Tests of Unitary Symmetry in Nuclei by Meson-Nucleus Reactions*

L. S. KISSLINGER†
Brookhaven National Laboratory, Upton, New York
and
Western Reserve University, Cleveland, Ohio

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Tests of unitary symmetry in nuclei by coherent reactions leading to hypernuclei are examined. Starting from the two-particle amplitudes for elastic and strangeness-changing meson-baryon reactions, a pseudopotential is derived which couples the nucleus and hypernucleus channels. Using experimental cross sections, numerical estimates for various processes are made in order to appraise the feasibility of experiments, and it is found that experimental tests are possible with present techniques.

I. INTRODUCTION

THE classification of baryons and mesons by the higher symmetries has had significant success, and much of the present experimental data on these systems can be incoprorated into the $SU(3)$ octet model.[1] It has been recognized that the many-baryon systems can be placed into $SU(3)$ supermultiplets, and that states of approximately the same special form will be connected by the generators of $SU(3)$.[2] The isobaric analog states are one example of such states.[3] If the strong baryon-baryon interaction is approximately a scalar under $SU(3)$, the hypernucleus states which are formed by the corresponding strangeness-changing process will also have narrow widths.[4] It is the purpose of this work to study meson-nucleus reactions in order to estimate the cross sections for the various processes and determine which of these, if any, are suitable for an experimental test of the existence of these new analog states as approximate eigenstates.

The $SU(3)$ group is of rank two and order eight. Thus there are six step-up and step-down operators whose directions in the two-dimensional I_3 and hypercharge space are given by the root diagram. In addition to the I spin, the two other (strangeness-changing) "angular-momentum" operators are referred to as U spin and V spin.[5] Although the problem of classifying a system of A baryons according to $SU(3)$ is a complex one,[2] the classification of the ground states of nuclei is quite simple. Recognizing that neutrons and protons are both states of maximum z component in their respective U-spin or V-spin multiplets, we make the following observations: (a) The U spin and V spins of nuclei are unique; (b) the neutrons and protons can be uncoupled trivially; (c) the U and V spins are large, i.e., $N+\frac{1}{2}Z$ and $Z+\frac{1}{2}N$, respectively. Therefore, the reduced matrix elements of the strangeness-changing generators of the Lie group will be large and vary rapidly from nucleus to nucleus compared to the corresponding I-spin matrix elements [the ground-state I spin is $|(Z-N)/2|$ because of the dynamics].

The proposed tests are then quasielastic meson-nucleus reactions leading to the unitary analog states. The formalism for calculating the cross section from the two-body information, which was developed mainly for the study of elastic scattering by complex systems,[6] is reviewed in the next section. Application is made to various processes in Sec. III, where the width is briefly discussed.

II. METHOD

The reactions under consideration involve transitions from an initial nuclear state to a state of the hypernucleus which would be at the same energy except for mass differences and differences in interaction potentials. A pseudopotential (optical potential) can be derived for such scattering processes in which only the coordinates of the mesons appear along with operators which can change the $SU(3)$ quantum numbers of the mesons and of the nucleus as a whole. That is, we look for terms in the pseudopotential like $\mathbf{u}^m\cdot\mathbf{U}^N$ or $\mathbf{v}^m\cdot\mathbf{V}^N$, where $\mathbf{U}^N=\sum_{j=\text{neutrons}}\mathbf{U}_j$, $\mathbf{U}_j$ is the U-spin operator of the jth baryon, and u^m is the meson $\mathbf{U}$-spin operator. For example, $u_+{}^m|\pi^+\rangle=|K^+\rangle$, $u_+{}^m|K^-\rangle=|\pi^-\rangle$, $U_-|p\rangle=|\Sigma^+\rangle$, and $U_-|n\rangle=[|\Sigma^0\rangle+\sqrt{3}|\Lambda\rangle]/2$. The operators v^m and V^N are the V-spin operators defined in the analogous manner, with $v_+{}^m|\pi^-\rangle=|K^0\rangle$, $v_+{}^m|K^-\rangle=[|\pi^0\rangle+\sqrt{3}|\eta\rangle]/2$, $V_-|n\rangle=|\Sigma^-\rangle$, and $V_-|p\rangle=[|\Sigma^0\rangle+\sqrt{3}|\Lambda\rangle]/2$.[7] Thus the resulting psuedopotential is of the form similar to the phenomenological pseudopotential used in the calculations of isobaric analog states.[3,8]

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[1] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961); M. Gell-Mann, Caltech Report No. CTSL-10, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).

