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# Systematic Investigation of Ultrahigh-Energy Interactions. I

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Nuclear interactions in the energy range of 1000 GeV are studied by means of a large nuclear-emulsion stack. Particular emphasis is placed on collecting the samples of jets in an unbiased way, as well as on estimating the primary energy with better accuracy. For this purpose families of interactions initiated by heavy primary cosmic-ray particles are analyzed. A slightly modified Castagnoli method was applied to each family of jets to estimate the incident energy per nucleon. The method also enables one to select the quasi-monoenergetic nucleons. The distribution of $K_{ch}$, the fraction of the primary energy imparted to the charged secondaries, is obtained from the interactions caused by the quasimonoenergetic nucleons. The arithmetic mean of $K_{ch}$ is 0.28. The mean transverse momentum of the nucleons which have interacted in the parent collision is estimated to be $0.5_2 \pm 0.2$ GeV/$c$.

## I. INTRODUCTION

DURING the last fifteen years a number of experiments were performed on multiple meson production, certain features of the phenomenon have been revealed experimentally, and a number of models have been proposed to account for the observed facts. Salient features of the experimental findings can be summarized as follows. First, the mean transverse momentum of the shower particles produced, mostly mesons, is about (0.3–0.4) GeV/$c$ and depends on neither the primary energy nor the emission angle, except presumbaly in the extreme forward and/or backward directions.[1–9] This is verified also in the machine-energy range where the transverse momenta of the various kinds of particles were accurately measured individually.[10–16] Second, the inelasticity, i.e., the fraction of incident energy carried away by secondary particles, was estimated in the energy region up to 10 TeV, while in the still higher-energy region, estimation from air-shower experiments is less direct. The indication is that its mean value is approximately 0.5 in all energy regions so far investigated, with no detectable dependence on primary energy.[2,5–8,17,18] Third, the mean multiplicity of the particles produced can be fitted either by $aE_{\rm inc}^{1/4}$ or by $b\log(E_{\rm inc}/\epsilon)$[5,7,18–20], where $a$, $b$, and $\epsilon$ are suitably chosen constants, though there are some indications favoring the former relation.[21] Fourth, the proportion of the nonpion components of the shower particles is about 30% in the energy range of several hundred GeV to tens of thousands GeV, while in the accelerator energy range the proportion of kaons is about 10%.[16] Fifth, the energy spectrum of the secondary pions in the center-of-mass (c.m.) system[5] seems to be approximately proportional to $E^{-2}dE$ for $p \gtrsim 0.3$ GeV/$c$. Alternatively, the integral form of the energy spectrum in the laboratory system in the angular region $\theta \lesssim 2.5\times10^{-3}$ is represented by $N(>E) \sim E^{-1.30\pm0.25}$.[18] Sixth, the angular distribution of secondary particles is in many cases indicative of the existence of two emission centers moving apart in the c.m. system,[22–24] and the mean four-momentum transfer between the two groups of the particles produced is estimated to be at least about $-1.5$ (GeV/$c$)$^2$.[25] Seventh, there is no definite indication of a variation with energy of the total inelastic cross section. Eighth, the proportion of leptons produced directly can be estimated from the mean free path of the particles produced to be less than 10%. Furthermore, the fact that there has been no single case of a lepton identified among the jet secondaries puts this limit well below 1%.

In order to explain these features of the multiple-production process, various models have been proposed,

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[7] A. G. Barkow *et al.*, Phys. Rev. **122**, 617 (1961).
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[18] M. Koshiba *et al.*, Nuovo Cimento Suppl. **1**, 1091 (1963).
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[20] P. K. Malhotra, in *Proceedings of the 1963 Cosmic Ray Conference, Jaipur, India* (Commercial Printing Press, Ltd., Bombay, India, 1963), Vol. 5, p. 40.
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[22] P. Ciok *et al.*, Nuovo Cimento **10**, 741 (1958).
[23] K. Niu, Nuovo Cimento **10**, 994 (1958).
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TABLE I. The primary charge and energy of each family. The errors of the primary energies are estimated by the Castagnoli method applied to the combined angular distributions of the family jets with the assumption that the secondary particles are emitted isotropically in the c.m. system.

