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Neutral Vector Mesons and the Hadronic Electromagnetic Current*

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The question of whether the entire hadronic electromagnetic current operator can be identical with a linear combination of the renormalized field operators for the known neutral vector mesons ρ^0, ϕ^0, and ω^0 is investigated in the context of a Lagrangian field theory. It is found that such an identity is completely consistent with gauge invariance, provided that these mesons are coupled only to conserved currents. The general renormalization problem of the strong interactions of these vector mesons is discussed. It is shown that the proposed identity between the hadronic electromagnetic current and the renormalized meson fields can be related to the possible identity between the unrenormalized currents generating the neutral vector mesons and those generating the photon; furthermore, this proposed identity leads to an exact relation between the entire $O(e^2)$ hadronic contribution to the photon propagator and the renormalized propagators of the neutral vector mesons, and such a relation implies, among other consequences, that to $O(e^2)$ and neglecting leptonic contributions, the ratio of the unrenormalized charge e_0 and the renormalized charge e is finite. Various experimental applications are given. In particular, the analysis of ϕ-ω mixing and their leptonic decay rates is made independently of the approximate validity of the SU_3 symmetry.

I. INTRODUCTION

THAT vector mesons might play a dominant role in the description of the electromagnetic interactions of hadrons was first suggested by the interpretation of the electromagnetic form factors of the nucleon.[1] Subsequently, the idea of vector dominance has been extended to apply to all electromagnetic interactions of hadrons.[2] It is evident from a study of the literature of this subject that it is, at the least, of great heuristic value to treat the vector mesons as elementary particles in this context. The utility of this kind of treatment can be substantially augmented by the inclusion of an explicit statement of the meaning of vector dominance (as described in dispersion theory) in the language of a local Lagrangian field theory. The statement which we propose for this purpose is the following: "To a very good approximation the entire hadronic electromagnetic current operator is *identical* with a linear combination of the known neutral vector-meson fields." The principal purpose of this paper is to exhibit a Lagrangian field theory in which the approximate identification referred to above becomes exact, and to examine its various theoretical implications and practical consequences; in this theory, the Maxwell equation can be written as

$$\partial F_{\mu\nu}/\partial x_\mu = e(\lambda_\rho\rho_\nu+\lambda_\phi\phi_\nu+\lambda_\omega\omega_\nu)-e(j_\nu)_{\text{lepton}}\,, \quad (1.1)$$

where

$$F_{\mu\nu}=\partial A_\nu/\partial x_\mu-\partial A_\mu/\partial x_\nu. \quad (1.2)$$

A_ν, ρ_ν, ϕ_ν, and ω_ν are, respectively, the field operators of the photon, the neutral ρ meson, the ϕ meson and the ω meson, λ_ρ, λ_ϕ, and λ_ω are constants, e is the charge of the electron, and $(j_\mu)_{\text{lepton}}$ is the current operator of the charged leptons.

The question of gauge invariance is studied in detail in Sec. II; it is shown that Eq. (1.1) is completely consistent with the requirement of gauge invariance, provided that the currents generating the three neutral vector-meson fields are all conserved currents.

In Secs. III and IV we examine the general renormalization problem of the strong interaction of a single massive vector meson which can be either stable or unstable. It is shown that if the unrenormalized current generating such a meson is conserved, then in the limit that the unrenormalized mass m_0 of the vector meson is infinite, the renormalized meson field becomes necessarily proportional to its unrenormalized current operator. In addition, it can be shown that the nonzero observed mass implies that the unrenormalized mass $m_0\neq 0$, and m_0 must be ∞ if the theory is divergent. These considerations can be readily applied to the known neutral vector mesons. The proposed identity

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[1] Y. Nambu, Phys. Rev. **106**, 1366 (1957). W. R. Frazer and J. R. Fulco, *ibid.* **117**, 1603 (1960).

[2] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961); M. Gell-Mann, *ibid.* **125**, 1067 (1962); Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **8**, 79 (1962); M. Gell-Mann, D. Sharp, and W. G. Wagner, *ibid.* **8**, 261 (1962); G. Feldman and P. T. Mathews, Phys. Rev. **132**, 823 (1963); S. Bergman and S. Drell, *ibid.* **133**, B791 (1964); R. F. Dashen and D. H. Sharp, *ibid.* **133**, B1585 (1964); L. Stodolsky, *ibid.* **134**, B1099 (1964); G. Barton and B. G. Smith, Nuovo Cimento **36**, 436 (1965); R. Gatto, Ergeb. Exakt. Naturw., Vol. 39; M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966); D. S. Beder, *ibid.* **149**, 1203 (1966).

between the hadronic electromagnetic current and the renormalized meson fields is, then, related to the possible identity between the unrenormalized currents generating the vector mesons and those generating the photon.

Throughout the paper, the renormalizability of the strong interactions of these neutral vector mesons is assumed. As is well known, the interaction between a neutral vector meson and a conserved vector current composed only of bilinear products of spin-$\frac{1}{2}$ and spin-0 fields with the minimal order of derivatives can be shown to be renormalizable in a perturbation series.

As we shall see, Eq. (1.1) implies that the isovector part of the hadronic electromagnetic current $(J_\mu{}^\gamma)$ is given by

$$(J_\mu{}^\gamma)_{\text{isovector}}=-(m_\rho{}^2/g_\rho)\rho_\mu\,, \tag{1.3a}$$

where all symbols refer to the renormalized quantities; m_ρ is the observed mass of the neutral ρ meson, g_ρ is the renormalized coupling constant, etc. It will be shown that the unrenormalized mass $m_\rho{}^0$ of the neutral ρ meson must be greater than $2m_\pi$ where m_π is the observed pion mass. If the theory is divergent, then $m_\rho{}^0$ is ∞, and consequently (1.3a) becomes identical with an alternative proposal[3] that the unrenormalized isovector part of the hadronic electromagnetic current $(J_\mu{}^\gamma)^0$ is the same as the unrenormalized current $(J_\mu{}^\rho)^0$ which generates the ρ-meson field;

$$(J_\mu{}^\gamma)^0{}_{\text{isovector}}=(J_\mu{}^\rho)^0\,, \tag{1.3b}$$

where both currents are normalized so that the spatial integrals of their fourth components are all equal to i times the z component isospin operator. The converse statement is also true: (1.3b) becomes identical with (1.3a), provided the unrenormalized mass $m_\rho{}^0=\infty$.

On the other hand, these two proposals would be different if the unrenormalized mass $m_\rho{}^0$ were finite. For example, (1.3a) implies that the isovector part of any electromagnetic form factor $F_{AB}{}^\gamma(q^2)$, which can be arbitrarily defined, for any real or virtual transition $A\rightarrow B+\gamma$ is related to the similarly defined form factor $F_{AB}{}^\rho(q^2)$ for the corresponding virtual or real transition $A\rightarrow B+\rho^0$, at the same 4-momentum transfer q_μ, by

$$[F_{AB}{}^\gamma(q^2)]_{\text{isovector}}=\frac{m_\rho{}^2}{m_\rho{}^2+q^2}F_{AB}{}^\rho(q^2)\,, \tag{1.4a}$$

where A and B can be any two hadronic systems. However, the alternative proposal (1.3b) leads to the identity

$$[F_{AB}{}^\gamma(q^2)]_{\text{isovector}}=\frac{m_\rho{}^2}{m_\rho{}^2+q^2}\left[1+\frac{q^2}{(m_\rho{}^0)^2}\right]F_{AB}{}^\rho(q^2)\,, \tag{1.4b}$$

[3] See, e.g., J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); M. Gell-Mann and F. Zachariasen (Ref. 2).

which becomes the same as (1.4a) only if the unrenormalized mass $m_\rho{}^0$ becomes ∞. If $m_\rho{}^0$ were finite, then the alternative proposal (1.3b) implies that

$$[F_{AB}{}^\gamma(q^2)]_{\text{isovector}}=0 \quad\text{at}\quad q^2+(m_\rho{}^0)^2=0\,. \tag{1.5}$$

Consequently, these two proposals (1.3a) and (1.3b) can be distinguished by examining experimentally the *zeros* of any isovector electromagnetic form factor in the timelike q^2 region.

Similar conclusions can be obtained for the ϕ meson and the ω meson. The renormalization problem of the actual ϕ-ω complex is slightly complicated because of their decays and because of their mixing. The general mathematical analysis is given in Sec. V and is independent of any assumption of the approximate validity of SU_3. This is based upon the fact that the renormalized hypercharge and baryon number currents can be defined independently of SU_3. The renormalized $\phi_\mu(x)$ and $\omega_\mu(x)$ fields are defined in such a way that if the ϕ and ω mesons were stable, then $\langle\text{vac}|\phi_\mu(x)|\omega\rangle=\langle\text{vac}|\omega_\mu(x)|\phi\rangle=0$; i.e., the matrix elements of $\phi_\mu(x)$ do not carry the ω-meson pole, and those of $\omega_\mu(x)$ do not carry the ϕ-meson pole. We find it most convenient to characterize ϕ-ω mixing by means of the resolution of the currents which act as sources of these fields in terms of the hypercharge and baryon number currents. Two angles, which we designate by θ_Y and θ_N, are in general necessary to describe this resolution. The isoscalar hadronic electromagnetic current becomes related to the renormalized fields ϕ_μ and ω_μ and the angle θ_Y by

$$(J_\mu{}^\gamma)_{\text{isoscalar}}=-\tfrac{1}{2}g_Y{}^{-1}\times[(\cos\theta_Y)m_\phi{}^2\phi_\mu-(\sin\theta_Y)m_\omega{}^2\omega_\mu]\,, \tag{1.6}$$

where g_Y is the renormalized hypercharge coupling constant.

In general, the two angles θ_Y and θ_N are different even to first order in the SU_3 symmetry-breaking interaction. Of course, in the limit of SU_3 symmetry, one must have $\theta_Y=\theta_N=0$. The actual values of θ_Y and θ_N depend on the nature of the SU_3 symmetry-breaking interactions for which a number of models can be made. We shall see, for example, that if one makes the *ad hoc* assumption that all SU_3 symmetry-breaking effects are due to the off-diagonal matrix elements of the "bare" mass matrix M_0 between $\phi_\nu{}^0$ and $\omega_\nu{}^0$, then

$$\theta_Y=\theta_N\neq 0\,. \tag{1.7}$$

We call this model the "mass-mixing" model.[4] On the other hand, one may make the opposite assumption that the "bare" mass matrix M_0 is diagonal, but the

[4] The "mass-mixing" model is formally similar to the "particle-mixing" model considered by S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964). We note that both this and the current-mixing model are consistent with the transversality of the vector mesons (i.e., the vector mesons are coupled to conserved currents). See also S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963); J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

SU_3 symmetry is broken by certain current operators terms. Such a model[5] is called the "current-mixing" model; in this model $\theta_Y \neq \theta_N$, but

$$m_\omega^2 \tan\theta_Y = m_\phi^2 \tan\theta_N. \tag{1.8}$$

Within the model, this relation between θ_Y and θ_N holds to all orders of the SU_3 symmetry-breaking interaction.

If one makes the further assumption that the SU_3 symmetry-breaking interaction transforms like the isoscalar member of a SU_3 octet,[6] and equates the observed masses of the nine vector mesons with the first-order perturbation expressions, then one finds $\theta_Y \cong 33°$, $\theta_N \cong 21°$ in the "current-mixing" model, but $\theta_Y = \theta_N \cong 32°$ in the "mass-mixing" model (or, $\theta_Y = \theta_N \cong 39°$ in a variation of the same "mass-mixing" model). The details of these special models are given in Sec. V 3. It is important to note that, independently of the dynamical model, the actual values of g_Y, θ_Y, and θ_N can be determined (at least, to a good approximation) by using the known rate of $\phi^0 \to K^+ + K^-$ and by measuring the leptonic decay rates of $\phi^0 \to l^+ + l^-$ and $\omega^0 \to l^+ + l^-$.

In Sec. VI, a discussion of the photon propagator is given. The entire $O(e^2)$ hadronic contribution to the photon propagation is expressed explicity in terms of the renormalized propagators of the vector mesons. From this expression, it follows that to $O(e^2)$ and neglecting leptonic contributions, the ratio of the unrenormalized charge e_0 and the renormalized charge e is finite. The upper limit of $(e_0/e)^2$ can be estimated, and we find

$$1 < (e_0/e)^2 < 1 + \tfrac{1}{4} m_\pi^{-2} e^2 \times [g_\rho^{-2} m_\rho^2 + \tfrac{1}{9} g_Y^{-2} (\cos^2\theta_Y m_\phi^2 + \sin^2\theta_Y m_\omega^2)] \cong 1.03.$$

Various applications of the proposed identity between the hadronic electromagnetic current and the vector-meson fields are discussed in Sec. VII. Some of these results have already been extensively studied in the literature; they are included in this paper, but with particular emphasis on the underlying assumptions and approximations used in the derivations.

In this paper, we consider only the usual hadronic electromagnetic current which is odd under the particle-antiparticle conjugation operator C determined by the strong interaction. The question whether there does, or does not, exist an additional $C = +1$ part of the hadronic electromagnetic current is not discussed.

II. GAUGE INVARIANCE

To simplify our discussion we will consider first only the isovector part of the hadronic electromagnetic current. The corresponding part of the Maxwell equation becomes, according to Eq. (1.1),

$$(\partial F_{\mu\nu}/\partial x_\mu)_{\text{isovector}} = e\lambda_\rho \rho_\nu. \tag{2.1}$$

[5] The "current-mixing" model is similar to the vector-mixing model considered by S. Coleman and H. J. Schnitzer (Ref. 4).

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

The complete Lagrangian can be written as (neglecting the weak interaction)

$$\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{st}} + \mathcal{L}_\gamma. \tag{2.2}$$

In this section we assume that the free Lagrangian $\mathcal{L}_{\text{free}}$ and the strong interaction Lagrangian $\mathcal{L}_{\text{st}}$ are already given, but demonstrate that for arbitrary $\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{st}}$ a gauge-invariant Lagrangian $\mathcal{L}_\gamma$ can be constructed which will yield Eq. (2.1) as part of its dynamical equations, provided that the neutral ρ-meson field is coupled only to a conserved current.

As a consequence of the strong interaction, the neutral ρ-meson field operator satisfies the dynamical equation

$$\partial G_{\mu\nu}^\rho / \partial x_\mu - m_\rho^2 \rho_\nu = g_\rho J_\nu^\rho + O(e), \tag{2.3}$$

where

$$G_{\mu\nu}^\rho = \frac{\partial}{\partial x_\mu}\rho_\nu - \frac{\partial}{\partial x_\nu}\rho_\mu. \tag{2.4}$$

m_ρ is the *observed* mass of ρ_μ and g_ρ is a *finite* coupling constant, depending on the normalization of the current J_μ^ρ. Equation (2.3) is the "renormalized" field equation of the meson field. The term $O(e)$ shows that it is valid only if one neglects the electromagnetic interaction. [If one wishes, one may also regard Eq. (2.3) as the definition of $g_\rho J_\mu^\rho$.]

The current J_ν^ρ is assumed to be conserved:

$$\partial J_\nu^\rho / \partial x_\nu = 0;$$

therefore,

$$\partial \rho_\nu / \partial x_\nu = 0 \tag{2.5}$$

on account of Eq. (2.3). The detailed form of J_ν^ρ depends on the strong interaction Lagrangian and the renormalization process which will be discussed in the next section. Here, the discussion of gauge invariance can be made independently of the detailed form of J_ν^ρ, provided that it is conserved. It can be readily shown that since J_ν^ρ is a conserved vector current and it transforms like the $I_z = 0$ member of an isotriplet under the isospin rotation, its spatial integral must be proportional to the z component I_z of the isospin operator. (See Appendix A.) For convenience, we shall adopt the normalization convention

$$-i\int J_4^\rho d^3r = I_z. \tag{2.6}$$

The isovector part of the hadronic electromagnetic interaction is assumed to be given by the Lagrangian density

$$(\mathcal{L}_\gamma)_{\text{isovector}} = e(\lambda_\rho g_\rho / m_\rho^2) \times (J_\mu^\rho A_\mu + \tfrac{1}{2} g_\rho^{-1} G_{\mu\nu}^\rho F_{\mu\nu}) + O(e^2). \tag{2.7}$$

The term $O(e^2)$ depends on the derivatives of the charged fields in J_μ^ρ. It is zero, if J_μ^ρ does not contain such derivatives; otherwise, it can be easily generated, say, by the usual minimal principle. Equation (2.7) gives a gauge invariant[7] Lagrangian $\mathcal{L}_\gamma$. Upon varying with respect to A_μ, it gives

$$(\partial F_{\mu\nu}/\partial x_\mu)_{\text{isovector}} = e(J_\nu{}^\gamma)_{\text{isovector}},$$

where

$$(J_\nu{}^\gamma)_{\text{isovector}} = -\frac{\lambda_\rho g_\rho}{m_\rho{}^2}\left(J_\nu{}^\rho - g_\rho{}^{-1}\frac{\partial}{\partial x_\mu}G_{\mu\nu}{}^\rho\right) + O(e). \quad (2.8)$$

From Eqs. (2.3) and (2.8), it follows that

$$(J_\nu{}^\gamma)_{\text{isovector}} = \lambda_\rho \rho_\nu,$$

which is Eq. (2.1). By using the normalization condition, Eq. (2.6), and the fact that the total hadronic charge Q is given by

$$Q = I_z + \tfrac{1}{2}Y,$$

one finds

$$\lambda_\rho = -(m_\rho{}^2/g_\rho).$$

Thus,[8]

$$(J_\nu{}^\gamma)_{\text{isovector}} = J_\nu{}^\rho - g_\rho{}^{-1}(\partial G_{\mu\nu}{}^\rho/\partial x_\mu) = -(m_\rho{}^2/g_\rho)\rho_\nu. \quad (2.9)$$

Although we have established Eq. (2.9) only to the zeroth order in e, the inclusion of all orders in e is possible, but a full discussion will not be given in this paper. Furthermore we observe that the definition of g_ρ has not yet been completely given as it requires a specification of a normalization condition on ρ_μ, to be given in Sec. III. For the present, however, we point out that the combinations $g_\rho{}^{-1}G_{\mu\nu}{}^\rho$, $g_\rho{}^{-1}\rho_\mu$, and $\lambda_\rho g_\rho$ which appear in Eqs. (2.7) and (2.9) are all independent of this specification.

For any real or virtual photon process $A \to B + \gamma$, the isovector part of the matrix element of $J_\mu{}^\gamma$ is then related to the matrix element of $J_\mu{}^\rho$ for the corresponding virtual or real process $A \to B + \rho^0$ provided the 4-momentum q_λ of γ is the same as that of ρ^0. By using Eqs. (2.3), (2.5), and (2.9), we find

$$\langle B|J_\mu{}^\gamma(x)|A\rangle_{\text{isovector}} = \frac{m_\rho{}^2}{q^2 + m_\rho{}^2}\langle B|J_\mu{}^\rho(x)|A\rangle, \quad (2.10)$$

where A and B can be any single- or multiple-particle states of the hadrons. Sometimes, it is convenient to express the matrix elements of $J_\mu{}^\gamma$ and $J_\mu{}^\rho$ in terms of a sum of form factors:

$$\langle B|J_\mu{}^\gamma(x)|A\rangle = \sum_i F_{AB}{}^\gamma(q^2) u_B{}^\dagger \Gamma_\mu{}^i u_A \exp(iq_\lambda x_\lambda),$$

and (2.11)

$$\langle B|J_\mu{}^\rho(x)|A\rangle = \sum_i F_{AB}{}^\rho(q^2) u_B{}^\dagger \Gamma_\mu{}^i u_A \exp(iq_\lambda x_\lambda),$$

where $u_B{}^\dagger\Gamma_\mu{}^i u_A$ denotes the appropriate choice of some spin-momentum functions. If A and B are, say, single nucleon states, then $\Gamma_\mu{}^i$ can be either the usual $\gamma_4\gamma_\mu$ or $\gamma_4\sigma_{\mu\nu}q_\nu$ and u_A, u_B the corresponding spinor functions. If A and B are multiparticle states, then $u_B{}^\dagger\Gamma_\mu{}^i u_A$ would depend not only on the spin-momentum variables but also on all other dynamical parameters that characterize the states A and B. The definitions of the form factors $F_{AB}{}^\gamma(q^2)$ and $F_{AB}{}^\rho(q^2)$, of course, depend on the explicit forms of $u_B{}^\dagger\Gamma_\mu{}^i u_A$ and therefore also on the index i. Equation (2.10) states that for whatever choice of such definitions, the form factor $F_{AB}{}^\rho(q^2)$ is related to the corresponding isovector part of $F_{AB}{}^\gamma(q^2)$ at the same q^2 by Eq. (1.4a):

$$[F_{AB}{}^\gamma(q^2)]_{\text{isovector}} = \frac{m_\rho{}^2}{q^2 + m_\rho{}^2}F_{AB}{}^\rho(q^2). \quad (1.4a)$$

Thus, compared to $F_{AB}{}^\rho(q^2)$, the electromagnetic form factor $[F_{AB}{}^\gamma(q^2)]_{\text{isovector}}$ always vanishes more rapidly[9] at $|q^2| = \infty$.