[2] R. J. Oakes, Phys. Rev. **131**, 2239 (1963); V. I. Ogievetski and H. Ting-Lhang, Phys. Letters **9**, 354 (1964); I. S. Gerstein, Nuovo Cimento **32**, 1706 (1964); H. Lipkin, Phys. Rev. Letters **14**, 18 (1965); N. Panchapa, Nuovo Cimento **39**, 581 (1965); Y. Tomozawa, Phys. Rev. **138**, B1558 (1965).

[3] A. M. Lane, Nucl. Phys. **35**, 676 (1962).

[4] H. Feshbach and A. Kerman, *Preludes in Theoretical Physics* (North-Holland Publishing Company, Amsterdam, 1966), p. 260.

[5] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

[6] For references see M. I. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), Chap. 11.

[7] Note that for n or $p\to\Lambda$, the Λ part of the $|(\Lambda,\Sigma^0)U=1, U=0\rangle$ state must be projected out.

[8] D. Robson, Phys. Rev. **137**, B535 (1965).

This is similar to the proposed tests of I spin in nuclei by meson reactions.[9]

The potential is defined in terms of the coherent scattering operator for the scattering of a projectile by a complex system. Using the impulse approximation for the scattering in the complex system, the psuedopotential in coordinate space has the form[6]

$$\langle x|V_c|\varphi_m\rangle=\frac{1}{(2\pi)^3}\sum_j\int\int\int\langle q'|t_j|q\rangle e^{-i(\mathbf{q}'-\mathbf{q})\cdot Z_j}\rho(Z_j)\times e^{-iq'\cdot x}\varphi_m(q)dq^3dq'^3d^3Z_j$$
$$+\text{absorptive terms}+\text{correlations terms}. \quad (1)$$

The nuclear particle density $\rho(Z_j)$ is normalized to unity, φ_m is the meson wave function, and $\langle\mathbf{q}'|t_j|\mathbf{q}\rangle$ is the scattering amplitude for the projectile scattered from the jth nucleon with momentum transfer $\mathbf{q}'-\mathbf{q}$. The "correlation terms" arise from correlations in the many-body system and are neglected. Making a partial-wave expansion of the scattering amplitude, one obtains

$$\langle q'|t_j|q\rangle=\sum_l\sum_{\mathcal{Q}}[(l+1)T_{l,\,j=l+1/2}(\mathcal{Q})+lT_{l,\,j=l-1/2}(\mathcal{Q})]\times P_l(\cos\theta)\mathcal{P}(\mathcal{Q}), \quad (2)$$

where θ is the scattering angle and $\mathcal{P}(\mathcal{Q})$ is the operator which projects out the quantum number $\mathcal{Q}$, chosen for convenience in the various processes. The spin-flip terms have been omitted, since only spinless nuclear states will be considered. The partial-wave scattering amplitudes $T_{lj}(\mathcal{Q})$ are defined in terms of the phase shifts as

$$T_{lj}(\mathcal{Q})=-[\exp i\delta_{lj}(\mathcal{Q})\sin\delta_{lj}(\mathcal{Q})]/\pi Eq(\text{diagonal}) \quad (3)$$
$$=i[\exp 2i\delta_{lj}(\mathcal{Q})]/\pi Eq(\text{off diagonal}).$$

Using experimental values for the phase shifts, Eqs. (1)–(3) give the pseudopotential, except that the absorptive terms must be separately estimated.

