Form Factors and the Absorptive One-Pion-Exchange Model*

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The one-pion-exchange model, corrected for absorption in the initial and final states, is examined for those two-body reactions involving the production of high-spin particles. It is found that the inclusion of form factors brings the theory into agreement with experiment. By consideration of a number of reactions, explicit expressions for the $N\pi N^*$ (1238) and $\pi\pi f$ vertex function form factors are obtained. It is shown that these form factors do not require the existence of any as yet undiscovered mesons. Implications of these form factors in two-pion-exchange calculations are also discussed.

I. INTRODUCTION

RECENTLY, a large number of two-body reactions at high energy have been quite successfully analyzed in terms of a simple one-pion-exchange model corrected for initial- and final-state interactions in the lower partial waves. The model was first proposed by Sopkovich[1] and subsequently discussed by many authors, notably Gottfried and Jackson,[2] Ross and Shaw,[3] and Durand and Chiu.[4] The model is based on the assumption that for those two-body inelastic reactions in which the one-pion-exchange mechanism is not forbidden by any conservations laws, the partial-wave amplitudes are given by the expression

$$T_{i,f}{}^{J}=(S_{i,i}{}^{J})^{1/2}T_{i,f}{}^{J,\mathrm{Born}}(S_{f,f}{}^{J})^{1/2}. \quad (1)$$

In Eq. (1), $T_{i,f}{}^{J,\mathrm{Born}}$ denotes the Born approximation to the appropriate one-pion-exchange amplitude of renormalized phenomenological field theory, and $S_{i,i}{}^{J}$ and $S_{f,f}{}^{J}$ denote the initial- and final-state elastic amplitudes. For clarity we omit indices referring to the individual particle helicities. Equation (1) expresses the fact that the large-impact-parameter collisions are dominated by the one-pion-exchange force, since, in the realm of strong interactions, this is the longest-range force. We shall refer hereafter to this equation as the distorted-wave Born approximation (DWBA). It has been found that this DWBA leads to good agreement between theory and experiment for a number of reactions. The agreement is generally obtained with the shape of the differential cross section, with the decay angular distributions of the resonances produced in the reaction, and with the over-all magnitude of the cross section. However, there is the following important exception. For those reactions in which either of the final-state particles has spin greater than unity, the predicted cross section is generally found to exceed the observed cross section by a factor typically of the order of 3. It is just this failure of the DWBA that we shall discuss in this paper.

The plan of the paper is as follows. In Sec. II we discuss the reactions which have been previously analyzed in terms of the DWBA. The modifications introduced by the inclusion of a form factor in the one-pion-exchange amplitude are given in Sec. III, and the results compared with experiment. Section IV contains a discussion of the form factors obtained in Sec. III, and also some remarks concerning their effect in two-pion-exchange processes. Our conclusions are summarized in Sec. V.

II. REVIEW OF PREVIOUS DWBA CALCULATIONS

In Table I we present some of the more relevant results of previous analyses in terms of the DWBA. Before discussing our modifications to the DWBA, we briefly review the results referred to in this table.

The first reaction, $\pi p \to \rho p$, was one of the original processes to be studied in terms of the DWBA. It was found that good agreement was obtained provided that one assumed a somewhat larger ρp cross section than is observed in πp scattering. It is worthwhile to point out here that such a behavior need not be considered a failure of the DWBA but in fact might be expected on the basis of the multiperipheral model of Amati, Fubini, and Stanghellini (AFS).[5] In their model the absorptive elastic amplitude is calculated by summing over all two-pion-exchange ladder diagrams. In πp scattering the dominant type of process is expected to be that shown in Fig. 1(a). In ρp scattering, two varieties of diagrams may be expected to dominate; they are those with a π at the top rung and those with an ω at the top rung [Fig. 1(b)]. Since the elastic absorptive contributions necessarily interefere constructively, we may thus expect the ρp cross section to exceed the πp cross section. Analyses of ρ photoproduction data at a few

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[1] N. J. Sopkovich, Nuovo Cimento **26**, 186 (1962).

[2] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

[3] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).

[4] L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964); **13**, 45 (1964); Phys. Rev. **137**, B1530 (1965).

[5] D. Amati, S. Fubini, and S. Stanghellini, Nuovo Cimento **5**, 896 (1962).

TABLE I. Predictions of the DWBA for a number of reactions. In each case N^* refers to the pion-nucleon resonance at 1238 MeV. References to the experimental works are given with the theoretical analyses quoted below. References marked with an asterisk denote DWBA calculations done in the older impact-parameter integral formalism. References without an asterisk denote exact partial-wave summation calculations. Note that in some cases use of partial-wave sums instead of the impact-parameter representation causes a reduction by as much as 50% in the forward differential cross section. This point is discussed in Ref. 16.

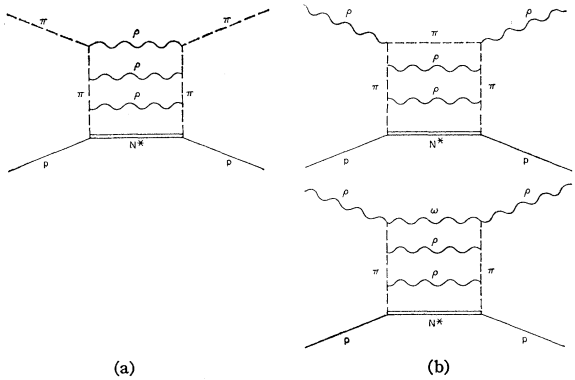
Reaction	Accuracy of the DWBA	Range of incident momenta studied (BeV/c)	References to DWBA calculation
$\pi p \to \rho p$	good	2–8	a*, b, c*
$\pi p \to \rho N^*$	~2 times too large	4, 8	c*, d*
$pp \to pp$ [high partial waves only]	good	$\lesssim 1$	e
$pp \to pN^{*+}$	2–3 times too large	3–15	f, g*
$pp \to nN^{*++}$	~2 times too large without extra absorption	5.5	h
$\bar{p}p \to \bar{N}^*N^*$	at least 5 times too large	3.25–7	i*
$Kp \to K^*N$	good	2–5	c*, j
$Kp \to K^*N^*$	~2 times too large	3, 3.5	c*
$\gamma p \to \omega p$	good	2–4	k, l*
$\gamma p \to \omega N^*$	probably too large	2–4	k
$\pi N \to fN$	2–4 times too large	4–10	m

[a] Reference 2.
[b] Reference 3.
[c] Reference 15.
[d] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965).
[e] G. Breit, Rev. Mod. Phys. **34**, 766 (1962).
[f] Reference 25.
[g] B. Margolis and A. Rotsstein, Nuovo Cimento **45A**, 1010 (1966).
[h] Reference 11.
[i] B. E. Y. Svensson, Nuovo Cimento **39**, 667 (1965).
[j] J. H. Friedman and R. R. Ross, Phys. Rev. Letters **16**, 485 (1966).
[k] Reference 17.
[l] G. Kramer and K. Schilling, Z. Physik. **191**, 51 (1966).
[m] Reference 16.

