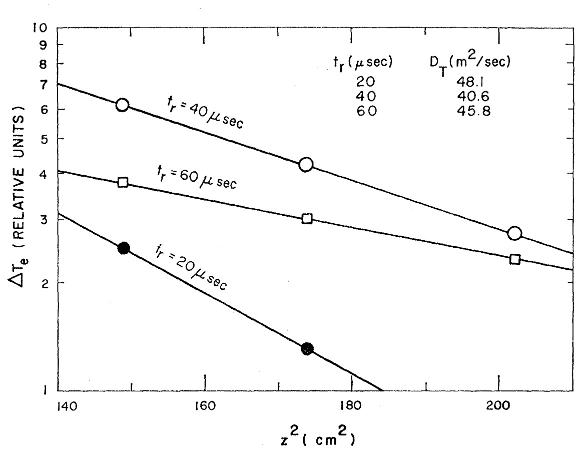


FIG. 6. The temperature increase as a function of $z^2$ at various times after the application of the microwave pulse. The plasma parameters are the same as in Fig. 4. The inserted table shows the diffusivities as calculated at different times $t_r$.

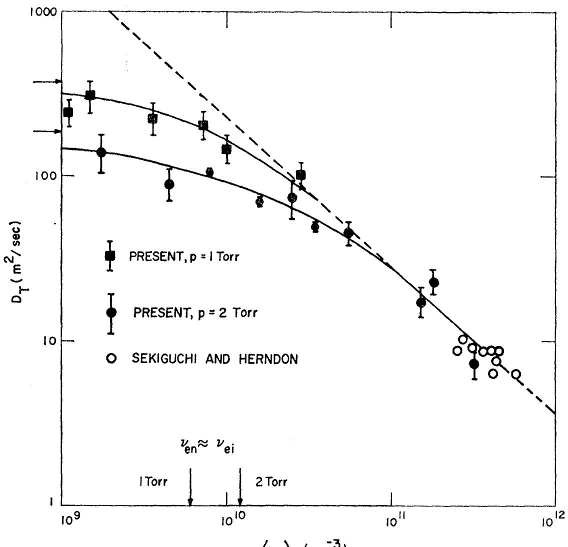


FIG. 7. Electron thermal diffusivity in neon as a function of average electron density. The measurements by Sekiguchi and Herndon (Ref. 8) were made at pressures between 1 and 21 Torr. The arrows on the vertical axis indicate values of $D_T$ at 1 and 2 Torr as calculated from Shkarofsky's theory. The vertical arrows on the abscissa show the densities at which $\nu_{en} \approx \nu_{ei}$.

Torr, respectively. For the electron momentum-transfer cross section we used the value $4.5\times10^{-17}$ cm$^2$, as calculated from Eq. (5b) in Chen's work[22] at a temperature of 300°K. The agreement between the theoretical prediction and experimental data is very good. Since $\nu_{en}/\nu_{ei} \approx 5$–$10$ for the lowest densities investigated, whereas a factor of the order of 100 is required to define a plasma as weakly ionized, one would expect the experimental points to lie below the calculated limiting values. We have measured $D_T$ at pressures up to 5 Torr (not included in Fig. 7), and at very low electron densities we found $D_T \propto 1/p$, as expected from Eq. (23).

As the electron density increases and the plasma becomes more and more ionized, the experimental inaccuracy increases due to nonuniform heating of the electrons, changes in the electron-density profile, and assumptions made in the heat-transfer analysis. In spite of these deficiencies, when we extend the use of our method to fully ionized plasmas, we found reasonable agreement with the high-density measurements of Sekiguchi and Herndon[8] and with the theory of Spitzer and Härm.[9] The broken line with slope approximately equal to $-1$ in Fig. 7 has been calculated from Spitzer and Härm's theory[9] for $T_e = 300$°K. The accuracy is of the order of $1/\ln\Lambda$ and amounts typically to 20% under the present experimental conditions. For electron densities above $5\times10^{10}$ cm$^{-3}$, which corresponds to a relative degree of ionization of the order of $10^{-6}$, there is good agreement between the experimental points and the theoretical prediction. Whenever $\nu_{ei} \gg \nu_{en}$, the plasma

[22] C. L. Chen, Phys. Rev. 135, A627 (1964).

behaves as if it were completely ionized, independent of the neutral gas pressure $p$, of course. The measurements of Sekiguchi and Herndon,[8] which were made for $1 \leq p \leq 21$ Torr, satisfy the above requirement and, therefore, agree with the theoretical curve.

