$\Delta S=0$ Partially Conserved Axial-Vector Current and $SU(3)$ Coupling Constants

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The meaning of the partially conserved axial-vector current (PCAC) hypothesis for the $\Delta S=0$ current is investigated. It is shown that PCAC for the $\Delta S=0$ current is not in good agreement with various semiphenomenological information. A new formulation of PCAC is given; the main point is that PCAC is valid (as an operator equation) only in the exact $SU(3)$ limit. It is shown that this form of PCAC is a consequence of field theory (and rather weak dynamical assumptions) for matrix elements between states with two photons and the vacuum state. We also find that in a mass-degenerate $SU(3)$ world, the strong-interaction mixing parameter α_S can be expressed in terms of the weak-interaction mixing parameter α_W. Using $0\leq\alpha_S\leq 1$, one finds that $0.145<\alpha_W<0.855$ (if the baryon masses are different) so that the allowed range for α_W is somewhat restricted by the baryon mass differences. Inserting a recent value for α_W in the formula for α_S, one obtains $\alpha_S=0.733\pm0.018$, in excellent agreement with other recent determinations of α_S.

1. INTRODUCTION

THE hypothesis[1] of partially conserved axial-vector current (PCAC) for the $\Delta S=0$ current has brought in a large number of consistency relations, relating various weak interaction quantities (e.g., g_A) to strong-interaction quantities (e.g., $g_{\pi N}$ and various cross sections) or giving consistency relations among strong interaction quantities.[2] Recently, Weisberger[3] has calculated the mixing parameter (or the F/D ratio) α by an extension of PCAC to the $\Delta S=1$ current. However, it has been pointed out by Martin[4] that the PCAC hypothesis for the $\Delta S=1$ current is almost certainly incompatible with experiment. In particular, Martin concludes that the PCAC hypothesis for the $\Delta S=1$ current does not lead to a reliable value for the $SU(3)$ mixing parameter α (which was calculated by Weisberger).[3]

At present, the situation with respect to the PCAC hypothesis for the $\Delta S=1$ current is therefore rather bad. In this paper we show that even the PCAC hypothesis for the $\Delta S=0$ current is not in good agreement with the existing semiphenomenological information. We then give dynamical arguments which indicate that PCAC for $\Delta S=0$ currents is valid in the exact $SU(3)$ limit, where all the octet-baryon masses are equal.

The contents of this paper are as follows:

Section 2. It has recently been pointed out[5] that the strong-interaction mixing parameter α_S is different from the weak-interaction mixing parameter α_W. This implies that the usual form[1] of PCAC for the $\Delta S=0$ current is maximally violated by 37%. We therefore propose a modified version of PCAC, where the divergence of the $\Delta S=0$ axial-vector current is proportional to the pion field only in the exact $SU(3)$ limit.

Section 3. Assuming the above-mentioned form of PCAC, we investigate the implications of PCAC for matrix elements between a state with two photons $|\gamma\gamma\rangle$ and the vacuum state $|0\rangle$.

Section 4. Using very weak dynamical assumptions, we show that PCAC is valid in the exact $SU(3)$ limit for matrix elements between the states $|\gamma\gamma\rangle$ and $|0\rangle$ i.e.,

$$\langle 0|\partial^\mu A_\mu(0)|\gamma\gamma\rangle \propto \langle 0|\varphi_\pi(0)|\gamma\gamma\rangle$$

in the exact $SU(3)$ limit. We thereby confirm our basic hypothesis for two rather different matrix elements, namely, the matrix element between two baryons and the vacuum-two-gamma matrix element. In addition, we find that in a mass-degenerate world there exists the following connection between the mixing parameters α_S and α_W:

$$\alpha_S=\tfrac{1}{2}+(M_\Xi/M)(\alpha_W-\tfrac{1}{2}), \tag{1}$$

where M is the nucleon mass and M_Ξ is the mass of the Ξ hyperon.

Section 5. We compare Eq. (1) with other (semiphenomenological) determinations of α_W and α_S. The formula (1) gives good agreement with the other determinations.

2. DIFFICULTIES WITH PCAC FOR $\Delta S=0$ CURRENTS

The PCAC hypothesis for $\Delta S=0$ currents consists of the following equations:

$$\partial_\mu A_l^{\mu}(x)=C_l\varphi_\pi^{\,l}(x), \quad l=1,2,3, \tag{2}$$

$$C_l=i2m_l^2Mg_A(0)/g_{\pi N}(0). \tag{2'}$$

Here $A_\mu^{\,l}(x)$ is the renormalized axial-vector current, $\varphi_\pi^{\,l}(x)$ is the renormalized Heisenberg field operator of the π mesons, $m=m_l$ is the pion mass, $g_{\pi N}(0)$ is the

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[2] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); Phys. Rev. **140**, 736 (1965); W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965).

[3] W. I. Weisberger, Phys. Rev. **143**, 1302 (1966).

[4] B. R. Martin, Nucl. Phys. **87**, 177 (1966).

[5] B. R. Martin, Phys. Rev. **138**, B1136 (1965); C. Jarlskog and H. Pilkuhn, Phys. Letters **20**, 428 (1966).

pion-nucleon coupling constant evaluated at zero momentum transfer, and $g_A(0)$ is the axial-vector-proton-proton coupling constant evaluated at zero momentum transfer.

Taking the matrix element of Eq. (2) between vacuum and the one-pion state, one obtains

$$\langle\pi^+|\partial^\mu A_\mu{}^l(0)|0\rangle = i2m^2 M g_A(0)/g_{\pi N}(0)\,, \qquad (3)$$

leading to the usual Goldberger-Treiman relation. The easy and rigorous derivation of the Goldberger-Treiman relation (3) from Eq. (2) is the main motivation for the PCAC hypothesis.[1] However, one has to define very clearly in which sense Eq. (2) has to be understood. It is tempting to take the simple point of view that Eq. (2) is a *definition* of the renormalized pion field operator $\varphi_\pi(x)$. This point of view leads, however, to difficulties, as one can see in the following way.

From the fit of the Cabibbo theory,[6] it is known that the axial-vector coupling constants can be determined in terms of the weak interaction $SU(3)$ mixing parameter α_W. For the vertex function at zero momentum transfer we thus have

$$\langle B_i|A_\mu{}^j(0)|B_k\rangle = -ig_A(0)\bar{u}_i\gamma_\mu\gamma_5 u_k[\alpha_W d_{ijk}+(1-\alpha_W)f_{ijk}]. \qquad (4)$$

From this equation we obtain

$$\langle B_i|\partial^\mu A_\mu{}^j(0)|B_k\rangle = g_A(0)(M_i+M_k) \times\bar{u}_i\gamma_5 u_k[\alpha_W d_{ijk}+(1-\alpha_W)f_{ijk}]. \qquad (5)$$

From recent fits of strong-interaction coupling constants,[5] it is known that it makes sense to introduce an $SU(3)$ mixing parameter α_S (with $\alpha_S\neq\alpha_W$) for the strong coupling constants. Thus we have

$$m_j^2\langle B_i|\varphi_\pi{}^j(0)|B_k\rangle = -ig_{\pi N}(0)\bar{u}_i\gamma_5 u_k[\alpha_S d_{ijk}+(1-\alpha_S)f_{ijk}]. \qquad (6)$$