| Family | Primary charge | Primary energy (TeV/nucleon) |
|---|---|---|
| 1010 | 8 | $20_{-3}^{+4}$ |
| 1013 | 13 | $1.4\pm0.2$ |
| 1067 | 6 | $2.0\pm0.2$ |
| Ja1 | 13 | $1.9_{-0.6}^{+0.8}$ |
| 1003 | 4 | $1.5\pm0.2$ |
| 1039 | 2 | $9.3_{-1.5}^{+1.9}$ |
| 1115 | 15 | $1.6\pm0.1$ |

such as Heisenberg's nonlinear field theory,[26] bremsstrahlung theory,[27] the Fermi model,[28] the hydrodynamical model,[29] the isobar model,[30] the peripheral[31] and multiperipheral models,[32] the two-fire-ball model,[22,23] etc. None of these models is capable of explaining all the main experimental findings listed above.

Recently the relation between multiple meson production and high-energy $p$-$p$ diffraction scattering was discussed by several authors,[32–34] and the validity of some of the above models was studied from this viewpoint.[35] The identification of the surviving nucleons and the investigation of their behavior would then further clarify the dynamics of multiple meson production.

When one studies the multiple meson production in the TeV region, one of the difficulties encountered is the estimation of the energies of the particles involved in the reaction. Since cosmic rays are the only available source of incident particles, the analysis begins with the determination of their primary energy. Many authors proposed kinematical methods for this purpose. According to these methods, the incident energy is derived from the emission angles of the created particles in the laboratory system with the aid of some assumptions about the nature of the interaction. The assumption common to most of the kinematical methods is that there is fore-and-aft symmetry of particle emission in the c.m. system.

However, the frequency of occurrence of asymmetric distributions was reported by several authors[17,18] to exceed the rate expected from the independent emissions of the secondary particles. Hence, the incident-energy estimate obtained from this assumption can not be regarded as reliable for each individual interaction. We use a slightly modified Castagnoli method in order to obtain a more reliable estimate. Our method is applicable to a family of interactions initiated by an incident heavy nucleus.

The method of scanning and measurement is described in Sec. II. The modification of the Castagnoli method is described in Sec. III. This modification enables one at the same time to select the quasimonoenergetic nucleons. From the interactions initiated by these monoenergetic nucleons the $K_{ch}$ distribution is obtained. This procedure and the result are presented in Sec. IV. In Sec. V and Sec. VI, respectively, the mean transverse momentum of the nucleons which interacted in the parent collision and the multiplicity of the secondary particles are described.

## II. EXPOSURE AND MEASUREMENT

The stack originally consisted of 500 pellicles of Ilford K-5 emulsions, $0.06\times45\times60$ cm$^3$ each. The large size of this stack not only affords a large number of interactions, but also allows a more detailed analysis of the individual events. The stack was flown for 36 h at an altitude of 7 g/cm$^2$. The flight curve is given in Ref. 36. The processing of the pellicles and the scanning for the high-energy electromagnetic cascades were accomplished at the Ryerson Physical Laboratory at the University of Chicago.

From the variety of the interactions found, seven families of jets produced by ultrahigh-energy heavy nuclei have been selected for the present analysis. They all have a length per plate of over 2 mm. There are two advantages in analyzing the series of jets initiated by heavy primaries. First, in the evaluation of the primary energy one can utilize the fact that the fragment nucleons, i.e., the nucleons evaporated in the rest system of the incoming heavy nuclei, have approximately the same energy in the lab system. The other advantage is that the emission angles of the secondary particles can be determined with an accuracy approaching $10^{-5}$ rad by using the surviving fragments of multiple charges, or neighboring tracks, as reference in the displacement measurement. All tracks emitted within a cone of half-angle $10^{-3}$ rad were traced until they interacted or left the stack. The same region was also scanned for neutral interactions. From the way each individual track is traced and identified at various depths, the detection efficiency is almost 100% for the interactions caused by charged particles and, provided $n_s\geq3$, also for those caused by neutral particles. The families of interactions studied in this paper and their primary charge and energy are listed in Table I. Among

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them, 1010, 1013, 1067, and Ja1 were analyzed in this laboratory and the results were combined with those of 1003, 1039 and 1115[37] which were measured at the University of Chicago.

