Theory of Nonleptonic Decays. II. $K \to 2\pi$†

Y. T. CHIU AND J. SCHECHTER
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois

AND

Y. UEDA
Physics Department, Indiana University, Bloomington, Indiana

(Received 12 August 1966)

We attempt to calculate all three CP-conserving $K \to 2\pi$ decay amplitudes from the universal current-current picture of weak interactions. In this model the suppression of the $K^+ \to \pi^+\pi^0$ mode seems to come about as a result of a cancellation between the pseudoscalar-meson octet and effective scalar-meson singlet intermediate-state contributions to the current-current "spurion."

I. INTRODUCTION

We have previously[1] shown that the numerical magnitudes of the s-wave nonleptonic hyperon decay amplitudes may be calculated from the universal current-current picture of weak interactions without introducing any *arbitrary* parameters. To achieve this we "saturated" the current-current spurion obtained[2] by eliminating the final pion from the decay matrix element using the hypothesis of partial conservation of axial-vector current (PCAC).

In this paper we attempt to apply this method to the CP-conserving $K \to 2\pi$ decays. Although several authors[3] have already treated this problem, our approach is rather different.[4]

There are two reasons why this method is less reliable for the kaon 2π decays than for the hyperon decays. The first is that the elimination of one pion involves an analytic continuation[5] of its 4-momentum to zero which is not so "harmless" in the kaon case. The second reason is that the relevant form factors are not at all well known for the kaon case. Therefore, we expect our results to be only semiquantitative at best. In spite of this, we observe that a natural mechanism exists for the suppression of the $K^+ \to \pi^+\pi^0$ mode. This comes about from a cancellation between the contributions from the pseudoscalar meson intermediate states and *effective* scalar singlet intermediate state to the weak spurion. The strongest point of our argument is that the signs of these two contributions are uniquely predicted when $SU(3)$ invariance is assumed for the form factors. In addition, the numerical magnitudes for all these $K \to 2\pi$ modes can be well fitted with reasonable choices of parameters.

The general formalism is described in some detail in Sec. II. Section III contains the results of saturating the spurion with a number of low-lying intermediate states, while in Sec. IV a numerical estimate is made for the case when only the spin-0 meson intermediate states are included.

II. $K \to 2\pi$ AMPLITUDES

We shall discuss the relation between the amplitudes for the CP-conserving decays

$$K_1^0 \to \pi^+\pi^-, \tag{1a}$$

$$K_1^0 \to \pi^0\pi^0, \tag{1b}$$

$$K^+ \to \pi^+\pi^0, \tag{1c}$$

and the current-current "spurion," to be defined.

We start by assuming that all weak interactions, nonleptonic as well as leptonic, are described by the universal Hamiltonian density

$$H_W(x) = \tfrac{1}{2}\sqrt{2}G \times \tfrac{1}{2}[J_\mu(x), J_\mu^\dagger(x)]_+, \tag{2}$$

where $G \simeq 10^{-5}/M_p^2$ and the Cabibbo[6] current $J_\mu(x)$ is given by

$$J_\mu(x) = J_\mu^{(\text{leptonic})}(x) + \cos\theta[(V_2^1)_\mu + (P_2^1)_\mu] + \sin\theta[(V_3^1)_\mu + (P_3^1)_\mu]. \tag{3}$$

Here $\sin\theta \simeq 0.26$ and $(V_b^a)_\mu$ and $(P_b^a)_\mu$ are, respectively, the vector and pseudovector octet currents. The portion of Eq. (2) which is relevant for nonleptonic decays is

$$H_W^{\text{NL}} = \tfrac{1}{2}\sqrt{2}G \cos\theta \sin\theta \times \tfrac{1}{2} \{[(V_2^1 + P_2^1), (V_1^3 + P_1^3)]_+ + (2 \leftrightarrow 3)\}. \tag{4}$$

† Work supported in part by the U. S. Atomic Energy Commission.

[1] Y. T. Chiu and J. Schechter, Phys. Rev. Letters **16**, 1022 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966). This paper is designated I. See also Y. Hara, S. Biswas, A. Kumar, and R. Saxena, Phys. Rev. Letters **17**, 268 (1966).

[2] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997 (1965); M. Suzuki, *ibid.* **15**, 986 (1965).

[3] S. N. Biswas and S. K. Bose, Nuovo Cimento (to be published); E. Ferrari, V. S. Mathur, and L. K. Pandit, Phys. Letters **21**, 560 (1966).

[4] The main differences between our approach and that of Ferrari, Mathur, and Pandit are that we include the scalar singlet intermediate state and assume $SU(3)$ invariance for the form factors. Because of this last point the cancellation they postulate to give suppression of the K_+ mode can not take place in our scheme. Both works seem to find a very small vacuum contribution.

[5] M. Suzuki, Phys. Rev. **144**, 1154 (1966); Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966); B. D'Espaquat and J. Iliopoulos, Phys. Letters **21**, 232 (1966); S. G. Callen and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); S. K. Bose and S. N. Biswas, *ibid.* **16**, 330 (1966).

