data points. As a check on data consistency over a longer period, the absolute lifetime analysis using Eq. (1) with the anticoincidence ring gives $(\tau_+/\tau_-)-1=0.0060 \pm 0.0031$ over the same distance (0.20–0.78) lifetime. The corresponding value for the π^+ lifetime is also shown in Table II with a standard deviation that includes statistical and consistency errors as well as that in the absolute momentum. The latter result is in fair agreement with one of the two recent accurate determinations of τ_+, but not with the other.

As shown in Table II, the comparison of π^+ and π^- lifetimes agrees with the other two contemporaneous experiments.[4,5] The three experiments utilized quite different methods, and we should like to emphasize that the present experiment requires no corrections except for the very small one for a difference in the momenta. Using the same method with improved beam and counters, the experiment will be repeated shortly with greater precision[8].

ACKNOWLEDGMENTS

We wish to thank James Vale and the cyclotron personnel for assistance with the apparatus and for giving us such stable operating conditions, E. F. McLaughlin and R. V. Schafer for cryogenic design of the hydrogen Čerenkov counter, H. Weisberg for assistance with the digitized spark chamber, G. R. Farrar for aid in the analysis, and A. C. Helmholz and B. J. Moyer for support and encouragement. Particularly we want to thank R. D. Eandi and B. Macdonald for the extensive help they gave with the apparatus and in the early running of the experiment.

[4] M. Bardon, U. Dore, D. Dorfan, M. Krieger, L. Lederman, and E. Schwarz, Phys. Rev. Letters **16**, 775 (1966).

[5] F. Lobkowicz, A. C. Melissinos, Y. Nagashima, S. Tewksbury, H. von Briesen, Jr., and J. D. Fox, Phys. Rev. Letters **17**, 548 (1966). The two values given for lifetime difference arise from different methods of averaging the data.

PHYSICAL REVIEW VOLUME 157, NUMBER 5 25 MAY 1967

Excited-Droplet Model for $pp \rightarrow pN_{1/2}^*$ at High Energies*

RICHARD C. ARNOLD
High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois
(Received 27 June 1966; revised manuscript received 12 January 1967)

A surface-excitation droplet model for $N_{1/2}^*$ production in pp collisions at high energy and small momentum transfer is proposed which explains the Δ^2 dependence of the first two observed $I=\frac{1}{2}$ isobars.

In the coherent-droplet model of Byers and Yang,[1] high-energy exchange reactions with small momentum transfer are pictured in terms analogous to small-angle elastic scattering of partially absorbing nuclei. The reactions are described in terms of a density function $\rho(r)$ which represents the optical-model density of hadron "matter." In this paper, a picture is proposed in which the significance of $\rho(r)$ is extended to a distribution of mass subject to coherent excitation in modes analogous to collective excitation of complex nuclei.

The proton and its excited states with the same isospin will be described in their own center-of-mass (c.m.) system by state functions $f(r)Y_L^M(\theta,\phi)\Psi$, where Ψ is a spinor. It will first of all be postulated that the "intrinsic" helicity component associated with Ψ is conserved in high-energy collisions, while the "spatial" part of the wave function provides a description of the excited states of the proton appropriate for the high-energy limit of the S-matrix elements.[2] The spin functions Ψ will, therefore, be ignored in the following discussion of S-matrix elements.

The second postulate to be made is that of "surface" modes of excitation; the radial wave functions are assumed to be similar in shape for all excited states, and overlap integrals with ground-state wave functions will be peaked at a radius associated with an effective elastic scattering radius. This will enable use of the Austern-Blair[3] approach to connect inelastic and elastic channels.

A third postulate is the diffraction-excitation (or "dissociation")[4] mechanism which suggests that in

* Work performed under the auspices of the U. S. Atomic Energy Commission

[1] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966).

[2] This decoupling of intrinsic and collective excitation components of the angular momentum of the state presumably cannot be a symmetry of the S matrix at rest since in that case there would be a doubling of $N^*_{1/2}$ parities not observed experimentally. The spin and parity of each N^* depends on a coupling of intrinsic (ψ) and collective (Y_L^M) degrees of freedom.

[3] N. Austern and J. S. Blair, Ann. Phys. (N. Y.) **33**, 15 (1965); W. H. Bassichis and A. Dar, *ibid.* **36**, 130 (1966). The form of the matrix element in the model presented here corresponds to a δ-function interaction potential between projectile and the hadronic matter distribution.