In order to avoid derivative terms in the optical potential, we will stay within the energy regions where an S-wave treatment is satisfactory and we will use a constant-scattering-length approximation.[10] These regions are generally most favorable for carrying out experiments. For each application, the two-body data is studied to find this region of applicability. One detailed description will be given in the next section to illustrate the method.

III. APPLICATIONS

We restrict ourselves to reactions with charged meson beams, because of the experimental difficulty in using neutral beams. Only Λ-hypernuclear states are treated since these will have widths given by the $SU(3)$ violations. Since each process has its own personality, we will discuss them individually. The calculation for the first process is discussed in some detail to illustrate the method and display the approximations.

A. $\pi^++(Z,N)\rightarrow K^++(Z,N-1,\Lambda)$

This is a U-spin-flip process which arises from the basic process $\pi^++n\rightarrow K^++\Lambda$. Restricting ourselves to S waves, the π-nucleon scattering amplitudes are

$$\langle\pi^\pm n|t|\pi^\pm n\rangle=(1/4\pi)\langle\pi^\pm n|a+b\mathbf{u}^m\cdot\mathbf{U}^n|\pi^\pm n\rangle, \quad (4a)$$
$$\langle K^+\Lambda|t|\pi^+n\rangle=(1/4\pi)\langle K^+\Lambda|a+b\mathbf{u}^m\cdot\mathbf{U}^n|K^+\Lambda\rangle, \quad (4b)$$
$$\langle\pi^\pm p|t|\pi^\pm p\rangle=(1/4\pi)\langle\pi^\pm p|a-b\mathbf{u}^m\cdot\mathbf{U}^n|\pi^\pm p\rangle, \quad (4c)$$

where

$$a=2T_{l=0}{}^{I=3/2}+T_{l=0}{}^{I=1/2},$$
$$b=T_{l=0}{}^{I=3/2}-T_{l=0}{}^{I=1/2}. \quad (5)$$

From the π-nucleon scattering one can find the relationship between the I-spin and U-spin phase shifts. This has been done in obtaining the results of Eqs. (4) and (5) (note that $\pi^\pm n$ involves $U=\frac{1}{2}$ and $\frac{3}{2}$ while $\pi^\pm p$ involves $U=0$ and 1). The pseudopoential obtained by substituting Eqs. (4) and (5) into Eq. (1) leads to a coupled-channel Klein-Gordon equation. Neglecting terms of order $(V/E)^2$, the coupled set of differential equations is

$$\left\{-\frac{\nabla^2}{2E_\pi}+2\pi^2[Aa+b(Z-N)+ic]\rho(x)-\mathcal{E}_\pi\right\}\varphi_\pi(x)=2\pi^2(2N)^{1/2}b\rho(x)\varphi_K(x),$$
$$\left\{-\frac{\nabla^2}{2E_K}+2\pi^2[Aa+b(Z-N+1)+id]\rho(x)-\mathcal{E}_K\right\}\varphi_K(x)=2\pi^2(2N)^{1/2}b\rho(x)\varphi_\pi(x), \quad (6)$$

with the boundary conditions

$$\varphi_\pi(\mathbf{x})\underset{r\to\infty}{\longrightarrow}e^{ik_\pi Z}+f_\pi(\theta)\frac{e^{ik_\pi r}}{r},$$
$$\varphi_K(\mathbf{x})\underset{r\to\infty}{\longrightarrow}f_K(\theta)\frac{e^{ik_K r}}{r}. \quad (7)$$

In Eqs. (6) and (7), E_π, E_K, k_π, and k_K are the π- and K-meson total energies and three-momenta, $A=N+Z$ is the nuclear mass number, and $\mathcal{E}_\pi$ and $\mathcal{E}_K$ are defined by

$$\mathcal{E}_\pi=(E_\pi{}^2-m_\pi{}^2)/2E_\pi,$$
$$\mathcal{E}_K=(E_K{}^2-m_K{}^2)/2E_K.$$

The constants c and d represent purely absorptive processes and are neglected in the following estimates. Therefore the production amplitude for the scattering

[9] A. K. Kerman and R. K. Logan, in *Proceedings of the International Conference on Nuclear Spectroscopy with Direct Reactions*, edited by F. E. Throw (Argonne National Laboratory, Argonne, Illinois, 1964), Report No. ANL 6878, p. 236.