BeV have led Drell and Trefil,[6] and Ross and Stodolsky,[7] to a similar conclusion. The second reaction, $\pi p \to \rho N^*$, has been observed at several energies and in each case it is found that the DWBA prediction exceeds the observed data by a factor of the order of 2. Moreover, there appears to be a tendency for the predicted production angular distribution to exhibit somewhat less forward peaking than that which is observed.

Next we review the results on the proton-proton–induced reactions. The first of these, namely, elastic scattering, provided one of the early determinations of the pion-nucleon coupling constant. In this case just the high partial waves at relatively low energies were fitted by the one-pion-exchange force. In this respect we should note that in these proton-proton reactions many of the multi-pion resonances may be exchanged as well as the pion itself, and that these other one-particle-exchange mechanisms may contribute significantly to the long-range part of the interaction, especially at higher energies. For instance, it has previously been pointed out that the energy dependence, sign, and

[6] S. D. Drell and J. S. Trefil, Phys. Rev. Letters **16**, 552 (1966); **16**, 832 (1966).
[7] Marc Ross and Leo Stodolsky, Phys. Rev. **149**, 1172 (1966).



(a) (b)

FIG. 1. Typical two-pion-exchange ladder diagrams contributing to elastic scattering; (a) for πp scattering, (b) for ρp scattering.

magnitude of the real part of the forward elastic proton-proton amplitude at high energy (10–30 BeV/c) are all consistent with the behavior that would result from the exchange of vector mesons.[8,9] However, the energy dependences of the inelastic proton-proton induced reactions listed in Table I are observed to follow closely the law obtained with pion exchange, namely, $d\sigma/d|t| = f(t)/sp_i^2$.[10] Thus we assume the vector-particle exchange contributions to these reactions are negligible at the energies of the presently available data. The first of these inelastic reactions, $pp \to pN^{*+}$, has been observed by a number of groups over a wide energy range and the data are some two to three times smaller than the DWBA predictions. Further, the observed production angular distributions are significantly more peaked than the predicted ones. As noted in Table I, the reaction $pp \to N^{*++}n$ has been studied by Alexander *et al.*[11] at 5.5 BeV/c. Independently of any model, isospin conservation requires that

$$d\sigma(pp \to nN^{*++})/d\sigma(pp \to pN^{*+}) = 3. \qquad (2)$$

However, a comparison of the observed data suggests a ratio considerably larger than this, with the result that the DWBA is in closer agreement with the data for the reaction $pp \to nN^{*++}$ than it is for $pp \to pN^{*+}$. Further experimental data on these reactions would be of considerable interest. In this respect it may be worthwhile to remark on the different experimental techniques that have been used in studying these two reactions. In the reaction $pp \to nN^{*++}$ the decay products of the N^* were observed. In the reaction $pp \to pN^{*+}$ in all cases the (scattered) proton momentum spectrum has been observed, from which the desired cross section may be deduced. However, this latter procedure involves a large background subtraction which may account, at

[8] P. C. M. Yock, Ph.D. thesis, M.I.T., 1965 (unpublished).
[9] J. J. Sakurai, Phys. Rev. Letters **16**, 1181 (1966).
[10] Here s and t denote the customary Mandelstam variables and p_i is the projectile momentum in the center-of-mass system.
[11] G. Alexander, B. Haber, A. Shapira, G. Yekutieli, and E. Gotsman, Phys. Rev. **144**, 1122 (1966).

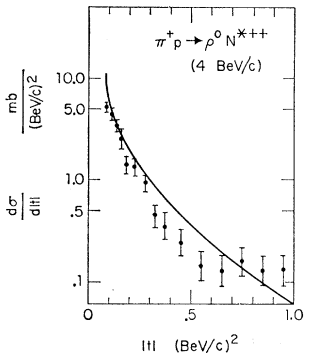


FIG. 2. Comparison between the data of [Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, 897 (1965)], and the DWOPE prediction for the reaction $\pi^+p \to \rho^0 N^{*++}$ at 4 BeV/c. The absorptive parameters are given in the text. The form factor cutoff is chosen to be $18m_\pi^2$.

least in part, for the discrepancy between the data for the two reactions. Finally, we mention the double-isobar production in $p\bar{p}$ collisions. In this case, with the insertion of extra absorption in the final state, the DWBA was found to exceed the observed data by a very nearly constant factor of 5.

The kaon-induced reactions mentioned in Table I have been analyzed in terms of a DWBA in which ρ exchange as well as π exchange is included in the basic interaction mechanism. The inclusion of the ρ exchange amplitude does not greatly effect the results for near-forward production[12] and for these events the DWBA with pion exchange only gives reasonable agreement for K^*N production but overestimates the K^*N^* production.

Referring to the photoproduction processes appearing in Table I, we mention the following facts. First, although the DWBA is in reasonable agreement with observations for the reaction $\gamma p \to \omega p$, the statistics of the presently available data are severely limited, so that a detailed comparison between theory and experiment is not yet possible. The double resonance photoproduction process, $\gamma p \to \omega N^*$, has not yet been observed. However, we may place an upper limit on its cross section at a few GeV. For the total cross section for $\gamma p \to$ anything is observed[13] to be about 80 μb, from which we may subtract at least 25 μb corresponding to photo-ρ production and other identified processes. Thus we obtain a generous upper limit of at most 55 μb for the process $\gamma p \to \omega N^*$. It turns out however, that the DWBA gives a cross section precisely of this order (see Fig. 14), so that we conclude that this model overestimates this reaction also.

Further discussion of these results will be given in the following sections. Summarizing the preceding discussion, though, we see that the DWBA consistently overestimates those reactions in which particles of high spin are produced. Moreover, the overestimation is greatest when two high-spin particles are produced.

[12] This follows as a result of the heaviness of the ρ compared to the π. See also J. H. Friedman and R. R. Ross, Phys. Rev. Letters **16**, 485 (1966).

[13] Cambridge Bubble Chamber Group, Phys. Rev. **146**, 994 (1966).

TABLE II. The couplings of the various amplitudes. In the table, U denotes the spin-$\frac{1}{2}$ spinor, U_α is the spin-$\frac{3}{2}$ spinor, ϵ_α is the transverse spin-one polarization vector, $\epsilon^{\alpha\beta}$ is the symmetric traceless transverse spin-two polarization tensor, and $\epsilon_{\alpha\beta\gamma\delta}$ is the completely antisymmetric fourth-rank tensor.