[4] M. Ross and L. Stodolsky, Phys. Rev. **149**, 1172 (1966).

collective excitation the S-matrix element for excitation is given simply by an overlap integral[5] between initial asymptotic two-particle (pp) state and final ($pN_{1/2}^*$) asymptotic state,[6] involving a δ-function interaction mechanism between the volume elements of the target and those of the projectile.

The initial state (over-all c.m.) two-particle wave function is given by a product of the "internal" wave functions of the two protons (both with $L=0$) and a plane wave describing the c.m. relative motion with initial momentum $\mathbf{p}$. The final state wave function will be a similar product but with final state relative momentum $\mathbf{p}'$, and one of the proton wave functions is replaced by that of an excited state. In the rest frame of the final state N^*, the overlap integral then becomes

$$T_L{}^M(\mathbf{p},\mathbf{p}')=\int d^3r\, e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}}Y_L{}^M(\theta,\phi)F_2{}^*(\mathbf{r})F_1(\mathbf{r})\,, \quad (1)$$

where F_1, F_2 are products of initial and final state radial internal wave functions, suitably boosted into the given frame. By the above assumptions, $T_L{}^M$ may be regarded as an S-matrix element for N^* excitation. If the z axis is chosen to lie along $\mathbf{p}'$, we can regard M as the "collective" component of the helicity of the final N^*, while L serves to index the collective component of the angular momentum of N^* in its rest frame.

A two-body reaction takes place in a plane and it is convenient to choose the y axis as the normal to this plane. Then after a change of variables to cylindrical coordinates (z,b,ϕ), the ϕ integration can be carried out explicitly, and $T_L{}^M$ can be written as

$$T_L{}^M=\int_0^\infty bdb\, J_M(\Delta b)\int_{-\infty}^{+\infty} dz\, e^{i\Delta P_{\parallel}z} \times Y_L{}^M(\theta,0)F_2{}^*(z,b)F_1(z,b)\,, \quad (2)$$

where $\cos\theta=z/b$, $(-t)^{1/2}\equiv\Delta=P_x-P_x'$, and $\Delta P_{\parallel}=P_z-P_z'$. In the high-energy limit where coherency over the entire interaction region is attained,

$$|\Delta P_{\parallel}|\ll R^{-1}, \quad (3)$$

(where R is a characteristic radius associated with the radial wave functions), this matrix element depends only on Δ. Furthermore, with a surface-excitation outlook, one assumes the z integration is peaked sharply for each impact parameter b at $|z|=0$. This enables factorization of $Y_L{}^M(\pi/2,0)$ in a way characteristic of a diffraction model. The result may be written as

$$T_L{}^M(\Delta)=Y_L{}^M(\pi/2,0)\int_0^\infty bdb\, J_M(\Delta b)H_{12}(b)\,. \quad (4)$$

[5] The field-theoretic significance of such an overlap integral is presumably similar (except for the phase relations) to the overlap matrix of Van Hove, Rev. Mod. Phys. **36**, 655 (1965).

[6] The N^* is treated as a stable particle for purposes of computing the high-energy S-matrix elements. The decay of N^* is connected with its rest-frame dynamics, which are assumed irrelevant to the excitation mechanism.

These approximations guarantee the relativistic covariance of the resulting matrix element.

In accordance with the second postulate, $H_{12}(b)$ is taken to differ only in normalization when comparing among excitation cross sections. Thus (4) may be written as

$$T_L{}^M(\Delta)=\bar{C}_L Y_L{}^M(\pi/2,0)\int_0^\infty bdb\, J_M(\Delta b)H(b)\,. \quad (5)$$

The expression (5) should now be considered as the defining equation of the model under consideration. There exist in fact no "exact" equations of motion from which this *Ansatz* can be derived by successive approximations, and hence the preceding discussion can be interpreted only as a heuristic guide to the physical significance of a model (for the 2-body matrix elements under consideration) defined by (5).