[10] L. S. Kisslinger, Phys. Rev. **98**, 761 (1955).

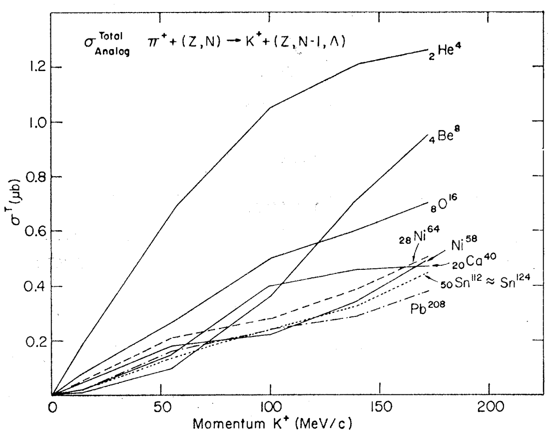

FIG. 1. Total cross sections integrated over the unitary analog peak for π^+ mesons on nuclei giving K^+ mesons plus Λ hypernuclei, neglecting absorption and regeneration.

into the unitary analog state is given by

$$f_K(\theta)=\pi E_K\int d^3r'\, e^{-i\mathbf{k}_K\cdot\mathbf{r}'}\rho(r')$$
$$\times\{-[Aa+b(Z-N+1)]\varphi_K+(2N)^{1/2}b\varphi_\pi\}\,, \quad (8)$$

where φ_K and φ_π are the solutions to Eqs. (6) with (7). Since even the S-wave part of the production amplitude corresponds to pion kinetic energies of over 750 MeV, the Born approximation can safely be used in the absence of absorption. Neglecting the term proportional to φ_K (a much smaller effect than the absorption), and taking a uniform distribution for the nuclear density $\rho(r)$ one obtains for the total cross section under the unitary analog peak:

$$\sigma^T=NSf(kR)/(k_\pi R)^2\,, \quad (9a)$$

where

$$f(kR)=j_1^2(q_0R)-j_1^2(q_1R)+j_0^2(q_0R)-j_0^2(q_1R)\,, \quad (9b)$$

with $q_0(q_1)$ the minimum (maximum) momentum transfer allowed, so that

$$q=|k_\pi-k_K|$$

and

$$q_1=k_\pi+k_K\,.$$

The constant S is evaluated by taking the limit $R\to 0$ and $N=1$ and by comparison with the experimental data.[11] Since a resonance is present at about 150 MeV/c,[12] the analysis cannot be relied on at or above that momentum.

It is interesting that the s-wave scattering lengths needed to fit the *low-energy π-nucleon scattering* fit the constant S which enters the calculation within a factor of 2, i.e., the $SU(3)$ description of $\pi^++n\to K^++\Lambda$ based on the elastic πn scattering at the same Q value is approximately correct, with the predicted magnitude of the strangeness-changing reaction somewhat too large. For most of the processes considered in this paper, the mass splittings introduce other channels which violate the symmetry badly, so the experimental amplitudes for the strangeness-changing two-body reactions must be used. The accuracy of the $SU(3)$ description for the two-body amplitudes is not necessary for the existence of a narrow unitary analog state. Considerations such as the baryon-meson coupling constants, force ranges, core sizes, and the structure of the hypernulcear state determine the width of the state.