Vertex	Coupling	Strength ($g^2/4\pi$)	Corresponding (partial) decay width in MeV
$p\pi^0p$	$g\bar{U}(p)\gamma_5 U(p)$	14.5	...
$p\pi^+N^{*++}$	$(g/m_\pi)\bar{U}_\alpha(N^*)U(p)p^\alpha(\pi)$	0.38	125
$\pi^+\pi^-\rho^0$	$2g\epsilon(\rho)\cdot p(\pi)$	2.0	100
$K^+\pi^-K^{*0}$	$2g\epsilon(K^*)\cdot p(\pi)$	1.5	30
$\gamma\pi\omega$	$(g/m_\omega)\epsilon_{\alpha\beta\gamma\delta}\epsilon^\alpha(\omega)\epsilon^\beta(\gamma)p^\gamma(\omega)p^\delta(\pi)$	4.4/137	1
$\pi^+\pi^-f^0$	$(4g/m_f)p_\alpha(\pi^+)\epsilon^{\alpha\beta}(f)p_\beta(\pi^-)$	2.1	80

Further, there appears to be a tendency for the DWBA to underestimate the forward peaking observed in these reactions. We mention also that, where data are available, the decay angular distributions are in reasonable agreement with the DWBA predictions.

III. ABSORPTIVE ONE-PION-EXCHANGE MODEL WITH FORM FACTORS

It is, of course, well known that for physical scattering values of the momentum transfer the Born amplitude, $T_{i,f}^{\text{Born}}(t)$, may not provide a reliable approximation to the exact one-pion-exchange (OPE) amplitude, $T_{i,f}^{\text{OPE}}(t)$. For the exact amplitude, which is the product of the pion propagator and two vertex functions, is known only at the unphysical scattering value of $t=m_\pi^2$. Away from the pion pole one may define a form factor $F(t)$ by equating the exact amplitude to the product $F(t)T_{i,f}^{\text{Born}}(t)$. It is then commonly assumed that this form factor does not differ greatly from unity for $|t| \lesssim 1$ (BeV/c)^2, since there are no states of low mass known to be strongly coupled to the pion. The inclusion of such slowly varying form factors in DWBA calculations has a relatively minor effect. This follows because such form factors affect the low partial waves only, and these are already damped by the initial- and final-state absorptive factors. However, the discrepancies between theory and experiment discussed in Sec. II are not large, so that we now wish to examine in some detail the exact nature of the form factors required to bring the theory into accord with the data for high-spin resonance production. We assume that, in contrast to Eq. (1),

$$T_{i,f}^J = (S_{i,i}^J)^{1/2} T_{i,f}^{J,\text{OPE}} (S_{f,f}^J)^{1/2}, \tag{3}$$

where

$$T_{i,f}^{J,\text{OPE}} = \int T_{i,f}^{\text{OPE}}(t) \mathfrak{D}^J(\Omega) d\Omega, \tag{4}$$

$$T_{i,f}^{\text{OPE}}(t) = T_{i,f}^{\text{Born}}(t) F(t), \tag{5}$$

with

$$F(m_\pi^2) = 1. \tag{6}$$

TABLE III. Initial-state absorption parameters used in the distorted-wave calculations. We disregard both the slight energy dependence and small real parts of the elastic-scattering amplitudes.

Initial state	a_i	R_i (F)
πp	0.76	0.8
pp	1.0	0.83
K^+p	0.55	0.7
γp	0.0	···

Calculations based on Eq. (3) shall be referred to as the distorted-wave one-pion-exchange model, (DWOPE). Note that such calculations have been performed previously by many authors, notably Ross and Shaw,[3] Bander and Shaw,[14] Jackson *et al.*,[15] Högaasen *et al.*,[16] and Maor and Yock.[17]

From Table I it is immediately apparent that to obtain agreement between theory and experiment we must include form factors associated with the $N\pi N^*(1238)$ and $\pi\pi f^0$ vertices only. For ease of calculation it is convenient, though not necessary, to assume a form factor of the form

$$F(t)=(m_c^2-m_\pi^2)/(m_c^2-t), \tag{7}$$

where m_c is an energy-independent "cutoff" mass. The couplings appropriate to the various vertices appearing in the calculations are tabulated in Table II. Calculational techniques for handling the form factor—inclusive analyses have been given elsewhere.[17] We merely point out here that the present calculations are based on partial-wave summations (typically for l to 100) as opposed to the commonly used impact-parameter integral method. The absorptive factors are parameterized in the usual manner:

$$S_{i,i}{}^J=1-a_i\exp[-(J-J_{\min}/p_iR_i)^2], \tag{8a}$$

$$S_{f,f}{}^J=1-a_f\exp[-(J-J_{\min}/p_fR_f)^2], \tag{8b}$$

where p_f denotes the final-state center-of-mass momentum. $J_{\min}=\frac{1}{2}$ for pion-, photon-, and kaon-induced reactions and $=0$ for proton-induced reactions. The parameters a_i and R_i are obtained by analysis of appropriate elastic-scattering data (see Table III). For the final-state absorptive parameters we adhere to the following convention throughout this paper:

$$a_f=1, \quad R_f=R_i. \tag{9}$$

The results do not depend too sensitively on the values of these parameters. We now examine, in terms of the DWOPE model, reactions involving N^* and f^0 production.

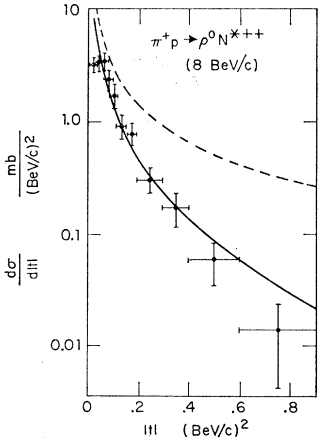


FIG. 3. The reaction $\pi^+p\to\rho^0N^{*++}$ at 8 BeV/c. The solid (broken) curve is the prediction of the DWOPE (DWBA) model, with parameters unchanged from those of Fig. 2. The data is from the Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965).

A. $\pi p\to\rho N^*$

In this case, good agreement with the data, as shown in Figs. 2 and 3, is obtained with the cutoff set at $m_c^2=18m_\pi^2$. The ρ and N^* spin-density matrix elements, as functions of momentum transfer, are given in Figs. 4 and 5. For comparison the DWBA predictions are included. As expected the DWOPE density matrix elements are generally closer to the pure OPE predictions, although there is still a substantial deviation. The agreement between the data, which are given with the figure captions, and the DWOPE model is satisfactory.