If the radial ground state wave functions were characteristically of square-well type with a sharp cutoff at $r=R$, the corresponding elastic scattering matrix element would take on the Fraunhofer black-disk diffraction pattern. At the same time, the surface-excitation postulate would lead to $H(b)\propto(b-R)$, effectively an illuminated-annulus model as in nuclear reactions with strongly absorbing nuclei. The empirical elastic scattering of hadrons at high energies suggests[1] not a sharp-cutoff model but a Gaussian distribution of density. Thus a smoothed-boundary $H(b)$ is desired; in the Austern-Blair nuclear theory,[3] which is successful if L is not too large, this is achieved by setting

$$H(b)=\frac{\partial}{\partial b}[1-e^{i\chi(b)}]\,, \quad (6)$$

where the elastic scattering amplitude is related to $\chi(b)$ as in the optical model.

A good empirical fit to pp elastic scattering data at small angles and high energies is provided by the choice

$$[1-e^{i\chi(b)}]=Ce^{-b^2/2R^2}\,, \quad (7)$$

implying a purely imaginary amplitude. With this expression, using the *Ansatz* (6), the integrals in (5) can be carried out in terms of confluent hypergeometric functions. The resulting differential cross sections for N^* production can be written as follows, putting $x=\Delta R$, $z=x^2/2$:

$$(d\sigma/dt)_L\bigg/\left(\frac{d\sigma}{dt}\right)_{EL} =C_L{}^2\sum_M[Y_L{}^M(\pi/2,0)]^2[G_M(x)]^2, \quad (8)$$

where

$$G_0(x)=(\pi/2)^{1/2}[{}_1F_1(-\tfrac{1}{2},1;z)]\,,\quad G_1(x)=x\,,$$
$$G_2(x)=(5/4)(\pi/2)^{1/2}z[{}_1F_1(\tfrac{1}{2},3;z)]\,,$$
$$G_3(x)=\tfrac{1}{3}xz[{}_1F_1(1,4;z)]\,.$$

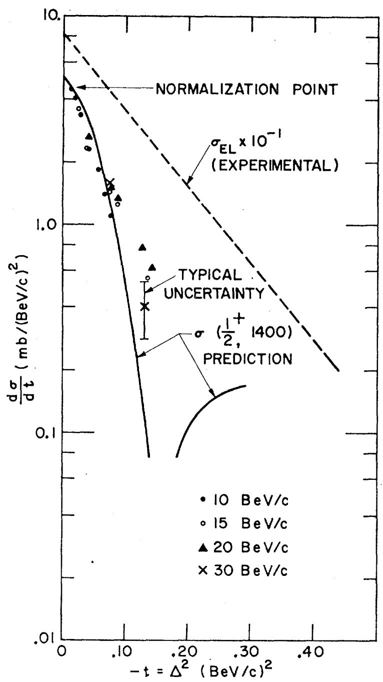

FIG. 1. Model predictions (σ_0) and data (from Ref. 7) on $pp \to pN_{1/2}^*(1400)$ cross sections. Elastic pp cross section at 30 BeV/c is shown for comparison.

The ratios (8) then contain only one parameter (C_L) for each $N_{1/2}^*$ channel which may be fixed by normalizing, at some value of Δ, to the empirical isobar production cross section. If R is independent of energy (a good qualitative approximation), then these ratios are also energy independent, as observed.[7]

Assuming that $N^*(1400)$ and $N^*(1520)$ are $L=0$ and $L=1$ excited states, respectively, the predicted cross sections are plotted in Figs. 1 and 2. The data shown are that of Anderson, *et al.*[7] for $pp \to pN_{1/2}^*$ over the energy region 15–30 BeV/c. All available data points are shown up to $\Delta^2=0.60$ (BeV/c)^2. In the regions where data were taken, the agreement in slopes of $d\sigma/dt$ is satisfactory. The second branch of the predicted curve for $N^*(1400)$ does not seem to be consistent with the lack of observed data beyond $\Delta^2=0.15$ (BeV/c)^2; this disagreement is presumably a failure of the model for this feature. A background could be responsible, or it may simply be impossible to separate the 1400-MeV and 1512-MeV isobars for $\Delta^2>0.15$ in this type of experiment.

The association of $L=0$ and $L=1$ with these two N^* states is supported by available evidence from πp scattering phase shift analyses,[8] which indicate certainly $J^p=\frac{3}{2}^-$ for a resonance at 1520 BeV/c, and if there is a resonance at 1400 BeV/c it is probably $\frac{1}{2}^+$. These J^p assignments have been indicated in Figs. 1 and 2.