[6] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

We define the amplitudes for processes (1) by the following matrix elements of Eq. (4):

$$A_{+-}=(8q_0q_0'p_0)^{1/2}\langle\pi^+(q)\pi^-(q')|H_W{}^{\rm NL}(0)|K_1{}^0(p)\rangle\,, \quad (4a)$$

$$A_{00}=(8q_0q_0'p_0)^{1/2}\langle\pi^0(q)\pi^0(q')|H_W{}^{\rm NL}(0)|K_1{}^0(p)\rangle\,, \quad (4b)$$

$$A_{+0}=(8q_0q_0'p_0)^{\frac{1}{2}}\langle\pi^+(q)\pi^0(q')|H_W{}^{\rm NL}(0)|K^+(p)\rangle\,. \quad (4c)$$

The amplitudes are related to the experimental lifetimes τ_{ij} by

$$\frac{1}{\tau_{ij}}=(1-\tfrac{1}{2}\delta_{ij})\frac{1}{16\pi M_K}\left[1-4\left(\frac{M_\pi}{M_K}\right)^2\right]^{1/2}|A_{ij}|^2\,, \quad (5)$$

where i and j can be $+$, 0, and $-$, and we have neglected the $\pi^+\pi^0$ mass difference for Eq. (4c). Experimentally[7] we have

$$|A_{+-}|=(28.2\pm0.3)\times10^{-7}\,M_{\pi^+}\,, \quad (6a)$$

$$|A_{00}|=(26.9\pm0.3)\times10^{-7}\,M_{\pi^+}\,, \quad (6b)$$

$$|A_{+0}|=(1.32\pm.01)\times10^{-7}\,M_{\pi^+}\,. \quad (6c)$$

Next we shall use the PCAC hypothesis[8] and the algebra of currents to eliminate one of the bosons in each of Eqs. (4) in order to convert the amplitudes A_{ij} into more easily evaluated matrix elements. We need the equal-time commutators

$$[B_b{}^a(0),V_d{}^c(\mathbf{x},0)]=\delta_d{}^aP_b{}^c(\mathbf{x},0)-\delta_b{}^cP_d{}^a(\mathbf{x},0)\,, \quad (7)$$

$$[B_b{}^a(0),P_d{}^c(\mathbf{x},0)]=\delta_d{}^aV_b{}^c(\mathbf{x},0)-\delta_b{}^cV_d{}^a(\mathbf{x},0)\,, \quad (8)$$

where the axial-vector "generator" $B_b{}^a(t)$ is defined by

$$B_b{}^a(t)=i\int d^3x(P_b{}^a)_4\,. \quad (9)$$

In our notation the PCAC hypothesis for the strangeness-conserving and strangeness-changing currents may be written as, for example,

$$\frac{\partial P_{1\mu}{}^2(x)}{\partial x_\mu}=-\left(\frac{\sqrt{2}M_Pg_A}{g_{\pi NN}}\right)M_\pi{}^2\pi_1{}^2(x)\,, \quad (10a)$$

$$\frac{\partial P_{1\mu}{}^3(x)}{\partial x_\mu}=-\left(\frac{\sqrt{2}M_Pg_A}{g_{\pi NN}}\right)M_K{}^2\pi_1{}^3(x)\,, \quad (10b)$$

where $\pi_a{}^b(x)$ is the renormalized pseudoscalar meson octet field operator, $g_A\simeq1.18$, and $(g_{\pi NN}{}^2/4\pi)\simeq14.6$.

The process of "removing" a pion from the $K\to2\pi$ matrix element is much more ambiguous[5] than the corresponding procedure in the case of the hyperon decays. One reason is that one pion here represents a large fraction of the total decay energy. Another reason, perhaps more serious, is that the generalized Bose statistics of the two-pion final state is disturbed. A possible way to avoid the second difficulty is to adopt the procedure of "removing" the initial K meson. There are indications[9] that the analytic continuation of the K 4-momentum to zero is not serious. To follow this approach we note that the reduction formula gives in this limit

$$A_{ij}=(4q_0q_0')^{1/2}iM_K{}^2\int d^4x\,\theta(-x_0)\langle\pi_i(q)\pi_j(q')|\times[H_W{}^{\rm NL}(0),\pi_b{}^a(x)]|0\rangle\,, \quad (11)$$

where $\pi_b{}^a$ stands for $\pi_3{}^1$ in the case of K^+ decay and $+(i/\sqrt{2})(\pi_3{}^2-\pi_2{}^3)$ in the case of $K_{10}{}^0$ decay.