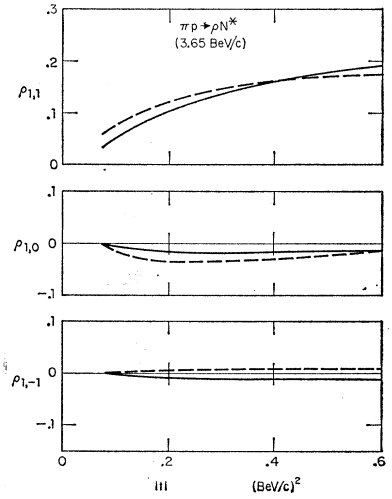


FIG. 4. ρ spin-density matrix elements in the reaction $\pi p\to\rho N^*$ at 3.65 BeV/c. The solid curves are the predictions of DWOPE model, and the broken curves are those of the DWBA. [G. Goldhaver, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. Marshall (University of Colorado Press, Boulder, Colorado, 1965), Vol. VII B, p. 343], reports the following experimental values for the density matrix averaged over $0<|t|<20m_\pi^2$: $\langle\rho_{1,1}\rangle=0.14\pm0.02$, $\langle\rho_{1,-1}\rangle=-0.003\pm0.04$, and $\langle\mathrm{Re}\rho_{1,0}\rangle=-0.044\pm0.03$. Note that these ρ spin-density matrix elements refer to that ρ rest frame with z axis along the incident π momentum. In each of the following reactions the spin-density matrix elements will be given in the analogous reference frame appropriate to the one-particle-exchange mechanism.

[14] M. Bander and G. L. Shaw, Phys. Rev. **139**, B956 (1965).
[15] J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. **139**, B428, (1965).
[16] H. Högaasen, J. Högaasen, R. Keyser, and B. E. Y. Svensson, Nuovo Cimento **42**, 323 (1966).
[17] U. Maor and P. C. M. Yock, Phys. Rev. **148**, 1542 (1966).

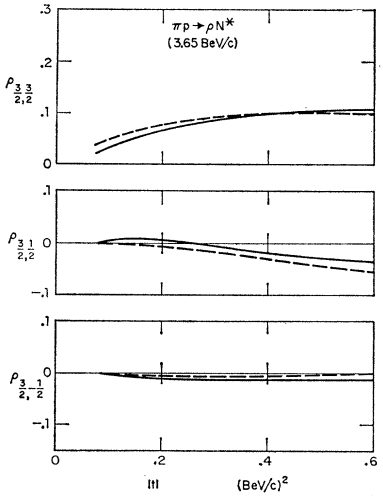


FIG. 5. N^* spin-density matrix elements in the reaction $\pi p \to \rho N^*$ at 3.65 BeV/c. The meaning of the curves are the same as those in Fig. 4. The experimental values [G. Goldhaber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. Marshall (University of Colorado Press, Boulder, Colorado, 1965), Vol. VII B, p. 343], again averaged over $0<|t|<20m_\pi^2$ are $\langle\rho_{3/2,3/2}\rangle=0.05\pm0.03$, $\langle \mathrm{Re}\rho_{3/2,-1/2}\rangle=0.013\pm0.03$, and $\langle \mathrm{Re}_{3/2,1/2}\rangle=-0.045\pm0.04$.

B. $pp \to pN^{*+}$ and $pp \to nN^{*++}$

Having set the cutoff mass at $18m_\pi^2$, the theory is now completely determined for these and the remaining reactions involving the $N^*(1238)$. The agreement bebetween the DWOPE model and experiment, as shown in Figs. 6–8, is satisfactory except for a moderate deviation from the data for the reaction $pp \to pN^{*+}$ (but not for $pp \to nN^{*++}$) at the intermediate energies. We remind the reader of the remarks in Sec. II alluding to these reactions. The N^* density matrix elements are given in Fig. 9 and are in fair agreement with the values quoted by Alexander *et al.*[11]

C. $Kp \to K^*N^*$

The predictions of the DWOPE model were compared with the presently available data for this reaction at 3 and 3.5 BeV/c.[18,19] As shown in Figs. 10 and 11, the data for the K^* and N^* spin-density matrix elements at 3.5 BeV/c are in good agreement with the theory. At 3 BeV/c the agreement was found to be satisfactory. The experimental cross section at 3 BeV/c, as shown in Fig. 12, is in good agreement with the predicted cross section. At 3.5 BeV/c the DWOPE differential cross section, as shown in Fig. 13, appears to be too small by a constant factor of about $\frac{2}{3}$. In this case the DWBA prediction, as shown by the broken curve, gives an equally good fit to the experimental cross section. Note that in all cases we have set $m_c^2=18m_\pi^2$, and that a ρ exchange term is not included. The cross sections in Figs. 12 and 13 are normalized to the following charge mode: $K^+p \to K^{*0}N^{*++} \to K^+\pi^-p\pi^+$.

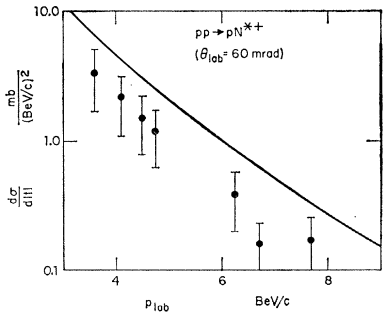


FIG. 6. Comparison between the CERN data [G. Cocconi, E. Lillethun, J. P. Scanlon, C. A. Stahlbrandt, C. C. Ting, J. Walters, and A. M. Wetherell, Phys. Letters **8**, 134 (1964)], and the DWOPE prediction for the reaction $pp \to pN^{*+}$ at $\theta_{\mathrm{lab}}=60$ mrad and intermediate energies. The absorptive parameters are given in the text, and the form-factor cutoff is kept, as in the previous figures, at $18m_\pi^2$.

D. $\gamma p \to \omega N^*$

In this case only final-state absorption is included, with $a_f=1$ and $R_f=0.8$ F. The results with and without the form factor are shown for the purpose of comparison in Fig. 14.