Using PCAC in connection with Eq. (**5**), we obtain from Eq. (6)

$$g_A(0)(M_i+M_k) = -ig_{\pi N}(0)C_j\frac{\alpha_S d_{ijk}+(1-\alpha_S)f_{ijk}}{\alpha_W d_{ijk}+(1-\alpha_W)f_{ijk}}, \qquad (7)$$

where C_j is given by Eq. (2′); hence

$$M_i+M_k = 2M\frac{\alpha_S d_{ijk}+(1-\alpha_S)f_{ijk}}{\alpha_W d_{ijk}+(1-\alpha_W)f_{ijk}}. \qquad (8)$$

Now let us define the exact $SU(3)$ limit as the limit where all the baryon masses are equal to the nucleon mass M. It is then seen that Eq. (8) gives

$$\alpha_S=\alpha_W. \qquad (9)$$

Thus, in the exact $SU(3)$ limit the mixing parameter is the same in the strong and in the weak interactions. Similar results have been obtained by Lee[7] and by Sakurai.[8]

However, if we introduce the physical masses in Eq. (8), we obtain the following relations:

$$\alpha_W/\alpha_S = 2M/(M_\Sigma+M_\Lambda)\,, \qquad (10)$$

$$(2\alpha_W-1)/(2\alpha_S-1) = M/M_\Xi\,, \qquad (11)$$

$$(\alpha_W-1)/(\alpha_S-1) = M/M_\Sigma. \qquad (12)$$

The various fits[5,6] to α_W and α_S are consistent with

$$\alpha_S=0.73\,,\quad \alpha_W=0.66 \qquad (13)$$

(A more detailed discussion is given in Sec. 5). Let us compare Eqs. (10)–(12) with Eq. (13) and the experimental mass ratios. One finds that Eq. (10) deviates from experiments by at least 10%, Eq. (11) is in agreement with experiments, and Eq. (12) deviates from experiments by at least 37%. Hence it is obvious that Eq. (8) is not in good agreement with experiments. Therefore, we conclude that PCAC cannot be an operator identity [i.e., Eq. (2) is incorrect] in the case where the baryon masses are different. This is, in fact, already clear from Eq. (7), since this equation requires that C_j should depend on i and k in order to be satisfied. However, Eq. (2) as an operator equation shows that C_j cannot depend on i and k.

A possible criticism of the above conclusion should be discussed. One may argue that the failure of Eq. (8) shows that one cannot use $SU(3)$ coupling constants (even if α_S is different from α_W). This criticism is, however, not reasonable, since the results obtained in Refs. 5 and 6 show that in semiphenomenological calculations one can use $SU(3)$ coupling constants [note that our PCAC hypothesis (2) refers to $\Delta S=0$ currents; hence our discussion does not include the kaon-baryon-baryon coupling constants, where an application of $SU(3)$ is more doubtful].

Another criticism is that the fits of the coupling constants in Refs. 5 and 6 depend on specific dynamical models. This criticism is, of course, true. However, we take the pragmatic point of view that we base our considerations on present-day physics. We would also like to point out that the only coupling constant which can be fitted in a (reasonable) model-independent way is the pion-nucleon coupling constant (using forward-dispersion relations), because most forward-dispersion relations involve unphysical cuts, where the discontinuity cannot be calculated from unitarity (i.e., cross sections). Thus, unless one has a dynamical theory which explains every detail in strong-interaction physics, one has to rely on model-dependent determinations of the coupling constants.

Guided by the discussion above we propose that the $\Delta S=0$ PCAC and the pion-baryon coupling constants should be treated according to the following assumptions: (i) The pion-baryon coupling constants in broken $SU(3)$ symmetry are calculated by using the exact $SU(3)$ coupling constants with a strong-interaction mixing parameter which is different from the weak-interaction parameter. (ii) The hypothesis of PCAC

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[7] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).

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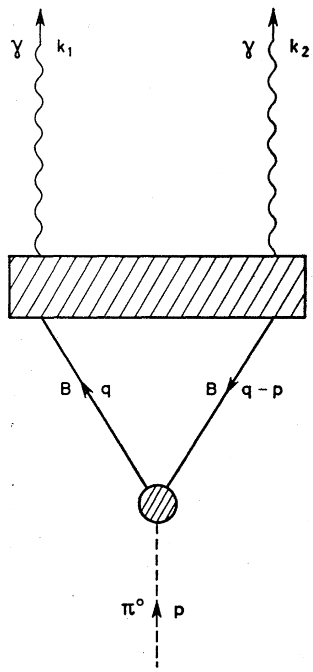

FIG. 1. Diagram for the decay of a neutral pion into two photons.

for $\Delta S=0$ currents reads

$$\partial^\mu A_\mu{}^l(x)=C_l\varphi_\pi{}^l(x)+R_l(x)\,,\quad l=1,2,3, \tag{14}$$

where

$$C_l=i2m_l{}^2Mg_A(0)/g_{\pi N}(0)\,. \tag{14'}$$

The operator $R_l(x)$ vanishes in the limit of exact $SU(3)$, i.e.,

$$\langle\alpha|R_l(0)|\beta\rangle_{SU(3)}=0\,, \tag{15}$$

where $|\alpha\rangle$ and $|\beta\rangle$ are two arbitrary states, and the index "$SU(3)$" indicates that the exact mass degenerate $SU(3)$ limit has to be taken in the evaluation of the matrix element. The mass degenerate $SU(3)$ limit is defined by

$$M_k=M\,,\quad k=1,\cdots,8\,. \tag{16}$$

Here, M is the proton mass, and is not the mean mass of the baryon octet.

The reason for taking the baryon masses equal to the proton mass in the exact $SU(3)$ limit is that Eq. (14) then allows one to derive the Goldberger-Treiman relation in the $SU(3)$ limit in exactly the same way as in Eqs. (2) and (3).

The advantage in having the operator $R_l(x)$ in Eq. (14) together with the condition (15) is that $R_l(x)$ is not a completely arbitrary operator. If one knows that the matrix element

$$\langle\alpha|\varphi_\pi{}^l(x)|\beta\rangle$$

is large in comparison with the relative mass differences in the baryon octet, it is reasonable to neglect the operator $R_l(x)$ in comparison with $C_l\varphi_\pi{}^l(x)$ (note that C_l is proportional to M). Thus the introduction of the operator $R_l(x)$ does not make PCAC void of content.

It should be noted that from a dynamical point of view it is reasonable to assume that the mass-degenerate $SU(3)$ limit corresponds to the high-energy limit (where the various mass differences can be neglected). Thus PCAC with $R_l(x)=0$ becomes valid in the high-energy limit. This feature is satisfactory for the derivation of the Adler-Weisberger formula,[2] where one finally lets an energy go to infinity.

In connection with Eqs. (14) and (15), it should be noted that in a general quark model $\partial_\mu A^\mu$ is proportional to the pion field plus $SU(3)$-breaking terms. Hence a quark model is consistent with assumption (ii).

3. PCAC AND THE DECAY OF NEUTRAL PSEUDOSCALAR MESONS

In this section we shall apply assumption (ii) (i.e., the modified form of PCAC) to a comparison of the axial-vector decay into two photons with the neutral pion decay into two photons.