Identical considerations can be applied to the isoscalar part. Let the dynamical equation of the ϕ and ω mesons be given by

$$\partial G_{\mu\nu}{}^\phi/\partial x_\mu - m_\phi{}^2\phi_\nu = \mathcal{J}_\nu{}^\phi,$$

[7] Under the gauge transformation $A_\mu \to A_\mu + \partial\Lambda/\partial x_\mu$ the electromagnetic interaction Lagrangian density (2.7) plus (2.20) (which is given below) transforms like $\mathcal{L}_\gamma \to \mathcal{L}_\gamma - eJ_\mu\partial\Lambda/\partial x_\mu$, where $\mathcal{L}_\gamma = (\mathcal{L}_\gamma)_{\text{isovector}} + (\mathcal{L}_\gamma)_{\text{isoscalar}}$ and $J_\mu = J_\mu{}^\rho + \frac{1}{2}Y_\mu$; for simplicity, all $O(e^2)$ terms are omitted. According to Eqs. (2.6) and (2.16), $-i\int J_4 d^3r = Q = I_z + \frac{1}{2}Y$. The difference $J_\mu{}'$ between the current J_μ and the minimal electromagnetic current operator $(J_\mu)_{\min}$, which satisfies the same normalization condition $-i\int(J_4)_{\min}d^3r = Q$, may not be zero, but it is obvious that this difference $J_\mu{}' = J_\mu - (J_\mu)_{\min}$ must satisfy $[\partial J_\mu{}'/\partial x_\mu] = 0$ and $\int J_4{}'d^3r = 0$. An implicit assumption is made here that $[\partial J_\mu{}'/\partial x_\mu] = 0$ is satisfied kinematically, *without* the use of dynamical equations. For example, $J_\mu{}'$ can be simply proportional to $(\partial G_{\mu\nu}{}^\rho/\partial x_\nu)$. Under the gauge transformation (of the second kind), the free Lagrangian and the strong interaction Lagrangian density transform like $(\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{st}}) \to (\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{st}}) + e(J_\mu)_{\min}(\partial\Lambda/\partial x_\mu)$. [If one wishes, one may also regard this as the definition of the minimal current $(J_\mu)_{\min}$.] Correspondingly, the total Lagrangian $\mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{st}} + \mathcal{L}_\gamma$ transforms according to $\mathcal{L} \to \mathcal{L} + eJ_\mu{}'(\partial\Lambda/\partial x_\mu)$. The gauge invariance of the action integral $\int\mathcal{L}d^4x$, and therefore also that of the equation of motion, is consequently guaranteed. For the ρ^0 meson, if we identify the unrenormalized current $(J_\mu{}^\rho)^0$, which enters in Eq. (3.2) below, as the minimal z-component isospin current, then the current $J_\mu{}^\rho$ is defined by Eq. (3.13), $J_\mu{}^\rho = (J_\mu{}^\rho)^0 + (\text{constant})\partial G_{\mu\nu}{}^\rho/\partial x_\nu$; therefore, the corresponding difference, $J_\mu{}^\rho - (J_\mu{}^\rho)^0$, is $J_\mu{}' = (\text{constant})\ \partial G_{\mu\nu}{}^\rho/\partial x_\nu$, which does satisfy $(\partial J_\mu{}'/\partial x_\mu) = 0$ in a purely kinematical way.

It is important to note that a single direct coupling $\rho_\mu A_\mu$ violates gauge invariance. This can be most easily seen by observing that such a term in the Lagrangian generates a non-gauge-invariant contribution to the ρ-meson current proportional to A_μ. This circumstance is associated with the fact that $(\partial\rho_\mu/\partial x_\mu) = 0$ is *not* a kinematical identity, but is derived only after using the dynamical equation and the condition that $(\partial J_\mu{}^\rho/\partial x_\mu) = 0$. This point has often been incorrectly stated in the literature. [See, e.g., M. Ross and L. Stodolsky (Ref. 2) and L. Stodolsky (Ref. 2).]

The same gauge-invariant Lagrangian densities (2.7) and (2.20) can also be cast into other different, but equivalent, forms. Some of these alternative forms are discussed in Appendix B.

[8] We emphasize that (2.7)–(2.9) are equations in renormalized quantities. The term $J_\mu{}^\rho A_\mu$ may be regarded as the "direct" photon-hadron coupling while the term $G_{\mu\nu}{}^\rho F_{\mu\nu}$ is the gauge-invariant photon-ρ meson mixing term discussed by Feldman and Matthews (Ref. 2). We note that Eq. (2.9) is achieved *not* via the introduction of a term of the form $\rho_\mu A_\mu$ in the Lagrangian, but by the assumption of a special relation between the "direct" source term and the mixing term. In the case of free photons, it is the direct source term rather than the mixing term which couples the photon to the hadrons, but nevertheless, Eq. (2.9) holds. This would seem to resolve the issue raised by Feldman and Matthews in their Ref. 14. A related resolution has been given by Barton and Smith (Ref. 2, Sec. 4.3) in the context of dispersion relations, with reference to the connection between $f_{\rho\pi\gamma}$ and $f_{\rho\pi\phi}$. See case 2 of Appendix B for further discussions.

[9] This property can be related to the Barton and Smith "boundary condition" [Ref. 2, Eq. (4.14)].

and

$$\partial G_{\mu\nu}{}^{\omega}/\partial x_\mu - m_\omega{}^2\omega_\nu = \mathcal{J}_\nu{}^{\omega}, \tag{2.12}$$

where ϕ_ν and ω_ν are, respectively, the "renormalized" field operators of the ϕ meson and the ω meson, m_ϕ and m_ω are the observed masses,

$$G_{\mu\nu}{}^{\phi} = \frac{\partial}{\partial x_\mu}\phi_\nu - \frac{\partial}{\partial x_\nu}\phi_\mu,$$

and

$$G_{\mu\nu}{}^{\omega} = \frac{\partial}{\partial x_\mu}\omega_\nu - \frac{\partial}{\partial x_\nu}\omega_\mu. \tag{2.13}$$

Both currents $\mathcal{J}_\nu{}^\phi$ and $\mathcal{J}_\nu{}^\omega$ are assumed to be conserved:

$$\partial \mathcal{J}_\nu{}^\phi/\partial x_\nu = \partial \mathcal{J}_\nu{}^\omega/\partial x_\nu = 0. \tag{2.14}$$

Consequently, as will also be proved in Appendix A, the hypercharge Y and the baryon number N must be linear functions of the spatial integrals of $\mathcal{J}_4{}^\phi$ and $\mathcal{J}_4{}^\omega$. Without any loss of generality, we may express this linear relationship in terms of four real constants g_Y, g_N, θ_Y, and θ_N:

$$g_Y Y = -i\int[\cos\theta_Y \mathcal{J}_4{}^\phi - \sin\theta_Y \mathcal{J}_4{}^\omega]d^3r,$$

and

$$g_N N = -i\int[\sin\theta_N \mathcal{J}_4{}^\phi + \cos\theta_N \mathcal{J}_4{}^\omega]d^3r. \tag{2.15}$$

It is convenient to define two conserved currents Y_μ and N_μ which satisfy the normalization conditions

$$-i\int Y_4 d^3r = Y, \tag{2.16}$$

and

$$-i\int N_4 d^3r = N; \tag{2.17}$$

these two currents are related to $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$ by

$$g_Y Y_\mu = \cos\theta_Y \mathcal{J}_\mu{}^\phi - \sin\theta_Y \mathcal{J}_\mu{}^\omega$$

and

$$g_N N_\mu = \sin\theta_N \mathcal{J}_\mu{}^\phi + \cos\theta_N \mathcal{J}_\mu{}^\omega. \tag{2.18}$$

The inverse relations are

$$\mathcal{J}_\mu{}^\phi = [\cos(\theta_Y-\theta_N)]^{-1}[\cos\theta_N g_Y Y_\mu + \sin\theta_Y g_N N_\mu]$$

and

$$\mathcal{J}_\mu{}^\omega = [\cos(\theta_Y-\theta_N)]^{-1}[-\sin\theta_N g_Y Y_\mu + \cos\theta_Y g_N N_\mu]. \tag{2.19}$$

It is important to note that the above definitions of g_Y, g_N, θ_Y, and θ_N depend only on the conservation of Y and N, and are *independent* of any assumption concerning the approximate validity of SU_3 symmetry. The constants g_Y and g_N are the renormalized coupling constants, and the angles θ_Y and θ_N relate the currents $g_Y Y_\mu$ and $g_N N_\mu$ to $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$. In general, these two angles are different, $\theta_Y \neq \theta_N$, even to the first order in the SU_3 symmetry-breaking interaction. Estimations of the actual values of θ_Y and θ_N can be made by making specific dynamical assumptions; these discussions will be given in Sec. V.

In order to identify the isoscalar part, $(J_\mu{}^\gamma)_{\text{isoscalar}}$, of the hadronic electromagnetic current with a linear sum of the renormalized field operators ϕ_μ and ω_μ, we assume that the corresponding isoscalar part of the electromagnetic interaction is given by the following Lagrangian density:

$$(\mathcal{L}_\gamma)_{\text{isoscalar}} = -\tfrac{1}{2}e\{J_\mu{}^Y A_\mu + \tfrac{1}{2}g_Y{}^{-1} \times[\cos\theta_Y G_{\mu\nu}{}^\phi - \sin\theta_Y G_{\mu\nu}{}^\omega]F_{\mu\nu}\}. \tag{2.20}$$

As a consequence, the isoscalar part of the electromagnetic field is given by

$$(\partial F_{\mu\nu}/\partial x_\mu)_{\text{isoscalar}} = e(J_\nu{}^\gamma)_{\text{isoscalar}}, \tag{2.21}$$

where $(J_\mu{}^\gamma)_{\text{isoscalar}}$ is given by Eq. (1.6).

For any real or virtual photon process $A \to B+\gamma$, the isoscalar part of the matrix element of $J_\mu{}^\gamma$ is then related to the matrix elements of $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$ for the corresponding virtual or real processes $A \to B+\phi^0$, and $A \to B+\omega^0$ at the same 4-momentum transfer. By using Eq. (1.6), we find

$$\langle B|J_\mu{}^\gamma|A\rangle_{\text{isoscalar}} = \tfrac{1}{2}g_Y{}^{-1}\left[\cos\theta_Y\left(\frac{m_\phi{}^2}{q^2+m_\phi{}^2}\right)\langle B|\mathcal{J}_\mu{}^\phi|A\rangle - \sin\theta_Y\left(\frac{m_\omega{}^2}{q^2+m_\omega{}^2}\right)\langle B|\mathcal{J}_\mu{}^\omega|A\rangle\right]. \tag{2.22}$$

The matrix elements of $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$ are, in turn, related to those of Y_μ and N_μ through Eq. (2.19). We have, then, the following alternative expression:

$$2\langle B|J_\mu{}^\gamma|A\rangle_{\text{isoscalar}} = \left(\frac{m_\phi{}^2}{q^2+m_\phi{}^2}\right)[C_Y\langle B|Y_\mu|A\rangle + C_N\langle B|N_\mu|A\rangle] + \left(\frac{m_\omega{}^2}{q^2+m_\omega{}^2}\right)[(1-C_Y)\langle B|Y_\mu|A\rangle - C_N\langle B|N_\mu|A\rangle], \tag{2.23}$$

where

$$C_Y = [\cos(\theta_Y-\theta_N)]^{-1}\cos\theta_Y\cos\theta_N,$$

and

$$C_N = [g_Y\cos(\theta_Y-\theta_N)]^{-1}g_N\cos\theta_Y\sin\theta_Y. \tag{2.24}$$

Equation (2.23) shows that at $q^2=0$,

$$\langle B|J_\mu{}^\gamma|A\rangle_{\text{isoscalar}} = \tfrac{1}{2}\langle B|Y_\mu|A\rangle.$$

III. RENORMALIZATION OF THE NEUTRAL ρ MESON

In this section, we will discuss the renormalization problem of the *strong* interaction of the neutral ρ

meson, but only in the absence of the electromagnetic interaction.

Let us assume that the part of the Lagrangian density describing the neutral ρ meson and its strong interaction is given by

$$\mathcal{L}_\rho=-\tfrac{1}{4}(G_{\mu\nu}{}^0)^2-\tfrac{1}{2}(m_\rho{}^0\rho_\mu{}^0)^2-g_\rho{}^0\rho_\mu{}^0(J_\mu{}^\rho)^0, \qquad (3.1)$$

where the superscript 0 denotes the unrenormalized quantities; thus, $\rho_\mu{}^0$ is the unrenormalized meson-field operator

$$G_{\mu\nu}{}^0=\frac{\partial}{\partial x_\mu}\rho_\nu{}^0-\frac{\partial}{\partial x_\nu}\rho_\mu{}^0,$$

$m_\rho{}^0$ is its unrenormalized mass, $(J_\mu{}^\rho)^0$ is the unrenormalized current and $g_\rho{}^0$ is the unrenormalized coupling constant. The equation of motion is given by

$$\partial G_{\mu\nu}{}^0/\partial x_\mu-(m_\rho{}^0)^2\rho_\nu{}^0=g_\rho{}^0(J_\nu{}^\rho)^0. \qquad (3.2)$$

In Eq. (3.1), we assume for simplicity that $(J_\nu{}^\rho)^0$ does not depend on $\rho_\mu{}^0$; otherwise, Eq. (3.1) has to be modified so that the equation of motion [Eq. (3.2)] remains valid.

The current operator $(J_\nu{}^\rho)^0$ transforms like the z component of an isospin triplet, and it is assumed to be conserved; i.e.,

$$\partial(J_\nu{}^\rho)^0/\partial x_\nu=0. \qquad (3.3)$$

Therefore, according to the general theorem established in Appendix A, the integral $\int(J_4{}^\rho)^0d^3r$ is proportional to the observed z-component isospin operator I_z, and we may, without any loss of generality, choose

$$\int(J_4{}^\rho)^0d^3r=iI_z. \qquad (3.4)$$

Since the normalization of $\rho_\mu{}^0$ is fixed by the Lagrangian density (3.1) and the canonical rules, condition (3.4) defines the unrenormalized coupling constant $g_\rho{}^0$.

To obtain renormalized equations we first set

$$\rho_\mu{}^0=Z^{1/2}\rho_\mu, \qquad (3.5)$$

where Z is a wave-function renormalization constant. The Lagrangian density $\mathcal{L}_\rho$ given by Eq. (3.1) may be written in terms of ρ_μ in the form

$$\mathcal{L}_\rho=\mathcal{L}_{\text{free}}+\mathcal{L}_{\text{st}}, \qquad (3.6)$$

where

$$\mathcal{L}_{\text{free}}=-\tfrac{1}{4}(G_{\mu\nu}{}^\rho)^2-\tfrac{1}{2}m_\rho{}^2\rho_\mu{}^2, \qquad (3.7)$$

$$\mathcal{L}_{\text{st}}=-\tfrac{1}{4}(Z-1)(G_{\mu\nu}{}^\rho)^2-\tfrac{1}{2}[(m_\rho{}^0)^2Z-m_\rho{}^2]\rho_\mu{}^2-g_\rho{}^0Z^{1/2}(J_\nu{}^\rho)^0\rho_\nu, \qquad (3.8)$$

and m_ρ is the observed mass of the ρ meson. The above expression of $\mathcal{L}_\rho$ is convenient for a perturbation series expansion in terms of $\mathcal{L}_{\text{st}}$, since the free Lagrangian is constructed so that it would have the correct energy spectrum if the neutral ρ meson were stable.

The equation of motion of the renormalized field ρ_μ is given by

$$\frac{\partial G_{\mu\nu}}{\partial x_\mu}-m_\rho{}^2\rho_\nu=Z^{-1/2}g_\rho{}^0Z_0(J_\nu{}^\rho)^0+(1-Z_0)\frac{\partial G_{\mu\nu}}{\partial x_\mu}\equiv g_\rho J_\nu{}^\rho, \qquad (3.9)$$

with

$$Z_0=(m_\rho/m_\rho{}^0)^2, \qquad (3.10)$$

and

$$J_\nu{}^\rho=\frac{g_\rho{}^0}{g_\rho}\frac{Z_0}{Z^{1/2}}(J_\nu{}^\rho)^0+(1-Z_0)\frac{1}{g_\rho}\frac{\partial G_{\mu\nu}}{\partial x_\mu}.$$

The renormalized current $J_\nu{}^\rho$ is the same current as that used in Eq. (2.3). Its normalization condition [Eq. (2.6)] fixes the relation between g_ρ and Z; the relation being

$$Z_0g_\rho{}^0=Z^{1/2}g_\rho. \qquad (3.11)$$

Hence,

$$\frac{\rho_\nu}{g_\rho}=\frac{1}{Z_0}\frac{\rho_\nu{}^0}{g_\rho{}^0},\quad \frac{G_{\mu\nu}}{g_\rho}=\frac{1}{Z_0}\frac{G_{\mu\nu}{}^0}{g_\rho{}^0} \qquad (3.12)$$

and

$$J_\nu{}^\rho=(J_\nu{}^\rho)^0+(1-Z_0)\frac{1}{g_\rho}\frac{\partial G_{\mu\nu}}{\partial x_\mu}. \qquad (3.13)$$

It follows from Eqs. (3.12) and (3.13) that the ratios ρ_ν/g_ρ and $G_{\mu\nu}/g_\rho$, and the renormalized current density $J_\nu{}^\rho$ are independent of Z as expected. The factor Z is, of course, of utility principally in connection with divergence difficulties and is introduced in order to make it possible to express the theory in terms of finite quantities. Apart from this requirement it can be chosen for convenience and has no physical consequences. Some convenient choices will be discussed later.

By using Eqs. (3.9), (3.10), and (3.13), one finds the important relation

$$(J_\nu{}^\rho)^0=-\left(\frac{m_\rho{}^2}{g_\rho}\right)\rho_\nu+g_\rho{}^{-1}\left(\frac{m_\rho}{m_\rho{}^0}\right)^2\frac{\partial G_{\mu\nu}{}^\rho}{\partial x_\mu}. \qquad (3.14)$$

While the precise value of $m_\rho{}^0$ depends on the form $(J_\nu{}^\rho)^0$ assumed in the particular strong-interaction theory, it can be shown that [see Eq. (4.7) in the next section] $m_\rho{}^0>2m_\pi$, and that

$$m_\rho{}^0=\infty, \qquad (3.15)$$

if the theory is divergent. Consequently, the right-hand side of Eq. (3.14) contains only finite quantities. In particular, if the theory is divergent, then $m_\rho{}^0=\infty$, and

$$(J_\nu{}^\rho)^0=-(m_\rho{}^2/g_\rho)\rho_\nu. \qquad (3.16)$$

In this case, the unrenormalized current $(J_\nu{}^\rho)^0$ becomes proportional to the renormalized field operator ρ_ν. The renormalized current $J_\nu{}^\rho$ is, of course, different from the field operator ρ_ν.

The proportionality between the unrenormalized current operator and the renormalized field operator is a general consequence of a vector field interacting with a conserved current, provided that the unrenormalized mass $=\infty$. If the unrenormalized mass were finite, then one would have Eq. (3.14) instead of Eq. (3.16).