## III. DETERMINATION OF PRIMARY ENERGY AND SELECTION OF NUCLEONS AMONG SECONDARIES

An ultrahigh-energy nucleus passing through a nuclear emulsion and interacting with an emulsion nucleus produces shower particles. Among these shower particles there are secondary particles created by the interaction, mostly pions and kaons, and two kinds of nucleons. One kind of nucleon includes those which actually participated in the production of the secondary particles, and the other kind includes those which evaporated or were ejected from the incident nucleus. The former will be referred to as degraded and the latter as fragment nucleons. In some cases there are also multiply charged fragments in the shower. Let us denote the charge of the incident particle by $Z_i$ and the sum of the charges of the multiply charged fragments by $\sum Z_f$. If the production of nucleon-antinucleon pairs is as small in the 1000-GeV region as it is at 23 GeV,[16] the number of emerging protons $N_p$ is expected to be

$$N_p=Z_i-\sum Z_f. \quad (3.1)$$

As a working hypothesis we assume that these $N_p$ protons are emitted with emission angles considerably smaller than those of the created particles and we denote the emission angle of the $N_p$th track by $\Theta(N_p)$. The observed values of $\Theta(N_p)$ for the analyzed families are all smaller than $10^{-3}$ rad. Within the respective cones of half-angle $\Theta(N_p)$, a total of 23 charged interactions and 20 neutral interactions have been located. These interactions, by our assumption, are to be regarded as due to nucleons. We shall now examine the validity of this assumption quantitatively. Since neutral pions decay into two photons with a lifetime of about $10^{-16}$ sec, they do not contribute to the production of neutral jets. The situation is as follows. The neutral-to-charged ratio of the primaries of the secondary interactions, $\kappa$, is given by

$$\kappa=\frac{\lambda_n n_n+\lambda_K n_{K^0,\bar{K}^0}}{\lambda_p n_p+\lambda_d n_d+\lambda_\pi n_{\pi^+,\pi^-}+\lambda_K n_{K^+,K^-}}, \quad (3.2)$$

where $n$ stands for the numbers of the respective particles, ($n_{K^0,\bar{K}^0}$ is the total number of neutral $K$ mesons, $K^0$ and $\bar{K}^0$, and similarly for $n_{\pi^+,\pi^-}$ and $n_{K^+,K^-}$), and the parameters $\lambda$ are the probabilities for these particles to initiate interactions. The triton contamination is small and we neglect it.[38]

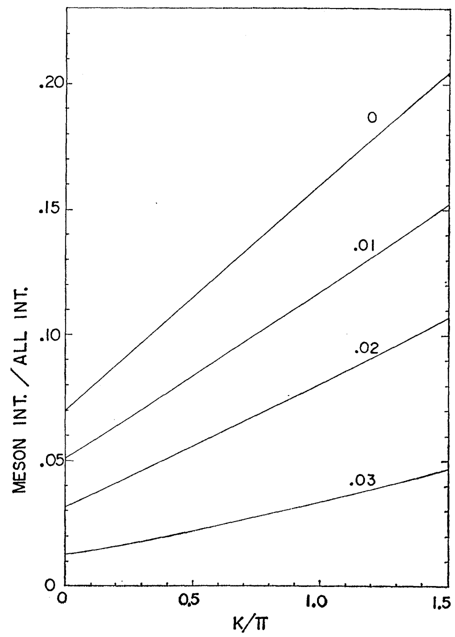


FIG. 1. The fractions of the meson interactions among all interactions are shown versus the ratio $K/\pi$ for several values of the deuteron-to-nucleon ratio $\alpha$.