Using standard reduction technique, one has

$$\langle k_1k_2|A_\mu(0)|0\rangle=T_\mu(A_\mu\to\gamma\gamma)\,, \tag{17}$$

where $T_\mu(A_\mu\to\gamma\gamma)$ is the decay amplitude for the process axial-vector → photon+photon. The four-momenta k_1 and k_2 are the four-momenta of the photons. From Eq. (17) we get

$$\langle k_1k_2|\partial^\mu A_\mu(0)|0\rangle=i(k_1+k_2)_\mu T^\mu(A_\mu\to\gamma\gamma)\,, \tag{18}$$

and from PCAC [Eq. (14)] we then obtain

$$i(k_1+k_2)_\mu T^\mu(A_\mu\to\gamma\gamma)=C\langle k_1k_2|\varphi_\pi(0)|0\rangle +\langle k_1k_2|R(0)|0\rangle\,. \tag{19}$$

Using a reduction technique, we have for the $\pi^0\to\gamma\gamma$ amplitude

$$T(\pi^0\to\gamma\gamma)=\langle k_1k_2|j_\pi(0)|0\rangle\,, \tag{20}$$

where $j_\pi(0)$ is the neutral pion current. With zero mass pions we have

$$(k_1+k_2)^2=0\,, \tag{21}$$

and Eq. (20) becomes, in the zero-pion-mass limit,

$$m^2\langle k_1k_2|\varphi_\pi(0)|0\rangle=T(\pi^0\to\gamma\gamma)\,. \tag{22}$$

Using Eqs. (19) and (21), we obtain

$$i(k_1+k_2)_\mu T^\mu(A_\mu\to\gamma\gamma)=(C/m^2)T(\pi^0\to\gamma\gamma) +\langle k_1k_2|R(0)|0\rangle\,. \tag{23}$$

In the exact $SU(3)$ limit we thus obtain

$$i(k_1+k_2)_\mu T^\mu(A_\mu\to\gamma\gamma)_{SU(3)} =(C/m^2)T(\pi^0\to\gamma\gamma)_{SU(3)}\,. \tag{24}$$

In Sec. 4 we shall show that within rather weak dynamical assumptions Eq. (24) follows from field theory (or Feynman diagrams).

4. DYNAMICAL CALCULATIONS

Let us consider the amplitude for neutral pion decay into two photons. Using field theory, this amplitude can be written as in Fig. 1, because the fundamental Lagrangian involves a pion-baryon-baryon coupling. In Fig. 1 the renormalized vertex function has been introduced at the pion-baryon-baryon vertex; it is seen that the amplitude is the product of the vertex function and a factor which essentially is the Compton amplitude for the baryons.

In writing the amplitude as in Fig. 1 we have neglected a possible $\lambda\varphi_\pi{}^4$ term in the Lagrangian. It is well known that such a term is necessary in general in order to have a renormalizable theory. However, it is not possible here to discuss the question of the renormalizability of the $T(\pi^0\to\gamma\gamma)$ amplitude in depth; we only remark that the first few orders in a perturbation expansion of $T(\pi^0\to\gamma\gamma)$ are finite without a $\lambda\varphi_\pi{}^4$ term in the Lagrangian; therefore, such a term seems to be unnecessary in order to have a renormalizable theory.

There is also a more intuitive reason for omitting the $\lambda\varphi_\pi{}^4$ term. If one takes this coupling into account, it is natural to expect that two of the pions in the intermediate state go together to form a ρ-meson state. Thus the diagram in Fig. 2 becomes essentially the product of the $\pi^0\pi^0\rho^0$ vertex function and the Compton amplitude for the process $\gamma\pi^0\to\gamma\rho^0$, and this Compton amplitude is negligible relative to, e.g., the charged Compton amplitude for the process $\gamma p\to\gamma p$ in the limit of zero-mass pions. Hence, for neutral pions in the intermediate states the $\lambda\varphi_\pi{}^4$ interaction can be neglected. With charged pions in the intermediate states the amplitude for $\pi^+\pi^0\to\pi^+\gamma$, e.g., will occur. However, from an analysis of photoproduction data, Donnachie and Shaw have concluded that the coupling constant for the $\gamma-3\pi$ coupling is consistent with zero.[9]

Let us now return to the diagram in Fig. 1. Writing the Compton amplitude as

$$T(B_i+\gamma\to B_k+\gamma)$$
$$=e^2\bar{u}_k(q)M_{\mu\nu}{}^{ik}(q,k_1,k_2)u_i(q-p)e_1{}^\mu e_2{}^\nu, \quad (25)$$

where e_1 and e_2 are the polarization vectors for the photons and e is the electric charge, we obtain

$$T(\pi^0\to 2\gamma)=e^2\sum_{ik} g_{ik}\int\frac{dq}{(2\pi)^4}$$
$$\times\mathrm{Tr}\left[\gamma_5\frac{1}{\boldsymbol{q}-\boldsymbol{p}-M_i}M_{\mu\nu}{}^{ik}(q,k_1,k_2)\frac{1}{\boldsymbol{q}-M_k}\right]$$
$$\times F_{ik}(q^2,qp,p^2)e_1{}^\mu e_2{}^\nu, \quad (26)$$

where the indices i and k refer to exchange of two baryons B_i and B_k between the vertex part and the Compton part of the diagram in Fig. 1. The quantity $F_{ik}(q^2,qp,p^2)$ is a form factor for the vertex normalized

[9] A. Donnachi and G. Shaw, Ann. Phys. (N. Y.) **37**, 333 (1966); see also in *Proceedings of the International Symposium on Electron and Photon Interaction at High Energies* (Deutsche Physics Gesellschaft, 1966), Vol. I, pp. 64 and 172–174.

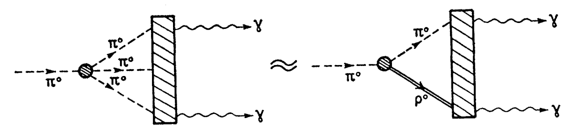

FIG. 2. Approximation for diagrams involving the $\lambda\varphi_\pi{}^4$ coupling.

such that

$$F_{ik}(M_i{}^2,0,0)=1. \quad (27)$$

The g_{ik}'s are the pion-baryon-baryon coupling constants evaluated at zero momentum transfer in accordance with the definition (27).

For the amplitude for $\partial_\mu A^\mu\to 2\gamma$ decay defined in Eq. (5), we get similarly

$$ip_\mu T^\mu(A_\mu\to 2\gamma)=ie^2\sum_{ik} g_A{}^{ik}\int\frac{dq}{(2\pi)^4}$$
$$\times\mathrm{Tr}\left[\boldsymbol{p}\gamma_5\frac{1}{\boldsymbol{q}-\boldsymbol{p}-M_i}M_{\mu\nu}{}^{ik}(q,k_1,k_2)\frac{1}{\boldsymbol{q}-M_k}\right]$$
$$\times F_{ik}{}^A(q^2,qp,p^2)e_1{}^\mu e_2{}^\nu, \quad (28)$$
$$F_{ik}{}^A(M_i{}^2,0,0)=1. \quad (29)$$

Here $g_A{}^{ik}$ are the axial-vector current coupling constants evaluated at zero momentum transfer and with the baryons on the mass shell.

The form factor introduced in Eq. (28) is the form factor corresponding to the $\gamma_\mu\gamma_5$ term in the axial-vector–baryon vertex function. In addition to this form factor, various "induced" form factors can occur. In fact, the most general expression for the off-shell axial-vector-baryon vertex, namely,

$$\langle B_i(q')|A_\mu{}^j(0)|B_k(q)\rangle=-ig_A{}^{ik}(0)\bar{u}_i(q)\sum_{r,r'=0}^{1}(\boldsymbol{q}'-M_i)^r$$
$$\times[\gamma_\mu F_{1A,ik}{}^{rr'}(q^2,q'^2,qq')+q_\mu F_{2A,ik}{}^{rr'}(q^2,q'^2,qq')$$
$$+q_\mu' F_{3A,ik}{}^{rr'}(q^2,q'^2,qq')]\gamma_5(\boldsymbol{q}-M_k)^{r'}u_k(q)$$

involves 12 form factors. However, all of these form factors have their dynamical origin in exchange of various particles between B_i and B_k, and we can therefore include these form factors in the amplitude $M_{\mu\nu}{}^{ik}(q,k_1,k_2)$. The reason why we have explicitly introduced the form factor

$$F_{ik}{}^A(q^2,qp,p^2)=F_{1A,ik}{}^{00}(q^2,q^2-2qp+p^2,q^2-qp)$$

in Eq. (28) is that we want $g_A{}^{ik}(0)$ to be the renormalized axial-vector coupling constants. Alternatively, we could have used unrenormalized coupling constants, and no form factor would occur in Eq. (28).