Next, we discuss the relation between the electromagnetic current J_ν^γ and the unrenormalized current $(J_\nu^\rho)^0$ of the ρ meson. If we assume Eq. (1.3a),

$$(J_\nu^\gamma)_{\text{isovector}}=-(m_\rho^2/g_\rho)\rho_\nu\,, \tag{1.3a}$$

then, by using Eq. (3.14), we find

$$(J_\nu^\gamma)_{\text{isovector}}=(J_\nu^\rho)^0-g_\rho^{-1}(m_\rho/m_\rho^0)^2(\partial G_{\mu\nu}^\rho/\partial x_\mu)\,, \tag{3.17}$$

which implies that in the case of an infinite unrenormalized mass m_ρ^0

$$(J_\nu^\gamma)_{\text{isovector}}=(J_\nu^\rho)^0. \tag{3.18}$$

Since we do not consider the renormalization problem of the electromagnetic interaction, there is no difference between J_ν^γ and the unrenormalized electromagnetic current operator $(J_\nu^\gamma)^0$.

There exists an alternative possibility in which one assumes that, instead of (1.3a),

$$(J_\nu^\gamma)^0{}_{\text{isovector}}=(J_\nu^\rho)^0\,, \tag{1.3b}$$

and therefore, neglecting higher-order electromagnetic corrections,

$$(J_\nu^\gamma)_{\text{isovector}}=-\left(\frac{m_\rho^2}{g_\rho}\right)\rho_\nu+g_\rho^{-1}\left(\frac{m_\rho}{m_\rho^0}\right)^2\frac{\partial G_{\mu\nu}^\rho}{\partial x_\mu}. \tag{3.19}$$

In the case of $m_\rho^0=\infty$, one has

$$(J_\nu^\gamma)_{\text{isovector}}=(-m_\rho^2/g_\rho)\rho_\mu. \tag{3.20}$$

It has already been mentioned in the introduction that these two views become identical if $m_\rho^0=\infty$; but if the unrenormalized mass m_ρ^0 turns out to be finite, then the proposal (1.3a) implies the form-factor relations given by Eq. (1.4a) while the proposal (1.3b) implies the alternative form-factor relation given by[10] Eq. (1.4b).

IV. THE ρ-MESON PROPAGATOR

Let us consider the usual spectral representation of the vacuum expectation value of the commutator $[\rho_\mu(x),\rho_\nu(0)]$ in the Heisenberg representation:

$$\langle[\rho_\mu(x),\rho_\nu(0)]\rangle_{\text{vacuum}}=\int\sigma_\rho(a)\left[\delta_{\mu\nu}-a^{-1}\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right]\Delta_a(x)da\,, \tag{4.1}$$

where

$$\Delta_a(x)=-i(2\pi)^{-3}\int\omega^{-1}\sin\omega t\exp(i\mathbf{q}\cdot\mathbf{r})d^3q\,,$$

and

$$\omega=(\mathbf{q}^2+a)^{1/2}.$$

The function $\sigma_\rho(a)$ is related to the matrix elements of the spatial components of ρ_μ by

$$\sigma_\rho(a)=\tfrac{1}{3}\sum_\Gamma\delta(a-m_\Gamma^2)|\langle\Gamma|\boldsymbol{\rho}(0)|\text{vac}\rangle|^2\geqq0\,, \tag{4.2}$$

where the sum extends over all eigenstates $|\Gamma\rangle$ of the strong interaction Hamiltonian that satisfy

$$(\text{momentum})_\Gamma=0,\quad(\text{energy})_\Gamma=m_\Gamma,\quad\text{and}\quad(\text{spin})_\Gamma=1\,, \tag{4.3}$$

and the factor $\frac{1}{3}$ is due to the further sum over all three components of $\boldsymbol{\rho}(0)$. Throughout the paper, all boldface letters denote 3-vectors.

By using the Lagrangian given by Eq. (3.6), one finds that the components of the canonical momentum $\mathbf{\Pi}(x)$ of the field variable $\boldsymbol{\rho}(x)$ are given by

$$\Pi_j=-iZG_{j4}^\rho.$$

From the equation of motion, $\mathbf{\Pi}$ is also related to ρ_4 by

$$\rho_4=m_\rho^{-2}[i(Z_0/Z)\boldsymbol{\nabla}\cdot\mathbf{\Pi}-g_\rho(J_4^\rho)^0]. \tag{4.4}$$

The comparison between the spectral representation [Eq. (4.1)] and the equal-time commutator between $\mathbf{\Pi}$ and $\boldsymbol{\rho}$ leads to the sum rule[11]

$$Z^{-1}=\int\sigma_\rho(a)da. \tag{4.5}$$

Similarly, by using the equal-time commutator between ρ_4 and $\boldsymbol{\rho}$, and assuming that $(J_4^\rho)^0$ commutes with $\boldsymbol{\rho}$ at equal time, one finds

$$Z^{-1}(m_\rho^0)^{-2}=(Z_0/Z)m_\rho^{-2}=\int a^{-1}\sigma_\rho(a)da\,, \tag{4.6}$$

where Z_0 is given by Eq. (3.10). For the physical ρ meson, the integrations in (4.5) and (4.6) extend from $4m_\pi^2$ to ∞, where m_π is the observed pion mass. By taking the ratio of (4.5) and (4.6), one obtains

$$(m_\rho^0)^2=\left[\int_{4m_\pi^2}^\infty a^{-1}\sigma_\rho(a)da\right]^{-1}\int_{4m_\pi^2}^\infty\sigma_\rho(a)da. \tag{4.7}$$

Since $\sigma_\rho\geqq0$, Eqs. (4.7), (4.5), and (4.6) imply, respectively,

$$(m_\rho^0)^2>4m_\pi^2\,,$$

$$Z^{-1}>0\,,\quad\text{and}\quad(Z_0/Z)>0.$$

In order that the theory be renormalizable [i.e., that the renormalized propagator given by Eq. (4.10) below exists], the behavior of $\sigma_\rho(a)$ as $a\to\infty$ must be such that

[10] Equation (1.4b) may be compared with Eq. (4.7) of the paper by Gell-Mann and Zachariasen (Ref. 2).

[11] Equations (4.5) and (4.6), together with Eq. (3.5), are the same as Eqs. (12) and (11), respectively, of K. Johnson, Nucl. Phys. 25, 435 (1961).

$\int a^{-1}\sigma_\rho(a)da$ is convergent. Thus, (Z_0/Z) must be finite. On the other hand, the integral $\int \sigma_\rho(a)da$ need not be convergent. If it in fact diverges, then both Z^{-1} and the unrenormalized mass m_ρ^0 must be infinite. Consequently we establish (3.15).

Combining Eqs. (4.5) and (4.6) we find

$$1-Z_0=Z\int a^{-1}(a-m_\rho^2)\sigma_\rho(a)da. \qquad (4.8)$$

For the hypothetical case of a stable ρ meson (i.e., if m_ρ were less than $2m_\pi$), Eq. (4.8) implies the inequality $0\leqq Z_0\leqq 1$, and hence $(m_\rho^0)^2\geqq m_\rho^2$ on account of Eq. (3.10).

The renormalized ρ-meson propagator

$$\hat{D}_{\mu\nu}(x)=-i(2\pi)^{-4}\int D_{\mu\nu}(q)\exp(iq_\lambda x_\lambda)d^4q \qquad (4.9)$$

is given in terms of $\sigma_\rho(a)$ through the relation

$$D_{\mu\nu}(q)=\int\frac{\delta_{\mu\nu}+a^{-1}q_\mu q_\nu}{q^2+a-i\epsilon}\sigma_\rho(a)da, \qquad (4.10)$$

where ϵ is a positive infinitesimal quantity, q_μ denotes the 4-momentum $(\mathbf{q},iq_0)$ and $q^2=\mathbf{q}^2-q_0^2$. Using (4.6), (4.1), and (4.2), one easily sees that $\hat{D}_{\mu\nu}(x)$ is related to the time-ordered product of the renormalized fields via

$$\hat{D}_{\mu\nu}(x)=\{T[\rho_\mu(x)\rho_\nu(0)]\}_{\text{vacuum}}-im_\rho^{-2}(Z_0/Z)\delta^4(x)\delta_{4\mu}\delta_{4\nu}. \qquad (4.11)$$

In our notation, both d^4q and $\delta^4(x)$ are real. The last term in Eq. (4.11) cancels the noncovariant part in $\{T[\rho_\mu(x)\rho_\nu(0)]\}_{\text{vacuum}}$. The appearance of this noncovariant term is due to the fact that the commutator $[\rho_\mu(x),\rho_\nu(0)]$ does not vanish for $x_0=0$.

Because the ρ meson is an unstable particle, it is convenient to define the renormalized mass m_ρ in terms of the inverse propagator. Writing

$$D_{\mu\nu}(q)=\delta_{\mu\nu}F(q^2)+q_\mu q_\nu G(q^2), \qquad (4.12)$$

and noting the relation

$$F+q^2G=(Z_0/Z)m_\rho^{-2}, \qquad (4.13)$$

which follows from Eqs. (4.10) and (4.6), we obtain

$$D_{\mu\nu}^{-1}(q)=[F(q^2)]^{-1}[\delta_{\mu\nu}-q_\mu q_\nu Z_0^{-1}Zm_\rho^2G(q^2)]. \qquad (4.14)$$

The renormalized mass is then defined as that value of $-q^2$ at which the real part of the $\delta_{\mu\nu}$ term of the inverse propagator vanishes; i.e., by the equation

$$\mathrm{Re}[1/F(-m_\rho^2)]=0. \qquad (4.15)$$

For the physical ρ meson, $[F(-m_\rho^2)]^{-1}$ is pure imaginary; the same must also hold for $F(-m_\rho^2)$, and we find

$$\mathrm{Re}F(-m_\rho^2)=\int_{4m_\pi^2}^\infty \sigma_\rho(a)\mathcal{P}\left[\frac{1}{a-m_\rho^2}\right]da=0,$$

where $\mathcal{P}$ denotes the principal value. If, as is usually assumed, the imaginary part of $(1/F)$ as well as the derivative of the real part can be regarded as constant within the resonance width, then the width is given by the formula

$$\gamma m_\rho=[Z\pi\sigma(m_\rho^2)]^{-1}Z_1, \qquad (4.16)$$

where we have written Z/Z_1 for the derivative of the real part. Thus

$$\frac{Z}{Z_1}\equiv\left[\frac{d}{dq^2}\mathrm{Re}\left(\frac{1}{F}\right)\right]_{q^2=-m_\rho^2}. \qquad (4.17)$$

The definitions of m_ρ^2 and γ given by Eqs. (4.15) and (4.16) correspond to those given by a pole approximation for F in the resonance region of the form

$$F(q^2)\cong(Z_1/Z)(q^2+m_\rho^2-i\gamma m_\rho)^{-1}. \qquad (4.18)$$

In the hypothetical case of a stable ρ meson, the renormalized mass may still be determined by Eq. (4.15), while the derivative designated in Eq. (4.17) determines the matrix element of the renormalized field ρ_μ between the vacuum state and the state of a ρ meson at rest; thus,

$$\langle\text{vac}|\boldsymbol{\rho}(0)|\rho\rangle=(2m_\rho)^{-1/2}(Z_1/Z)^{1/2}\mathbf{s}, \qquad (4.19)$$

where $\mathbf{s}$ is the polarization vector $(\mathbf{s}^2=1)$. The spectral function $\sigma_\rho(a)$ contains a delta function of the form $(Z_1/Z)\delta(a-m_\rho^2)$ so that Eq. (4.6) implies

$$1=\frac{Z_1}{Z_0}+m_\rho^2\frac{Z}{Z_0}\int_b^\infty a^{-1}\sigma_\rho(a)da, \qquad (4.20)$$

where b is the lower limit of the continuous spectrum in this hypothetical case. Hence one concludes

$$0<Z_1/Z_0<1. \qquad (4.21)$$

So far, except for the requirement that the renormalized ρ-meson propagator $D_{\mu\nu}(q)$ should be free from divergence difficulties, the choice of the wave-function renormalization constant Z is completely arbitrary. It is clear that different choices of Z can differ from each other only by a finite positive multiplicative factor, and such different choices all lead to exactly the same physical result. On account of Eq. (4.19), the conventional choice for the renormalization constant is $Z=Z_1$. On the other hand, the fact that the ρ meson is unstable makes this choice less compelling, and we note that the choice $Z=Z_0$ simplifies a number of formulas. For the remainder of this section and in the applications of Sec. VII, we shall use the convention

$$Z=Z_0=(m_\rho/m_\rho^0)^2, \qquad (4.22)$$

and therefore

$$\rho_\mu(x)=(m_\rho^0/m_\rho)\rho_\mu^0(x). \qquad (4.23)$$

With this convention, it follows from Eq. (3.11) that

$$g_\rho=(m_\rho/m_\rho^0)g_\rho^0, \qquad (4.24)$$

and from Eqs. (4.12) and (4.13) that

$$F(0)=m_\rho^{-2},$$

and

$$D_{\mu\nu}(q=0)=m_\rho^{-2}\delta_{\mu\nu}. \qquad (4.25)$$

The width γ can also be explicitly expressed in terms of the transition matrix elements. Let us define a modified current operator $\hat{J}_\mu{}^\rho(x)$ whose matrix element between any two eigenstates A and B of the strong interaction Hamiltonian is given by[12]

$$\langle B|\hat{J}_\mu{}^\rho(x)|A\rangle=-[g_\rho D_{\mu\nu}(q)]^{-1}\langle B|\rho_\nu(x)|A\rangle, \qquad (4.26)$$

where q_μ is the difference between the 4-momenta of the states A and B. This modified current operator $\hat{J}_\mu{}^\rho(x)$ does not satisfy either the usual Hermiticity condition or the locality condition. Nevertheless, since the integrated operator $\int\rho_4 d^3r$ is, according to Eqs. (3.4) and (3.14), $-i(g_\rho/m_\rho^2)I_z$, one has, by using Eqs. (4.25) and (4.26),

$$\int \hat{J}_4{}^\rho d^3r=iI_z. \qquad (4.27)$$

The modified current operator $\hat{J}_\mu{}^\rho(x)$ is useful since the matrix elements of $J_\mu{}^\rho(x)$ must, by definition [Eq. (3.9)], satisfy

$$\langle B|J_\mu{}^\rho(x)|A\rangle=0 \quad \text{at} \quad q^2+m_\rho^2=0, \qquad (4.28)$$

while those of $\hat{J}_\mu{}^\rho$ do not. By using Eqs. (4.2), (4.16), (4.18), and (4.26), we find that the partial decay width of the ρ^0 meson to the final channel Γ is given by

$$\text{rate}(\rho^0\to\Gamma)=\tfrac{2}{3}\pi g_\rho^2(Z_1/Z_0)|\langle\Gamma|\hat{\mathbf{J}}^\rho(0)|\text{vac}\rangle|^2(2m_\rho)^{-1} \times(\text{density of states})_\Gamma, \qquad (4.29)$$

where $\hat{\mathbf{J}}^\rho$ denotes the spatial component of $\hat{J}_\mu{}^\rho(x)$. The total width γ is

$$\gamma=\sum \text{rate}(\rho^0\to\Gamma),$$

where the sum extends over all different channels Γ. The factor (Z_1/Z_0) is due to our choice [Eq. (4.22)] of the wave-function renormalization.

Equation (4.29) can be readily used for calculating the various decay rates of the neutral ρ meson. These applications will be discussed in Sec. VII. It is clear that all above discussions can be applied to any vector meson provided its current is conserved. The detailed treatment of the ϕ-ω mixing problem will be given in the following section.

[12] For single-particle states A and B, $\langle B|\hat{J}|A\rangle$ is, of course, related to the conventional vertex operator $\Gamma_\mu(p_B,p_A)$ via the formula

$$\langle B|\hat{J}_\mu(x)|A\rangle=u_B{}^\dagger\Gamma_\mu(p_B,p_A)u_A\exp(iq_\lambda x_\lambda),$$

where p_B, p_A denote, respectively, the 4-momenta of the states B and A, and $q=p_B-p_A$.

V. RENORMALIZATION OF THE ω AND ϕ MESONS AND THE MIXING PROBLEM

1. Renormalized Equations

Although the renormalization problem of the ω-ϕ complex is complicated by the possible mixing (i.e., virtual transition $\omega\rightleftarrows\phi$) between these two fields, it can nevertheless be carried out in a manner which parallels our treatment of the ρ meson. Let $\phi_\mu(x)$ and $\omega_\mu(x)$ denote the renormalized fields, and $\phi_\mu{}^0(x)$, $\omega_\mu{}^0(x)$ the corresponding unrenormalized fields. We write

$$\psi_\mu(x)=\begin{pmatrix}\phi_\mu(x)\\ \omega_\mu(x)\end{pmatrix} \qquad (5.1)$$

and

$$\psi_\mu{}^0(x)=\begin{pmatrix}\phi_\mu{}^0(x)\\ \omega_\mu{}^0(x)\end{pmatrix}. \qquad (5.2)$$

The part of the Lagrangian density describing the ϕ, ω mesons and their strong interactions can always be written, in analogy to Eq. (3.1) for the ρ-meson problem, as

$$\mathcal{L}_{\phi-\omega}=-\tfrac{1}{4}\tilde{\mathcal{G}}_{\mu\nu}{}^0K_0\mathcal{G}_{\mu\nu}{}^0-\tfrac{1}{2}\tilde{\psi}_\mu{}^0M_0{}^2\psi_\mu{}^0-\tilde{\psi}_\mu{}^0g_0J_\mu{}^0, \qquad (5.3)$$

where

$$\mathcal{G}_{\mu\nu}{}^0=\begin{bmatrix}\dfrac{\partial}{\partial x_\mu}\phi_\nu{}^0-\dfrac{\partial}{\partial x_\nu}\phi_\mu{}^0\\[2ex] \dfrac{\partial}{\partial x_\mu}\omega_\nu{}^0-\dfrac{\partial}{\partial x_\nu}\omega_\mu{}^0\end{bmatrix},$$

and

$$J_\mu{}^0=\begin{pmatrix}Y_\mu{}^0\\ N_\mu{}^0\end{pmatrix}, \qquad (5.4)$$

which denotes the unrenormalized current operators; g_0 is a (2×2) real matrix, K_0, $M_0{}^2$ are both (2×2) real symmetric positive-definite matrices, and $\sim$ (the tilde) denotes transposition. The currents $Y_\mu{}^0$ and $N_\mu{}^0$ are conserved; they satisfy

$$-i\int(Y_4{}^0)d^3r=\text{hypercharge}=Y$$

and

$$-i\int(N_4{}^0)d^3r=\text{baryon number}=N, \qquad (5.5)$$

and will be referred to as the unrenormalized hypercharge current and baryon number current.

There is a certain arbitrariness in the matrices K_0, $M_0{}^2$, and g_0 so long as the normalization and orientation (in the internal space) of the fields $\psi_\mu{}^0$ is not specified. That is, by linearly transforming the fields,

$$\psi_\mu{}^0=A\psi_\mu{}', \qquad (5.6)$$

where A is an arbitrary nonsingular real matrix, and defining

$$\begin{aligned} \mathcal{G}_{\mu\nu}' &= A^{-1}\mathcal{G}_{\mu\nu}^0, \\ K_0' &= \tilde{A}K_0A, \\ M_0'^2 &= \tilde{A}M_0^2A, \\ g_0' &= \tilde{A}g_0, \end{aligned} \tag{5.7}$$

the Lagrangian density (5.3) becomes

$$\mathcal{L}_{\phi-\omega} = -\tfrac{1}{4}\tilde{\mathcal{G}}_{\mu\nu}'K_0'\mathcal{G}_{\mu\nu}' - \tfrac{1}{2}\tilde{\psi}_\mu'M_0'^2\psi_\mu' - \tilde{\psi}_\mu'g_0'J_\mu^0.$$

One may therefore specify the normalization and orientation by imposing certain restrictions on the forms of K_0, M_0^2, and g_0. The conventional specification is the "canonical form," defined by requiring that K_0 be the unit matrix and M_0^2 be diagonal. It will, however, prove to be more convenient in connection with SU_3 considerations to use what we will refer to as the "aligned form." The aligned form is defined by requiring g_0 to be diagonal,

$$g_0 = \begin{pmatrix} g_Y^0 & 0 \\ 0 & g_N^0 \end{pmatrix}, \tag{5.8}$$

and the diagonal elements of K_0 to be unity,

$$K_0 = \begin{pmatrix} 1 & \kappa \\ \kappa & 1 \end{pmatrix}, \tag{5.9}$$

where κ is a constant which may, or may not, be zero. Both the canonical and aligned forms are always possible and specify the normalization and orientation of ψ_μ^0 completely. We will for the present, however, leave the choice unspecified.