From Eq. (23) we draw the interesting conclusion that $D_T$ in neon is independent of the electron temperature since $\nu_{en} \propto v^2$. But for the fully ionized plasma $D_T$ is approximately proportional to $T_e^{5/2}$. Therefore, $T_e$ must be known in order to compare the experimental data with theoretical predictions. In a neon afterglow plasma at 2.4 Torr, Leiby[23] has measured $T_e$ as a function of time. He found that the electrons would be in thermal equilibrium with the neutral gas at about 300 $\mu$sec after the breakdown pulse. Our highest density point in Fig. 7 was measured 600 $\mu$sec after the breakdown, and at this time and later it is reasonable to assume that $T_e \approx 300°$K.

We must now consider how the electron temperature might have been influenced by mechanisms other than heat conduction. For $p < 1$ Torr, diffusion cooling[24] of the electrons has been suggested as an important energy-loss mechanism. In this process the faster electrons diffuse rapidly to the walls of the tube, resulting in an effective temperature decrease for the remaining electrons. Measurements of $\langle D_T \rangle$ for $0.1 \leq p \leq 0.5$ Torr yielded values less than theoretical estimates which neglected the effects of this process.

Finally, if metastable neon atoms had been produced during the active discharge period, they could have transferred energy to the plasma electrons and thereby provide a heating source prevailing for a long time in the afterglow. To investigate this possibility we also made some measurement of $\langle D_T \rangle$ in a Penning mixture consisting of 99.9% Ne and 0.1% Ar. The addition of argon produced no change in the data implying that heating produced by metastable neon atoms was not important in our experiment. The absence of such heating may be due to the short (20–30-$\mu$sec) duration of our high-voltage breakdown pulse, as contrasted with the 300-$\mu$sec pulse used by Biondi.[25]

In conclusion, we want to point out that the high temperature–sensitivity of the Tonks-Dattner resonances has enabled us to determine the thermal diffusivity over a wide range of electron densities. The experimental approach is strictly valid only at low plasma densities, but by applying it to higher densities we have obtained values of $D_T$ that agree with those of previous investigators,[8,9] thereby bridging the transition region between the limits of the weakly and fully ionized plasma.

## ACKNOWLEDGMENTS

The author is sincerely indebted to H. J. Schmitt who initiated this project. He also wishes to express his appreciation to I. P. Shkarofsky at RCA Victor Company, Ltd., Montreal, Canada, for helpful discussions regarding the transport properties of a weakly ionized plasma. Several of the author's colleagues in the Plasma Physics Department of the Sperry Rand Research Center, in particular G. Meltz and C. D. Lustig, have contributed with stimulating discussions and valuable comments on the manuscript. It is a special pleasure to thank J. E. Morris for his excellent assistance in the laboratory.

[23] C. C. Leiby, Jr., Sperry Rand Research Center Report, RR-63-60, (unpublished).

[24] M. A. Biondi, Phys. Rev. 93, 1136 (1954).

[25] M. A. Biondi, Phys. Rev. 83, 653 (1951).

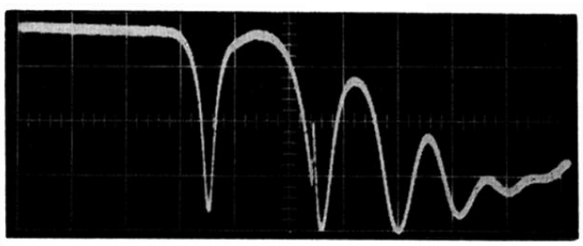

FIG. 2. Tonks-Dattner resonances in a 2-Torr neon afterglow. The resonances were excited by a frequency of 500 MHz. The time scale was 0.5 msec/cm. The little spike just before the second resonance was caused by a heating pulse in the waveguide. A detailed picture of this heating effect is shown in Fig. 4.