Using the identity

$$\boldsymbol{p}=(-\boldsymbol{q}+\boldsymbol{p}-M_i)+(\boldsymbol{q}-M_k)+(M_i+M_k), \quad (30)$$

Eq. (28) can be written

$$ip_\mu T^\mu(A_\mu \to 2\gamma) = T_1 + T_2 + T_3\,, \tag{31}$$

where

$$T_1 = ie^2 \sum_{ik} g_A{}^{ik} \int \frac{dq}{(2\pi)^4} \times \mathrm{Tr}\left[\gamma_5 M_{\mu\nu}{}^{ik}(q,k_1,k_2) \frac{1}{\boldsymbol{q} - M_k}\right] \times F_{ik}{}^A(q^2, qp, p^2) e_1{}^\mu e_2{}^\nu\,, \tag{31$'$}$$

$$T_2 = ie^2 \sum_{ik} g_A{}^{ik} \int \frac{dq}{(2\pi)^4} \times \mathrm{Tr}\left[\gamma_5 \frac{1}{\boldsymbol{q} - \boldsymbol{p} - M_i} M_{\mu\nu}{}^{ik}(q,k_1,k_2)\right] \times F_{ik}{}^A(q^2, qp, p^2) e_1{}^\mu e_2{}^\nu\,, \tag{31$''$}$$

$$T_3 = ie^2 \sum_{ik} (M_i + M_k) g_A{}^{ik} \int \frac{dq}{(2\pi)^4} \times \mathrm{Tr}\left[\gamma_5 \frac{1}{\boldsymbol{q} - \boldsymbol{p} - M_i} M_{\mu\nu}{}^{ik}(q,k_1,k_2) \frac{1}{\boldsymbol{q} - M_k}\right] \times F_{ik}{}^A(q^2, qp, p^2) e_1{}^\mu e_2{}^\nu\,. \tag{31$'''$}$$

Note that, apart from various constant factors, T_3 has exactly the same structure as the $\pi^0 \to 2\gamma$ decay amplitude in Eq. (26) if the substitution

$$F_{ik}{}^A(q^2, qp, p^2) \to F_{ik}(q^2, qp, p^2)\,,$$

is made in Eq. (31$'''$). According to Eq. (24), which was based on PCAC, the amplitude $ip_\mu T^\mu(A_\mu \to 2\gamma)$ should be proprotional to the amplitude $T(\pi^0 \to 2\gamma)$ in the limit of zero-mass pions. Since the terms T_1 and T_2 are manifestly different from the $T(\pi^0 \to 2\gamma)$ amplitude, it is obvious that the terms T_1 and T_2 should vanish (at least in the zero-pion-mass limit) in order that PCAC be valid. Thus, in order that the PCAC hypothesis be consistent with field theory, the $ip^\mu T_\mu(A_\mu \to 2\gamma)$ amplitude should be equal to T_3 given in Eq. (31$'''$). Inspection of Eqs. (26) and (31$'''$) shows that the requirement of PCAC implies that the pseudoscalar meson coupling is equivalent to the pseudovector meson coupling (involving the derivative of the pion field), since Eq. (26) is just the expression for the pseudovector coupling and Eq. (31$'''$) is the expression for the pseudoscalar coupling. Since such an equivalence between the two meson couplings was shown many years ago, the terms T_1 and T_2 vanish in the limit of zero mass pions. [The reader who is prepared to believe the above argument by words in favor of $T_1 = T_2 = 0$ can omit the following calculations and read the text following Eq. (38).]

In order to treat the terms T_1 and T_2 we need the most general form of a trace involving the off-shell Compton amplitude. This amplitude is evidently a 4×4 matrix in the space of the γ matrices, and since every 4×4 matrix can be written as a linear combination of the matrices $1, \gamma_\mu, \gamma_\mu\gamma_\nu, \gamma_\mu\gamma_\nu\gamma_\sigma$, (where μ, ν, and σ are all different), and γ_5 we get (using invariance under parity transformations) that the off-shell Compton amplitude $M^{ik}(q,k_1,k_2)$ contains the factors

$$q_\mu q_\nu,\ q_\mu k_{1\nu},\ q_\mu k_{2\nu},\ k_{1\mu} k_{2\nu};$$
$$\gamma_\mu q_\nu,\ \gamma_\mu k_{1\nu},\ \gamma_\mu k_{2\nu},\ \boldsymbol{q} q_\mu q_\nu, \text{ etc.};$$
$$\gamma_\mu \gamma_\nu,\ \boldsymbol{q}\boldsymbol{k}_1 q_\mu q_\nu, \text{ etc.};$$
$$\gamma_\mu \boldsymbol{q} \gamma_\nu,\ \gamma_\mu \boldsymbol{k}_1 \gamma_\nu,\ \gamma_\mu \boldsymbol{k}_2 \gamma_\nu,\ \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2 q_\mu q_\nu,\ \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2 q_\mu k_{1\nu},\ \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2 q_\mu k_{2\nu},\ \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2 k_{1\mu} k_{2\nu};$$

where each of these factors should be multiplied by invariant functions. The invariant functions depend on the invariants q^2, qk_1, and qk_2 or, alternatively, on the invariants

$$s_1 = q^2\,,\quad s_2 = (q-k_1)^2\,,\quad s_3 = (q-p)^2\,,$$

since $p = k_1 + k_2$. Using the fact that the trace of γ_5 and one, two, and three γ matrices vanishes we have

$$\begin{aligned}\mathrm{Tr}[\gamma_5 M_{\mu\nu}{}^{ik}(q,k_1,k_2)(\boldsymbol{q}+M_k)] &= \mathrm{Tr}[\gamma_5\{\gamma_\mu[\boldsymbol{q}C_1{}^{ik}(s_i) + \boldsymbol{k}_1 C_2{}^{ik}(s_i) + \boldsymbol{k}_2 C_3{}^{ik}(s_i)]\gamma_\nu \\ &\quad + \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2[q_\mu q_\nu D_1{}^{ik}(s_i) + q_\mu k_{1\nu} D_2{}^{ik}(s_i) \\ &\quad + q_\mu k_{2\nu} D_3{}^{ik}(s_i) + k_{1\mu} k_{2\nu} D_4{}^{ik}(s_i)]\}(\boldsymbol{q}+M_k)]\,.\end{aligned} \tag{32}$$

Here $C_j{}^{ik}$ and $D_j{}^{ik}$ are the off-shell scalar Compton amplitude (crossing symmetry, gauge invariance, and other symmetries give restrictions on the number of independent scalar amplitudes) contributing to the trace.