The integral on the right-hand side of Eq. (11) may be rewritten as

$$-\frac{1}{M_K{}^2}\left(\frac{g_{\pi NN}}{\sqrt{2}M_Pg_A}\right)\int d^4x\,\theta(-x_0)\langle\pi_i(q)\pi_j(q')|\left[H_W{}^{\rm NL}(0),\frac{\partial}{\partial x_\mu}P_{b\mu}{}^a(x)\right]|0\rangle \quad (12a)$$

$$=\frac{i}{M_K{}^2}\left(\frac{g_{\pi NN}}{\sqrt{2}M_Pg_A}\right)\int_{-\infty}^0dt\int d^3x\langle\pi_i(q)\pi_j(q')|\left[H_W{}^{\rm NL}(0),\frac{d}{dt}(P_b{}^a)_4(x)\right]|0\rangle \quad (12b)$$

$$=(i/M_K{}^2)(g_{\pi NN}/\sqrt{2}M_Pg_A)\Big\{-i\langle\pi_i(q)\pi_j(q')|[H_W{}^{\rm NL}(0),B_b{}^a(0)]|0\rangle-\int d^3x\langle\pi_i(q)\pi_j(q')|[H_W{}^{\rm NL}(0),(P_b{}^a)_4(x,-\infty)]|0\rangle\Big\} \quad (12c)$$

$$=(1/M_K{}^2)(g_{\pi NN}/\sqrt{2}M_Pg_A)\langle\pi_i(q)\pi_j(q')|[H_W{}^{\rm NL}(0),B_b{}^a(0)]|0\rangle\,, \quad (12d)$$

where we used Eq. (10b) in obtaining (12a), neglect of terms at spatial infinity in obtaining (12b), Eq. (9) in obtaining (12c), and, following Okubo,[10] the observation that because no physical intermediate state has the same energy as the vacuum the second, "surface"

[7] A. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965).

[8] M. Gell-Mann and Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

[9] See, for example, R. H. Dalitz, Varenna Lectures on Weak Interactions, 1964 (to be published), where the validity of the PCAC hypothesis for strangeness changing currents is disscussed. The analog of the Adler-Weisberger relation obtained by continuing the K 4-momentum to zero seems to hold.

[10] S. Okubo, Nuovo Cimento **16**, A586 (1966).

term in Eq. (12c) drops. Thus we finally have

$$A_{ij}=-i(4q_0q_0')^{1/2}(g_{\pi NN}/\sqrt{2}M_Pg_A)\times\langle\pi_i(q)\pi_j(q')|[B_b{}^a(0),H_W{}^{\rm NL}(0)]|0\rangle. \quad (13)$$

We note that the equal-time commutator in Eq. (13) can be evaluated by using (7) and (8). By CP invariance there will remain only matrix elements of parity-conserving current products, i.e., forms like VV and PP.

For the purpose of making dynamical estimates, it is convenient to approximate the Lorentz-invariant matrix element of Eq. (13) as

$$(4q_0q_0')^{1/2}\langle\pi_i(q)\pi_j(q')|[B_b{}^a(0),H_W{}^{\rm NL}(0)]|0\rangle$$
$$=(4q_0q_0')^{1/2}\langle\pi_i(q)|[B_b{}^a(0),H_W{}^{\rm NL}(0)]\times|\bar\pi_j(-q')\rangle \quad (14a)$$
$$\simeq(2\mu)\langle\pi_i(0,\mu)|[B_b{}^a(0),H_W{}^{\rm NL}(0)]\times|\bar\pi_j(0,\mu)\rangle, \quad (14b)$$

where μ is a degenerate mass for the pseudoscalar meson octet. Equation (14a) follows from crossing symmetry, while in Eq. (14b) we have made the approximation of assuming no change in the matrix element when analytically continuing the momentum of π_i from $-q'$ to $+q'$. For simplicity we have considered the matrix element to be taken between particles at rest.

The right-hand side of Eq. (14b) can be written in terms of the $SU(3)$ "spurions"[11]:

$$V_{bd}{}^{ac}=\langle\pi|[V_b{}^a,V_d{}^c]_+|\pi\rangle, \quad (15a)$$

$$P_{bd}{}^{ac}=\langle\pi|[P_b{}^a,P_d{}^c]_+|\pi\rangle. \quad (15b)$$

It is also convenient to introduce the total spurion

$$S_{bd}{}^{ac}=V_{bd}{}^{ac}+P_{bd}{}^{ac}. \quad (16)$$

This has the following decomposition into irreducible $SU(3)$ tensors:

$$S_{bd}{}^{ac}=\tau T_{bd}{}^{ac}+\delta[(\delta_d{}^aD_b{}^c+\delta_b{}^cD_d{}^a)-\tfrac{2}{3}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a)]+\sigma(\delta_d{}^a\delta_b{}^c-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c)\langle\pi\pi\rangle, \quad (17)$$

where

$$T_{bd}{}^{ac}=(\pi_b{}^a\pi_d{}^c+\pi_d{}^a\pi_b{}^c)-\tfrac{1}{5}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a+\delta_d{}^aD_b{}^c+\delta_b{}^cD_d{}^a)-\tfrac{1}{12}(\delta_b{}^a\delta_d{}^c+\delta_d{}^a\delta_b{}^c)\langle\pi\pi\rangle, \quad (17a)$$

$$D_b{}^a=\pi_e{}^a\pi_b{}^e-\tfrac{1}{3}\delta_b{}^a\langle\pi\pi\rangle, \quad (17b)$$

$$\langle\pi\pi\rangle=\pi_n{}^m\pi_m{}^n. \quad (17c)$$

The coefficients τ, δ, and σ correspond, respectively, to the $\{27\}$, $\{8_s\}$ and $\{1\}$ $SU(3)$ representations. Only the first two contribute to $K\to 2\pi$ decays.