As in the case of the ρ meson, we assume that J_μ^0 does not depend upon ψ_μ^0. The equation of motion implied by Eq. (5.3) is then

$$K_0(\partial/\partial x_\mu)\mathcal{G}_{\mu\nu}^0 - M_0^2\psi_\nu^0 = g_0J_\nu^0. \tag{5.10}$$

By setting $g_0=0$ in (5.10), one easily sees that the squares of the mechanical (i.e., unrenormalized) masses of the ϕ and ω are the eigenvalues of the matrix M_{mech}^2,

$$M_{\text{mech}}^2 \equiv K_0^{-1/2}M_0^2K_0^{-1/2}. \tag{5.11}$$

Under the transformation (5.7), we have

$$\begin{aligned} M_{\text{mech}}'^2 &= K_0'^{-1/2}M_0'^2K_0'^{-1/2} \\ &= (K_0^{1/2}AK_0'^{-1/2})^{-1}M_{\text{mech}}^2(K_0^{1/2}AK_0'^{-1/2}), \end{aligned} \tag{5.12}$$

which is a similarity transformation. Hence, as expected, the unrenormalized masses are unaffected by the transformation.

To obtain renormalized equations we set

$$\psi_\mu^0 = S\psi_\mu, \tag{5.13}$$

where the matrix S corresponds to the renormalization factor $Z^{1/2}$ introduced by Eq. (3.5) for the ρ meson. We may call S the renormalization matrix. Similarly to Eqs. (3.6) to (3.8), the Lagrangian density $\mathcal{L}_{\phi-\omega}$ given by Eq. (5.3) may be rewritten in terms of ψ_μ in the form

$$\mathcal{L}_{\phi-\omega} = \mathcal{L}_{\phi-\omega\ \text{free}} + \mathcal{L}_{\phi-\omega\ \text{st}}, \tag{5.14}$$

where

$$\mathcal{L}_{\phi-\omega\ \text{free}} = -\tfrac{1}{4}\tilde{\mathcal{G}}_{\mu\nu}\mathcal{G}_{\mu\nu} - \tfrac{1}{2}\tilde{\psi}_\mu M^2\psi_\mu, \tag{5.15}$$

$$\begin{aligned} \mathcal{L}_{\phi-\omega\ \text{st}} = &-\tfrac{1}{4}\tilde{\mathcal{G}}_{\mu\nu}(\tilde{S}K_0S-1)\mathcal{G}_{\mu\nu} \\ &-\tfrac{1}{2}\tilde{\psi}_\mu(\tilde{S}M_0^2S-M^2)\psi_\mu - \tilde{\psi}_\mu\tilde{S}g_0J_\mu^0, \end{aligned} \tag{5.16}$$

$$M^2 = \begin{pmatrix} m_\phi^2 & 0 \\ 0 & m_\omega^2 \end{pmatrix}; \tag{5.17}$$

$\mathcal{G}_{\mu\nu} = S^{-1}\mathcal{G}_{\mu\nu}^0$, and m_ϕ^2, m_ω^2 are the observed masses of the designated mesons.

The equation of motion of the renormalized field ψ_μ can, in analogy to (3.9), be written as

$$\begin{aligned} \frac{\partial\mathcal{G}_{\mu\nu}}{\partial x_\mu} - M^2\psi_\nu &= (M^2S^{-1}M_0^{-2}g_0)J_\nu^0 \\ &\quad + (1-M^2S^{-1}M_0^{-2}K_0S)\frac{\partial\mathcal{G}_{\mu\nu}}{\partial x_\mu} \\ &= \mathcal{J}_\nu \equiv gJ_\nu, \end{aligned} \tag{5.18}$$

where $\mathcal{J}_\nu$ and J_ν are related to the currents $\mathcal{J}_\nu^\phi$, $\mathcal{J}_\nu^\omega$, Y_ν, and N_ν introduced in Sec. II by

$$\mathcal{J}_\nu = \begin{pmatrix} \mathcal{J}_\nu^\phi \\ \mathcal{J}_\nu^\omega \end{pmatrix}, \quad \text{and} \quad J_\nu = \begin{pmatrix} Y_\nu \\ N_\nu \end{pmatrix}.$$

We recall that Y_ν and N_ν satisfy the normalization Eqs. (2.16) and (2.17) and they represent, respectively, the renormalized hypercharge current and the renormalized baryon-number current. The matrix g is, according to Eq. (2.15),

$$g = T^{-1}g_D, \tag{5.19}$$

where T and g_D are, respectively, related to the angles θ_Y, θ_N and the renormalized coupling constants g_Y, g_N by

$$T = \begin{pmatrix} \cos\theta_Y & -\sin\theta_Y \\ \sin\theta_N & \cos\theta_N \end{pmatrix}, \tag{5.20}$$

and

$$g_D = \begin{pmatrix} g_Y & 0 \\ 0 & g_N \end{pmatrix}. \tag{5.21}$$

Similarly to Eqs. (3.11) to (3.13), we have

$$M_0^{-2}g_0 = SM^{-2}g. \tag{5.22}$$

Hence

$$\begin{aligned} g^{-1}M^2\psi_\mu &= g_0^{-1}M_0^2\psi_\mu^0 = g_0'^{-1}M_0'^2\psi_\mu', \\ g^{-1}M^2\mathcal{G}_{\mu\nu} &= g_0^{-1}M_0^2\mathcal{G}_{\mu\nu}^0 = g_0'^{-1}M_0'^2\mathcal{G}_{\mu\nu}', \end{aligned} \tag{5.23}$$

and

$$J_\nu = J_\nu^0 + g^{-1}\frac{\partial\mathcal{G}_{\mu\nu}}{\partial x_\mu} - \bar{M}_{\text{mech}}^{-2}g^{-1}M^2\frac{\partial\mathcal{G}_{\mu\nu}}{\partial x_\mu}, \tag{5.24}$$

where

$$\bar{M}_{\text{mech}}^{-2}=(K_0^{-1/2}g_0)^{-1}M_{\text{mech}}^{-2}(K_0^{-1/2}g_0)$$
$$=(K_0'^{-1/2}g_0')^{-1}M_{\text{mech}}'^{-2}(K_0'^{-1/2}g_0'). \quad (5.25)$$

The matrix $\bar{M}_{\text{mech}}^{-2}$ is not necessarily symmetric, but it differs from M_{mech}^{-2} only by a similarity transformation and hence has the same eigenvalues. Equations (5.23) and (5.25) show that $g^{-1}M^2\psi_\mu$, $g^{-1}M^2\mathcal{G}_{\mu\nu}$, and $\bar{M}_{\text{mech}}^2$ are independent of the choice of the renormalization matrix S and also of the normalization and orientation conditions imposed on the unrenormalized fields. We shall refer to this property as *normalization-orientation invariance*. The current J_ν is, because of the presence of the term $g^{-1}(\partial\mathcal{G}_{\mu\nu}/\partial x_\mu)$, normalization-orientation invariant only for the hypothetical case of degenerate M^2. This is to be expected, since application of a linear transformation to both sides of Eq. (5.18) in general does not leave M^2 diagonal.

The isoscalar electromagnetic current is given by Eq. (1.6), which can also be written as

$$(J_\mu{}^\gamma)_{\text{isoscalar}}=-\tfrac{1}{2}(g^{-1}M^2\psi_\mu)_1, \quad (5.26)$$

and is therefore normalization-orientation invariant. The notation $(\ )_1$ means the upper component of the enclosed column matrix.

Equations (5.18) and (5.22) imply that, in analogy to Eq. (3.14) for the ρ-meson case

$$J_\nu{}^0=-g^{-1}M^2\psi_\nu+\bar{M}_{\text{mech}}^{-2}g^{-1}M^2(\partial\mathcal{G}_{\mu\nu}/\partial x_\mu). \quad (5.27)$$

For a divergent theory, the mechanical (i.e., unrenormalized) masses of the ϕ and ω fields are infinite, and hence $\bar{M}_{\text{mech}}^{-2}$ vanishes. Comparison with (5.26) and (5.27) then shows

$$(J_\nu{}^\gamma)_{\text{isoscalar}}=\tfrac{1}{2}(J_\nu{}^0)_1=\tfrac{1}{2}Y_\nu{}^0, \quad (5.28)$$

which plays the same role as Eq. (3.18) for the isovector current. Hence the alternative identifications of the isoscalar electromagnetic current with the *unrenormalized* hypercharge current on the one hand, and the renormalized ϕ-ω fields on the other become identical hypotheses for a divergent theory.

2. The ϕ-ω Propagator and Normalization-Orientation Conditions for the Renormalized Fields

Let us consider the spectral representation of the vacuum expectation value of the commutator $[\psi_\mu{}^0(x),\tilde{\psi}_\nu{}^0(0)]$ in the Heisenberg representation:

$$\langle[\psi_\mu{}^0(x),\tilde{\psi}_\nu{}^0(0)]\rangle_{\text{vacuum}}$$
$$=\int\sigma_{\phi\omega}{}^0(a)\left[\delta_{\mu\nu}-a^{-1}\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right]\Delta_a(x)da. \quad (5.29)$$

The function $\Delta_a(x)$ has already been used in Eq. (4.1), and $\sigma_{\phi\omega}{}^0(a)$ is a (2×2) Hermitian matrix given by

$$\sigma_{\phi\omega}{}^0(a)=\tfrac{1}{3}\sum_\Gamma\delta(a-m_\Gamma{}^2)\langle\text{vac}|\boldsymbol{\psi}^0(0)|\Gamma\rangle$$
$$\times\langle\Gamma|\boldsymbol{\psi}^{0\dagger}(0)|\text{vac}\rangle, \quad (5.30)$$

where the boldface letters denote the space components and the sum extends over all $I=0$ eigenstates $|\Gamma\rangle$ of the strong interaction Hamiltonian which satisfy Eq. (4.3). From time-reversal invariance, or CPT invariance, it follows that $\sigma_{\phi\omega}{}^0(a)$ is also a real matrix. Similarly to the derivation of Eqs. (4.5) and (4.6), one can show that the following two sum rules hold for the unrenormalized spectral function $\sigma_{\phi\omega}{}^0(a)$:

$$\int\sigma_{\phi\omega}{}^0(a)da=K_0{}^{-1}, \quad (5.31)$$

and

$$\int a^{-1}\sigma_{\phi\omega}{}^0(a)da=M_0{}^{-2}. \quad (5.32)$$

For the realistic case of unstable ϕ meson and ω meson, both integrations extend from $9m_\pi{}^2$ to ∞. It follows from these two sum rules that the squares of the unrenormalized masses $(m_\phi{}^0)^2$ and $(m_\omega{}^0)^2$ (i.e., the eigenvalues of $M_{\text{mech}}{}^2$) satisfy

$$(m_\phi{}^0)^2>9m_\pi{}^2 \quad\text{and}\quad (m_\omega{}^0)^2>9m_\pi{}^2. \quad (5.33)$$

The above inequalities can be most easily established by choosing the canonical form $K_0=1$ and $M_0{}^2=M_{\text{mech}}{}^2$ diagonal.

We now discuss the question of a convenient specification of normalization and orientation conditions for the renormalized fields. It is sometimes convenient to specify normalization and orientation separately. We note that the renormalization matrix S is a real nonsingular matrix and that any arbitrary real nonsingular matrix can always be uniquely decomposed in the form

$$S=T_S{}^{-1}R, \quad (5.34)$$

where the matrix R is diagonal and positive,

$$R=\begin{pmatrix}R_\phi & 0\\ 0 & R_\omega\end{pmatrix}, \quad (5.35)$$

and

$$T_S=\begin{pmatrix}\cos\alpha_1 & -\sin\alpha_1\\ \sin\alpha_2 & \cos\alpha_2\end{pmatrix}, \quad (5.36)$$

with α_1, α_2 real. We shall refer to R as the normalization matrix and T_S as the orientation matrix of S. As an application of the above separation, we note that the renormalized current $J_\mu(x)$, defined by Eq. (5.18), is independent of the choice of the normalization matrix R. This can be verified by using Eq. (5.24) and noting that the only normalization-orientation noninvariant term in the right-hand side of (5.24) is

$$g^{-1}(\partial \mathcal{G}_{\mu\nu}/\partial x_\mu)=g_0^{-1}M_0{}^2SM^{-2}S^{-1}(\partial \mathcal{G}_{\mu\nu}{}^0/\partial x_\mu),$$

which is independent of the normalization matrix R since M^2 is diagonal by definition.

In the hypothetical case of a stable ϕ and ω meson, a convenient orientation may be defined by the requirement

$$\langle \text{vac}|\phi_\mu|\omega\rangle=\langle \text{vac}|\omega_\mu|\phi\rangle=0, \tag{5.37}$$

where $|\phi\rangle$ and $|\omega\rangle$ are the "physical" ϕ-meson and ω-meson states in the hypothetical case. Thus, the angles α_1 and α_2 in the orientation matrix T_S are given by

$$\tan\alpha_1=\langle \text{vac}|\phi_\mu{}^0|\omega\rangle/\langle \text{vac}|\omega_\mu{}^0|\omega\rangle,$$

and (5.38)

$$\tan\alpha_2=-\langle \text{vac}|\omega_\mu{}^0|\phi\rangle/\langle \text{vac}|\phi_\mu{}^0|\phi\rangle;$$

Equation (5.37) and the equation of motion (5.18) then imply

$$\langle \text{vac}|J_\mu|\phi\rangle=\langle \text{vac}|J_\mu|\omega\rangle=0, \tag{5.39}$$

and, therefore, the matrix elements $\langle A|J_\mu(0)|B\rangle$ are free of poles at $q^2=(p_A-p_B)^2=-m_\phi{}^2$ or $-m_\omega{}^2$ for arbitrary hadronic states A and B. The converse is also true; i.e., the requirement that $\langle A|J_\mu(0)|B\rangle$ be free of ϕ-ω poles implies Eq. (5.39) and, consequently, Eq. (5.37). The normalization matrix R can be fixed by specifying values for $\langle \text{vac}|\boldsymbol{\phi}|\phi\rangle$ and $\langle \text{vac}|\boldsymbol{\omega}|\omega\rangle$, the conventional choice, analogous to $Z=Z_1$ in Eq. (4.19), being

$$(2m_\phi)^{1/2}\mathbf{s}\cdot\langle \text{vac}|\boldsymbol{\phi}|\phi\rangle=(2m_\omega)^{1/2}\mathbf{s}\cdot\langle \text{vac}|\boldsymbol{\omega}|\omega\rangle=1, \tag{5.40}$$

where $|\phi\rangle$ and $|\omega\rangle$ refer to the ϕ-meson and ω-meson states in their respective rest systems and $\mathbf{s}$ is the polarization vector ($\mathbf{s}^2=1$).

Next, we turn to the realistic case of unstable ϕ and ω; it is convenient to discuss the normalization and orientation conventions with reference to the propagator. The unrenormalized propagator $\mathfrak{D}_{\mu\nu}{}^0(q)$ is related to $\sigma_{\phi\omega}{}^0(a)$, defined in Eq. (5.30), by

$$\mathfrak{D}_{\mu\nu}{}^0(q)=\int \frac{\delta_{\mu\nu}+a^{-1}q_\mu q_\nu}{q^2+a-i\epsilon}\sigma_{\phi\omega}{}^0(a)da. \tag{5.41}$$

Its inverse can be written as

$$(\mathfrak{D}_{\mu\nu}{}^0)^{-1}=M_0{}^2\delta_{\mu\nu}+(q^2\delta_{\mu\nu}-q_\mu q_\nu)\Pi_0(q^2), \tag{5.42}$$

where $\Pi_0(q^2)$ is a (2×2) matrix related to $\sigma_{\phi\omega}{}^0(a)$ by

$$\Pi_0(q^2)=\left[\int \frac{\sigma_{\phi\omega}{}^0(a)da}{q^2+a-i\epsilon}\right]^{-1} \times\left[\int \frac{\sigma_{\phi\omega}{}^0(a)}{a(q^2+a-i\epsilon)}da\right]M_0{}^2, \tag{5.43}$$

which, in view of Eq. (5.32), is also equal to

$$\Pi_0(q^2)=q^{-2}\left\{\left[\int \frac{\sigma_{\phi\omega}{}^0(a)da}{q^2+a-i\epsilon}\right]^{-1}-M_0{}^2\right\}. \tag{5.44}$$

From (5.43), we find that $\Pi_0(q^2)$ at $q^2=0$ is given by

$$\Pi_0(0)=M_0{}^{-2}\int a^{-2}\sigma_{\phi\omega}{}^0(a)da\, M_0{}^2,$$

and from (5.44) it follows that $\Pi_0(q^2)$, like $\sigma_{\phi\omega}{}^0(a)$, is a symmetric matrix.

The observed masses of the ϕ and ω mesons are determined by the two roots $q^2=-m_\phi{}^2$ and $q^2=-m_\omega{}^2$ of the equation

$$\det|M_0{}^2+q^2\,\text{Re}\Pi_0(q^2)|=0, \tag{5.45}$$

where $\text{Re}\Pi_0(q^2)$ denotes the real part of $\Pi_0(q^2)$.

At $q^2=-m_\omega{}^2$ and $q^2=-m_\phi{}^2$, the vanishing of the determinant requires that the matrix $M_0{}^2+q^2\Pi_0(q^2)$ must be, respectively, of the forms

$$M_0{}^2-m_\omega{}^2\,\text{Re}\Pi_0(-m_\omega{}^2)=N_\omega{}^2u(\omega)\tilde{u}(\omega),$$

and (5.46)

$$M_0{}^2-m_\phi{}^2\,\text{Re}\Pi_0(-m_\phi{}^2)=-N_\phi{}^2u(\phi)\tilde{u}(\phi),$$

where N_ω and N_ϕ are real numbers, and $u(\omega)$ and $u(\phi)$ are two real column [i.e., (2×1)] matrices, both normalized to unity,

$$\tilde{u}(\omega)u(\omega)=\tilde{u}(\phi)u(\phi)=1. \tag{5.47}$$

The difference in signs in the two equations in (5.46) corresponds to the assertion that the diagonal elements of the left-hand sides of these two equations are, respectively, nonnegative and nonpositive. This is rigorously true in the case of the pole approximation (which will be discussed in detail later), since $(q^2+m_\phi{}^2)>0$ at $q^2=-m_\omega{}^2$ but $(q^2+m_\omega{}^2)<0$ at $q^2=-m_\phi{}^2$. We regard these sign assignments as highly probable in the actual case. The treatment which follows can obviously be tailored to fit whatever signs actually occur.

A general orientation convention [that reduces to Eq. (5.37) in the hypothetical case of stable ϕ and ω mesons] can be obtained by choosing the orientation matrix T_S, defined by Eqs. (5.34) and (5.36), as

$$T_S=\begin{pmatrix} u_1(\omega) & u_2(\omega) \\ u_1(\phi) & u_2(\phi)\end{pmatrix}, \tag{5.48}$$

where $u_1(i)$, $u_2(i)$ are the two matrix elements of the (2×1) matrix $u(i)$, and $i=\phi$ or ω.