In the following we consider only T_1, since T_2 can be treated quite analogously (by a simple change of integration variable). We have

$$\begin{aligned}\mathrm{Tr}\left[\gamma_5 M_{\mu\nu}{}^{ik}(q,k_1,k_2)\frac{1}{\boldsymbol{q}-M_k}\right] e_1{}^\mu e_2{}^\nu &= \frac{1}{q^2 - M_k{}^2} \times \mathrm{Tr}[\gamma_5\{\boldsymbol{e}_1[\boldsymbol{q}C_1{}^{ik} + \boldsymbol{k}_1 C_2{}^{ik} + \boldsymbol{k}_2 C_3{}^{ik}]\boldsymbol{e}_2 + \boldsymbol{q}\boldsymbol{k}_1\boldsymbol{k}_2 \\ &\quad \times [(qe_1)(qe_2)D_1{}^{ik} + (qe_1)(qk_2)D_2{}^{ik} + (qe_1)(k_2e_2)D_3{}^{ik} \\ &\quad + (k_1e_1)(k_2e_2)D_4{}^{ik}]\}(\boldsymbol{q}+M_k)]\,.\end{aligned} \tag{33}$$

Using the well-known formula

$$\mathrm{Tr}[\gamma_5 \boldsymbol{abcd}] = 4[a,b,c,d]\,, \tag{34}$$

with

$$[a,b,c,d] = \epsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma\,, \tag{34$'$}$$

$$\epsilon_{0123} = +1\,, \tag{34$''$}$$

we get

$$\begin{aligned}\mathrm{Tr}[\gamma_5 M_{\mu\nu}{}^{ik}(q,k_1,k_2)(\boldsymbol{q}-M_k)^{-1}]e_1{}^\mu e_2{}^\nu &= [4/(q^2-M_k{}^2)]\{[e_1, qC_1{}^{ik} + k_1 C_2{}^{ik} + k_2 C_3{}^{ik}, e_2, q] \\ &\quad + [q,k_1,k_2,q]((qe_1)(qe_2)D_1{}^{ik} + (qe_1)(qk_2)D_2{}^{ik} \\ &\quad + (qe_1)(k_2e_2)D_3{}^{ik} + (k_1e_1)(k_2e_2)D_4{}^{ik})\} \\ &= [4/(q^2-M_k{}^2)][e_1, k_1C_2{}^{ik} + k_2C_3{}^{ik}, e_2, q]\,,\end{aligned} \tag{35}$$

where we have used the antisymmetry of the product $[a,b,c,d]$. The invariant amplitudes $C_2{}^{ik}$ and $C_3{}^{ik}$ depend on the invariants s_1, s_2, and s_3.

Let us now study T_1 (and T_2) in the limit of zero mass pions. Here we have by energy-momentum conservation in the rest system of the pion

$$\mathbf{p}=0=\mathbf{k}_1+\mathbf{k}_2\,,\quad \text{i.e.,}\quad |\mathbf{k}_1|=|\mathbf{k}_2|\,,$$

and

$$p_0=|\mathbf{k}_1|+|\mathbf{k}_2|\to 0\,.$$

In the limit of zero mass pions we can thus treat the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ as infinitesimal and we shall work only to order k^2. (k^2 is the order of magnitude of e.g., $\mathbf{k}_1\mathbf{k}_2$.) Since the Compton amplitudes are functions of q^2, qk_1, and qk_2, we have in the limit $qk_1\to 0$, $qk_2\to 0$ that the Compton amplitude (depending only on q^2 in the limit) becomes the off-shell Compton amplitude for scattering of zero-frequency photons on baryons. In this limit we can therefore apply the well-known low-energy theorem[10] for the Compton amplitude. The fact that the baryons are off-shell is not important, since the on-shell condition in the proof of the low-energy theorem[10] is of no essential importance (the essential points are charge conservation and gauge invariance). We have

$$\lim_{qk_1\to 0,\, qk_2\to 0} C_j{}^{ik}(q^2,qk_1,qk_2)=C_j{}^{ik}(q^2,0,0)^{\text{Born}} +D_j{}^{ik}(q^2,0,0)O(k)+O(k^2/q^2)\,. \quad (36)$$

The second term on the right-hand side of Eq. (36) is proportional to the anomalous magnetic moment of the baryon and contains terms of the form

$$\boldsymbol{\sigma}\cdot[(\mathbf{e}_1\times\mathbf{k}_1)\times(\mathbf{e}_2\times\mathbf{k}_2)](1/\omega_1)\,,$$

multiplied by factors of the order 1. With no anomalous magnetic moment, $D_j{}^{ik}(q^2,0,0)$ vanishes exactly. Let us neglect the anomalous magnetic moment, since it will only have a very small influence in comparison with $(C_j{}^{ik})^{\text{Born}}$. Hence $D_j{}^{ik}$ vanishes, and substituting Eq. (36) in Eq. (35), we get

$$\mathrm{Tr}[\gamma_5 M_{\mu\nu}{}^{ik}(q,k_1,k_2)(\boldsymbol{q}-M_k)^{-1}]e_1{}^\mu e_2{}^\nu=[4/(q^2-M_k{}^2)] \times[e_1,k_1C_2{}^{ik}(q^2)^{\text{Born}}+k_2C_3{}^{ik}(q^2)^{\text{Born}},e_2,q]+O(k^3)\,. \quad (37)$$

In order to obtain T_1 we should integrate (37) over q. However, since (37) is proportional to the *vector* q (times functions of q^2), the integral over q vanishes identically by symmetry, so that $T_1=O(k^3)$. (In Appendix A we show by an explicit calculation that the Born approximation gives $T_1=T_2=0$ for arbitrary pion mass.) A similar argument holds for T_2. Hence, if we neglect the order k^3, Eq. (30) becomes

$$ip_\mu T^\mu(A_\mu\to 2\gamma)=ie^2\sum_{ik}(M_i+M_k)g_A{}^{ik}\int\frac{dq}{(2\pi)^4} \times\mathrm{Tr}\left[\gamma_5\frac{1}{\boldsymbol{q}-\boldsymbol{p}-M_i}M_{\mu\nu}{}^{ik}(q,k_1,k_2)\frac{1}{\boldsymbol{q}-M_k}\right] \times F_{ik}{}^A(q^2,0,0)e_1{}^\mu e_2{}^\nu+O(k^3)\,. \quad (38)$$

It is a very satisfactory feature in the derivation of Eq. (38) that the equality holds only to the order k^3, because the "equivalence" between the pseudoscalar and the pseudovector cannot be true for all energies. (The equivalence is one between a renormalizable and a nonrenormalizable theory, and as such, is evidently rather formal.)

Note that the calculations leading from Eq. (31) to Eq. (38) are needed in order to establish Eq. (38) to the order k^3. We cannot simply let $k\to 0$, because then every amplitude vanishes and gives the trivial identity $0=0$; this follows, e.g., from the fact that the $\pi^0\to 2\gamma$ amplitude for invariance reasons contains the factor

$$[e_1,e_2,k_1,k_2]\,,$$

i.e., the amplitude is of order k^2. Therefore, in order to avoid trivialities we have to keep $O(k^2)$ terms and neglect only $O(k^3)$ terms. In the final result the k dependence will of course disappear.