The number of deuterons is expressed as

$$n_d=\alpha(n_p+n_n)=2\alpha n_p, \quad (3.3)$$

where $\alpha$ is a parameter and where $n_n$ was set equal to $n_p$. On the assumption of charge-independent production of mesons, the numbers of the various mesons are written as

$$\begin{aligned} n_{\pi^+,\pi^-}&=\tfrac{2}{3}\beta(n_p+n_n)=\tfrac{4}{3}\beta n_p,\\ n_{K^+,K^-}&=\tfrac{1}{2}\beta\gamma(n_p+n_n)=\beta\gamma n_p,\\ n_{K^0,\bar{K}^0}&=\tfrac{1}{2}\beta\gamma(n_p+n_n)=\beta\gamma n_p, \end{aligned} \quad (3.4)$$

where $\beta$ is the ratio of the pion number to the nucleon number and $\gamma$ is the ratio of the kaon number to the pion number. The $\lambda$'s are proportional to the cross sections of the respective particles. Using the total cross sections measured at machine energy, we have

$$\begin{aligned} \frac{\lambda_d}{\lambda_N}&=\frac{\sigma_A(d-N)}{\sigma_t(N-N)}\simeq\frac{74}{38},\\ \frac{\lambda_{\pi^\pm}}{\lambda_N}&=\frac{\sigma_t(\pi^\pm-N)}{\sigma_t(N-N)}\simeq\frac{24}{38},\\ \frac{\lambda_{K^{\pm 0}}}{\lambda_N}&=\frac{\sigma_t(K^{\pm 0}-N)}{\sigma_t(N-N)}\simeq\frac{20}{38}. \end{aligned} \quad (3.5)$$

[37] We thank Dr. W. Wolter, then at the University of Chicago, for sending his results before publication. See also *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and The Physical Society, London, 1966). We acknowledge also the contribution by Dr. H. Aizu in analyzing these events.

[38] Dahanayake *et al.*, Nuovo Cimento 1, 888 (1955).

Substituting (3.4) and (3.5) into (3.2), we obtain

$$\kappa=\frac{1+0.53\beta\gamma}{1+3.90\alpha+0.84\beta+0.53\beta\gamma}. \quad (3.6)$$

Inserting the observed value $\kappa=0.87$, the fraction $m$ of the observed events to be regarded as meson interactions is given by

$$m\simeq\frac{0.11+0.14\gamma-2.86\alpha-3.60\alpha\gamma}{1.57-3.84\alpha\gamma}. \quad (3.7)$$

The fraction $m$ is shown versus $\gamma$ for several values of $\alpha$ in Fig. 1.

If we take 0.37 for the value of $\gamma$,[39] the proportion of the pion and kaon interactions in our sample is bounded by an upper limit of 10%. When Kim's result is taken, i.e., $\gamma=1.34$ in the extreme forward direction,[9] the upper bound of the fraction of mesons becomes 19%. In the case when there exist at least a few percent deuterons, a situation which is quite reasonable in view of the evaporation process, the meson contamination is reduced to a negligible amount. Moreover, we observed only a few cases of high-energy electromagnetic cascades within the cone of half-angle $\Theta(N_p)$. This further supports our conclusion.

From the observed 43 interactions we select the jets with $n_s\geqslant 5$ and $N_H\leqslant 20$, where $n_s$ is the number of the shower particles, and $N_H$ is the number of the heavy or grey tracks. In the case of a neutral jet the emission angle of the primary neutral particle cannot be determined unless one knows which preceding interaction produced this particle. Only those neutral jets were selected in which estimates of the emission angle of the primary neutral particle did not differ by more than a factor of 2 for different choices of candidates for the parent interactions. These selection criteria left 20 charged events and 13 neutral ones for further analysis.

In first approximation the incident energies per nucleon of nucleus-nucleus interactions are estimated by the Castagnoli method[40] from the relation

$$\langle\log\tan\theta\rangle=-\log\gamma_c+\langle\log\tan(\theta^*/2)\rangle=-\log\gamma_c, \quad (3.8)$$

where $\gamma_c$ is the Lorentz factor of the c.m. system in the lab system.

When the masses of the incident particle and the target particle are known, the total energy of the incident particle in the lab system, $E_i$, is evaluated as

$$E_i=M_t\{\gamma_c^2-1+\gamma_c[\gamma_c^2-1+(M_i/M_t)^2]^{1/2}\}, \quad (3.9)$$

where $M_i$ is the mass of the incident particle and $M_t$ is the mass of the target particle. In the extremely relativistic case, (3.9) reduces approximately to

$$E_i=2M_t\gamma_c^2. \quad (3.10)$$

If we consider a nucleus-nucleus interaction to be simply a superposition of nucleon-nucleon collisions and if $M_t$ is taken to be the nucleon mass, then $E_i$ in Eq. (3.10) gives the energy per nucleon of the incident nucleus.

