and

$$Ia(J+I)-(I+1)a(J+I-1)$$
$$=-Ib(I-J-1)+(I+1)b(I-J-2). \quad \text{(A8)}$$

On adding (A7) and (A8), we get

$$xa(x)-(x+2)a(x-1)$$
$$=-[(y-1)b(y-1)-(y+1)b(y-2)], \quad \text{(A9)}$$

where

$$x=J+I, \quad y=I-J,$$

while subtracting (A7) from (A8), we get

$$\frac{a(x)-a(x-1)}{x+1}=-\frac{b(y-1)-b(y-2)}{y}. \quad \text{(A10)}$$

The Eqs. (A9) and (A10) lead to

$$b(z)=-a(z), \quad \text{(A11)}$$

and

$$xa(x)-(x+2)a(x-1)=\text{constant}, \quad \text{(A12)}$$

$$a(x)-a(x-1)=\text{constant}\cdot(x+1). \quad \text{(A13)}$$

Hence,

$$a(x)=\lambda^2(x+1-p)(x+2+p)\cdots, \lambda^2>0. \quad \text{(A14)}$$

Therefore,

$$A(J,I)=\eta\lambda\left(\frac{(J+I+1-p)(J+I+2+p)}{(2J+1)(2J+2)(2I+1)(2I+2)}\right)^{1/2}, \quad \text{(A15)}$$

where η is a phase factor. The condition that the radical vanishes somewhere so that negative values of $J+I$ do not occur in the I.R. of E_4 leads to

$$p=0, \tfrac{1}{2}, 1, \cdots. \quad \text{(A16)}$$

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Back Electromagnetic Scattering of Leptons from Nucleons*

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The expression for the cross section for the back electromagnetic scattering of leptons and antileptons from nucleons in the laboratory system is given in the first-order Born approximation. The expression includes the dependence on spin states of both particles, which are described by the corresponding electromagnetic form factors. It is shown that the spin flip of leptons is a result of their electromagnetic structure.

I. INTRODUCTION

HIGH-energy electron scattering on nucleons is one of the important sources of information about the electromagnetic structure of nucleons.[1,2] Now that polarized targets are available,[3] the study of spin dependence in the electron-proton and muon-proton interactions has theoretical as well as practical importance.

Electron-nucleon scattering is a particular case of the scattering of two nonidentical fermions. Though the $g-2$ experiments on electrons and muons show that their anomalous magnetic moments are of the order of e^2/hc, the Pauli form factor $f_2(q^2)$ associated with them is not a monotonic decreasing function of q^2, and effects of the form factor might be measurable at high momentum transfer. This necessitates the inclusion of $f_2(q^2)$ in the first-order Born calculations; and the cross section for electron-nucleon scattering, including correlations between various pairs of spin states and in convenient frames of reference, has previously been given.[4–7] Possible effects of muon structure in muon-proton scattering were studied in detail by Barnes.[4]

In a laboratory system, it is very lengthy and cumbersome to calculate the cross section if one includes the dependence on arbitrary initial and final spin states of both particles described by their electromagnetic form factors.[8] We have therefore selected the physically interesting case of back scattering of leptons (electrons or muons) from nucleons. For a given energy of incident leptons, the momentum transfer is largest in this case. Since the electromagnetic structure of the lepton will influence the cross section considerably in the high-

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energy region,[9,10] we restrict ourselves to the consideration of the longitudinal polarizations of the leptons.

II. EXPRESSION FOR THE CROSS SECTION

Covariant expressions for the nucleon and lepton currents are given by

$$J_n = ie\bar{u}_n[F_1(q^2)\gamma_\mu + (\kappa_n/2M)F_2(q^2)\sigma_{\mu\nu}q_\nu]u_n$$

and

$$J_l = -ie\bar{u}_l[f_1(q^2)\gamma_\mu + (\kappa_l/2m)f_2(q^2)\sigma_{\mu\nu}q_\nu]u_l.$$

The electromagnetic nucleon form factors F_E and F_M are related to F_1 and F_2 by

$$F_E = F_1 - (q^2/4M^2)\kappa_n F_2\,,$$
$$F_M = F_1 + \kappa_n F_2\,.$$

The cross section is expressed in terms of form factors F_E and F_M.