Performing the indicated commutation in Eq. (13) and employing the "spurion" notation just introduced gives the results

$$A_{+-}=-\sqrt{2}i(\delta+\tfrac{4}{5}\tau)A_0, \quad (18)$$

$$A_{00}=-\sqrt{2}i[\delta-(6/5)\tau]A_0, \quad (19)$$

$$A_{+0}=-\sqrt{2}\tau A_0, \quad (20)$$

where

$$A_0=-i\tfrac{1}{4}\sqrt{2}G\sin\theta\cos\theta\,(2\mu)(g_{\pi NN}/\sqrt{2}M_Pg_A). \quad (21)$$

This completes the job of expressing the $K\to 2\pi$ amplitudes in terms of the weak spurion. An alternate approach, due to Hara and Nambu,[5] leads to the same result except that the amplitudes are to be multiplied by a common factor or $(M_\pi{}^2-M_K{}^2)/\mu^2$, where μ is a degenerate pseudoscalar meson octet mass.

Still another way of obtaining essentially the same result is to "remove" the two pions separately and symmetrize[12] the spurions, the only difference being an over-all factor of $\frac{1}{2}$ compared to our removal of the K meson. We shall see in Sec. IV that the present lack of knowledge about meson-meson form factors prevents us from choosing any of the above three approaches and that such over-all factors can be compensated for by a slight adjustment of our form factor parameters.

We note from Eq. (20) that the $\Delta I=\frac{3}{2}$, $K_+\to\pi_+\pi_0$ decay only receives a contribution from the $\{27\}$ representation, as it must. The following (up to) $\Delta I=\frac{3}{2}$ relation[13] can be read off immediately as

$$A_{+-}-A_{00}=+2iA_{+0}. \quad (22)$$

The comparison with experiment[14] is read off, with suitable adjustment of phases, from Eqs. (6) to be

$$(28.2\pm0.3)-(26.9\pm0.3)=2.64\pm0.03. \quad (23)$$

Finally, we remark that although we are assuming $SU(3)$ symmetry in our treatment of the spurion, we did not assume $SU(3)$ invariance to arrive at the spurion. Indeed, it is well known[15] that all $K\to 2\pi$ decays vanish in this limit when we assume crossing symmetry also.

III. SATURATING THE SPURION

We now set up the formalism for saturating the current-current spurion with a number of low-lying intermediate states. We assume that $SU(3)$ invariance holds for the various form factors involved. We do not necessarily assume chiral $SU(3)\times SU(3)$ representations for the mesons, however. The choice of intermediate states to be initially considered may be summarized by the *approximate* completeness relation

$$1\simeq|0\rangle\langle0|+\sum(|\pi_8\rangle\langle\pi_8|+|\sigma_1\rangle\langle\sigma_1|+|\rho_9\rangle\langle\rho_9|), \quad (24)$$

[11] Note that we are using exactly the same notation here for the meson spurion as we used for the baryon spurion in I. This should cause no confusion.

[12] See M. Suzuki, Ref. 5.

[13] E. C. G. Sudarshan, Nuovo Cimento **41**, A283 (1966); T. Das and K. Mahanthappa, *ibid.* **41**, 618 (1966).

[14] We note that Eq. (22) is *not* quite satisfied within the quoted experimental errors, neglecting final-state interactions. However, it is not so far wrong as to make us question its exact validity yet.

[15] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964); S. Okubo, Phys. Letters **8**, 362 (1964).

where π_8 stands for the usual pseudoscalar-meson octet, ρ_9 for the vector meson nonet, and σ_1 for a unitary singlet scalar meson or strong s-wave π-π interaction. The inclusion of this latter effect is prompted by the observation of several authors[16] that it seems to dominate the saturation of states in the Adler-Weisberger sum rule[17] for pion-pion scattering. The vector-vector spurion receives contributions only from π_8 and ρ_9, while the pseudovector-pseudovector spurion receives contribution from all except π_8. The relevant form factors are defined by

$$(4p_0p_0')^{1/2}\langle\pi(p')V_{b\alpha}{}^a(0)|\pi(p)\rangle = f(q^2)(p+p')_\alpha F_b{}^a, \quad (25)$$