In the case of $N_{1/2}^*(1690)$, there is some doubt as to the spin-parity assignment of the peak seen in the experiments of Ref. 7. The more obvious assignment is $J^p=\frac{5}{2}^+$, since at least at 1688 MeV there is certainly such a resonant state in πp scattering. However, detailed analyses[8] indicate also the strong possibility of a highly inelastic $J^p=\frac{5}{2}^-$ resonance at a nearby energy. If the former state were to dominate the $pp \to pN_{1/2}^*$ data, the model prediction would have the $L=2$ shape shown in Fig. 3 as $\sigma_2(\frac{5}{2}^+)$. This shape is in clear disagreement with the 1690-MeV experimental data, indicated in Fig. 3. If, instead, a $\frac{5}{2}^-$ resonance dominated the 1690 region, the $d\sigma/dt$ curve would have an $L=1$ shape, similar to $\sigma_1(\frac{3}{2}^-)$[1520], as in Fig. 2. This curve also does not fit the data. Thus the model fails for the bump at 1690.

Finally, for $N_{1/2}^*(2190)$ there is rather good evidence from πp elastic scattering and polarization data[9] that $J^p=\frac{7}{2}^-$ is appropriate. This leads to $L=3$ in the droplet model, and the prediction of Eq. (8) is shown as $\sigma_3(\frac{7}{2}^-)$ in Fig. 4. The corresponding data points may be consistent for $\Delta^2<0.20$ (BeV/c)^2, but the data definitely diverge from the prediction for $\Delta^2>0.20$, indicating a poor approximation for $H(b)$ in this case.

The model for $N^*(\frac{7}{2}^-)$ has, however, an attractive

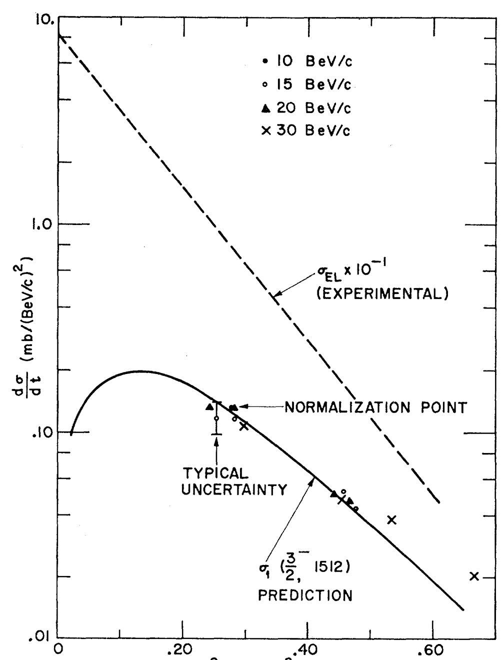

FIG. 2. Model predictions (σ_1) and data (from Ref. 7) on $pp \to pN_{1/2}^*(1512)$ cross sections. Elastic pp cross section at 30 BeV/c is shown for comparison.

---

[7] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters **16**, 855 (1966); and J. Menes (private communication). Data have also been given by G. Cocconi *et al.* [Phys. Letters **8**, 134 (1964)] but these data seem to be inconsistent with the former data.

[8] P. Bareyre, C. Brickman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965).

[9] A. Yokosawa, S. Suwa, R. E. Hill, R. J. Esterling, and N. E. Booth, Phys. Rev. Letters **16**, 714 (1966).

feature which may be associated with the idea of Regge recurrences.[10] Since this N^* may be a rotational-level recurrence of $N^*(\frac{3}{2}^-)[1520]$, it may be hypothesized that the radial wave functions may have the same normalization in the excited-droplet model, i.e., $C_3=C_1$. This equality was used in plotting σ_3 in Fig. 4, and it is apparent that it provides a reasonably good estimate of the magnitude of the cross section for $0.10<\Delta^2<0.25$ $(\text{BeV}/c)^2$.