As for the nucleon-nucleus interactions, an additional correction factor proposed by Lohrmann *et al.*[8] has been applied to the Castagnoli method. We modified their procedure slightly for our sample, and took the corrected incident-nucleon energy $E$ to be related to the quantity $E_i$, derived from Eq. (3.10), by

$$\begin{aligned} E&=(1/1.3)E_i, \quad (N_H<3),\\ E&=1.8E_i, \quad (N_H\geqslant 3). \end{aligned} \quad (3.11)$$

The identification of the fragment nucleons among the primaries of the 33 secondary interactions has been attempted as follows. First we assume the normalized momentum distribution of the fragment nucleons, $n^*(p^*)$, in the rest system of the evaporating nucleus to be given by

$$n^*(p^*)dp^*=\frac{1}{\pi^{3/2}}\frac{p^{*2}}{p_F^3}\exp\left[-\frac{p^{*2}}{p_F^2}\right]dp^*, \quad (3.12)$$

where $p^*$ is the momentum of the fragment nucleons and $p_F$ is taken to be 120 MeV/$c$.[41] Momentum and angle distribution of the fragment nucleons in the lab system, $n(p,\Omega)$, is obtained by transforming Eq. (3.12). Namely,

$$\begin{aligned} n(p,\Omega)&=n^*(p^*,\Omega^*)\frac{\partial(p^*,\Omega^*)}{\partial(p,\Omega)}\\ &=\frac{1}{\pi^{3/2}p_F^3}\frac{\gamma_0p^2(E-\beta_0p\cos\theta)}{E}\\ &\quad\times\exp\left[-\frac{\gamma_0^2}{p_F^2}(E-\beta_0p\cos\theta)^2+\frac{M^2}{p_F^2}\right], \end{aligned} \quad (3.13)$$

where $\beta_0$ is the velocity of the evaporating nucleus in the lab system, $\gamma_0$ is the Lorentz factor corresponding to $\beta_0$, and $M$ is the nucleon mass. Integration with respect to the angle gives, for the energy distribution in a given

[39] The following data are combined and the estimated contribution of the surviving nucleons is corrected: B. Edwards *et al.* (Ref. 2); E. Lohrmann and M. W. Teucher, Phys. Rev. **112**, 587 (1958); A. G. Barkow *et al.* (Ref. 7); Daniel *et al.*, Phil. Mag. **43**, 753 (1952); J. Mulvey, Proc. Roy. Soc. (London) **221**, 367 (1954); Naugle *et al.*, Phil. Mag. **92**, 1986 (1953); Kaplon *et al.*, Phys. Rev. **93**, 1424 (1954); Lal *et al.*, Nuovo Cimento Suppl. **12**, 347 (1954).

[40] C. Castagnoli *et al.*, Nuovo Cimento **10**, 1261 (1953).

[41] This value is obtained from the mean energy of $p$, $d$, and $t$ which were taken from the graph of C. F. Powell, P. H. Fowler, and D. H. Perkins, *The Study of the Elementary Particles by the Photographic Method* (Pergamon Press, Inc., New York, 1959), p. 442.

angular interval,

$$N(E,\theta_1\leqslant\theta<\theta_2)dE=[k(E,\theta_1)-k(E,\theta_2)]dE,$$

$$k(E,\theta)=\frac{1}{\pi^{1/2}p_F\gamma_0\beta_0}\times\exp\left[\frac{M^2}{p_F^2}-\frac{\gamma_0^2}{p_F^2}(E-\beta_0 p\cos\theta)^2\right]. \quad (3.14)$$

Considering that

$$\gamma_0\gg 1,\quad \theta\ll 1,$$

$k(E,\theta)$ can be represented approximately by the following equation by changing the variable from $E$ to $\gamma$ which is the Lorentz factor of the fragment nucleon in the lab system:

$$k(\gamma,\theta)=\frac{1}{\pi^{1/2}p_F\gamma_0}\times\exp\left\{\frac{M^2}{p_F^2}\left[1-\frac{1}{4}\left(\frac{\gamma_0}{\gamma}+\frac{\gamma}{\gamma_0}+\gamma\gamma_0\theta^2\right)^2\right]\right\}. \quad (3.15)$$