$$f(0)=+1, \quad (25')$$

$$(4p_0p_0')^{1/2}\langle\rho_9(p')|V_{b\alpha}{}^a(0)|\pi(p)\rangle = -ig(q^2)\epsilon_{\alpha\beta\gamma\delta}p_\beta p_\gamma'\epsilon_\delta D_b{}^a, \quad (26)$$

$$(2p_0)^{1/2}\langle 0|P_{b\alpha}{}^a(0)|\pi_{a'}{}^{b'}(p)\rangle = i(\sqrt{2}M_P g_A/g_{\pi NN})p_\alpha\delta_{a'}{}^a\delta_b{}^{b'}, \quad (27)$$

$$(4p_0p_0')^{1/2}\langle\sigma_1(p')|P_{b\alpha}{}^a(0)|\pi_{a'}{}^{b'}(p)\rangle = im(q^2)\{(p+p')_\mu + [(\mu^2-M_\sigma{}^2)/(q^2+\mu^2)]q_\mu\}\delta_{a'}{}^a\delta_b{}^{b'}, \quad (28)$$

$$(4p_0p_0')^{1/2}\langle\rho_9(p')|P_{b\alpha}{}^a(0)|\pi(p)\rangle = i\{n_1(q^2)\epsilon_\alpha + n_2(q^2)\times p_\alpha'(p\cdot\epsilon) + n_3(q^2)p_\alpha(p\cdot\epsilon)\}F_b{}^a, \quad (29)$$

where $D_b{}^a$ and $F_b{}^a$ are, respectively, the symmetric and antisymmetric $SU(3)$ matrices and we have used Nambu's form[8] of the PCAC hypothesis in arriving at Eq. (28). In each equation $q=p-p'$, while in Eqs. (26) and (29) ϵ is the vector-meson polarization.

With these form factors and the approximation of Eq. (24) we easily compute the spurion coefficients[18] τ, δ, and σ;

$$\tau = I_\pi{}^V + I_\rho{}^V + \tfrac{1}{2}I_\sigma{}^P - I_\rho{}^P + \tfrac{1}{2}I_0{}^P, \quad (30a)$$

$$\delta = -(9/5)I_\pi{}^V + \tfrac{1}{5}I_\rho{}^V + \tfrac{3}{5}I_\sigma{}^P + (9/5)I_\rho{}^P + \tfrac{3}{5}I_0{}^P, \quad (30b)$$

$$\sigma = -\tfrac{3}{4}I_\pi{}^V + (7/12)I_\rho{}^V + \tfrac{1}{8}I_\sigma{}^P + \tfrac{3}{4}I_\rho{}^P + \tfrac{1}{8}I_0{}^P, \quad (30c)$$

where

$$I_\pi{}^V = \frac{1}{(2\pi)^2}\int_\mu^\infty dk_0(k_0{}^2-\mu^2)^{1/2}(k_0+\mu)[f(q^2)]^2, \quad (31)$$

$$q^2 = 2\mu(k_0-\mu), \quad (31')$$

$$I_\rho{}^V = \frac{\mu}{(2\pi)^2}\int_{M_\rho}^\infty dk_0(k_0{}^2-M_\rho{}^2)^{1/2}(k_0{}^2-M_\rho{}^2)[g(q^2)]^2, \quad (32)$$

$$q^2 = 2\mu k_0 - \mu^2 - M_\rho{}^2, \quad (32')$$

with M_ρ = degenerate vector-meson mass.

$$I_\sigma{}^P = -\frac{1}{(2\pi)^2}\frac{1}{2\mu}\int_{M_\sigma}^\infty dk_0(k_0{}^2-M_\sigma{}^2)^{1/2}[m_1(q^2)]^2 \times\{(M_\sigma{}^2+\mu^2+2\mu k_0)+(\mu^2+2\mu k_0-M_\sigma{}^2)\times((M_\sigma{}^2-\mu^2)/(M_\sigma{}^2-2\mu k_0))^2\}, \quad (33)$$

$$q^2 = 2\mu k_0 - \mu^2 - M_\sigma{}^2, \quad (33')$$

$$I_\rho{}^P = \frac{1}{(2\pi)^2}\frac{1}{2\mu}\int_{M_\rho}^\infty dk_0(k_0{}^2-M_\rho{}^2)^{1/2}\{-3[n_1(q^2)]^2 + (k_0{}^2-M_\rho{}^2)(\mu^2[n_2(q^2)]^2 + (\mu^4/M_\rho{}^2)[n_3(q^2)]^2 + (2\mu^2/M_\rho{}^2)[n_1(q^2)n_3(q^2)] - (2\mu^3k_0/M_\rho{}^2)[n_2(q^2)n_3(q^2)])\}, \quad (34)$$

$$q^2 = 2\mu k_0 - \mu^2 - M_\rho{}^2, \quad (34')$$

$$I_0{}^P = -\mu(M_P g_A/g_{\pi NN})^2. \quad (35)$$

The quantity $I_\pi{}^V$, for example, denotes the integral which comes from the pion intermediate state contribution to the vector-vector spurion.