The form factors in Eqs. (26) and (38) are off the mass shell, and nothing is known about them. However, the integrals (26) and (38) have denominators of the form

$$1/[(q-p)^2-M_i{}^2]\quad\text{and}\quad 1/(q^2-M_i{}^2)\,,$$

which should be multiplied by the various invariant Compton amplitudes. Hence it is reasonable to expect that the integrals in Eqs. (26) and (38) are rather rapidly convergent. In fact, due to the "low-energy" theorem for the Compton amplitude,[10] the invariant Compton amplitudes are essentially given by the Born approximation, and it is therefore relevant to compare the convergence of the integrals (26) and (38) with the convergence of the Born approximation with the form factors equal to unity. In this case, Eq. (26) becomes[11]

$$T(\pi^0\to 2\gamma)^{\text{Born}} =2ie^2\sum_i g_i\int\frac{dq}{(2\pi)^4} \times\mathrm{Tr}\left[\gamma_5\frac{1}{\boldsymbol{q}-\boldsymbol{p}-M_i}\boldsymbol{e}_1\frac{1}{\boldsymbol{q}-\boldsymbol{k}_1-M_i}\boldsymbol{e}_2\frac{1}{\boldsymbol{q}-M_i}\right] =\frac{e^2}{4\pi^2}\sum_i\frac{g_i}{M_i}\left(\frac{2M_i\arcsin((\sqrt{p^2})/2M_i)}{\sqrt{p^2}}\right)^2 \times[e_1,e_2,k_1,k_2]\,. \quad (39)$$

[10] W. Thirring, Phil. Mag. **41**, 1193 (1950); F. E. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954).

[11] R. J. Finkelstein, Phys. Rev. **72**, 415 (1947); J. Steinberger, *ibid.* **76**, 1180 (1949); J. Schwinger, *ibid.* **82**, 664 (1951).

In the limit of zero mass pions ($p^2 \to 0$), this expression tends to the finite limit

$$T(\pi^0 \to 2\gamma)^{\text{Born}} = \frac{e^2}{4\pi^2} \sum_i \frac{g_i}{M_i} [e_1, e_2, k_1, k_2]. \quad (40)$$

The convergences of Eq. (39) is essentially due to gauge invariance, which can be seen in the following way. Gauge invariance means that replacing e_1 and e_2 by k_1 and k_2, respectively, the amplitude vanishes. This, in turn, implies that the trace in Eq. (39) has to give a product of k_1, k_2, e_1, and e_2. With two powers of momenta outside the integral, the integrand has to converge better than expected (*a priori*, one would expect the integral to diverge because it only involves spin-$\frac{1}{2}$ propagators).

Hence it appears reasonable to assume that the integrals (26) and (38) are also rather rapidly convergent, especially since the form factors are expected to vanish for $q^2 \to \infty$. Since the normalization of F_{ik} and $F_{ik}{}^A$ is the same, we may approximate

$$F_{ik}(q^2,0,0) \approx F_{ik}{}^A(q^2,0,0), \quad (41)$$

under the integrals in view of the expected rather rapid convergence. In other words, we expect that an eventual difference in the structure of F_{ik} and $F_{ik}{}^A$ shows up so far from the normalization point $q^2 = M_i^2$ that the contributions to the integrals from these distant regions in the space of integration are negligible. This assumption makes sense only if the form factors do not vary too rapidly (for a more detailed discussion, see Appendix B).

Collecting the results exhibited in Eqs. (26) and (38), we find the following formula in the exact $SU(3)$ limit:

$$i p_\mu T^\mu (A_\mu \to 2\gamma)_{SU(3)} = 2M(g_A(0)/g_{\pi N}(0)) T(\pi^0 \to 2\gamma)_{SU(3)}, \quad (42)$$

which is just the result expected from PCAC, as is seen by comparison with Eq. (24). Hence we have shown that PCAC is correct (within weak dynamical assumptions) for the amplitude $T_\mu(A_\mu \to 2\gamma)$.

If, on the other hand, we assume that with degenerate masses PCAC holds, i.e.,

$$\langle k_1 k_2 | R_l(0) | 0 \rangle \approx 0,$$

also in the case of mass degeneration, we get instead of Eq. (42)

$$\sum_{ik} (M_i + M_k) g_A{}^{ik} I_{\mu\nu}{}^{ik} e_1{}^\mu e_2{}^\nu = \frac{2M g_A(0)}{g_{\pi N}(0)} \sum_{ik} g_{ik} I_{\mu\nu}{}^{ik} e_1{}^\mu e_2{}^\nu, \quad (43)$$

$$I_{\mu\nu}{}^{ik} = \int \frac{dq}{(2\pi)^4} \operatorname{Tr}\left[\gamma_5 \frac{1}{\boldsymbol{q} - \boldsymbol{p} - M_i} \times M_{\mu\nu}{}^{ik}(q,k_1,k_2) \frac{1}{\boldsymbol{q} - M_k}\right] F_{ik}(q^2,0,0), \quad (44)$$

which is valid in the limit of zero-mass pions.

Now we can use the fact that in the zero-mass limit the Compton amplitudes are essentially the Born approximation amplitudes; the latter are proportional to the square of the (renormalized) charge. Hence all integrals $\boldsymbol{I}^{ik}$ involving neutral intermediate particles vanish. Thus many terms disappear in Eq. (43). Using $SU(3)$ coupling constants, one finds that (43) reduces to

$$I_{\mu\nu}{}^N + (M_\Xi/M)(2\alpha_W - 1) I_{\mu\nu}{}^\Xi = I_{\mu\nu}{}^N + (2\alpha_S - 1) I_{\mu\nu}{}^\Xi.$$

Hence the important feature emerges that the integrals $I_{\mu\nu}$ cancel completely (which is very advantageous since the $I_{\mu\nu}$'s depend on the details in the dynamics of strong and weak interactions through the form factor). We obtain the simple formula

$$\alpha_S = \tfrac{1}{2} + (\alpha_W - \tfrac{1}{2}) M_\Xi / M. \quad (45)$$

From Eq. (45) one can obtain limits on α_W by using $0 \leq \alpha_S \leq 1$. One obtains

$$\tfrac{1}{2}(1 - M/M_\Xi) \leq \alpha_W \leq \tfrac{1}{2}(1 + M/M_\Xi), \quad (46a)$$

or

$$0.145 \leq \alpha_W \leq 0.855. \quad (46b)$$

From (46a) it is seen that in the equal-mass limit one has $0 \leq \alpha_W \leq 1$. But in the case where the masses are not equal, the allowed range for α_W becomes more restricted. It is interesting that the allowed range for the weak-interaction parameter α_W is determined by the strong $SU(3)$ breaking interactions.

5. DISCUSSION

First we note that in the limit of complete mass degeneracy, Eq. (45) reduces to the identity $\alpha_S = \alpha_W$. This is a satisfactory feature since one knows (see Sec. 2) that in the limit of exact $SU(3)$ $\alpha_S = \alpha_W$. With broken $SU(3)$ there is still a possibility that $\alpha_S = \alpha_W$, but then

$$\alpha_S = \alpha_W = \tfrac{1}{2}. \quad (47)$$

Since we know that $\alpha_W \neq \frac{1}{2}$ it follows that $\alpha_S \neq \alpha_W$.

Secondly, by using the most recent value of α_W found by Brene *et al.*,[6]

$$\alpha_W = 0.665 \pm 0.018, \quad (48)$$

Eq. (45) yields

$$\alpha_S = 0.733 \pm 0.018, \quad (49)$$

where we have taken into account that the error in the determination of α_W is essentially a fitting error.