The production cross sections are given in Fig. 1. For nuclei heavier than He^4 they are found to be microbarns, and cross sections as a function of energy are approximately the same for nuclei from Ni to Pb. The minimum momentum transfer is about 2.7 F^{-1}, so the increase in the reduced matrix element $(2N)^{1/2}$ is compensated by the decrease in the Bessel functions as the radius increases [see Eq. (9b)]. The kinematic analysis resembles that in the recent studies of coherent processes by Stodolsky.[13] The absorption can easily be included in Eq. (8) in the distorted-wave approximation if reliable estimates of the absorptive potentials can be made. The absorptive processes will further reduce the cross sections; however, if the resonance is narrow an experimental test might be possible with present techniques.

B. $\pi^-+(Z,N)\to K^0+(Z-1,\,N,\,\Lambda)$

The V-spin analysis of this process is similar to the U-spin analysis in Sec. III A. The cross sections differ only in kinematics by a factor of Z/N, and Coulomb energy differences. Thus the results are qualitatively given in Fig. 1.

C. $K^-+(Z,N)\to \eta+(Z-1,\,N,\,\Lambda)$

This process at first seems promising because of the small K-η mass difference and the large cross section for $K^-+p\to\eta+\Lambda$ just above threshold.[14] However, the

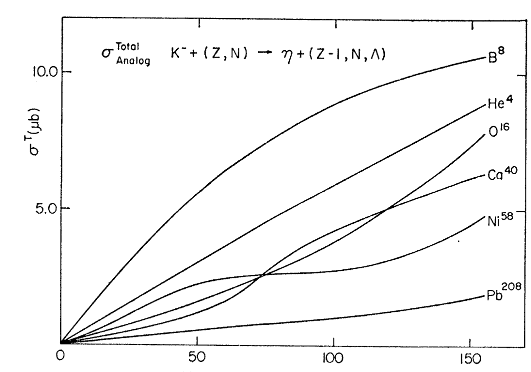

FIG. 2. $K^-+(A,N)\to\eta+(Z-1,\,N,\,\Lambda)$. See caption for Fig. 1.

[11] L. Bertanza *et al.*, Phys. Rev. Letters **8**, 322 (1962).
[12] G. T. Hoff, Phys. Rev. **139**, B671 (1965).
[13] L. Stodolsky, Phys. Rev. **144**, 1145 (1966); see also H. Lipkin, Ref. 2; H. Feshbach and A. Kerman, Ref. 4.
[14] D. Berley *et al.*, Phys. Rev. Letters **15**, 641 (1965).

cross sections turn out to be only a few μb, as can be seen in Fig. 2. This is an overestimate not only because of the neglect of absorption, but also because of the rapid drop in the two-body amplitude above 100 MeV/c[14] which is not included in the present analysis.

D. $K^- + (Z,N) \rightarrow \pi^0(Z-1, N, \Lambda)$

Except for kinematics this process can be obtained fromt he results of the process described in Sec. III E, to be considered next, by a factor of $Z/2N$.

E. $K^- + (Z,N) \rightarrow \pi^- + (Z, N-1, \Lambda)$

Using the analysis of the process described in Sec. III A, the constant S is evaluated by comparison to the experimental results[15] (see Fig. 3) for $K^- + p \rightarrow \pi^0 + \Lambda$.[16] A reasonable fit for $k_K - > 50$ MeV/c for $S = 6.35$ mb is found. The results are shown in Fig. 4. This reaction is quite favorable due to the low momentum transfer possible at rather low energy.[4,12] There are interesting

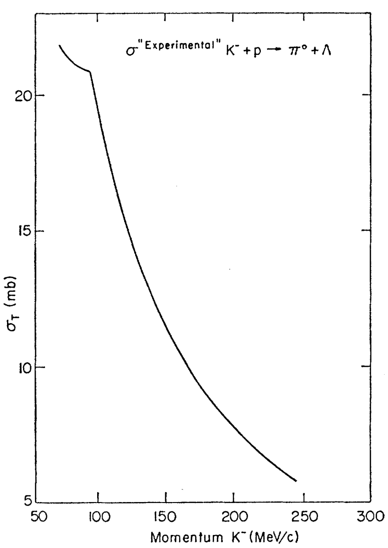

FIG. 3. Cross sections for $K^- + p \rightarrow \pi^0 + \Lambda$ used in the calculation. These were obtained by an analysis (see references contained in Ref. 16) of the entire K^-p data [M. Sakitt (private communication).