Let $\mathfrak{D}_{\mu\nu}{}^{-1}(q)$ be the inverse of the renormalized propagator, related to the inverse of the unrenormalized propagator by

$$\mathfrak{D}_{\mu\nu}{}^{-1}(q)=\tilde{S}[\mathfrak{D}_{\mu\nu}{}^0(q)]^{-1}S. \tag{5.49}$$

By using Eqs. (5.46) to (5.48), we find that, at the two roots $q^2=-m_\phi^2$ and $q^2=-m_\omega^2$ of Eq. (5.45),

$$\tilde{S}[M_0^2+q^2\,\mathrm{Re}\Pi_0(q^2)]S=\begin{bmatrix} \dfrac{R_\phi^2N_\omega^2}{m_\phi^2-m_\omega^2}(q^2+m_\phi^2) & 0 \\ 0 & \dfrac{R_\omega^2N_\phi^2}{m_\phi^2-m_\omega^2}(q^2+m_\omega^2)\end{bmatrix}. \tag{5.50}$$

Correspondingly, the real part of the renormalized propagator becomes

$$\mathrm{Re}[\mathfrak{D}_{\mu\nu}^{-1}(q)]=\begin{bmatrix} \dfrac{R_\phi^2N_\omega^2}{m_\phi^2-m_\omega^2}(q^2+m_\phi^2) & 0 \\ 0 & \dfrac{R_\omega^2N_\phi^2}{m_\phi^2-m_\omega^2}(q^2+m_\omega^2)\end{bmatrix}\delta_{\mu\nu}+\cdots, \tag{5.51}$$

at the two roots $q^2=-m_\phi^2$ and $-m_\omega^2$ where $\cdots$ denotes the part of $\mathfrak{D}_{\mu\nu}^{-1}$ that is proportional to $q_\mu q_\nu$. [In the hypothetical case of the stable ϕ and ω mesons, Eq. (5.51) implies that, independently of the constants N_ϕ and N_ω the orientation condition, Eq. (5.37), is satisfied.]

So far as the renormalization problem is concerned, the only requirement on the renormalization matrix S is that the renormalized propagator

$$\mathfrak{D}_{\mu\nu}(q)=S^{-1}\mathfrak{D}_{\mu\nu}^{0}(q)\tilde{S}^{-1} \tag{5.52}$$

should be free from divergence difficulties. Thus, just as for the renormalization constant Z in the ρ-meson case, R_ϕ, or R_ω, is determined only up to an arbitrary finite multiplicative factor; all different choices of such finite multiplicative factors clearly lead to the same physical results. As we shall see below, a particularly convenient choice of R_ϕ and R_ω is to set simply

$$R_\phi^2N_\omega^2=R_\omega^2N_\phi^2=(m_\phi^2-m_\omega^2); \tag{5.53}$$

consequently, Eq. (5.51) becomes

$$\mathrm{Re}[\mathfrak{D}_{\mu\nu}^{-1}(q)]=\begin{pmatrix} q^2+m_\phi^2 & 0 \\ 0 & q^2+m_\omega^2\end{pmatrix}\delta_{\mu\nu}+\cdots, \tag{5.54}$$

at the two roots $q^2=-m_\phi^2$ and $-m_\omega^2$.

For practical applications, it is reasonable to assume that in the resonance region from $q^2\cong-m_\omega^2$ to $q^2\cong-m_\phi^2$ the propagator is dominated by the two poles; i.e., only these two-pole contributions are included in $\mathfrak{D}_{\mu\nu}(q)$. [With this approximation the normalization choice (5.53) becomes, for the hypothetical case of stable ϕ-ω mesons, the same as the normalization condition given by Eq. (5.40)]. Thus, Eq. (5.54) should hold for the entire resonance region $m_\omega^2\lesssim-q^2\lesssim m_\phi^2$ and, in particular, the transformation matrix S satisfies

$$M^2=\tilde{S}M_0^2S=\begin{pmatrix} m_\phi^2 & 0 \\ 0 & m_\omega^2\end{pmatrix}, \tag{5.55}$$

and

$$\tilde{S}\,\mathrm{Re}\Pi_0(-m_\phi^2)S=\tilde{S}\,\mathrm{Re}\Pi_0(-m_\omega^2)S=\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}. \tag{5.56}$$

We note that Eqs. (5.55) and (5.56) can also be derived by making a weaker approximation in which one neglects only the difference between the real part of $\Pi_0(q^2)$ at $q^2=-m_\omega$ and that at $q^2=-m_\phi^2$; i.e., we assume

$$\mathrm{Re}\Pi_0(-m_\phi^2)=\mathrm{Re}\Pi_0(-m_\omega^2). \tag{5.57}$$

[The imaginary parts of $\Pi_0(q^2)$ at $q^2=-m_\phi^2$ and $-m_\omega^2$ are clearly very different.] From Eq. (5.56), it follows that the pole approximation implies this weaker condition (5.57). Equation (5.57) is, in fact, equivalent to the pole approximation, for a hypothetical stable ϕ-ω system, but does not require the full extent of the pole approximation for the realistic case.

Finally, we wish to relate the renormalization matrix S with the matrices g_0 and g occurring in Eqs. (5.3) and (5.18), respectively. If in the resonance region, $m_\phi^2\gtrsim-q^2\gtrsim m_\omega^2$, the pole dominance is a good approximation, then by combining (5.22) and (5.55) one obtains

$$gg_0^{-1}=\tilde{S}. \tag{5.58}$$

If, in addition, the aligned form is chosen for the unrenormalized fields, then according to Eq. (5.8) g_0 is diagonal and, using (5.19) and (5.21), we have

$$S^{-1}=\tilde{T}\begin{pmatrix} g_Y^0/g_Y & 0 \\ 0 & g_N^0/g_N\end{pmatrix}. \tag{5.59}$$

We recall that the matrix T depends on two angles θ_Y and θ_N which are introduced in Sec. II [Eq. (2.18)] to relate the currents $\mathcal{J}_\mu^\phi$ and $\mathcal{J}_\mu^\omega$ to g_YY_μ and g_NN_μ; Eqs. (5.59) and (5.13) show that the same matrix also transforms the unrenormalized field ψ_μ^0, after multiplication by a diagonal matrix $g_0g_D^{-1}$, into the renormalized field ψ_μ. Note, however, that T is not equal to the T_S defined by Eqs. (5.34) and (5.36); these two matrices are related to each other by

$$R^{-1}T_S=\tilde{T}g_0g_D^{-1}, \tag{5.60}$$

where R, g_0, g_D are all diagonal matrices.

3. Special Models

Hitherto, our discussions have been completely general. In this section, we will discuss two specific models. As we shall see, the angles θ_Y and θ_N become related, though in different ways, in each of these two models. For the definitions of these models it is convenient to adopt the aligned form for the unrenormalized fields. Accordingly g_0 has the diagonal form (5.8) and K_0 the form (5.9). (In the limit of SU_3 symmetry, the aligned form also implies $K_0=1$ and M_0^2 diagonal. Therefore, in this limit the aligned form becomes the same as the canonical form.)

Current-Mixing Model

We consider first the special model in which the matrix M_0^2 is assumed to be diagonal:

$$M_0^2=\begin{pmatrix}(\mu_1^0)^2 & 0\\ 0 & (\mu_2^0)^2\end{pmatrix}. \tag{5.61}$$

The matrix K_0 may, or may not, be diagonal. If K_0 is also diagonal, then according to Eq. (5.9), K_0 is a unit matrix, and $(\mu_1^0)^2$, $(\mu_2^0)^2$ become, respectively, the (unrenormalized mass)^2 of the ϕ meson and the ω meson.

The SU_3 symmetry is assume *not* to be valid; as a result, the matrix $\Pi_0(q^2)$ is not diagonal. A likely mechanism is that in the absence of the SU_3 symmetry, the vacuum expectation value $\langle\mathrm{vac}|Y_\mu^0(x)N_\nu^0(0)|\mathrm{vac}\rangle$, for example, is no longer zero. Such mixed-current matrix elements can give rise to the off-diagonal matrix element of $\Pi_0(q^2)$, but *not* of M_0^2. For convenience, we call any model, in which M_0^2 is diagonal but $\Pi_0(q^2)$ is not diagonal, the "current-mixing" model. For our analysis, the precise mechanism of the SU_3 symmetry-breaking interaction is, however, immaterial.

Assuming the validity of (5.55), it is easy to verify that Eqs. (5.55) and (5.61) imply that S has the general form

$$S=\begin{pmatrix}(\mu_1^0)^{-1}m_\phi\cos\theta & -(\mu_1^0)^{-1}m_\omega\sin\theta\\ (\mu_2^0)^{-1}m_\phi\sin\theta & (\mu_2^0)^{-1}m_\omega\cos\theta\end{pmatrix}, \tag{5.62}$$

where the angle θ is real and its value depends on the matrix $\mathrm{Re}\Pi_0(-m_\phi^2)=\mathrm{Re}\Pi_0(-m_\omega^2)$. Upon comparing (5.62) with (5.59), we find

$$\left(\frac{g_Y^0}{g_Y}\right)^2=\left(\frac{\mu_1^0}{m_\phi}\right)^2\left(\cos^2\theta+\frac{m_\phi^2}{m_\omega^2}\sin^2\theta\right), \tag{5.63}$$

$$\left(\frac{g_N^0}{g_N}\right)^2=\left(\frac{\mu_2^0}{m_\omega}\right)^2\left(\cos^2\theta+\frac{m_\omega^2}{m_\phi^2}\sin^2\theta\right), \tag{5.64}$$

$$\tan\theta_Y=(m_\phi/m_\omega)\tan\theta, \tag{5.65}$$

$$\tan\theta_N=(m_\omega/m_\phi)\tan\theta, \tag{5.66}$$

and, consequently

$$m_\omega^2\tan\theta_Y=m_\phi^2\tan\theta_N. \tag{5.67}$$

It is important to note that these results hold to *all* orders in the SU_3 symmetry-breaking interaction, provided that the "current-mixing" model is valid.

An estimation of the values of θ_Y and θ_N can be made if one assumes that the SU_3 symmetry-breaking interaction transforms like the $I=0$ member of an octet under the SU_3 transformations. To simplify our discussions, we will further approximate $\mathrm{Re}\Pi_0(q^2)$ by a constant matrix, as is the case in the pole approximation.

In the limit of SU_3 symmetry, the matrix $\Pi_0(q^2)$ must be a diagonal matrix. Let us denote its real part by

$$\lim_{SU_3\,\mathrm{sym}}\mathrm{Re}\Pi_0=\begin{pmatrix}(\lambda_1^0)^2 & 0\\ 0 & (\lambda_2^0)^2\end{pmatrix}. \tag{5.68}$$

The renormalized masses of the octet and singlet vector mesons in the SU_3 symmetry limit are, respectively, given by

$$m_{\mathrm{octet}}=(\mu_1^0/\lambda_1^0)$$
and
$$m_{\mathrm{singlet}}=(\mu_2^0/\lambda_2^0). \tag{5.69}$$

The inclusion of the SU_3 symmetry-breaking interaction in the current-mixing model does not change M_0^2, but it alters $\mathrm{Re}\Pi_0$ from (5.68) to

$$\mathrm{Re}\Pi_0=\begin{pmatrix}(\lambda_1)^2 & \eta\\ \eta & (\lambda_2)^2\end{pmatrix}. \tag{5.70}$$

If the SU_3 symmetry-breaking interaction is assumed to transform like the $I=0$ member of an octet, then to the first order of such a symmetry-breaking interaction one must have

$$\lambda_2=\lambda_2^0. \tag{5.71}$$

Adopting the notations of Coleman and Schnitzer,[5] we define ϵ and β by

$$\lambda_1=\lambda_1^0(1+2\epsilon)^{1/2}$$
and
$$(\lambda_1^0\lambda_2^0)^{-1}\eta=\beta. \tag{5.72}$$

To first order in the SU_3 symmetry-breaking interaction, the observed masses of K^*, ρ, ϕ, and ω are related to the zeroth-order renormalized masses m_{octet}, m_{singlet} and the two parameters ϵ and β by

$$m_{K^*}=(1+\epsilon)^{-1/2}m_{\mathrm{octet}}, \tag{5.73}$$

$$m_\rho=(1-2\epsilon)^{-1/2}m_{\mathrm{octet}}, \tag{5.74}$$

$$m_\phi^{-2}+m_\omega^{-2}=m_{\mathrm{singlet}}^{-2}+(1+2\epsilon)m_{\mathrm{octet}}^{-2}, \tag{5.75}$$

and

$$(m_\phi m_\omega)^{-2}(m_{\mathrm{octet}}m_{\mathrm{singlet}})^2=1+2\epsilon-\beta^2. \tag{5.76}$$

Thus, one finds

$$m_{\mathrm{octet}}=839\text{ MeV},\quad m_{\mathrm{singlet}}=817\text{ MeV},$$
$$\epsilon=-0.115.$$

and

$$\beta=-0.18. \tag{5.77}$$

From Eqs. (5.75) and (5.76), it is clear that $\beta=+0.18$ is also a solution. However, since under the transformation $\phi_\mu{}^0 \to +\phi_\mu{}^0$, $\omega_\mu{}^0 \to -\omega_\mu{}^0$, β transforms $\to -\beta$, we may, without any loss of generality, choose β to be negative (so that $\tan\theta$ becomes positive). The angle θ in Eq. (5.62) is related to these parameters by

$$\tan\theta=(\beta m_\omega{}^2 m_{\text{singlet}})^{-1} m_{\text{octet}}(m_\omega{}^2-m_{\text{singlet}}{}^2), \tag{5.78}$$

which together with Eqs. (5.65) and (5.66) yields

$$\theta\cong 26°,$$
$$\theta_Y\cong 33°,$$

and

$$\theta_N\cong 21°. \tag{5.79}$$

For convenience, we have chosen θ, and also θ_Y and θ_N, to be in the first quadrant, instead of the third quadrant. (Under $\phi_\mu \to -\phi_\mu$, $\omega_\mu \to -\omega_\mu$, but $\phi_\mu{}^0 \to +\phi_\mu{}^0$ and $\omega_\mu{}^0 \to +\omega_\mu{}^0$, one finds $\theta \to \theta+180°$, $\theta_Y \to \theta_Y+180°$ and $\theta_N \to \theta_N+180°$.)

Mass-Mixing Model

Next, we consider a different model in which the ϕ-ω mixing is assumed to be due entirely to the off-diagonal matrix elements of $M_0{}^2$. In general, if the matrix $M_0{}^2$ is nondiagonal, then $\Pi_0(q^2)$ would also contain nonzero off-diagonal matrix elements. However, in the mass-mixing model, we make the *ad hoc* assumption that

$$\operatorname{Re}\Pi_0(-m_\phi{}^2)=\operatorname{Re}\Pi_0(-m_\omega{}^2)=\begin{pmatrix}(\lambda_1)^2 & 0\\ 0 & (\lambda_2)^2\end{pmatrix}, \tag{5.80}$$

but $M_0{}^2$ can be any arbitrary (2×2) real symmetric matrix. By using Eqs. (5.55) and (5.56), one finds that the matrix S is given by

$$S=\begin{pmatrix}\lambda_1{}^{-1} & 0\\ 0 & \lambda_2{}^{-1}\end{pmatrix}U, \tag{5.81}$$

where U is the real orthogonal matrix which diagonalizes

$$\begin{pmatrix}\lambda_1{}^{-1} & 0\\ 0 & \lambda_2{}^{-1}\end{pmatrix}M_0{}^2\begin{pmatrix}\lambda_1{}^{-1} & 0\\ 0 & \lambda_2{}^{-1}\end{pmatrix}. \tag{5.82}$$

By comparing (5.81) with (5.49), one derives in this mass-mixing model

$$\theta_Y=\theta_N, \tag{5.83}$$

$$(g_Y{}^0/g_Y)=\lambda_1, \tag{5.84}$$

and

$$(g_N{}^0/g_N)=\lambda_2. \tag{5.85}$$

The matrix $M_0{}^2$ is real symmetric matrix; therefore, it can be diagonalized by a real orthogonal matrix, say V:

$$M_0{}^2=\tilde{V}\begin{pmatrix}(\mu_1)^2 & 0\\ 0 & (\mu_2)^2\end{pmatrix}V, \tag{5.86}$$

where

$$V=\begin{pmatrix}\cos a & \sin a\\ -\sin a & \cos a\end{pmatrix}, \tag{5.87}$$

and $\mu_1{}^2$, $\mu_2{}^2$ are the two eigenvalues of $M_0{}^2$. In the limit of SU_3 symmetry, $a=0$; therefore, to the first order in the SU_3 symmetry-breaking interaction, we can neglect a^2, and (5.82) becomes

$$\begin{pmatrix}(\mu_1/\lambda_1)^2 & \xi\\ \xi & (\mu_2/\lambda_2)^2\end{pmatrix}, \tag{5.88}$$

where

$$\xi=(\lambda_1\lambda_2)^{-1}(\mu_1{}^2-\mu_2{}^2)\sin a\cos a\cong(\lambda_1\lambda_2)^{-1}(\mu_1{}^2-\mu_2{}^2)a.$$

The values of (μ_1/λ_1), (μ_2/λ_2), and ξ can be determined from the known vector-meson masses, if we make the following further dynamical assumptions:

(i) The eigenvalues of $M_0{}^2$ are *not* changed by the introduction of the SU_3 symmetry-breaking interaction; i.e., in the limit of SU_3 symmetry, one has from Eq. (5.86), $V=1$ and

$$\lim_{SU_3\,\text{sym}} M_0{}^2=\begin{pmatrix}(\mu_1)^2 & 0\\ 0 & (\mu_2)^2\end{pmatrix}. \tag{5.89}$$

This assumption appears natural if one imagines the unrenormalized theory to be in the canonical form, instead of the aligned form which is used here. In the canonical form this theory is characterized by a nondiagonal g_0' given by $g_0'=Vg_0$.

In addition, we approximate the matrix $\operatorname{Re}\Pi_0(q^2)$ by a constant matrix, and denote its limiting form by

$$\lim_{SU_3\,\text{sym}} \operatorname{Re}\Pi_0(q^2)=\begin{pmatrix}(\lambda_1{}^0)^2 & 0\\ 0 & (\lambda_2{}^0)^2\end{pmatrix}. \tag{5.90}$$

The renormalized octet and singlet masses in the SU_3 symmetry limit are, therefore, given respectively by

$$m_{\text{octet}}=(\mu_1/\lambda_1{}^0),$$

and (5.91)

$$m_{\text{singlet}}=(\mu_2/\lambda_2{}^0).$$

(ii) The SU_2 symmetry-breaking interaction transforms like the isoscalar member of an octet. Thus, to first order in the SU_3 symmetry-breaking interaction,

$$\lambda_2{}^0=\lambda_2. \tag{5.92}$$

The parameter λ_1 is assumed to be different from its SU_3 symmetry-limiting value $\lambda_1{}^0$. We write

$$\lambda_1=\lambda_1{}^0(1+2\epsilon)^{1/2}. \tag{5.93}$$

The SU_3 symmetry-breaking interaction mixes ϕ-ω through the angle a, or the parameter ξ in (5.88), but the difference between the observed vector-meson masses and their zeroth-order masses $m_{\rm octet}$ and $m_{\rm singlet}$ depend on both ξ and ϵ. We have, as in Eqs. (5.73) and (5.74),

$$m_{K^*}=(1+\epsilon)^{-1/2}m_{\rm octet}\,, \qquad (5.94)$$

$$m_\rho=(1-2\epsilon)^{-1/2}m_{\rm octet}\,, \qquad (5.95)$$

but, instead of Eqs. (5.75) and (5.76),

$$m_\phi^2+m_\omega^2=(1+2\epsilon)^{-1}m_{\rm octet}^2+m_{\rm singlet}^2\,,$$

and (5.96)

$$m_\phi^2 m_\omega^2=(1+2\epsilon)^{-1}(m_{\rm octet}m_{\rm singlet})^2-\xi^2.$$

From the known masses m_{K^*}, m_ρ m_ϕ, and m_ω, we determine

$$m_{\rm octet}=839\text{ MeV}\,,$$

$$m_{\rm singlet}=859\text{ MeV}\,,$$

$$\epsilon=-0.115\,,$$

and

$$\xi=1.93\times10^5\,(\text{MeV})^2. \qquad (5.97)$$

The angles θ_Y and θ_N are given by

$$\tan\theta_Y=\tan\theta_N=\xi^{-1}(m_{\rm singlet}^2-m_\omega^2)\,, \qquad (5.98)$$

and therefore

$$\theta_Y=\theta_N\cong32°. \qquad (5.99)$$

For convenience, we have chosen ξ positive and $\theta_Y=\theta_N$ in the first quadrant.