The strong-interaction mixing parameter α_S has been determined by Jarlskog and Pilkuhn[5] using a baryon-exchange model to explain the observed backward peaks in the reactions $K^- p \to \Lambda\pi^0$, $K^- p \to \Sigma^- \pi^+$, $\pi^- p \to \Sigma^- K^+$, and $\pi^- p \to \Lambda^0 K^0$. These authors find

$$\alpha_S = 0.71_{-0.05}{}^{+0.04}, \quad (50)$$

in excellent agreement with the result (49). Another determination has been made by Martin[5] from a semi-phenomenological study of the first hyperon resonance

Y_1^* (1385), using a one-channel approximation. Contributions from Σ and Y_1^* exchange, and the exchange of a low-energy S-wave pion-pion pair, to dispersion relations for the $P_{3/2}\pi$-Λ scattering amplitude were evaluated. The coupling constant $g_{\Sigma\Lambda}$ and a parameter related to the $P_{3/2}\pi$-Λ scattering length were varied, so that the amplitude calculated from the dispersion relation was consistent with the input resonant amplitude. This could be achieved for a value of $g_{\Sigma\Lambda\pi}$ corresponding to a strong mixing parameter

$$\alpha_S=0.75. \tag{51}$$

This value is also in excellent agreement with Eq. (49).

Thus we have seen that methods based on rather different models lead to a value of α_S which is given by Eq. (49). It is therefore natural to conclude that there is strong evidence against the assumption that $\alpha_S=\alpha_W$ (when the baryon masses are different).

The good agreement between the formula (45) and semiphenomenological fits follows already by a comparison of Eq. (45) with the Eqs. (10)–(12) in Sec. 2. Equations (10)–(12) were derived from the primitive form of PCAC in Eq. (2) by taking matrix elements between one-baryon states. Equation (45), which was derived on the basis of our rather weak dynamical assumptions and PCAC, is identical with Eq. (11). Equations (10) and (12) were in disagreement with experiments, while Eq. (11) was in agreement with experiments, for some unknown reason. Our dynamical calculation shows why Eq. (11) is in agreement with experiments: because the dynamical calculation leads to Eq. (11) and not to Eq. (10) or (12).

The strongest dynamical assumption in our calculation leading to Eq. (45) is the use of PCAC and $SU(3)$ coupling constants. The fact that Eq. (45) agrees with other recent determinations of α_S makes it likely that the PCAC hypothesis (used for $\Delta S=0$ currents) in its modified form (ii) can be used together with the exact $SU(3)$ coupling constants with $\alpha_S\neq\alpha_W$ to take into account broken $SU(3)$. This statement has, of course, only been shown to be valid as far as the pion-baryon vertices are concerned.

The dynamical picture which emerges from the above considerations is *that the pion-baryon coupling constants can be calculated in broken $SU(3)$ by using $SU(3)$ coupling constants with $\alpha_S\neq\alpha_W$ and with α_S given by Eq. (45)*. As is clearly exhibited in Eq. (45), the fact that α_S becomes different from α_W is due to the mass splitting. This point of view is not necessarily consistent with that of Freund and Nambu[12]; however, we would like to point out that the arguments presented in the present paper are only valid for pions coupled to baryons (and not, e.g., for mesons coupled to excited baryon states).

Recently, the $KN\Lambda$ coupling constant has been determined by use of forward dispersion relations for KN scattering.[13] An upper limit for the $KN\Sigma$ coupling constant was also found. In the evaluation of the dispersion integral over the unphysical region it was assumed that the only contribution other than S wave comes from a pole term at Y_1^* (1385). It turns out that the K coupling constant is a factor 2 or 3 smaller than the $SU(3)$-invariant value. However, this result does not contradict our conclusion, since we have applied PCAC (or the Goldberger-Treiman relations) to $\Delta S=0$ currents. The results of Refs. 4 and 13 show, however, that we cannot generalize PCAC to $\Delta S=1$ currents. (At least, the predictions of such a generalization would be violated experimentally.)

Also, our method gives a result, even if we do not use $SU(3)$ coupling constants, for the pion-baryon-baryon vertices. Without $SU(3)$, Eq. (45) becomes

$$g_{\pi\Xi}/g_{\pi N}=(2\alpha_W-1)M_\Xi/M\,, \tag{52}$$

from which we find

$$g_{\pi\Xi}^2/4\pi=2.85\,, \tag{53}$$

where $g^2/4\pi=14.8$. It is interesting to compare the value (53) with the value obtained in the exact $SU(3)$ limit $\alpha_S=\alpha_W$. One obtains

$$(g_{\pi\Xi}^2/4\pi)_{SU(3)}=1.42\,. \tag{54}$$

The difference between (53) and (54) is a factor of 2, which is due to the occurrence of the squared mass ratio M_Ξ/M.

In the derivation of Eq. (45) we have neglected electromagnetic mass differences. If these are taken into account we obtain, instead of Eq. (45),

$$2(1-\alpha_W)I^\Sigma(M_{\Sigma^+}-M_{\Sigma^-})/M \\ +(2\alpha_W-1)(M_{\Xi^-}/M)I^\Xi=(2\alpha_S-1)I^\Xi\,, \tag{55}$$

where, e.g.,

$$I^\Sigma=\int\frac{dq}{(2\pi)^4}\mathrm{Tr}\left[\gamma_5\frac{1}{\boldsymbol{q}-\boldsymbol{p}-M_\Sigma}\right. \\ \left.\times M_{\mu\nu}{}^{\Sigma\gamma\to\Sigma\gamma}(q,k_1,k_2)\frac{1}{\boldsymbol{q}-M_\Sigma}\right]F_{\Sigma\Sigma}(q^2,0,0)e_1{}^\mu e_2{}^\nu\,, \tag{56}$$

and where we have assumed

$$I^{\Sigma^+}\approx I^{\Sigma^-}. \tag{57}$$

From Eq. (55) we obtain

$$\alpha_S=\tfrac{1}{2}+(\alpha_W-\tfrac{1}{2})M^\Xi/M \\ +(1-\alpha_W)((M_{\Sigma^+}-M_{\Sigma^-})/M)I^\Sigma/I^\Xi. \tag{58}$$

Thus, it is seen that the electromagnetic correction to Eq. (45) contains the ratio I^Σ/I^Ξ, i.e., this correction depends on the details in the dynamics. In order to obtain a rough estimate of the numerical influence of

[12] P. G. O. Freund and Y. Nambu, Phys. Rev. Letters **13**, 221 (1964).

[13] M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters **21**, 229 (1966).

the correction term we calculate the ratio I^{Σ}/I^{Ξ} in the Born approximation with the form factors equal to one. We obtain

$$(I^{\Sigma}/I^{\Xi})^{\text{Born}}=M_{\Xi}/[\tfrac{1}{2}(M_{\Sigma^+}+M_{\Sigma^-})]. \tag{59}$$

Hence the correction term in Eq. (58) becomes

$$2(1-\alpha_W)\frac{M_{\Sigma^+}-M_{\Sigma^-}}{M_{\Sigma^+}+M_{\Sigma^-}}\frac{M_{\Xi}}{M}=-0.31\times10^{-2}. \tag{60}$$

Instead of the value (49) for α_S we therefore obtain

$$\alpha_S=0.730\pm0.018. \tag{61}$$

The electromagnetic correction has obviously no influence on the above discussion.

From the point of view of principles, it is interesting to note that the proof given in Sec. 4 shows that the neutral pion decay into two photons is given by the Born approximation with two intermediate baryon states, provided that the pion mass vanishes. Hence the lifetime of the neutral, massless pion can be calculated quite trivially. Assuming that the lifetime is a slowly varying function of the mass, it follows that a reasonable approximation to the lifetime is given by the two-baryon approximation. This calculation has been done in Ref. 14.