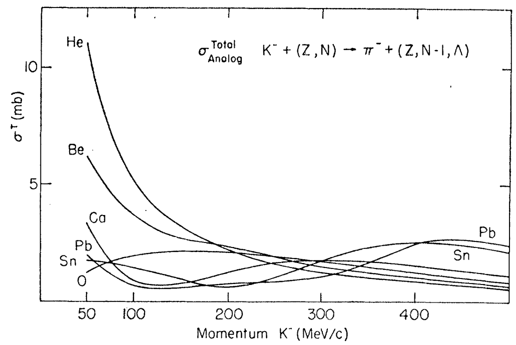

FIG. 4. $K^- + (Z,N) \rightarrow \pi^- + (Z, N-1, \Lambda)$. See caption for Fig. 1.

effects of coherency and the magnitude of the cross sections make experimental searches for this process entirely reasonable with present techniques.[17] Presumably, a coincidence experiment would be necessary to obtain adequate energy resolution with the K^- beams now available.

It is not easy to make a reliable estimate of the position and width of the resonance. One important consideration, of course, is the $SU(3)$ invariance of the force. One guide is the magnitude of the $f_{NN\pi}$, $f_{\Sigma\Sigma\pi}$, and $f_{\Sigma\Lambda\pi}$ coupling constants, which account for the long-range part of the N-N and Λ-N force. This is not given directly by $SU(3)$, since the D/F ratio for the symmetric and antisymmetric coupling of the two baryons with the mesons is needed.[18] Choosing a D/F ratio of $\frac{3}{2}$, one finds that $f_{\Sigma\Sigma\pi} = 0.24$, $f_{\Sigma\Lambda\pi} = 0.20$ (using $f_{NN\pi} = 0.28$). These values are not inconsistent with the coupled-channel calculations for the Λ-N scattering lengths,[19] although they correspond to a rather small core. The magnitude of the width, however, remains the major uncertainty of the calculation.

In conclusion, the experimental detection and systematic study of these interesting new states of strongly interacting systems is possible if unitary symmetry applies and would be extremely valuable for our understanding of these systems. The corrections due to absorption and regeneration are now being carried out. Details of the method and results will be published shortly.

ACKNOWLEDGMENTS

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[15] W. E. Humphrey and R. R. Ross, Phys. Rev. **127**, 1305 (1962); M. Sakitt *et al.*, *ibid.* **139**, B719 (1965); J. K. Kim, Phys. Rev. Letters **14**, 29 (1965).

[16] From isospin considerations the cross section needed in the calculation, $\sigma(k^- + n \rightarrow \pi^- + \Lambda)$, is twice the cross section $\sigma(K^- + p \rightarrow \pi^0 + \Lambda)$, since the process takes place entirely in an isospin-1 state. Since the experimental cross section for the latter process is not nearly so reliable as for the other channels measured in the $K^- + p$ experiments (Ref. 15), the needed cross section can be more reliably obtained from theoretical analysis of all the data. Therefore, the "experimental" cross sections shown in Fig. 3 are obtained by a Dalitz-Tuan [R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960); J. D. Jackson, D. G. Ravenhall, and H. W. Wild, Nuovo Cimento **10**, 834 (1958); M. H. Ross and G. L. Shaw, Ann. Phys. (N. Y.) **9**, 391 (1960)] analysis of the data [M. Sakitt (private communication)].

[17] Estimates of this process for forward scattering by the impulse approximation have also been made by A. K. Kerman and B. Rouben (private communication).

[18] M. Gell-Mann, Ref. 1, Table 3; or J. J. deSwart, Rev. Mod. Phys. **35**, 916 (1963).

[19] J. J. deSwart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962).