Now if the current-current picture is a good one, it must predict a great suppression for the decay $K^+\to\pi^+\pi^0$. In other words, the coefficient τ must be small, (octet dominance). Of the contributions to τ we note that $I_\rho{}^P$ is negligible[19] and that $I_\rho{}^P$ is probably small compared to $I_\sigma{}^P$ since, as previously noted, a σ-type contribution appears to dominate the ρ contribution to the Adler-Weisberger sum rule for $\pi\pi$ scattering. Thus only $I_\pi{}^V$, $I_\rho{}^V$, and $I_\sigma{}^P$ remain. From Eqs. (31) and (32) the first two are seen to be positive, independently of the details of integration over form factors. In addition, $I_\sigma{}^P$ can be seen from Eq. (33) to be negative (the last term is actually negligible in our case), independently of the details of the form factor. Thus a natural mechanism for cancellation appears to exist. This is perhaps the main conclusion of our paper. In Sec. IV we show that all the $K\to 2\pi$ decays can be fit in such a scheme with reasonable estimates of the form factors and coupling parameters.

IV. NUMERICAL RESULTS

Substitution of Eqs. (30a) and (30b) into Eqs. (18), (19), and (20) gives the theoretical predictions for the $K\to 2\pi$ amplitudes in terms of integrals over form factors. It seems plausible, as mentioned above, to neglect $I_\rho{}^P$ and $I_0{}^P$. For simplicity, we shall also neglect $I_\rho{}^V$. Then, we have

$$A_{+-} = -\sqrt{2}i(I_\sigma{}^P - I_\pi{}^V)A_0, \quad (36a)$$

$$A_{00} = -\sqrt{2}i(3I_\pi{}^V)A_0, \quad (36b)$$

[16] S. L. Adler, Phys. Rev. **140**, B736 (1965); K. Kawarabayashi, W. D. McGlinn, and W. W. Wada, Phys. Rev. Letters **15**, 897 (1965). See also C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966).

[17] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); S. L. Adler, *ibid.* **14**, 1051 (1965).

[18] The contributions from a hypothetical octet scalar meson intermediate state to τ, δ, and σ would be $\frac{1}{2}I_{\sigma_8}{}^P$, $-\frac{3}{5}I_{\sigma_8}{}^P$, and $(5/12)I_{\sigma_8}{}^P$, respectively. $I_{\sigma_8}{}^P$ is given by an expression like Eq. (33) and is similarly expected to be negative.

[19] From Eq. (35) we compute $I_0{}^P = -0.87\ M_\pi{}^3$ for $\mu = 350$ MeV. This is to be compared with Eqs. (40).

$$A_{+0}=-\sqrt{2}(I_\pi{}^V+\tfrac{1}{2}I_\sigma{}^P)A_0. \tag{36c}$$

$I_\pi{}^V$ involves the pseudoscalar-meson vector form factor $f(q^2)$, while $I_\sigma{}^P$ involves the pseudoscalar-meson–scalar-meson axial-vector transition form factor $m(q^2)$. We assume that these have the standard forms similar to nucleon form factors.

$$m(q^2)/m(0)=f(q^2)=(1+q^2/\beta)^{-2}, \tag{37}$$

with $\beta=0.71$ when q is expressed in BeV/c. We shall regard $m(0)$ as a parameter. With the form (37) $I_\pi{}^V$ and $I_\sigma{}^P$ may be integrated analytically. This process yields

$$I\pi^V=\frac{1}{3}\frac{1}{(2\pi)^2}\left(\frac{\beta}{2\mu}\right)^3\left(\frac{\beta}{\beta-2\mu^2}\right)(\mathrm{CF})_1 \tag{38}$$

$$I_\sigma{}^P\simeq-\frac{1}{3}\frac{[m(0)]^2}{(2\pi)^2}\left(\frac{\beta}{2\mu}\right)^3\left(\frac{\beta}{\beta-\mu^2-M_\sigma{}^2}\right)(\mathrm{CF})_2, \tag{39}$$

where the "correction-factors" $(\mathrm{CF})_1$ and $(\mathrm{CF})_2$ are given in the Appendix and are numerically close to unity. In the evaluation of $I_\sigma{}^P$ the last term of Eq. (33) was neglected because it is very small. (We only consider values of $\mu>\frac{1}{2}M_\sigma$.) We note the rather sensitive dependence of these expressions on β and the degenerate pseudoscalar meson mass μ. Thus it does not seem worthwhile to attempt anything more elaborate than a rough fit. In particular, we see that we could accommodate a ρ contribution or a different method of analytic continuation by suitable variation of parameters.