If the recurrence equality $(C_3=C_1)$ is taken more seriously than the assumptions (6) and (7) leading to a specific shape for $H(b)$, a shape-independent inequality may be obtained between σ_3 and σ_1, which reads

$$\sigma_3/\sigma_1 \geqslant \tfrac{3}{8} \quad (\text{equality in the limit } \Delta^2 \to 0). \qquad (9)$$

This lower bound for σ_3 is shown as the lower dashed line in Fig. 4. The data points (3) for $N^*(2190)$ lie just above this curve; this indicated that the failure of the naive model in this case may be caused by a choice of $H(b)$ which does not drop off rapidly enough for $b\to 0$ (corresponding to the larger Δ^2 behavior of $d\sigma/dt$).[11]

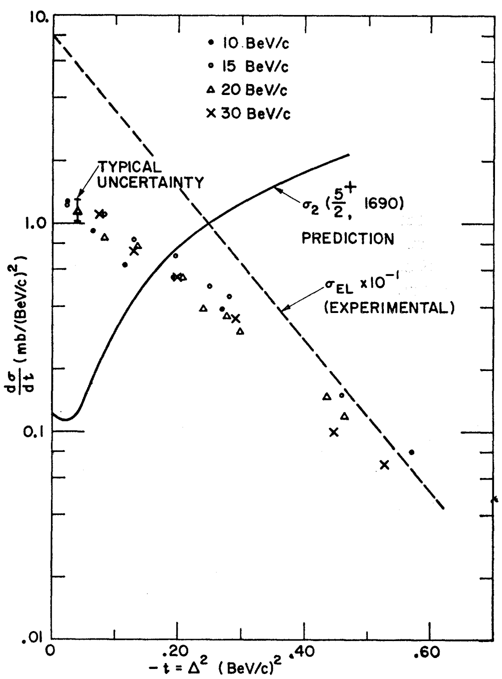

FIG. 3. Model predictions (σ_2) and data (from Ref. 7) on $pp\to pN_{1/2}^*(1690)$ cross sections. Elastic pp cross section at 30 BeV/c is shown for comparison. Normalization of model prediction based on Regge recurrence hypothesis and σ_0 normalization as in Fig. 1.

[10] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966).
[11] In nuclear physics calculations, the approximation (6) is often bad for $L\geqslant 3$; see, for example, Austern and Blair, Ref. 3.

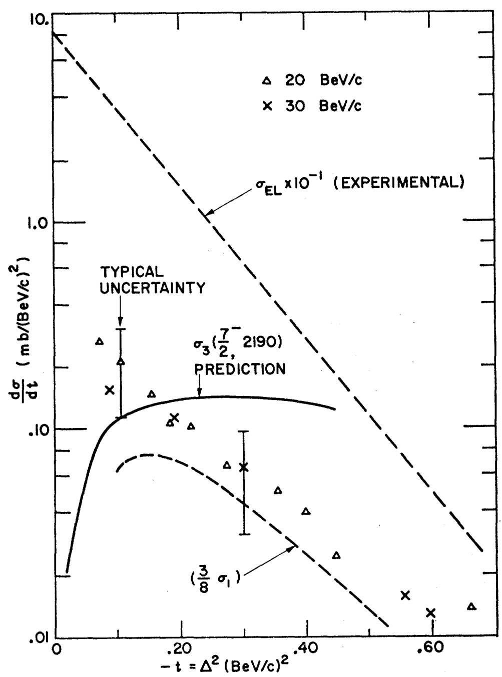

FIG. 4. Model predictions (σ_3) and data (from Ref. 7) on $pp\to pN_{1/2}^*(2190)$ cross sections. Elastic pp cross section at 30 BeV/c is shown for comparison. Normalization of model prediction (solid curve) based on Regge recurrence hypothesis and σ_1 normalization as in Fig. 2. Values of $\frac{3}{8}\sigma_1$ are shown for comparison with the "shape-independent" sum rule $\sigma_3\geqslant\frac{3}{8}\sigma_1$ discussed in text.

Correcting this feature may also allow σ_2 to come into better agreement with data, especially if mixtures of $L=1$ and $L=2$ matrix elements are considered.

Summarizing, the model in its most naive form [using (6) and (7)] provides a satisfactory explanation of the difference in slope of $d\sigma/dt$ observed in $N^*(1400)$ and $N^*(1520)$ excitation compared to elastic scattering, within the framework of an excited-droplet viewpoint. The cross section for $N^*(1690)$ is not explained, but this is probably not a simple single-resonance inelastic mechanism. The predictions for $N^*(2190)$ are not very good from the naive model, but the absolute normalization in this case can be computed from a Regge recurrence hypothesis, and the associated inequality (9) is satisfied. The correspondence between coherent processes (no exchange of quantum numbers other than angular momentum) and asymptotic constancy with energy of cross sections is a natural feature of such a model.

I would like to acknowledge a private communication of numerical data from J. Menes, and an informative conversation concerning relativistic kinematics with F. Coester.