The differential cross-section for the scattering of leptons from nucleons through an angle $\theta = 180°$ is given by ($\hbar = c = 1$)

$$\sigma(\xi, \xi', \mathbf{s}, \mathbf{s}', m, \theta = 180°) = \frac{1}{4}\frac{e^4\beta'^2K'}{\beta q^4 K[\beta'(M+K)+\beta K]}\Phi\,.$$

Here

$$\Phi = (1-\xi\xi')(1+\mathbf{s}\cdot\mathbf{s}')2AF_E^2 + \{(1+\xi\xi')\times[1-(\mathbf{s}\cdot\mathbf{k}^0)(\mathbf{s}'\cdot\mathbf{k}^0)] + \epsilon(\xi+\xi')[(\mathbf{s}\cdot\mathbf{k}^0) - (\mathbf{s}'\cdot\mathbf{k}^0)]\}2q_0BF_M^2\,,$$

where

$$A = f_1^2[E(KK'-kk') + m^2M - p(kK'-k'K)] - 4\epsilon q_0M^2\kappa_l f_1f_2 + (2/m^2)q_0M^2\kappa_l^2f_2^2(KK'+kk'-m^2)\,,$$

$$B = f_1^2(KK'+kk'-m^2) + 4\epsilon q_0M\kappa_l f_1f_2 + (2/m^2)q_0\kappa_l^2f_2^2 \times[E(KK'-kk') + m^2M - p(kK'-k'K)]\,;$$

ξ, $\xi' = \pm 1$ are the longitudinal spin states of the incident and scattered leptons; $\xi = +1$ indicates positive helicity and $\xi = -1$, negative helicity; $\mathbf{s}$ and $\mathbf{s}'$ are unit polarization vectors of the target and recoil nucleons[11]; β and β' are the velocities of the incident and scattered leptons; K, K', E are the energies, and k, k', p are the magnitudes of the 3-momenta, of the incident and the scattered leptons and the recoil nucleons, respectively; M (m) is the mass of the nucleon (lepton); κ_n (κ_l) is the anomalous magnetic moment of the nucleon (lepton); $\mathbf{k}^0$ is a unit vector along the incident direction of the leptons. $q_0 = E - M$; q is the 4-momentum transfer.

In the above expression, with $\epsilon = +1$, we get the cross section for scattering of leptons (electrons, negative muons) by nucleons, and with $\epsilon = -1$ that for scattering of antileptons (positrons, positive muons). Averaging over the initial, and summing over the final, spin states of the particles leads to a result in agreement with those of Barnes[4] and Nikishov,[5] taken for the particular case of back scattering under consideration.

In the case of scattering of very high-energy leptons (k, $k' \gg m$), the expression for the differential cross section is much simplified and assumes the form

$$\sigma(\xi, \xi', \mathbf{s}, \mathbf{s}', \theta = 180°) = \tfrac{1}{2}(e^2/2k)^2(1+2\omega)^{-2}\{(1-\xi\xi')(1+\mathbf{s}\cdot\mathbf{s}')aF_E^2 + [(1+\xi\xi')(1-(\mathbf{s}\cdot\mathbf{k}^0)(\mathbf{s}'\cdot\mathbf{k}^0)) + \epsilon(\xi+\xi')(\mathbf{s}\cdot\mathbf{k}^0 - \mathbf{s}'\cdot\mathbf{k}^0)]bF_M^2\,,$$

where

$$a = (1+2\omega)\epsilon\kappa_l f_2(-2f_1 + (q^2/4m^2)2\epsilon\kappa_l f_2)\,,$$
$$b = \omega^2(f_1^2 + 4\epsilon\kappa_l f_1f_2 + 2\kappa_l^2 f_2^2)\,,$$
$$\omega = k/M\,.$$

III. DISCUSSION

If the lepton is considered as a structureless point particle (i.e., $f_1 = 1$ and $\kappa_l f_2 = 0$), back scattering is exclusively due to the magnetic form factor F_M^2 of the nucleons. This term accompanies the lepton-spin correlation coefficients $(1+\xi\xi')$ and $(\xi+\xi')$, the presence of which shows that the scattering cross section is nonzero for $\xi = +1$ and $\xi' = +1$ or $\xi = -1$ and $\xi' = -1$, i.e., spin flip is not possible.[7] However, if the lepton has a form factor $\kappa_l f_2$, an additional term with spin correlation $(1-\xi\xi')$ is introduced into the expression and is exclusively associated with the charge form factor F_E^2 of the nucleon. This shows that scattering is also possible for $\xi = +1$ and $\xi' = -1$ or $\xi = -1$ and $\xi' = +1$. This spin flip of back scattered leptons, therefore, is a result of the electromagnetic structure of the leptons.

In recent experiments of Barber *et al.*,[12] the differential cross section for electron-electron scattering at $E_e = 600$ MeV has been measured in the center-of-mass system. A single form factor $f = (1+q^2/\Lambda^2)^{-1}$ is attributed to the electron and the limiting value $\Lambda > 1.1$ GeV/c is found. The experiments give the form factor at lower q^2, and the effect indicated above may need data at higher momentum transfer and also a knowledge of the function $f_2(q^2)$.

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