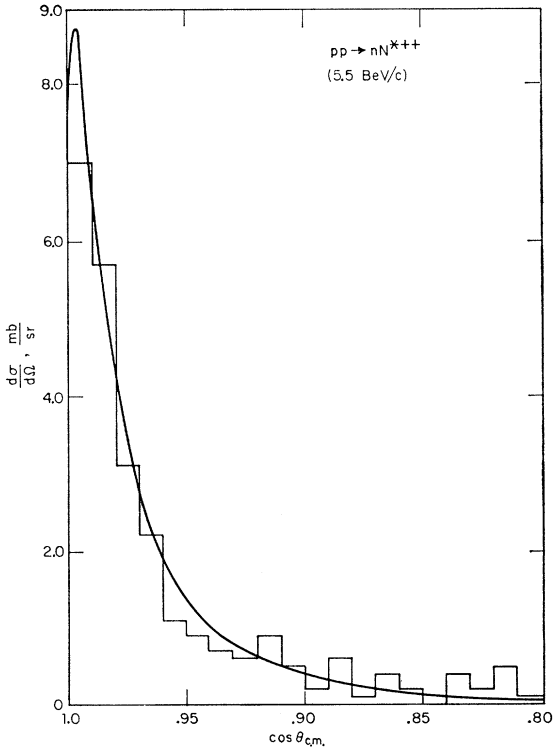


FIG. 7. Comparison between the data of Alexander *et al.* (Ref. 11) and the DWOPE prediction for the reaction $pp \to nN^{*++}$ at 5.5 BeV/c. The DWOPE model parameters are the same as those in Fig. 6.

[18] F. Gard, J. Debaisieux, J. Heughebaert, L. Pape and R. Windmolders, in *Proceedings of the Twelfth Annual International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[19] M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. Leith, G. Lynch, F. Muller, and J. M. Perreau, Nuovo Cimento **39**, 417 (1965).

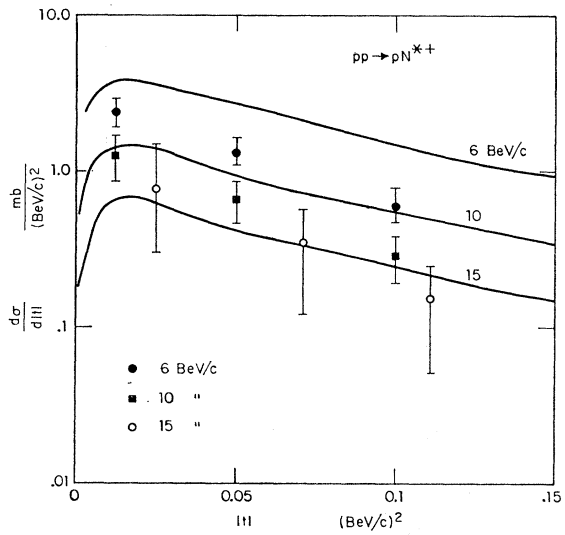


FIG. 8. Comparison between the Brookhaven data [E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hein, T. J. McMahon, and I. Hadelhaft, Phys. Rev. Letters **16**, 855 (1966)], and the DWOPE predictions for the reaction $pp \to pN^{*++}$ at 6, 10, and 15 BeV/c. The DWOPE-model parameters are unchanged from those in Figs. 6 and 7.

E. $\pi N \to f^0 N$

Data for this reaction are now available at 4,[20] 6,[21] 8,[22] and 10 BeV/c.[23] Good agreement as shown in Fig. 15 is obtained with the production differential cross section at 6 BeV/c by setting $m_c^2 = 18m_\pi^2$. With this value of m_c, the predicted cross sections at 4 and 8 BeV/c were found to be in equally good agreement with the data.

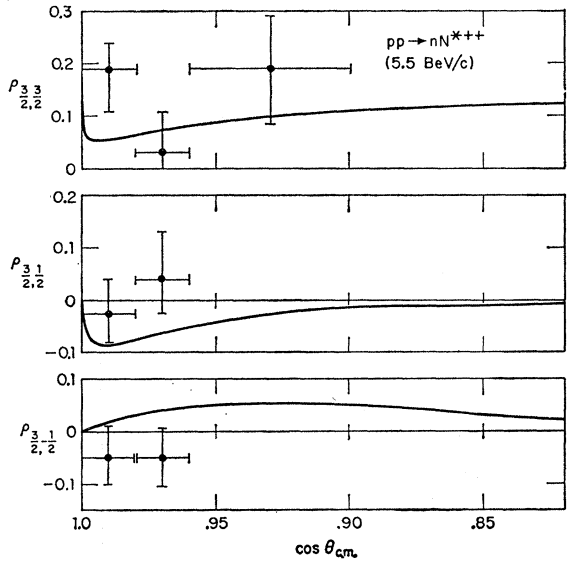


FIG. 9. N^* spin-density matrix elements in the reaction $pp \to nN^{*++}$. The curves are the DWOPE predictions and the experimental data those of Alexander *et al.* (Ref. 11).

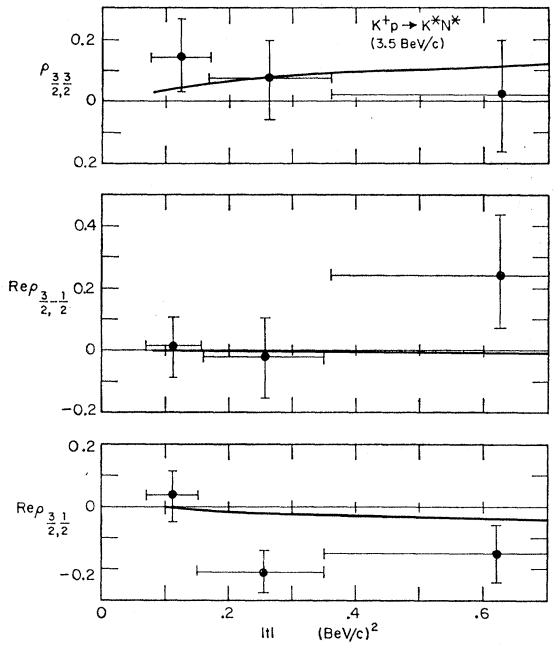


FIG. 10. K^* spin-density matrix elements in the reaction $K^+p \to K^*N^*$ at 3.5 BeV/c. The absorptive parameters are given in the text, and the form-factor cutoff is kept, as in the previous reactions, at $18m_\pi^2$. The data is from Ref. 18. Note that a ρ exchange term is not included in the basic interaction mechanism.

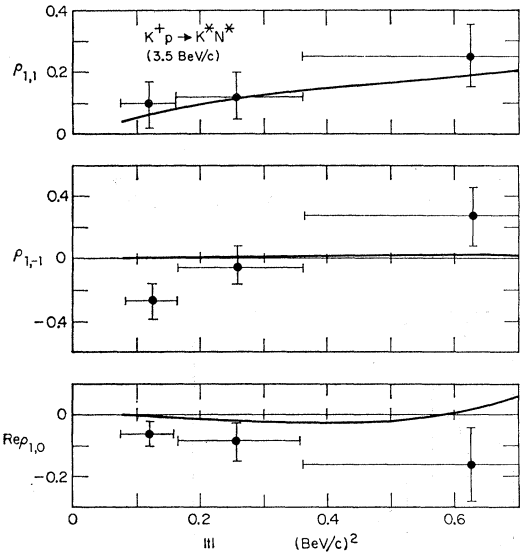


FIG. 11. N^* spin-density matrix elements in the reaction $K^+p \to K^*N^*$ at 3.5 BeV/c. The DWOPE-model parameters are unchanged from those of Fig. 10 and the data is from Ref. 18.