Mass-Mixing Model (A Variation)

The mass-mixing model implies $\theta_Y=\theta_N$, but the above estimation $\theta_Y=\theta_N\cong32°$ is based on further *ad hoc* assumptions (i) and (ii). In this section, we will give a variation of the same mass-mixing model in which all the above formulas (5.80)–(5.88) are assumed to remain applicable. The only change is that, instead of the previous additional assumption (i), one assumes[4]:

(i)′. The matrix $\text{Re}\Pi_0(q^2)$ is approximated by a constant matrix, and it is, for some unspecified dynamical reason, *not* changed by the SU_3 symmetry-breaking interaction. Thus, one may use Eq. (5.80), but set

$$\lambda_1=\lambda_1^0\,,$$

and (5.100)

$$\lambda_2=\lambda_2^0.$$

In place of Eq. (5.89), one may write

$$\lim_{SU_3\,\rm sym} M_0^2=\begin{pmatrix}(\mu_1^0)^2 & 0\\ 0 & (\mu_2^0)^2\end{pmatrix}. \qquad (5.101)$$

Instead of by Eq. (5.91), the renormalized octet and singlet masses in the SU_3 symmetry limit are given by

$$m_{\rm octet}=(\mu_1^0/\lambda_1^0)$$

and (5.102)

$$m_{\rm singlet}=(\mu_2^0/\lambda_2^0).$$

We make the same assumption (ii) as in the previous case. By using Eqs. (5.86), (5.87), and neglecting $O(a^2)$, we find, in order to conform to the assumption (ii),

$$\mu_2=\mu_2^0. \qquad (5.103)$$

The masses of K^*, ρ, ϕ, and ω are related to their SU_3-symmetry limits $m_{\rm octet}$, $m_{\rm singlet}$, and the ratio

$$(\mu_1/\mu_1^0)=1+2\delta\,,$$

by

$$m_{K^*}^2=m_{\rm octet}^2(1+\delta)\,, \qquad (5.104)$$

$$m_\rho^2=m_{\rm octet}^2(1-2\delta)\,, \qquad (5.105)$$

$$m_\phi^2+m_\omega^2=(1+2\delta)m_{\rm octet}^2+m_{\rm singlet}^2\,, \qquad (5.106)$$

and

$$m_\phi^2 m_\omega^2=(1+2\delta)(m_{\rm octet}m_{\rm singlet})^2-\xi^2\,, \qquad (5.107)$$

where ξ is given by Eq. (5.88). Equation (5.98) remains applicable. By using the known vector-meson masses, one finds, in place of Eqs. (5.97) and (5.99),

$$m_{\rm octet}=850\text{ MeV}\,,$$

$$m_{\rm singlet}=884\text{ MeV}\,,$$

$$\delta=0.103\,,$$

$$\xi=2.07\times10^5\,(\text{MeV})^2\,,$$

and

$$\theta_Y=\theta_N\cong39°. \qquad (5.108)$$

We note that Eqs. (5.104)–(5.107) reduce to Eqs. (5.94)–(5.96) if we set $\epsilon=-\delta$ and neglect $O(\epsilon^2)$. The difference between the two estimations [(5.99) and (5.108)] of $\theta_Y=\theta_N$ in the mass-mixing model lies, therefore, only in the higher-order terms of the SU_3 symmetry-breaking interaction.

VI. PHOTON PROPAGATOR

The hadronic contribution to the electromagnetic current may influence purely leptonic processes through its effect on the photon propagator. Indeed, discussions of contributions arising from this source to the anomalous magnetic moment of leptons[13] and to electron-positron scattering[14] have already appeared in the literature. We wish, however, to exhibit here the fact that our considerations imply an exact (in the strong interactions) connection between the order e^2 hadronic contributions to the photon propagator and the vector-meson propagators. Such a connection can be derived directly by considering the set S_γ of all e^2 order Feynman graphs for the photon propagator (to all orders in the strong interaction, but neglecting leptonic contribu-

[13] L. Durand III, Phys. Rev. **128**, 441 (1962); C. Bouchiat and L. Michel, J. Phys. Radium **22**, 121 (1961).
[14] R. Gatto, Nuovo Cimento **28**, 658 (1963).

tions) and the set S_{meson} of all Feynman graphs for the vector-meson propagators (to all orders in the strong interactor, but only zeroth order in e). By using the strong interaction Lagrangian (3.7), (3.8), (5.15), and (5.16), and the gauge-invariant electromagnetic interaction Lagrangian (2.7) and (2.20) [or, more simply, the alternative identities (1.3b) and (5.28)], it can be readily seen that each of the graphs in the set S_γ corresponds to a subset of graphs in the set S_{meson}. It is convenient to include also the free photon propagator in S_γ. One finds, then, that there exists a homomorphism between the set S_γ and the set S_{meson}. While it is straightforward to convert this homomorphism into algebraic relations, and to derive the results that are given by Eqs. (6.9) and (6.15) below, the detailed description of such a graphic procedure turns out to be somewhat unnecessarily complicated. Thus, we shall give, instead, a formal analytic proof in the following.

For convenience, we begin with the general expression first used by Källén,[15] and write for the vacuum expectation value of the photon commutator

$$\hat{K}_{\mu\nu}{}^{\gamma}(x)=\langle 0|[A_\mu(x),A_\nu(0)]|0\rangle$$
$$=-i\delta_{\mu\nu}\Delta_0(x)-L\frac{\partial^2}{\partial x_\mu\partial x_\nu}\Delta_0(x)+\int d^4y d^4z$$
$$\times\langle\text{vac}|[\mathcal{J}_\mu{}^\gamma(y),\mathcal{J}_\nu{}^\gamma(z)]|\text{vac}\rangle D_R(x-y)D_R(-z)\,,\quad(6.1)$$

where $\mathcal{J}_\mu{}^\gamma$ is the total electromagnetic current defined by

$$\partial F_{\mu\nu}/\partial x_\mu=-\mathcal{J}_\nu{}^\gamma\,,\qquad(6.2)$$

$\Delta_0(x)$ is the same $\Delta_a(x)$ function given by Eq. (4.1) with $a=0$, and D_R is the retarded Green's function satisfying

$$\Box^2 D_R(x)=-\delta^4(x)\,.\qquad(6.3)$$

The constant L is to be chosen so as to guarantee $\hat{K}_{\mu\nu}{}^\gamma=0$ for x spacelike. To order e^2 we may split $\hat{K}_{\mu\nu}{}^\gamma$ into a zeroth order plus a lepton part and a hadron part; thus

$$\hat{K}_{\mu\nu}{}^\gamma(x)=-i\delta_{\mu\nu}\Delta_0(x)+\hat{K}_{\mu\nu}{}^{\gamma l}(x)+\hat{K}_{\mu\nu}{}^{\gamma h}(x)\,,\quad(6.4)$$

with

$$\hat{K}_{\mu\nu}{}^{\gamma h}=e^2\int d^4y d^4z\langle\text{vac}|[J_\mu{}^\gamma(y),J_\nu{}^\gamma(z)]|\text{vac}\rangle$$
$$\times D_R(x-y)D_R(-z)-L_h\frac{\partial^2}{\partial x_\mu\partial x_\nu}\Delta_0(x)\,,\quad(6.5)$$

and $J_\nu{}^\gamma$ denotes the hadronic electromagnetic current. For simplicity of notation we include only the ρ contribution to $J_\nu{}^\gamma$ in the following discussion. The ϕ and ω contributions will be added to the final formula. Thus, applying Eq. (2.9), we write for the ρ contribution to $\hat{K}_{\mu\nu}{}^{\gamma h}$

$$\hat{K}_{\mu\nu}{}^{\gamma\rho}=\frac{e^2m_\rho{}^4}{g_\rho{}^2}\int d^4yd^4z\langle\text{vac}|[\rho_\mu(y),\rho_\nu(z)]|\text{vac}\rangle$$
$$\times D_R(x-y)D_R(-z)-L_\rho\frac{\partial^2}{\partial x_\mu\partial x_\nu}\Delta_0(x)$$
$$=\frac{e^2m_\rho{}^4}{g_\rho{}^2}\int d^4yd^4z\left[\delta_{\mu\nu}-a^{-1}\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right]\Delta_a(y-z)\sigma_\rho(a)$$
$$\times D_R(x-y)D_R(-z)da-L_\rho\frac{\partial^2}{\partial x_\mu\partial x_\nu}\Delta_0(x)\,,\quad(6.6)$$

on account of Eq. (4.1).

Taking Fourier transforms, we find

$$K_{\mu\nu}{}^{\gamma\rho}(q)=i\int\hat{K}_{\mu\nu}{}^{\gamma\rho}(x)e^{-iq_\lambda x_\lambda}d^4x$$
$$=2\pi i\frac{e^2m_\rho{}^4}{g_\rho{}^2}\int_{4m_\pi{}^2}^{\infty}\frac{\sigma_\rho(a)}{a^2}da\,\epsilon(q_0)$$
$$\times\left[\delta_{\mu\nu}\delta(q^2+a)+\frac{q_\mu q_\nu}{a}(\delta(q^2+a)-\delta(q^2))\right],\quad(6.7)$$

where $\epsilon(q_0)=q_0/|q_0|$. The term in $\delta(q^2)$ is the contribution of the L_ρ term. The ρ contribution to the photon propagator

$$D_{\mu\nu}{}^\gamma(q)=q^{-2}\delta_{\mu\nu}+D_{\mu\nu}{}^{\gamma h}(q)+D_{\mu\nu}{}^{\gamma l}(q)\qquad(6.8)$$

is obtained from (6.7) via the correspondence[16]

$$2\pi i\epsilon(q_0)\delta(q^2+a)\rightarrow 1/(q^2+a-i\epsilon)\,,$$

yielding the ρ contribution to $D_{\mu\nu}{}^{\gamma h}(q)$:

$$D_{\mu\nu}{}^{\gamma\rho}(q)=\frac{e^2m_\rho{}^4}{g_\rho{}^2}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\int da\frac{\sigma_\rho(a)}{a^2}\frac{1}{q^2+a-i\epsilon}\,.\quad(6.9)$$

It is of interest to note that

$$\lim_{q^2\to\infty}D_{\mu\nu}{}^{\gamma\rho}=\frac{e^2m_\rho{}^4}{g_\rho{}^2}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\frac{1}{q^2}\int da\frac{\sigma_\rho(a)}{a^2},\quad(6.10)$$

so that the ρ contribution to the order e^2 part of the charge renormalization is given by[16a]

$$\frac{\delta(e_0{}^2)_\rho}{e^2}=\frac{e^2m_\rho{}^4}{g_\rho{}^2}\int_{4m_\pi{}^2}^{\infty}\frac{\sigma_\rho(a)}{a^2}da\,,\qquad(6.11)$$

[15] G. Källén, Helv. Phys. Acta **25**, 417 (1952). The L used in this paper corresponds to M in Källén's notation.

[16] Some additional details and references to some of the relevant literature are given by L. M. Brown and F. Calogero, Phys. Rev. **120**, 653 (1960), Appendix.

[16a] *Note added in proof.* In writing (6.11) we use the ratio

$$\lim_{q^2\to\infty}q^2D^\gamma(q^2)/\lim_{q^2\to 0}q^2D^\gamma(q^2)$$

as the definition of $(e_0/e)^2$ where $D^\gamma(q^2)$ is the coefficient of $\delta_{\mu\nu}$ in $D_{\mu\nu}{}^\gamma$. An alternative definition of $(e_0/e)^2$ is given simply by the limit

$$\lim_{q^2\to 0}q^2[D^\gamma(q^2)]_{\text{unren}}$$

where $[D^\gamma(q^2)]_{\text{unren}}$ is the coefficient of $\delta_{\mu\nu}$ in the unrenormalized photon propagator $[D_{\mu\nu}{}^\gamma(q^2)]_{\text{unren}}$. It is interesting to note that for the present case these two definitions give the same result only if the unrenormalized mass $m_\rho{}^0$ is infinite. For details see T. D. Lee and B. Zumino [Nuovo Cimento (to be published)].

where e is the observed charge, and it is related to the unrenormalized charge e_0 by $e_0^2=e^2+(\delta e_0^2)_\rho+\cdots$. From Eq. (4.6) and $\sigma_\rho(a)\geqq 0$, it follows that

$$0<\frac{\delta(e_0^2)_\rho}{e^2}<\frac{1}{4}\left(\frac{e^2m_\rho^4}{g_\rho^2m_\pi^2}\right)\int_{4m_\pi^2}^\infty\frac{\sigma_\rho(a)}{a}=\frac{1}{4}\frac{Z_0e^2m_\rho^2}{Zg_\rho^2m_\pi^2}. \quad (6.12)$$

We note that, on account of Eq. (3.11), the product Zg_ρ^2 is independent of the choice of the renormalization constant Z. For definiteness, we may adopt the renormalization convention $Z=Z_0$ given by Eq. (4.22). As we shall see in the next section, this choice leads to $(4\pi)^{-1}g_\rho^2\cong 2.3$. Thus, (6.12) becomes

$$\frac{\delta(e_0^2)_\rho}{e^2}<\frac{1}{4}\frac{e^2m_\rho^2}{g_\rho^2m_\pi^2}\cong 2.4\%. \quad (6.13)$$

$D_{\mu\nu}{}^{\gamma\rho}$ can be expressed explicitly in terms of the ρ-propagator $D_{\mu\nu}(q)$. Recalling Eqs. (4.10) and (4.12), we write

$$D_{\mu\nu}(q)=\int\frac{\delta_{\mu\nu}+a^{-1}q_\mu q_\nu}{q^2+a-i\epsilon}\sigma_\rho(a)da=\delta_{\mu\nu}F(q^2)+q_\mu q_\nu G(q^2),$$

and obtain for the ρ contribution to the photon propagator

$$D_{\mu\nu}{}^{\gamma\rho}(q)=(e^2m_\rho^4/g_\rho^2)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)$$
$$\times\left\{\frac{1}{q^2-i\epsilon}[F(q^2)-F(0)-(q^2-i\epsilon)F'(0)]\frac{1}{q^2-i\epsilon}\right\}. \quad (6.14)$$

This expression is, for all physical processes, equivalent to that which would be derived by straightforward application of Feynman rules to the form of the theory given in Appendix B, Eq. (B8). The apparent photon mass squared appearing there, $(e''m_\rho^0/g_\rho^0)^2$, is equal to the term $(e^2m_\rho^4/g_\rho^2)F(0)$ of Eq. (6.14) to lowest order in e^2.

The complete expression for the hadronic contribution to the photon propagator, obtained from Eqs. (5.26) and (6.9), is

$$D_{\mu\nu}{}^{\gamma h}=e^2\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\int\frac{da}{a^2(q^2+a-i\epsilon)}$$
$$\times\{g_\rho^{-2}m_\rho^4\sigma_\rho(a)+\tfrac{1}{4}[g^{-1}M^2\sigma_{\phi\omega}(a)M^2\tilde{g}^{-1}]_{11}\}, \quad (6.15)$$

where g and M^2 are given by Eqs. (5.19) and (5.17), respectively, the subscript 11 denotes the (1,1)th matrix element, and $\sigma_{\phi\omega}(a)$ is the renormalized spectral-function matrix, related to the unrenormalized spectral-function matrix $\sigma_{\phi\omega}{}^0$ of Eq. (5.30) by

$$\sigma_{\phi\omega}(a)=S^{-1}\sigma_{\phi\omega}{}^0\tilde{S}^{-1}, \quad (6.16)$$

and to the renormalized ϕ-ω propagator $\mathfrak{D}_{\mu\nu}$ by

$$\mathfrak{D}_{\mu\nu}(q)=\int_{9m_\pi^2}^\infty\frac{\delta_{\mu\nu}+a^{-1}q_\mu q_\nu}{q^2+a-i\epsilon}\sigma_{\phi\omega}(a)da. \quad (6.17)$$

To order e^2 and neglecting leptonic contributions, we find that the entire hadronic contribution to charge renormalization is finite; the unrenormalized charge e_0 is given by

$$(e_0/e)^2=1+e^2\int a^{-2}da$$
$$\times\{g_\rho^{-2}m_\rho^4\sigma_\rho(a)+\tfrac{1}{4}[g^{-1}M^2\sigma_{\phi\omega}(a)M^2\tilde{g}^{-1}]_{11}\}$$
$$<1+\tfrac{1}{4}(g_\rho m_\pi)^{-1}e^2m_\rho^4\int_{4m_\pi^2}^\infty a^{-1}\sigma_\rho(a)da$$
$$+\frac{1}{36}\left[g^{-1}M^2\int_{9m_\pi^2}^\infty a^{-1}\sigma_{\phi\omega}(a)daM^2\tilde{g}^{-1}\right]_{11}m_\pi^2. \quad (6.18)$$

The value of the above upper limit of $(e_0/e)^2$ can be estimated by using Eqs. (6.13), (5.32), (5.55), and the numerical values found in the next section. We find

$$1<(e_0/e)^2<1+\tfrac{1}{4}e^2m_\pi^{-2}$$
$$\times[g_\rho^{-2}m_\rho^2+\tfrac{1}{9}g_Y^{-2}(\cos^2\theta_Ym_\phi^2+\sin^2\theta_Ym_\omega^2)]$$
$$\cong 1.03. \quad (6.19)$$

The finiteness of the hadronic contribution to charge renormalization arises from the fact that the spectral representation of the photon propagator is a factor a^{-2} more convergent than that of the vector-meson propagator, and corresponds to the circumstance exhibited in Eqs. (2.10) and (2.22) that the matrix elements of the hadronic part of the electromagnetic current are more convergent, by a factor q^2, than the corresponding matrix elements for the vector-meson currents.

VII. APPLICATIONS

In this section, we list the various applications[17] of the above general discussions, and in particular the experimental implications that follow from the proposed identity between the hadronic current operator $J_\mu{}^\gamma$ and the vector-meson fields.

1. $\rho^0\to\pi^++\pi^-$

This decay depends only on the strong interaction. It has already been pointed out in Sec. IV, Eq. (4.28), that all matrix elements of the renormalized current $J_\mu{}^\rho(x)$ vanish at $q^2=-m_\rho^2$; therefore, it is useful to introduce a modified current operator $\hat{J}_\mu{}^\rho(x)$ defined by

[17] The various applications given in this section have almost all been discussed in one form or another in the literature. See Refs. 2, 4, and 21 for a partial list of these references.

Eq. (4.26). By using Eq. (4.29), one finds ($\hbar=c=1$)

$$\text{rate}(\rho^0\to\pi^++\pi^-)=(48\pi m_\rho^2)^{-1}(m_\rho^2-4m_\pi^2)^{3/2}\times[g_\rho\hat{F}_{\pi\pi}{}^\rho(-m_\rho^2)]^2(Z_1/Z_0)\,,\quad(7.1)$$

where $\hat{F}_{\pi\pi}{}^\rho$ is the vertex function (or, the $\pi\pi$-ρ form factor), defined by

$$\langle(\pi^+\pi^-)^{\text{in}}|\hat{\mathbf{J}}^\rho(0)|\text{vac}\rangle=\mathbf{s}(p_\pi/\omega_\pi)\hat{F}_{\pi\pi}{}^\rho(q^2)\,,\quad(7.2)$$

$|(\pi^+\pi^-)^{\text{in}}\rangle$ is the two-pion "incoming" p-wave eigenstate of the strong-interaction Hamiltonian in the center-of-mass system, $\mathbf{s}$ is the polarization vector (chosen to be real), ω_π is the pion energy$=\frac{1}{2}(-q^2)^{1/2}$, and p_π is its corresponding momentum.

If inelastic processes are neglected, then the state $|(\pi^+\pi^-)^{\text{in}}\rangle$ differs from the corresponding stationary state by a multiplicative factor $e^{-i\delta_p}$ where δ_p is the two-pion p-wave phase shift. In this approximation, on account of time-reversal invariance, the phase of the matrix element $\langle(\pi^+\pi^-)^{\text{in}}|\varrho(0)|\text{vac}\rangle$ is δ_p, or $\delta_p+180°$. If, in addition, the nonresonant background is neglected for q^2 near $-m_\rho^2$, the resonance factor $[q^2+m_\rho^2-i\gamma m_\rho]^{-1}$ in the ρ-propagator $D_{\mu\nu}(q^2)$ is proportional to

$$-i(e^{2i\delta_p}-1)=2e^{i\delta_p}\sin\delta_p\,.$$

By using Eq. (4.26), we see that $\langle(\pi^+\pi^-)^{\text{in}}|\hat{\mathbf{J}}^\rho(0)|\text{vac}\rangle$ and, consequently, $\hat{F}_{\pi\pi}{}^\rho(q^2)$ are real at $q^2=\text{real}\cong-m_\rho^2$. Furthermore, in the same region $\hat{F}_{\pi\pi}{}^\rho(q^2)$ is expected to be a relatively slowly varying function of q^2.