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APPENDIX A: THE BORN APPROXIMATION TO AXIAL-VECTOR DECAY

In this Appendix it is shown that the arguments in Eqs. (31)-(38) are very straightforward and simple in the Born approximation with the form factors equal to unity. Using standard technique we have

$$ip_\mu T^\mu(A_\mu\to 2\gamma)^{\text{Born}}=-2e^2g_A\int\frac{dq}{(2\pi)^4}\times\text{Tr}\left[p\gamma_5\frac{1}{q-k_1-M}e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right], \tag{A1}$$

where we have considered only a single kind of baryon in the intermediate states to save writing, and where the factor 2 comes from Bose statistics. Using again the identity

$$\begin{aligned}p&=k_1+k_2\\&=(-q+k_1-M)+(q+k_2-M)+2M,\end{aligned} \tag{A2}$$

Eq. (A1) becomes

$$\begin{aligned}ip_\mu T^\mu(A_\mu\to 2\gamma)^{\text{Born}}=-2e^2g_A\int\frac{dq}{(2\pi)^4}&\times\left\{\text{Tr}\left[\gamma_5e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right]\right.\\&+\text{Tr}\left[\gamma_5\frac{1}{q-k_1-M}e_1\frac{1}{q-M}e_2\right]\\&\left.+2M\,\text{Tr}\left[\gamma_5\frac{1}{q-k_1-M}e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right]\right\}.\end{aligned} \tag{A3}$$

Now

$$\text{Tr}\left[\gamma_5e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right]=\frac{4[e_1,q,e_2,k_2]}{(q^2-M^2)[(q+k_2)^2-M^2]}. \tag{A4}$$

Using Feynman parametrization one has

$$\begin{aligned}\int\frac{dq}{(2\pi)^4}\text{Tr}\left[\gamma_5e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right]&=4\int_0^1d\alpha\int\frac{dq}{(2\pi)^4}\frac{[e_1,q,e_2,k_2]}{(q^2-M^2)^2}\\&=4\epsilon_{\mu\nu\rho\sigma}e_1{}^\mu e_2{}^\rho k_2{}^\sigma\int_0^1d\alpha\int\frac{dq}{(2\pi)^4}\frac{q^\nu}{(q^2-M^2)^2}.\end{aligned} \tag{A5}$$

The q integral obviously vanishes by symmetry.[15] Similar arguments apply to the second term in Eq. (A3). Hence we are left with

$$ip_\mu T^\mu(A_\mu\to 2\gamma)^{\text{Born}}=-4e^2g_AM\int\frac{dq}{(2\pi)^4}\times\text{Tr}\left[\gamma_5\frac{1}{q-k_1-M}e_1\frac{1}{q-M}e_2\frac{1}{q+k_2-M}\right]. \tag{A6}$$

Apart from a constant factor, the result (A6) is the amplitude for neutral pseudoscalar meson decay in the Born approximation. The result (A6) corresponds to Eq. (38).

The calculation leading from Eq. (A1) to Eq. (A6) reflects the well known "equivalence" between a pseudoscalar and a pseudovector coupling. In the text

[14] B. Lautrup and P. Olesen, Phys. Letters **22**, 342 (1965).

[15] In this argument we have tacitly assumed that integrals like the q integral in Eq. (A5) are convergent. Strictly speaking, this is not true; however, from field theory it is well known that by using some covariant cutoff procedure (e.g., regularization) integrals like the q integral in Eq. (A5) vanish.

this equivalence was shown to be generally valid only in the limit of zero-pion mass. We have seen here, however, that the equivalence is valid for the Born approximation without any restriction.

APPENDIX B: ESTIMATE OF THE INFLUENCE OF THE FORM FACTORS

In Sec. 4 it was argued that if the off-shell form factors did not vary too much with q^2, the difference between $F_{ik}(q^2,0,0)$ and $F_{ik}{}^A(q^2,0,0)$ could be neglected in the integrals involving the Compton amplitude. Clearly, it is impossible to estimate the degree of validity of this assumption in the realistic case, since nothing is known about off-shell form factors. In order to obtain some insight into the mathematical structure of this assumption we consider here a model-world where the form factors have the following form:

$$F(q^2)=1+(M^2-q^2)/(q^2-M_R{}^2)\,, \tag{B1}$$

where M_R is some mass describing a structure effect. In our model-world there exists only one baryon with mass M (this is an unessential assumption introduced to save writing), and we use the Born approximation since we consider the limit of zero-mass pions.

The model-world version of the integral (43) becomes

$$I'=2i\int\frac{dq}{(2\pi)^4}\left(1+\frac{M^2-q^2}{q^2-M_R{}^2}\right)\times\frac{\mathrm{Tr}[\gamma_5(\boldsymbol{q}-\boldsymbol{k}_1+M)\boldsymbol{e}_1(\boldsymbol{q}+M)\boldsymbol{e}_2(\boldsymbol{q}+\boldsymbol{k}_2+M)]}{[(q-k_1)^2-M^2][q^2-M^2][(q+k_2)^2-M^2]}=-8iM[e_1,e_2,k_1,k_2]I\,, \tag{B2}$$

where

$$I=\int\frac{dq}{(2\pi)^4}\left(1+\frac{M^2-q^2}{q^2-M_R{}^2}\right)\times\frac{1}{[(q-k_1)^2-M^2][q^2-M^2][(q+k_2)^2-M^2]}\,. \tag{B3}$$

The integral (B3) is easily evaluated using Feynman parametrization. One obtains in the limit of zero-mass pions

$$I=(1/32\pi^2iM^2)[1+K(M_R/M)]\,, \tag{B4a}$$

$$K(x)=\frac{-4}{(1-x^2)^2}\ln x-\frac{2}{1-x^2}\,. \tag{B4b}$$

Here $K(M_R/M)$ is the correction to the Born approximation without the form factor (B1).

In order to satisfy our condition that the form factor (B1) does not vary too much (which is a common assumption in weak-interaction physics) we require that $F(q^2)$ does not vary more than 10% between $q^2=0$ and $q^2=M^2$. This requires

$$M_R{}^2\geq 10M^2\,. \tag{B5}$$

Let us now assume that there is a difference in the structure of the axial-vector vertex and the corresponding pion vertex. This means that M_R is different for the two vertices. We take M_R equal to M_{AN} and $M_{\pi N}$, respectively.

Furthermore, it is reasonable to expect that the masses M_{AN} and $M_{\pi N}$ are of the order of magnitude of the pion-nucleon resonances. The limit (B5) is then a bit high, but let us take as an example

$$M_{AN}=(\sqrt{10})M=3.0\text{ GeV}, \tag{B6a}$$

$$M_{\pi N}=(\sqrt{20})M=4.2\text{ GeV}. \tag{B6b}$$

Then the relative difference between the integrals of the type (B4a) is

$$\Delta I/I=0.15\,, \tag{B7}$$

which corresponds to an error in the formula (45) (expressing α_S in terms of α_W) given by

$$(\alpha_W-\tfrac{1}{2})(M_\Xi/M)\Delta I/I=3\%. \tag{B8}$$

Thus we have seen that the rather large difference in structure exhibited in Eq. (B6) only gives an error of 3% in the value for α_S.

If we assume that our model-world gives a significant measure of the relative accuracy of the approximation $F_{ik}(q^2,0,0)\approx F_{ik}{}^A(q^2,0,0)$ in integrands like (43) we can conclude that this approximation is very good. Using Eq. (B8), the value for α_S becomes

$$\alpha_S=0.73\pm0.03\,, \tag{B9}$$

where we have included the electromagnetic correction from Eq. (61).