[20] Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento **31**, 729 (1964).

[21] F. Bruyant, M. Goldberg, C. Vegni, S. H. Winzeler, P. Fleury, J. Hune, R. Lestienne, G. de Rosny, and R. Vanderhaghen, in *Proceedings of the Twelfth Annual International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965).

[22] H. Yuta *et al.*, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

[23] M. Wahlig, E. Shibata, D. Gordon, D. Frisch, and I. Mannelli, Phys. Rev. **147**, 941 (1966).

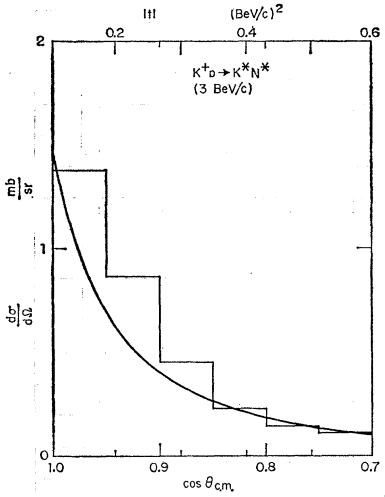


FIG. 12. Comparison between the data of Ref. 19 and the DWOPE prediction for the reaction $K^+p \to K^{*0}N^{*++} \to K^+\pi^-p\pi^+$ at 3 BeV/c. The parameters of the theory are unchanged from those in Fig. 10.

However, at 10 BeV/c the predicted cross section (as shown in Fig. 16) exceeds the reported cross section by a factor of about 2. This figure also shows the DWBA cross section for this reaction.

We remark that for the 10 BeV/c data the f^0 is observed by means of its $\pi^0\pi^0$ decay mode, whereas for the other data the f^0 is observed by means of its $\pi^+\pi^-$ decay modes. To obtain the experimental f^0 cross section it is crucial to obtain a reliable estimate of the background beneath the f^0 peak, a subtraction which was difficult at 10 BeV/c. In data involving the charged decay of f^0, two aspects facilitate an easier estimation of the required background subtraction. (1) The geometry of the experiments permitted full 4π solid-angle detection efficiency. This was not the case in the 10-BeV/c spark chamber experiment. (2) The familiar ρ peak is observed in the $\pi^+\pi^-$ histogram to the left of the f^0, which could in principle be advantageous in elucidating the amount of background. This again is not the case in the 10-BeV/c data. There, to the left of the f^0, is a low-mass S-wave enhancement stretching from threshold to about 800 MeV, whose precise cross section

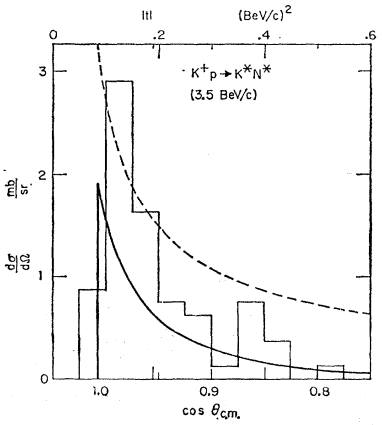


FIG. 13. The reaction $K^+p \to K^{*0}N^{*++} \to K^+\pi^-p\pi^+$ at 3.5 BeV/c. The solid (broken) curve is the DWOPE (DWBA) prediction and the data are from Ref. 18.

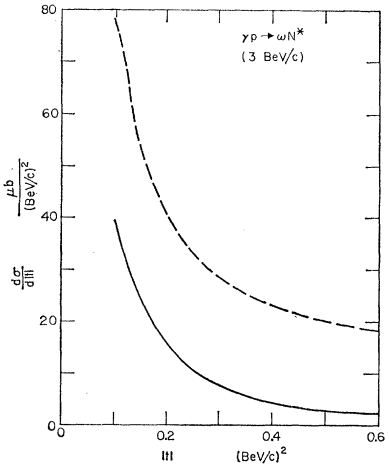


FIG. 14. Comparison between the DWOPE (solid curve) and DWBA (broken curve) for the reaction $\gamma p \to \omega N^*$ at 3 BeV/c. Note that absorption is included in the final state only, and that the form-factor cutoff is set at $18m_\pi^2$. The cross sections are normalized to a width of 1 MeV for the decay $\omega \to \pi\gamma$.

proved difficult to determine. This is due to contamination (15–40%), in that mass region only, of $3\pi^0$ events in which one π^0 escapes detection. Because of points (1) and (2) we believe that the discrepancy at 10 BeV/c cannot be taken as a serious argument against the DWOPE model.

Density matrix elements are given in Fig. 17. Present published data on these decay parameters are too poor to enable a definitive distinction to be made among the predictions of the DWOPE, DWBA, and the pure OPE model. This is again related to the problem of back-

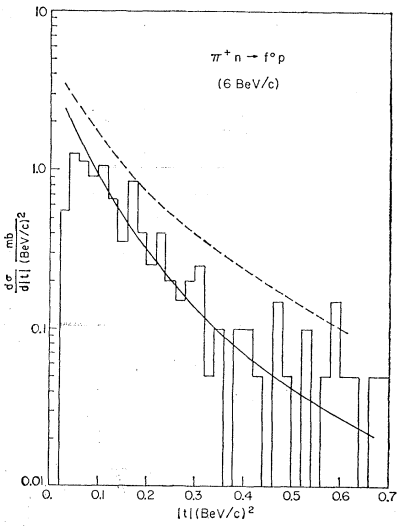


FIG. 15. Comparison between the data of Ref. 21 and the DWOPE prediction for the reaction $\pi^-p \to f^0n$ at 6 BeV/c. The absorption parameters are given in the text and the form-factor cutoff is set at $18m_\pi^2$. The normalization of the data is taken from Ref. 16.

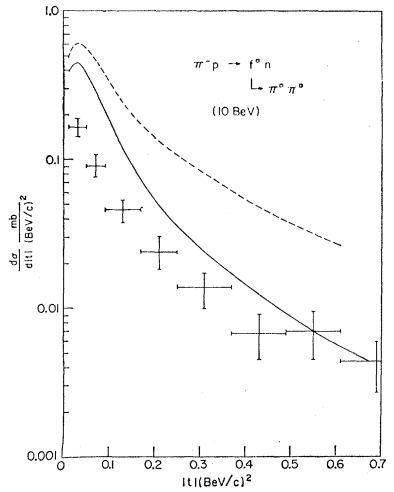


FIG. 16. The reaction $\pi^- p \to f^0 n$ at 10 BeV/c. The solid (broken) curve is the prediction of the DWOPE (DWBA) model, with parameters unchanged from those of Fig. 15. The data are from Ref. 23.