In Eq. (7.1), the factor (Z_1/Z_0) is due to our normalization convention [Eq. (4.22)]. The same convention also leads to Eq. (4.27), which implies that

$$\langle\pi^+|\hat{J}_4{}^\rho(0)|\pi^+\rangle=i\,;\quad(7.3)$$

therefore, the analytic continuation of the form factor $\hat{F}_{\pi\pi}{}^\rho(q^2)$ satisfies

$$\hat{F}_{\pi\pi}{}^\rho(q^2)=1\quad\text{at}\quad q^2=0\,.\quad(7.4)$$

By using Eq. (7.1) and the experimental values that

$$\text{rate}(\rho^0\to\pi^++\pi^-)=115.5\pm8.2\text{ MeV}\quad\text{and}\quad m_\rho=756.4\pm3.2\text{ MeV}\,,$$

one finds

$$(4\pi)^{-1}[g_\rho\hat{F}_{\pi\pi}{}^\rho(-m_\rho^2)]^2(Z_1/Z_0)=2.28\pm0.16\,.\quad(7.5)$$

According to Eqs. (4.17), (4.22), and (4.25), the factor (Z_1/Z_0) depends on the variation of $(q^2+m_\rho^2)^{-1}\times\text{Re}F^{-1}(q^2)$ from $q^2=0$ to $q^2=-m_\rho^2$, where F is defined by Eq. (4.12). As we shall see, while both (Z_1/Z_0) and the value of $\hat{F}_{\pi\pi}{}^\rho(q^2)$ at $q^2=-m_\rho^2$ are not known, the product $(Z_1/Z_0)\hat{F}_{\pi\pi}{}^\rho(-m_\rho^2)$ can be determined by using (7.1) and the leptonic decay rates of ρ^0.

2. $\rho^0\to l^++l^-$

By using Eqs. (1.3a), (4.19), and (4.22), it can be easily verified that

$$\text{rate}(\rho^0\to l^++l^-)=\tfrac{1}{3}\alpha^2[(g_\rho^2/4\pi)(Z_0/Z_1)]^{-1}\times m_\rho^{-2}(m_\rho^2-4m_l^2)^{1/2}(m_\rho^2+2m_l^2)\,,\quad(7.6)$$

where $l=e$ or μ and $\alpha=(137)^{-1}$. The present experimental value[18] of the branching ratio of $\rho^0\to\mu^++\mu^-$ is $(4.3\pm1.4)\times10^{-5}$. Thus, we find

$$(4\pi)^{-1}g_\rho^2(Z_0/Z_1)=(2.5\pm0.8)\,.\quad(7.7)$$

It is interesting to note that the rate of $\rho^0\to\pi^++\pi^-$ depends on $g_\rho^2(Z_1/Z_0)$, but the rate $\rho^0\to\mu^++\mu^-$ (or, e^++e^-) depends on $g_\rho^2(Z_0/Z_1)$. By taking the ratio of Eqs. (7.5) and (7.7), one finds

$$[\hat{F}_{\pi\pi}{}^\rho(-m_\rho^2)(Z_1/Z_0)]^2=0.9\pm0.3\,,\quad(7.8)$$

which is consistent with the approximation that both the vertex function and $(q^2+m_\rho^2)^{-1}\,\text{Re}F^{-1}(q^2)$ do not change much from $q^2=0$ to $q^2=-m_\rho^2$; i.e.,

$$\hat{F}_{\pi\pi}{}^\rho(q^2)\cong1\quad\text{for}\quad-q^2\lesssim m_\rho^2\,,\quad(7.9)$$

and

$$(Z_1/Z_0)\cong1\,.\quad(7.10)$$

Under the same approximation, the decay $\rho\to\pi^++\pi^-$ determines the coupling constant g_ρ^2 to be

$$(4\pi)^{-1}g_\rho^2\cong2.3\,.\quad(7.11)$$

3. ϕ-ω Decays and the Determination of θ_Y and θ_N

From Eq. (2.12) it follows that any matrix element of the renormalized current operator $\mathcal{J}_\mu{}^\phi(x)$ vanishes at $q^2=-m_\phi^2$, and that of $\mathcal{J}_\mu{}^\omega(x)$ vanishes at $q^2=-m_\omega^2$. It is useful to define the modified current operator

$$\hat{\mathcal{J}}_\mu(x)=\begin{pmatrix}\hat{\mathcal{J}}_\mu{}^\phi(x)\\ \hat{\mathcal{J}}_\mu{}^\omega(x)\end{pmatrix}\quad(7.12)$$

by

$$\langle B|\hat{\mathcal{J}}_\mu(x)|A\rangle=-[\mathfrak{D}_{\mu\nu}(q)]^{-1}\langle B|\psi_\nu(x)|A\rangle\,,\quad(7.13)$$

where $\psi_\mu(x)$ is the renormalized ϕ-ω field operator given by Eq. (5.1), and $\mathfrak{D}_{\mu\nu}(q)$ is the renormalized ϕ-ω propagator given by Eq. (5.52). In analogy with Eq. (4.29), the decay rate of the ϕ meson into a state Γ can be expressed in terms of the matrix element $\langle\Gamma|\hat{\mathcal{J}}_\mu{}^\phi(0)|\text{vac}\rangle$, and the decay rate of the ω meson into a state Γ can be expressed in terms of the matrix element $\langle\Gamma|\hat{\mathcal{J}}_\mu{}^\omega(0)|\text{vac}\rangle$. The fact that these two decay rates involve $\hat{\mathcal{J}}_\mu{}^\phi$ and $\hat{\mathcal{J}}_\mu{}^\omega$ separately is a consequence of the orientation condition [Eq. (5.48)] that we have used.

In the following, we will assume the validity of the pole dominance approximation in the resonance region and choose the normalization convention given by Eq. (5.53). Thus, from Eqs. (5.55) and (5.56), it follows that, in the region from $q^2=-m_\phi^2$ to $-m_\omega^2$, the real part of the inverse of the renormalized ϕ-ω propagator is given by

$$\text{Re}[\mathfrak{D}_{\mu\nu}(q)]^{-1}=(q^2+M^2)\delta_{\mu\nu}-q_\mu q_\nu\,,\quad(7.14)$$

[18] A. Wehmann *et al.*, Phys. Rev. Letters **17**, 1113 (1966). See also R. Weinstein, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

where

$$M^2=\begin{pmatrix} m_\phi^2 & 0 \\ 0 & m_\omega^2 \end{pmatrix}.$$

We recall that at $q^2=0$, $\mathfrak{D}_{\mu\nu}^{-1}(q)$ is always equal to $\tilde{S}M_0{}^2S\delta_{\mu\nu}$, and therefore

$$\mathfrak{D}_{\mu\nu}^{-1}(0)=M^2\delta_{\mu\nu}, \quad (7.15)$$

on account of Eq. (5.55), even though the pole approximation is only assumed for the resonance region $-m_\omega^2 \geqq q^2 \geqq -m_\phi^2$ (which does not include $q^2=0$). By using Eqs. (5.18) and (7.13), one finds that the spatial integral of $\mathfrak{J}_4(x)$ is related to the hypercharge Y and the baryon number N by

$$-i\int \mathfrak{J}_4(x)d^3r = g\begin{pmatrix} Y \\ N \end{pmatrix}, \quad (7.16)$$

which together with Eqs. (5.19), (5.20), and (5.21) gives

$$-i\int \mathfrak{J}_4{}^\phi(x)d^3r=[\cos(\theta_Y-\theta_N)]^{-1} \times[\cos\theta_N g_Y Y+\sin\theta_Y g_N N],$$

and (7.17)

$$-i\int \mathfrak{J}_4{}^\omega(x)d^3r=[\cos(\theta_Y-\theta_N)]^{-1} \times[-\sin\theta_N g_Y Y+\cos\theta_Y g_N N].$$

The decay rate of $\phi^0 \to K^+ + K^-$, or $K_1{}^0+K_2{}^0$ depends on the vertex function $\hat{F}_{KK}{}^\phi(q^2)$ at $q^2=-m_\phi^2$; $\hat{F}_{KK}{}^\phi(q^2)$ is defined by

$$\langle (2K)^{\text{in}} | \mathfrak{J}_i{}^\phi(0) | \text{vac}\rangle = \sqrt{2}\mathbf{s}_i(p_K/\omega_K)\hat{F}_{KK}{}^\phi(q^2), \quad i=1,2,3, \quad (7.18)$$

where $|(2K)^{\text{in}}\rangle$ denotes the two-kaon $I=0$, $S=$ strangeness $=0$, p-wave "incoming" eigenstate of the strong interaction Hamiltonian, $\mathbf{s}$ is the polarization vector (chosen to be real), ω_K is the kaon energy $=\frac{1}{2}(-q^2)^{1/2}$ and $p_K=(\omega_K{}^2-m_K{}^2)^{1/2}$. Just as in the case of $\hat{F}_{\pi\pi}{}^\rho(-m_\rho^2)$, the function $\hat{F}_{KK}{}^\phi(q^2)$ is real at $q^2=-m_\phi^2$, and $\hat{F}_{KK}{}^\phi(q^2)$ is expected to be a slowly varying function of q^2 near $q^2=-m_\phi^2$. From Eq. (7.17), we know that the diagonal matrix element of $\mathfrak{J}_4{}^\phi$ for a K^+ at rest is

$$\langle K^+ | \mathfrak{J}_4{}^\phi(0) | K^+\rangle = i[\cos(\theta_Y-\theta_N)]^{-1}\cos\theta_N g_Y, \quad (7.19)$$

which implies that the analytic continuation of the same vertex function $\hat{F}_{KK}{}^\phi(q^2)$, at $q^2=0$, becomes

$$\hat{F}_{KK}{}^\phi(0)=[\cos(\theta_Y-\theta_N)]^{-1}\cos\theta_N g_Y. \quad (7.20)$$

The formulas for the rates $\phi^0\to K^++K^-$ (and $K_1{}^0+K_2{}^0$), $\phi^0\to l^++l^-$, and $\omega^0\to l^++l^-$ can be obtained by following the same derivations of Eqs. (7.1) and (7.6). In order to use these rates to determine the two mixing angles θ_Y and θ_N, we assume that the vertex function $\hat{F}_{KK}{}^\phi(q^2)$ at $q^2=-m_\phi^2$ can be approximated by its value at $q^2=0$, i.e.;

$$\hat{F}_{KK}{}^\phi(-m_\phi)\cong\hat{F}_{KK}{}^\phi(0) = [\cos(\theta_Y-\theta_N)]^{-1}\cos\theta_N g_Y. \quad (7.21)$$

By using Eqs. (7.14) and (7.21), one finds

$$\text{rate}(\phi^0\to K^++K^-)=(48\pi m_\phi^2)^{-1}(m_\phi^2-4m_K^2)^{3/2} \times[\cos(\theta_Y-\theta_N)]^{-2}g_Y^2\cos^2\theta_N, \quad (7.22)$$

where m_K is the mass of $K^\pm$. The same expression applies to rate$(\phi^0\to K_1{}^0+K_2{}^0)$, provided that m_K refers to the mass of the neutral K meson.

Similarly, by using Eqs. (1.6) and (5.40), one finds

$$\text{rate}(\phi^0\to l^++l^-)=(1/12)\alpha^2(g_Y^2/4\pi)^{-1}m_\phi^{-2} \times(m_\phi^2-4m_l^2)^{1/2}(m_\phi^2+2m_l^2)\cos^2\theta_Y, \quad (7.23)$$

and

$$\text{rate}(\omega^0\to l^++l^-)=(1/12)\alpha^2(g_Y^2/4\pi)^{-1}m_\omega^{-2} \times(m_\omega^2-4m_l^2)^{1/2}(m_\omega^2+2m_l^2)\sin^2\theta_Y. \quad (7.24)$$

From the known rate of $\phi^0\to K^++K^-=1.7\pm0.4$ MeV, one obtains

$$\left[\frac{g_Y\cos\theta_N}{\cos(\theta_Y-\theta_N)}\right]^2\frac{1}{4\pi}=1.4\pm0.3. \quad (7.25)$$

The coupling constant g_Y^2 and the two mixing angles θ_Y and θ_N can then be determined by measuring the leptonic decay rates of ϕ^0 and ω^0. These leptonic decay rates can also be calculated theoretically by assuming the particular values of θ_Y and θ_N determined in the Sec. V (3. *Special Models.*) The results are given in Table I.

TABLE I. The coupling constant $(g_Y^2/4\pi)$ and the leptonic decay rates are calculated by assuming the rate $(\phi^0\to K^++K^-)$ is 1.7 (MeV/$\hbar$). [In the limit of SU_3 symmetry, $(g_Y^2/4\pi)=\frac{3}{4}(g_\rho^2/4\pi)\cong 1.7$.] To the accuracy given, there is no difference between the decay rates to μ pair and to e pair. The mean-square radii $R^2(K^0)$ and $R^2(\bar{K}^0)$ of the charge distributions of K^0 and $\bar{K}^0$ are calculated by using Eq. (7.42).

	Current-mixing model	Mass-mixing model	Mass-mixing model (variation)
θ_Y	33°	32°	39°
θ_N	21°	32°	39°
$(g_Y^2/4\pi)$	1.5	1.9	2.2
Rate $(\phi\to e^+e^-)$	2.2 (keV/$\hbar$)	1.7 (keV/$\hbar$)	1.2 (keV/$\hbar$)
Rate $(\omega\to e^+e^-)$	0.7 (keV/$\hbar$)	0.5 (keV/$\hbar$)	0.6 (keV/$\hbar$)
$R^2(K^0)$	-7.6×10^{-28} cm²	-7.0×10^{-28} cm²	-6.1×10^{-28} cm²
$R^2(\bar{K}^0)$	$+7.6\times10^{-28}$ cm²	$+7.0\times10^{-28}$ cm²	$+6.1\times10^{-28}$ cm²

4. Comparison Between $A\to B+\gamma$ and $A\to B+\rho^0$ (or, ϕ^0 and ω^0)

The identity (1.3a) between the hadronic electromagnetic current and the renormalized ρ-meson field implies that the isovector part of the electromagnetic form factor $F_{AB}{}^\gamma(q^2)$ of any real or virtual transition

$A \to B+\gamma$ is related to the corresponding form factor of $A \to B+\rho^0$ by Eq. (1.4a),

$$[F_{AB}{}^{\gamma}(q^2)]_{\text{isovector}} = \frac{m_\rho{}^2}{q^2+m_\rho{}^2} F_{AB}{}^{\rho}(q^2). \quad (1.4a)$$

Similarly, by using Eq. (2.22), one finds that the identity (1.6) implies that the isoscalar part of $F_{AB}{}^{\gamma}(q^2)$ is given by

$$[F_{AB}{}^{\gamma}(q^2)]_{\text{isoscalar}} = \tfrac{1}{2} g_Y{}^{-1}\left[\cos\theta_Y\left(\frac{m_\phi{}^2}{q^2+m_\phi{}^2}\right)F_{AB}{}^{\phi}(q^2) - \sin\theta_Y\left(\frac{m_\omega{}^2}{q^2+m_\omega{}^2}\right)F_{AB}{}^{\omega}(q^2)\right], \quad (7.26)$$

where $F_{AB}{}^{\phi}(q^2)$ and $F_{AB}{}^{\omega}(q^2)$ are related to the matrix elements of the renormalized currents $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$ by

$$\langle B|\, \mathcal{J}_\mu{}^\phi(x)\,|A\rangle = \sum_i F_{AB}{}^{\phi}(q^2) u_B{}^\dagger \Gamma_\mu{}^i u_A \exp(iq_\lambda x_\lambda),$$
$$\langle B|\, \mathcal{J}_\mu{}^\omega(x)\,|A\rangle = \sum_i F_{AB}{}^{\omega}(q^2) u_B{}^\dagger \Gamma_\mu{}^i u_A \exp(iq_\lambda x_\lambda), \quad (7.27)$$

and $u_B{}^\dagger\Gamma_\mu{}^i u_A$ denotes the same spin-momentum function used in Eq. (2.11). In the following, we shall discuss these form-factor relations in three separate regions of q^2:

(i) At $q^2=0$, the validity of Eqs. (1.4a) and (7.26) follows directly from the properties that the spatial integrals of $J_4{}^\rho(x)$ and $[\cos\theta_Y\, \mathcal{J}_4{}^\phi(x) - \sin\theta_Y\, \mathcal{J}_4{}^\omega(x)]$ are, respectively, iI_z and $ig_Y Y$. [See Eqs. (2.6) and (2.15).]

(ii) Near the resonance, we have, on account of Eqs. (2.3) and (2.12), for any hadronic states A and B,

$$F_{AB}{}^{a}(q^2)=0 \quad \text{at} \quad q^2=-m_a{}^2, \quad (7.28)$$

where a can be either ρ, or ϕ, or ω. Although $(q^2+m_a{}^2)^{-1} \times F_{AB}{}^a(q^2)$ and, therefore, also Eqs. (1.4a) and (7.26) remain well defined at $q^2+m_a{}^2=0$, it is much more convenient to use the modified current operators $\hat{J}_\mu{}^\rho(x)$, $\hat{\mathcal{J}}_\mu{}^\phi(x)$, $\hat{\mathcal{J}}_\mu{}^\omega(x)$, and their related form factors $\hat{F}_{AB}{}^\rho(q^2)$, $\hat{F}_{AB}{}^\phi(q^2)$, and $\hat{F}_{AB}{}^\omega(q^2)$, instead of $J_\mu{}^\rho(x)$, $\mathcal{J}_\mu{}^\phi(x)$, and $\mathcal{J}_\mu{}^\omega(x)$ [or $F_{AB}{}^\rho(q^2)$, $F_{AB}{}^\phi(q^2)$, and $F_{AB}{}^\omega(q^2)$], in the q^2 region near the vector-meson resonances.

A direct consequence of the identity (1.3a) is that for any $|\Delta\mathbf{I}|=1$ transition at $q^2=-m_\rho{}^2$, the ratio between the rates of

$$A \to B+\rho^0 \quad (7.29)$$

and

$$A \to B+l^+ +l^- \quad (l=e, \text{ or } \mu) \quad (7.30)$$

is independent of the initial and final complexes A and B. This independence is supposed to be an *exact* one, provided that the higher-order radiative corrections are neglected; furthermore, in taking this ratio, one should use directly the observed rate of the $|\Delta\mathbf{I}|=1$ transition $A\to B+l^+ +l^-$ at $q^2=-m_\rho{}^2$ without any background subtraction.

Since the neutral ρ meson is unstable against the strong interaction, reaction (7.29) can only be observed by studying its decay products, such as 2π or 4π. Instead of (7.29), one may use, e.g.,

$$A \to B+\pi^+ +\pi^-. \quad (7.31)$$

The two-pion p-wave amplitude has the familiar resonance behavior $[q^2+m_\rho{}^2-i\gamma m_\rho]^{-1}$ at $q^2\cong -m_\rho{}^2$. By using only the resonant part of the two-pion amplitude, the ratio

$$r(q^2) \equiv \frac{\text{rate}[A\to B+(\pi^+ +\pi^-)_{p\text{-state}}]}{\text{rate}[A\to B+l^+ +l^-]} \quad (7.32)$$

can be measured, and it should be independent of A and B for any $|\Delta\mathbf{I}|=1$ transitions in the region $q^2\cong -m_\rho{}^2$.

By following the same arguments that led to Eqs. (7.1) and (7.6), but without setting $q^2=-m_\rho{}^2$, one finds, for q^2 near $-m_\rho{}^2$,

$$r(q^2) = g_\rho{}^4\left[\frac{(m_\rho{}^2-4m_\pi{}^2)^{3/2}}{16\alpha^2(m_\rho{}^2-4m_l{}^2)^{1/2}(m_\rho{}^2+2m_l{}^2)}\right] \times \left[\frac{-q^2}{m_\rho{}^2}\right]^2 [\hat{F}_{\pi\pi}{}^\rho(q^2)]^2, \quad (7.33)$$

which is independent of A and B, and, in addition, is independent of the wave-function normalization factor (Z_1/Z_0). The functional dependence of the vertex function $\hat{F}_{\pi\pi}{}^\rho(q^2)$ at q^2 near $-m_\rho{}^2$ can be determined by measuring $r(q^2)$.