We shall use a value of degenerate pseudoscalar meson mass, $\mu=350$ MeV. For the scalar meson mass, we take the conventional[20] value $M_\sigma=390$ MeV. However, we note from Eq. (39) that the dependence of $I_\sigma{}^P$ on M_σ is small. Then taking[21] $m(0)=1.46$ and $\beta=0.71$ BeV^2, we find the results

$$I_\pi{}^V=5.25M_\pi{}^3, \tag{40a}$$

$$I_\sigma{}^P=-12.0M_\pi{}^3, \tag{40b}$$

$$|A_{+-}|=29.6\times10^{-7}\,M_\pi, \tag{41a}$$

$$|A_{00}|=27.0\times10^{-7}\,M_\pi, \tag{41b}$$

$$|A_{+0}|=1.3\times10^{-7}\,M_\pi. \tag{41c}$$

The agreement of Eqs. (41) with the experimental values, Eqs. (6), is of course impressive. We cannot obtain exact agreement because the present experimental amplitudes do not exactly satisfy the $\Delta I=\frac{3}{2}$ rule given by Eq. (22).

Thus we see that a good fit to the experimental amplitudes can be made with reasonable choice of physical parameters in our model. Our conclusion is that the universal current-current picture of weak interactions may be consistent with (CP conserving) $K\to 2\pi$ decays.

Finally, we stress that in this section a specific approximation to the more general equations of Sec. III has been used. In particular, if more definite information were available about the form factors of Eqs. (26) and (29), it would be very desirable to substitute these results into Eqs. (30), (32), and (34) to obtain the contribution of the vector meson intermediate states. If it were to turn out that these intermediate states dominate, a cancellation between $I_\rho{}^P$ and $I_\rho{}^V$ of Eq. (30a) must occur in order for this current-current picture to explain the suppression of the K^+ mode. In any case, we have demonstrated one possible mechanism (that of π_8 and σ_1 dominance in the saturation of states) which naturally appears to give the appropriate suppression. A possible justification for this mechanism has been noted to be the fact that the $\pi\pi$ Adler sum rule appears to require the existence of a low-energy s-wave resonance. As pointed out by Weinberg,[22] this is not necessarily inconsistent with a small s-wave scattering length. Another point of ambiguity in this connection concerns the possibility that the $\pi\pi$ Adler sum rule may not be correct. In this case, we might expect the vector-meson dominance mechanism mentioned above to hold.

ACKNOWLEDGMENT

We wish to thank Professor Y. Nambu for helpful discussions and encouragement.

APPENDIX

The correction factors to Eqs. (38) and (39) are

$$(\mathrm{CF}_1)=(3\alpha^3)\left\{\frac{1}{3\alpha^3}+\frac{1}{6\alpha^2}-\frac{(\alpha^3+2\alpha^2-4\alpha+1)}{6\alpha(1-\alpha^2)^2}-\frac{(1-\alpha)}{2(1-\alpha^2)^{5/2}}\ln\frac{1+(1-\alpha^2)^{1/2}}{\alpha}\right\}, \tag{A1}$$

$$\alpha=1/(\beta/2\mu^2-1)<1, \tag{A1'}$$

[20] L. M. Brown and P. Singer, Phys. Rev. **133**, B812 (1964). The conventional width is taken to be 90 MeV. Even if the σ represents a strong attractive interaction rather than an actual resonance, the parameters given still serve to almost saturate the Adler-Weisberger pion-pion sum rule of Ref. 16.

[21] To get an estimate of what $m(0)$ should be, we may relate it by the PCAC hypothesis to the $\sigma\pi\pi$ coupling constant. Then, calculating this coupling constant from the parameters of Ref. 20 gives $m(0)=1.14$. We note that working backwards, our larger value of $m(0)$ would give even better agreement for the value of g_A predicted from the Adler-Weisberger rule of Ref. 16. The explicit formulas used for the above procedure are as follows: The $\sigma\pi\pi$ coupling constant $g_{\sigma\pi\pi}$ is defined in terms of the width Γ_σ by

$$\Gamma_\sigma=\frac{(g_{\sigma\pi\pi})^2}{4\pi}\left(\frac{M_\pi}{M_\sigma}\right)^2\frac{3M_\sigma}{8}\left[1-4\left(\frac{M_\pi}{M_\sigma}\right)^2\right]^{1/2}.$$

Furthermore, $m(0)$ is related to $g_{\sigma\pi\pi}$ by

$$m(0)=\frac{\sqrt{2}Mg_A}{g_{\pi NN}}\frac{g_{\sigma\pi\pi}M_\pi}{M_\sigma{}^2-M_\pi{}^2}.$$