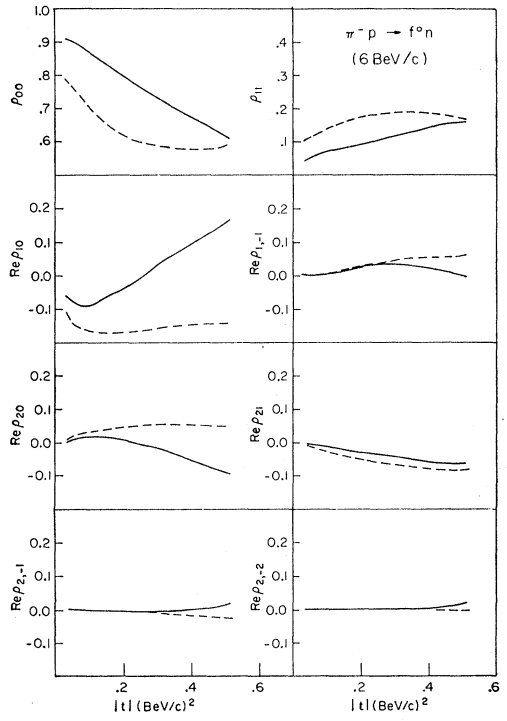


FIG. 17. f^0 spin-density matrix elements in the reaction $\pi^- p \to f^0 n$ at 8 BeV/c. The solid (broken) curve is the predictions of the DWOPE (DWBA) model. The DWOPE-model parameters are again unchanged from those in Fig. 15. {Density matrix elements from preliminary data [K. W. Lai and J. M. Scarr (private communication)], at 6 BeV/c appear to be in better agreement with the DWBA predictions than the DWOPE predictions. However, as in the data of Yuta *et al.* at 8 BeV/c, ρ_{22} becomes negative due to background effects, thus precluding a definitive distinction. We further mention that the 8-BeV/c data (Ref. 22) are in fair agreement with ρ_{00} but not with any other matrix element.}

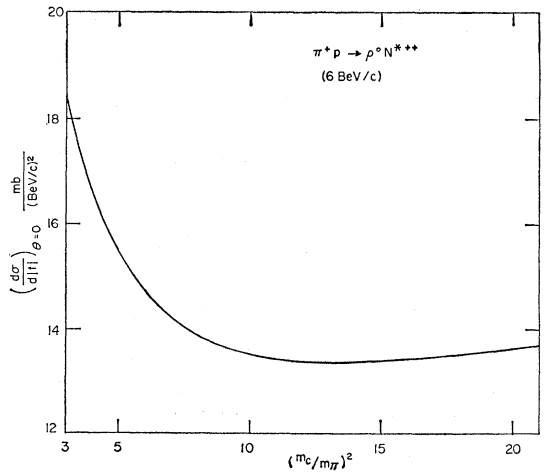


FIG. 18. The forward differential cross section for the reaction $\pi^+ p \to \rho^0 N^{*++}$ at 6 BeV/c in a DWOPE model with a step-function form factor as given in Eq. (10). Note that, at 6 BeV/c, $|t|_{\theta=0^\circ} = 2m_\pi^2$.

ground. This is indicated in the 8 BeV/c data of Yuta *et al.*[22] in which ρ_{22}, a parameter which must be positive-definite, is negative. (For $|t| \leq 0.3$ BeV/c^2 they obtain $\rho_{22} = -0.17 \pm 0.04$.)[22]

IV. DISCUSSION

The first point to be made concerning the preceeding analysis concerns the range of t values over which the form factor, Eq. (7), with m_c^2 set equal to $18m_\pi^2$ for both the $N\pi N^*$ and $\pi\pi f^0$ vertices, may be considered to have been reliably determined in terms of the DWOPE theory. We have already remarked that since the insertion of absorption reduces the lower partial-wave amplitudes, the form factor will have little effect for large $|t|$. To see this quantitatively consider the reaction $\pi^+ p \to \rho^0 N^{*++}$ and, instead of inserting the Ferrari-Selleri form factor, Eq. (7), insert a step function form factor

$$F(t) = \theta(t + m_c^2). \tag{10}$$

The forward differential cross section so obtained (at 6 BeV/c) is shown in Fig. 18 as a function of m_c^2. From the figure it is apparent that for $|t| \gtrsim 7m_\pi^2$ the absorptive OPE model is insensitive to the inclusion of form

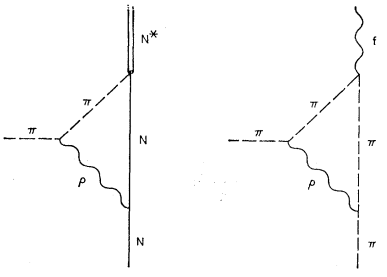


FIG. 19. Typical triangle Feynman diagrams which include a $\pi\rho$ intermediate state, contributing to the physical $N\pi N^*$ and $\pi\pi f^0$ vertices.

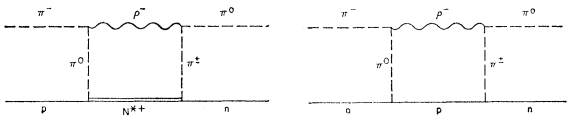


FIG. 20. Two-pion-exchange processes contributing to pion-nucleon charge-exchange scattering.

factors. We thus conclude that the form factors take the values

$$F_{N\pi N^*}(t)\simeq F_{\pi\pi f}(t)\simeq(18/19)(1+t/18m_\pi^2) \quad (11)$$

for $0\lesssim -t\lesssim 7m_\pi^2$. Outside this range of t values no information on these form factors can be gotten from the DWOPE model. Thus the analytic structure, $F(t)=(m_c^2-m_\pi^2)(m_c^2-t)^{-1}$, used in the calculations by no means implies the existence of a $T=1$ pseudoscalar meson at a mass around 600 MeV.

In this respect we mention that many triangle Feynman diagrams may be expected to contribute to the full $N\pi N^*$ and $\pi\pi f^0$ vertices. Of these, two typical ones are shown below in Fig. 19. However, following the prescription given by Landau *et al.*,[24] it was found that none of the triangle diagrams involving a $\pi\rho$ intermediate state has an anomalous threshold.