We note that the q^2-dependent factor in Eq. (7.33) must be expected to produce a shift[19] in the ρ peak observed in the process $A\to B+l^+ +l^-$ from that observed in $A\to B+\pi^+ +\pi^-$. The main shift may be expected to arise from the factor $(q^2)^2$; from this source alone the shift in the ρ peak is from $q^2=-m_\rho{}^2\cong -(756 \text{ MeV})^2$ to

$$q^2 = -\tfrac{1}{4}m_\rho[3m_\rho+(m_\rho{}^2-8\gamma^2)^{1/2}] \cong -(745 \text{ MeV})^2. \quad (7.34)$$

Identical arguments can be applied to any $|\Delta\mathbf{I}|=0$ transition

$$A\to B+l^+ +l^-$$

by comparing its rate with that of

$$A\to B+\phi^0 \text{ (or } \omega^0)$$

at $q^2=-m_\phi{}^2$ (or $-m_\omega{}^2$). The rates of the latter reactions can be measured by, e.g., using the *resonant* part of $A\to B+K^+ +K^-$ for the ϕ meson and $A\to B+3\pi$ for the ω meson. The ratios of these rates to the corresponding lepton pair production rates are, again, independent of A and B, provided Eq. (1.6) holds.

(iii) For q^2 away from the vector-meson resonances, it is more convenient to use Eqs. (1.4a) and (7.26).

[19] One of us (N.M.K.) wishes to acknowledge a discussion of this point with M. Good and A. Silverman.

Although the form factors $F_{AB}{}^{\rho}(q^2)$, $F_{AB}{}^{\phi}(q^2)$, and $F_{AB}{}^{\omega}(q^2)$ are not known, one may assume some simple analytic functions for these vector-meson form factors. The phenomenological parameters contained in these functions can, then, be determined by using the experimental results on the electromagnetic form factor $F_{AB}{}^{\gamma}(q^2)$.

As an example, we may consider the special case $A=B$ and assume, for q^2 spacelike (or, for any q^2 away from the resonances),

$$F_{AB}{}^{\rho}(q^2)/F_{AB}{}^{\rho}(0)=[1+(q^2/\Lambda_1{}^2)]^{-1} \tag{7.35}$$

and

$$\frac{F_{AB}{}^{\phi}(q^2)}{F_{AB}{}^{\phi}(0)}=\frac{F_{AB}{}^{\omega}(q^2)}{F_{AB}{}^{\omega}(0)}=\left[1+\left(\frac{q^2}{\Lambda_0{}^2}\right)\right]^{-1}, \tag{7.36}$$

where Λ_1 and Λ_0 are phenomenological parameters characterizing the overall q^2 dependence of the $I=1$ and $I=0$ vertex functions. At $q^2=0$, the values of these form factors $F_{AA}{}^{\rho}(0)$, $F_{AA}{}^{\phi}(0)$, and $F_{AA}{}^{\omega}(0)$ are known; they can be readily determined by using (2.11), (7.27), and the identities

$$\int J_4{}^{\rho}(x)d^3r=iI_z\,,$$

$$\int \mathcal{J}_4{}^{\phi}(x)d^3r=i[\cos(\theta_Y-\theta_N)]^{-1}\times[\cos\theta_N g_Y Y+\sin\theta_Y g_N N]\,, \tag{7.37}$$

$$\int \mathcal{J}_4{}^{\omega}(x)d^3r=i[\cos(\theta_Y-\theta_N)]^{-1}\times[-\sin\theta_N g_Y Y+\cos\theta_Y g_N N]\,.$$

Thus, the resulting electromagnetic form factor $F_{AA}{}^{\gamma}(q^2)$ in the spacelike q^2 region (or, for any q^2 away from the resonances) becomes dependent only on Λ_0, Λ_1, θ_Y, θ_N, and, if A has nonzero baryon number, the ratio (g_N/g_Y); among these, the angles θ_Y and θ_N can be either directly measured, or theoretically calculated by using special models.

Such a study for $A=B=$ single nucleon has been made by Massam and Zichichi[20]; they assumed $\theta_Y=\theta_N\cong35°$ and found that the existing data in the spacelike q^2 region is consistent with $\Lambda_0\cong\Lambda_1\cong1$ BeV.

At present, it is not possible to make a similar study for the timelike q^2 region away from the resonance. In this connection, we may recall the possibility of the alternative proposal [Eq. (1.3b)] which implies Eq. (1.4b) instead of Eq. (1.4a). Thus, it seems particularly interesting to investigate reactions such as $A\rightarrow B+l^++l^-$ for large $-q^2$ and see whether $[F_{AB}{}^{\gamma}(q^2)]_{\text{isovector}}$ can be zero at some $q^2=-(m_\rho{}^0)^2$. If $F_{AB}{}^{\gamma}(q^2)$ has a zero, then this could be regarded as a confirmation of Eq. (1.3b), and the value of the mechanical mass $m_\rho{}^0$ would become measurable. Otherwise, it is consistent with the assumption that $m_\rho{}^0$ is infinite, and the two different proposals (1.3a) and (1.3b) are the same.

5. Electromagnetic Form Factor of K^0 and $\bar{K}^0$

Let $|K^0,p\rangle$ and $|\bar{K}^0,p\rangle$ denote, respectively, the state of a neutral K^0 and $\bar{K}^0$ with 4-momentum p_μ. From Lorentz invariance and current conservation, one has

$$\langle K^0,p'|J_\mu{}^{\gamma}(0)|K^0,p\rangle=-\langle\bar{K}^0,p'|J_\mu{}^{\gamma}|\bar{K}^0,p\rangle=\tfrac{1}{2}(\omega\omega')^{-1/2}(p'+p)_\mu F_{KK}{}^{\gamma}(q^2)\,, \tag{7.38}$$

where $i\omega$ and $i\omega'$ are, respectively, p_4 and p_4'. By using Eqs. (1.4a), (7.26), (7.37), and (7.38), one finds

$$F_{KK}{}^{\gamma}(q^2)=-\tfrac{1}{2}f_{KK}{}^{\rho}(q^2)\left(\frac{m_\rho{}^2}{q^2+m_\rho{}^2}\right)+\frac{1}{2}\frac{1}{\cos(\theta_Y-\theta_N)}\left[\frac{m_\phi{}^2\cos\theta_Y\cos\theta_N f_{KK}{}^{\phi}(q^2)}{q^2+m_\phi{}^2}+\frac{m_\omega{}^2\sin\theta_Y\sin\theta_N f_{KK}{}^{\omega}(q^2)}{q^2+m_\omega{}^2}\right], \tag{7.39}$$

where

$$f_{KK}{}^{a}(q^2)=F_{KK}{}^{a}(q^2)/F_{KK}{}^{a}(0)\,, \tag{7.40}$$

and $a=\rho$, ϕ, and ω. At $q^2=0$, $f_{KK}{}^{a}(q^2)=1$.

The mean-square radius of the change distribution of K^0 is, by definition,

$$R^2(K^0)=-6(d/dq^2)F_{KK}{}^{\gamma}(q^2) \tag{7.41}$$

at $q^2=0$; the corresponding mean-square radius of $\bar{K}^0$ is $R^2(\bar{K}^0)=-R^2(K^0)$. If the differences between the three derivatives $(d/dq^2)f_{KK}{}^{a}(q^2)$ at $q^2=0$ can be neglected, then one finds

$$R^2(K^0)=3\{-m_\rho{}^{-2}+[\cos(\theta_Y-\theta_N)]^{-1}\times[m_\phi{}^{-2}\cos\theta_Y\cos\theta_N+m_\omega{}^{-2}\sin\theta_Y\sin\theta_N]\}\,. \tag{7.42}$$

[If one assumes (7.35) and (7.36), then this expression is valid, provided one neglects $(\Lambda_1{}^{-2}-\Lambda_0{}^{-2})$.] The numerical value of $R^2(K^0)$ can be estimated by using either the current-mixing model, or the mass-mixing model. The results, which are given in Table I, are about a factor of 30 smaller than the estimate given by Zeldovich.[21] While the existing experimental evidence in support of vector dominance seems substantial enough to make a value as large as that given by Zeldovich rather unlikely, nevertheless a measurement[22] of the charge radius of K^0 and $\bar{K}^0$ could constitute a

[20] T. Massam and A. Zichichi, Nuovo Cimento 43, 1137 (1966).

[21] Y. B. Zeldovich, Zh. Eksperim. i Teor. Fiz. 36, 782 (1959) [English transl.: Soviet Phys.—JETP 9, 984 (1959)]. Some earlier discussions have been given by G. Feinberg, Phys. Rev. 109, 1381 (1958). A recent calculation by W. Frazer (private communication) gives results of the same order as those in Table I.

[22] One of us (N.M.K.) is grateful to O. Piccioni for a stimulating discussion of this point.

further relatively sensitive test of the theory, and it would also serve as a measure of the possible difference between the derivatives of the different strong interaction form factors $f_{KK}{}^{a}$ at $q^2=0$.

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APPENDIX A: AN ELEMENTARY THEOREM

The current $J_\mu{}^\rho(x)$ is first introduced in Eq. (2.3). From its definition, we know that (i) $J_\mu{}^\rho(x)$ transforms like the $I_z=0$ member of an $I=1$ triplet, (ii) $J_\mu{}^\rho(x)$ commutes with the baryon number operator N, and (iii) $J_\mu{}^\rho(x)$ anticommutes with the particle antiparticle conjugation operator C. The following theorem can be easily established:

Theorem. If $J_\mu{}^\rho(x)$ is conserved under the strong interaction, then

$$-i\int J_4{}^\rho(x)d^3r=\lambda I_z\,, \tag{A1}$$

where λ is a constant.

Proof. Let $A_z=-i\int J_4{}^\rho d^3r$, and define

$$[I_j,A_k]=i\epsilon_{jkl}A_l\,, \tag{A2}$$

where the subscripts j, k, and l can be either x, or y, or z, and ϵ_{jkl} is the usual third-rank constant antisymmetric tensor. Since A_z and I_j commute with the strong interaction Hamiltonian $H_{\rm st}$, the other two components A_x and A_y must also commute with $H_{\rm st}$. We recall that the only single-particle eigenstate of $H_{\rm st}$ that is degenerate with $|\Lambda^0\rangle$ is $|\bar\Lambda^0\rangle$. The state $A_j|\Lambda^0\rangle$ must, therefore, be a linear function of $|\Lambda^0\rangle$ and $|\bar\Lambda^0\rangle$. From Eq. (A2) and $I_j|\Lambda^0\rangle=I_j|\bar\Lambda^0\rangle=0$, we find

$$A_j|\Lambda^0\rangle=A_j|\bar\Lambda^0\rangle=0\,. \tag{A3}$$

Similarly, we can show that in the sector of the single nucleon and single antinucleon states

$$\begin{bmatrix} p \\ n \\ \bar n \\ -\bar p \end{bmatrix}, \tag{A4}$$

the operator A_j must be of the form

$$\begin{pmatrix} a\sigma_j & b\sigma_j \\ c\sigma_j & d\sigma_j \end{pmatrix},$$

where σ_j is the usual (2×2) Pauli matrix, and a, b, c, d are constants. Now, $[A_z,N]=0$ implies $b=c=0$, and $\{A_z,C\}=0$ implies $a=d$. Thus, in the single nucleon-antinucleon sector,

$$A_z=a\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}. \tag{A5}$$

Since all known hadrons are connected through $H_{\rm st}$ to some multiple-particle states of the nucleon, antinucleon, Λ^0, $\bar\Lambda^0$ system, the theorem is, then, proved by using Eqs. (A3) and (A5), and by setting $\lambda=2a$ in Eq. (A1).

In exactly the same way, we can also prove that the spatial integral of $(J_4{}^\rho)^0$, which satisfies Eqs. (3.2) and (3.3), must also be proportional to the operator I_z.

Similar considerations can also be applied to currents $\mathcal{J}_\mu{}^\phi$ and $\mathcal{J}_\mu{}^\omega$ which are defined by Eq. (2.12). Both currents (i) are isoscalars, (ii) commute with N, and (iii) anticommute with C. Let

$$Q_\alpha\equiv-i\int \mathcal{J}_4{}^\alpha d^3r\,, \tag{A6}$$

where $\alpha=\phi$ or ω. The conservation law of $\mathcal{J}_\mu{}^\alpha$ implies that $[Q_\alpha,H_{\rm st}]=0$. By using the above properties (i)–(iii), it is easy to see that Q_α must be of the form

$$a_\alpha\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A7}$$

in the sector

$$\begin{pmatrix} \Lambda^0 \\ \bar\Lambda^0 \end{pmatrix},$$

and Q_α is of the form

$$b_\alpha\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{A8}$$

in the nucleon-antinucleon sector (A4) where a_α and b_α are constants. Since all known hadrons are connected through $H_{\rm st}$ to some multiple-particle states of the nucleon, antinucleon, Λ^0, $\bar\Lambda^0$ system, the operator Q_α must, therefore, be related to the hypercharge operator Y and the baryon number operator N by

$$Q_\alpha=a_\alpha N+(b_\alpha-a_\alpha)Y\,. \tag{A9}$$

This establishes Eq. (2.15).

APPENDIX B: ALTERNATIVE FORMS OF ρ-γ COUPLING

For simplicity, we will consider in this Appendix only the isovector part of the electromagnetic interaction. The Lagrangian density which includes both such an interaction and the strong interaction of the ρ^0 meson is given by

$$\mathcal{L}=-\tfrac14(F_{\mu\nu}{}^0)^2-\tfrac14(G_{\mu\nu}{}^0)^2-\tfrac12(m_\rho{}^0\rho_\mu{}^0)^2 -(g_\rho{}^0\rho_\mu{}^0+e^0A_\mu{}^0)(J_\mu{}^\rho)^0-\tfrac12(e^0/g_\rho{}^0)G_{\mu\nu}{}^0F_{\mu\nu}{}^0\,, \tag{B1}$$

where the superscript zero denotes the unrenormalized quantities. If we neglect the renormalization problem of the electromagnetic interaction, then $e^0=e$, $A_\mu{}^0=A_\mu$, and $F_{\mu\nu}{}^0=F_{\mu\nu}$. The above Lagrangian density $\mathcal{L}$ contains both the Lagrangian densities (2.7) and (3.1) which are used in Secs. II and III. By using the definition of $J_\mu{}^\rho$ [Eq. (3.13)] one can show that apart from a trivial partial integration,

$$\mathcal{L}=-\tfrac{1}{4}(F_{\mu\nu}{}^0)^2+(2.7)+(3.1). \tag{B2}$$

(1) We consider first the transformation

$$\rho_\mu{}^0=[1-(e^0/g_\rho{}^0)^2]^{-1/2}\rho_\mu{}',$$

and

$$A_\mu{}^0=-(e^0/g_\rho{}^0)[1-(e^0/g_\rho{}^0)^2]^{-1/2}\rho_\mu{}'+A_\mu{}'. \tag{B3}$$

It is easy to see that in terms of $\rho_\mu{}'$ and $A_\mu{}'$, (B1) can be expressed in an alternative form

$$\mathcal{L}=-\tfrac{1}{4}(F_{\mu\nu}{}')^2-\tfrac{1}{4}(G_{\mu\nu}{}')^2-\tfrac{1}{2}(m_\rho{}'\rho_\mu{}')^2 -(g'\rho_\mu{}'+e^0A_\mu{}')(J_\mu{}^\rho)^0, \tag{B4}$$

where

$$m_\rho{}'=[1-(e^0/g_\rho{}^0)^2]^{-1/2}m_\rho{}^0, \tag{B5}$$

$$g'=g_\rho{}^0[1-(e^0/g_\rho{}^0)^2]^{1/2}, \tag{B6}$$

$$G_{\mu\nu}{}'=\frac{\partial}{\partial x_\mu}\rho_\nu{}'-\frac{\partial}{\partial x_\nu}\rho_\mu{}',$$

and

$$F_{\mu\nu}{}'=\frac{\partial}{\partial x_\mu}A_\nu{}'-\frac{\partial}{\partial x_\nu}A_\mu{}'.$$

According to Eq. (B4), the currents which generate the new (unrenormalized) fields $\rho_\mu{}'$ and $A_\mu{}'$ are, respectively, $g'(J_\mu{}^\rho)^0$ and $e^0(J_\mu{}^\rho)^0$. In contrast to Eq. (3.17) these two currents are now proportional to each other; therefore, apart from the coupling constants g' and e^0, they formally satisfy Eq. (1.3b), even though the Lagrangian (B1) is derived based on the identity (1.3a). In terms of these new field variables, the difference between the proposals (1.3a) and (1.3b) lies now in the form of other electromagnetic interactions which are not included in (B1). We note that if (1.3a) holds, then, for example, the lepton current j_μ interacts with $A_\mu{}'$ through the Lagrangian density

$$e^0j_\mu A_\mu{}^0=e^0j_\mu A_\mu{}'-[(e^0)^2/g']j_\mu\rho_\mu{}'.$$

(2) Next, we consider a different transformation:

$$\rho_\mu{}''=\rho_\mu{}^0+(e^0/g_\rho{}^0)A_\mu{}^0,$$

and

$$A_\mu{}''=[1-(e^0/g_\rho{}^0)^2]^{1/2}A_\mu{}^0. \tag{B7}$$

The Lagrangian density (B1) now becomes

$$\mathcal{L}=-\tfrac{1}{4}(F_{\mu\nu}{}'')^2-\tfrac{1}{4}(G_{\mu\nu}{}'')^2-\tfrac{1}{2}(m_\rho{}^0\rho_\mu{}'')^2-g_\rho{}^0\rho_\mu{}''(J_\mu{}^\rho)^0 +(m_\rho{}^0)^2(e''/g_\rho{}^0)\rho_\mu{}''A_\mu{}''-\tfrac{1}{2}(m_\rho{}^0)^2 \times(e''/g_\rho{}^0)^2(A_\mu{}'')^2, \tag{B8}$$

where

$$e''=e^0[1-(e^0/g_\rho{}^0)^2]^{-1/2},$$

$$F_{\mu\nu}{}''=\frac{\partial}{\partial x_\mu}A_\nu{}''-\frac{\partial}{\partial x_\nu}A_\mu{}'', \tag{B9}$$

and

$$G_{\mu\nu}{}''=\frac{\partial}{\partial x_\mu}\rho_\nu{}''-\frac{\partial}{\partial x_\nu}\rho_\mu{}''.$$

In terms of the transformed fields, a gauge transformation means that

$$A_\mu{}''\to A_\mu{}''+\partial\Lambda/\partial x_\mu$$

and

$$\rho_\mu{}''\to\rho_\mu{}''+(e''/g_\rho{}^0)\partial\Lambda/\partial x_\mu. \tag{B10}$$

In Eq. (B8), the only photon-matter coupling is given by $(m_\rho{}^0)^2(e''/g_\rho{}^0)\rho_\mu{}''A_\mu{}''$. Such a term, by itself, clearly violates gauge invariance; but the combination

$$-\tfrac{1}{2}(m_\rho{}^0\rho_\mu{}'')^2+(m_\rho{}^0)^2(e''/g_\rho{}^0)\rho_\mu{}''A_\mu{}'' -\tfrac{1}{2}(m_\rho{}^0)^2(e''/g_\rho{}^0)^2(A_\mu{}'')^2 \tag{B11}$$

is invariant under the gauge transformation (B10). Consequently, the Lagrangian density (B8) is also gauge-invariant. [Under the gauge transformation (B10), expression (B8) $\to$ (B8) $-e''(J_\mu{}^\rho)^0\partial\Lambda/\partial x_\mu$. This additional term $-e''(J_\mu{}^\rho)^0\partial\Lambda/\partial x_\mu$ is, as usual, canceled by a corresponding term generated by the free Lagrangian and the strong interaction Lagrangian of the charged particles under the same gauge transformation.]

In the language of Feynman graphs, (B8) shows that there is a direct ρ-photon coupling vertex given by $(m_\rho{}^0)^2(e''/g_\rho{}^0)\rho_\mu{}''A_\mu{}''$. The application of such vertices in the photon propagator would lead to a non-gauge-invariant and negative term for (photon mass)^2, which is, however, completely canceled by the additional term $-\tfrac{1}{2}(m_\rho{}^0)^2(e''/g_\rho{}^0)^2A_\mu{}^2$ in (B8).