[22] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

$$(\mathrm{CF})_2=\frac{M_\sigma}{2\mu}(3\kappa^3)\left\{\frac{2\mu}{M_\sigma}\left(\frac{1}{3\kappa^3}-\frac{(1+2\kappa^2)}{6\kappa(1-\kappa^2)^2}\right)+\left[1+\left(\frac{\mu}{M_\sigma}\right)^2\right]\left(\frac{1}{6\kappa^2}+\frac{4-\kappa^2}{6(1-\kappa^2)^2}\right)\right.$$
$$\left.+\frac{1}{2(1-\kappa^2)^{5/2}}\left\{\frac{2\mu}{M_\sigma}\kappa-\left[1+\left(\frac{\mu}{M_\sigma}\right)^2\right]\right\}\ln\frac{1+(1-\kappa^2)^{1/2}}{\kappa}\right\}, \quad \text{(A2)}$$

$$\kappa=2\mu M_\sigma/(\beta-\mu^2-M_\sigma^2)<1. \quad \text{(A2')}$$

PHYSICAL REVIEW VOLUME 157, NUMBER 5 25 MAY 1967

Lie Group of the Strong-Coupling Theory. III. Dynamical Generation of Extra Internal Symmetry for a Class of Representations

P. BABU,* A. RANGWALA, AND VIRENDRA SINGH†
Tata Institute of Fundamental Research, Bombay, India

(Received 12 October 1966)

The strong-coupling dynamics, with isospin $SU(2)_I$ as the given internal symmetry group, leads to the generation of a larger internal symmetry group $SU(2)_{I_1}\otimes SU(2)_{I_2}$ ($\mathbf{I}_1+\mathbf{I}_2=\mathbf{I}$) for the hyperon isobar-pion coupling constants, the hyperon isobar series having isospin-spin $I=J\pm\frac{1}{2}=0, 1, 2, \cdots, \infty$. In this connection we describe a "modified contraction" method for constructing the irreducible representations of the strong-coupling group $\mathcal{G}=[SU(2)_I\otimes SU(2)_J]\times T_9$, which starts from the "uncontracted" group $SU(2)\otimes SU(4)$ and provides a transparent formulation of the dynamical generation of extra symmetry for a class of irreducible representations of group $\mathcal{G}$. One more method for constructing the representations of the group $\mathcal{G}$ is also described which starts from a knowledge of the irreducible representations of inhomogeneous Euclidean group in four dimensions.

I. INTRODUCTION

In the first two papers of this series the two irreducible representations (I.R.), which are suitable for describing the nucleon and hyperon isobars, of the strong coupling group $\mathcal{G}\equiv[SU(2)_{\mathrm{isospin}}\otimes SU(2)_{\mathrm{spin}}]\times T_9$ of the symmetric pseudoscalar meson theory were explicitly constructed.[1,2] The isobar-isobar-pion coupling constants and the magnetic-moment predictions obtained were found to be in very good agreement with experimental data.

The most surprising physical result which emerged from the calculation of hyperon coupling constants in LG II was as follows. Even though one started with the internal symmetry group $SU(2)_I\otimes SU(2)_J$, one found that the coupling constants for this I.R. came out as if the internal symmetry group was bigger and was at least as large as $SU(2)\otimes SU(2)\otimes SU(2)$ which contains as a subgroup $SU(2)_I\otimes SU(2)_J$. Thus in this case we find that the strong-coupling Chew-Low dynamics leads to the generation of more symmetry. One may emphasize that this extra symmetry is not present in *all* the solutions of strong-coupling dynamics but only for a class of them. One would like to express the dynamical generation of this symmetry in a transparent fashion. We are able to do this by using a "modified contraction" procedure which we describe in the next section.

As we pointed in LG II, the irreducible representations (I.R.) of $\mathcal{G}$, corresponding to hyperon isobar with isospin-spin content $I=J\pm\frac{1}{2}=0, 1, 2, \cdots, \infty$ cannot be obtained by group contraction method used by Cook, Goebel, and Sakita starting with the group $SU(4)$, as all such I.R.'s are characterized by having only isobars with $I-J=$ integers.[3] The mathematically more powerful technique of "induced representation" method does, of course, lead to such I.R.'s, but it does not bring out the dynamical generation of the extra symmetry which is there for *some* of the I.R.'s of $\mathcal{G}$.[4,5]

We also describe in the following an alternative method which depends on knowing the unitary I.R.'s of the inhomogeneous Euclidean group in four dimensions, E_4. The algebraic structure of E_4 is isomorphic to $[SU(2)\otimes SU(2)]\times T_4$, where the four translation operators transform as the $(\frac{1}{2},\frac{1}{2})$ representation of the compact subgroup $SU(2)\otimes SU(2)$. The I.R.'s of this group

* Present address: Department of Physics, California Institute of Technology, Pasadena, California.

† Present address: Department of Physics, The Rockefeller University, New York, New York.

[1] Virendra Singh, Phys. Rev. **144**, 1275 (1966).

[2] Virendra Singh and B. M. Udgaonkar, Phys. Rev. **149**, 1164 (1966) (to be referred to as LG II).

[3] T. Cook, C. J. Goebel, and B. Sakita, Phys. Rev. Letters **15**, 35 (1965).

[4] C. J. Goebel, in *Proceedings of 1966 Conference on "Noncompact Groups in Particle Physics"* edited by Y. Chow (W. A. Benjamin, Inc., 1966).

[5] T. Cook and B. Sakita, Argonne report, 1966 (unpublished).