A number of other approaches to the problem of the failure of the DWBA in reactions involving high-spin particles were examined. First, the straightforward insertion of extra absorption beyond that suggested by elastic-scattering data was considered. However, this approach may be criticized on the grounds that the absorption radii R_i and R_f are required to be ~ 1.5 F to bring the DWBA predictions down to the observed data, and this range is about as great as that of the one-pion-exchange interaction itself, which does not give rise to absorptive scattering.[11,25] Moreover it is not obvious how a strong correlation could exist between spins of resonances and the widths of their elastic-scattering diffraction peaks. On the other hand, form-factor effects may well be most apparent in high-spin resonance production, since the Born amplitudes for such reactions depend strongly on the spins of the particles involved.

Durr and Pilkuhn[26] have given a prescription for calculating at least a part of the vertex functions, arising solely from kinematical factors. However, the vertex functions we have found to be necessary are considerably more damped than those obtained by Durr and Pilkuhn. In this respect we mention the well-known method of Ferrari and Selleri,[27] which involves no absorption but very strongly damped ($m_c^2\simeq 6m_\pi^2$) form factors. We believe the present DWOPE method may be preferable because of the less violent character of the form factors used in the latter model, and also because of the significant departure observed in the various final-state resonant decay distributions from those predicted by the pure OPE mechanism.

Further indirect evidence for the necessity of including these vertex-function form factors is provided by a consideration of the absorptive part of the π^-p charge-exchange amplitude, which is known experimentally. A major contribution to this amplitude may be expected to arise from the two-pion-exchange (TPE) process depicted in Fig. 20(a). Note that one-pion exchange is forbidden in this process. The absorptive part of the TPE diagram may be calculated by the method proposed by Maor and Yock.[17] However, this procedure leads to a result which, without the inclusion of form factors, exceeds the observed amplitude by a factor of the order of 5 at projectile momenta of a few BeV/c. Moreover, the predicted absorptive amplitude is of the wrong sign. It is then not apparent which diagrams could cancel the contribution from the diagram. However, if one includes the previously determined $N\pi N^*$ form factor, the amplitude becomes comparable, in magnitude, to that which arises from the analogous TPE diagram shown in Fig. 20(b), with a proton at the bottom rung. Thus, since the latter diagram gives a contribution of the opposite sign to that of the former, we obtain the result that the two leading contributory TPE diagrams partially cancel. One is then lead to consider, as dominant interaction mechanisms, exchanges of heavier systems such as 4 π's and ρ's. These interaction mechanisms have relatively short interaction ranges, so that a somewhat broader differential cross section might be expected to result. Qualitative evidence supporting this reasoning is indeed provided by the rather flat shape observed in the differential cross section near the forward direction.[28] Further details of these and other TPE calculations will be published elsewhere. Note, though, that a similar cancelation also occurs in the leading TPE contributions to the reaction $\pi^-p\rightarrow\omega n$, for which the observed differential cross section shows very little peaking.

V. CONCLUSIONS

As shown in Table I, the absorptive one-pion-exchange model (without form factors) consistently overestimates the production of particles of spin greater than unity. Moreover, for each process, the actual amount of overestimation appears to be approximately proportional to the number of vertices containing high-spin particles. The theory may be brought into agreement with experiment by the inclusion of momentum-

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[25] P. C. M. Yock, Nuovo Cimento **44**, 777 (1966).
[26] H. P. Durr and H. Pilkuhn, Nuovo Cimento **40A**, 899 (1965).
[27] E. Ferrari and F. Selleri, Phys. Rev. Letters **7**, 387 (1961). See also E. Ferrari (to be published). In this paper the author raises criticism against the absorption model.
[28] G. Backenstoss, B. Hyams, W. Männer, A. Reale, and U. Stierlin, Nuovo Cimento **47**, 814 (1966); I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters **14**, 408 (1965); A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzasio, and B. Amblard, *ibid.* **14**, 763 (1965). Each of these groups reports a flat differential cross section for $|t|<5m_\pi^2$.

transfer-dependent vertex-function form factors. Form factors so obtained for the $N\pi N^*(1238)$ and $\pi\pi f^0$ vertices are given in Eqs. (11).[29]

[29] Note that in most of the reactions discussed in this paper, better fits to the data may be obtained by slight adjustment of the absorption and form-factor parameters. However, the interpretation of these modifications is ambiguous. Moreover, we believe that the present status of experimental and theoretical knowledge concerning these reactions is insufficient to warrant such refinements of the model.

ACKNOWLEDGMENTS

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Interference Effects in ρ-Meson Decay*

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In this paper, the asymmetry in the decay of the ρ meson produced in the reaction $\pi^-+N\to\rho+N$ is studied in terms of interference effects due to the coherent production of pairs of pions in other isospin states. The hypotheses of a ρ-ϵ (s-wave, $T=0$ pion pair) interference, a constant π-π phase shift, and other mechanisms are studied. The theoretical predictions resulting from these models are compared with recent experimental results on the decay of the neutral and charged ρ meson.

I. INTRODUCTION

THE asymmetry[1] which is observed in the decay of the neutral ρ meson has led to considerable speculation about the $T=0$, s-wave pion-pion phase shift. Vavious hypotheses have been suggested to explain this phenomenon. One explanation[2] assumes a resonance with a mass and width approximately of that of the ρ meson; this resonance is known as the ϵ. A second hypothesis[3,4] has the s-wave phase shift going down through $\pi/2$ at a dipion mass of approximately 0.76 BeV, while skeptics have suggested that a constant phase shift might be a simpler explanation. New experimental results[5,6] with much improved statistics are now available making a more detailed theoretical study worthwhile.

Such a comparison requires a model for ρ and ϵ production (for convenience we shall refer to a $T=0$, s-wave pair of pions as an ϵ whether or not they are in a resonant state). We shall use the absorptive peripheral model with one-pion exchange (OPE), as described in Sec. II.

In this paper, we shall study the decay of the neutral ρ meson in terms of the ρ-ϵ interference, and compare various theoretical predictions for the s-wave phase shift with the new experimental results. This is done in Sec. III. We shall also consider the effect of $T=2$ dipion production on the asymmetry of both the neutral and the charged ρ decay.

II. GENERAL FORMALISM

The absorptive peripheral model with OPE has had many successes, in particular in explaining the production cross sections and the spin-density matrices for ρ production.[7] It should be remarked here that in such production processes the exchange pion is always off the mass shell. Thus, any phase shift deduced from

* This work is partially supported by the U. S. Atomic Energy Commission and the National Science Foundation.

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[2] L. Durand and Y. Chiu, Phys. Rev. Letters **14**, 329 (1965); **14**, (E) 680 (1965).

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[6] D. R. Clear, T. F. Johnston, J. Pilcher, J. D. Prentice, N. R. Steenberg, E. West, T. S. Yoon, W. A. Cooper, W. Manner, L. Voyvodic, and W. D. Walker, Nuovo Cimento (to be published).

[7] For a detailed study, see J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Svensson, Phys. Rev. **139**, B428 (1965).