The $\gamma$ distribution in a given angular interval is given by

$$N(\gamma,\theta_1\leqslant\theta<\theta_2)d\gamma=[k(\gamma,\theta_1)-k(\gamma,\theta_2)]Md\gamma. \quad (3.16)$$

From (3.15) and (3.16), one can see that the energy distribution of the fragment nucleons has its peak nearly at the incident energy $E_i$ with a half-width of $\sim\frac{1}{10}E_i$. The transverse momentum distribution function $n(p_t)$ for the fragment nucleons is obtained from Eq. (3.12) as

$$n(p_t)dp_t=dp_t\int_0^\infty dp^*\int d\Omega^*\,\delta(p_t-p^*\sin\theta^*)n^*(p^*)$$

$$=\frac{2p_t}{p_F^2}\exp\left[-\frac{p_t^2}{p_F^2}\right]dp_t. \quad (3.17)$$

The mean $p_t$ value for the fragment nucleons is

$$\langle p_{t\,\mathrm{frag}}\rangle=\tfrac{1}{2}(\sqrt{\pi})p_F\simeq 110\ \mathrm{MeV}/c \quad (3.18)$$

when $p_F=120$ MeV/$c$.

Let us arrange the secondary interactions in order of the emission angles of their primaries in the parent-heavy-nucleus interaction. The mean energy $\bar{E}(\Theta(n))$ of the primaries within the cone of half-angle $\Theta(n)$ containing $n$ interactions is calculated by the following procedure.

(i) First the Castagnoli method is utilized for each interaction to obtain the Lorentz factor.

For a nucleus-nucleus interaction the Lorentz factor is obtained by excluding $N_p$ tracks from the forward direction. Namely,

$$\log\gamma^{(-N_p)}=-\frac{1}{n_s-N_p}\sum_{i=N_p+1}^{n_s}\log\tan\theta_i, \quad (3.19)$$

where $N_p$ is given by Eq. (3.1), $\theta_i$ is the emission angle of the $i$th track relative to the primary direction from the forward, and $n_s$ is the number of shower particles.

For a nucleon-nucleus interaction, two Lorentz factors are calculated: one from all the shower particles and the other by excluding the most forward track. They are denoted by $\gamma^{(-0)}$ and $\gamma^{(-1)}$, respectively. Furthermore, the correction corresponding to equation (3.11) is applied at this step by incorporating a constant $C$:

$$\log\gamma^{(-0)}=-\frac{1}{n_s}\sum_{i=1}^{n_s}\log\tan\theta_i+C, \quad (3.20)$$

$$\log\gamma^{(-1)}=-\frac{1}{n_s-1}\sum_{i=2}^{n_s}\log\tan\theta_i+C, \quad (3.21)$$

where

$C=-0.057$ for a nucleon-nucleus interaction with $N_H<3$

$=+0.128$ for a nucleon-nucleus interaction with $N_H\geqslant 3$. (3.22)

(ii) We obtain the weighted mean of the Lorentz factor $\bar{\gamma}_{\Theta(n)}$ from $n$ interactions in the cone of half-angle $\Theta(n)$ on the assumption that the two charge states of the nucleon after interaction are equally probable and that the nucleons are emitted in the most forward direction, an assumption we scrutinized earlier in this section. Namely,

$$\log\bar{\gamma}_{\Theta(n)}=\frac{\sum_j(n_{sj}-N_{pj})\log\gamma_j^{(-N_{pj})}+\frac{1}{2}\sum_k[n_{sk}\log\gamma_k^{(-0)}+(n_{sk}-1)\log\gamma_k^{(-1)}]}{\sum_j(n_{sj}-N_{pj})+\sum_k(n_{sk}-\frac{1}{2})}, \quad (3.23)$$

where $j$ and $k$ specify the type of the interaction, $i$ for nucleus-nucleus interactions including primary interaction and $j$ for nucleon-nucleus interactions.

(iii) The mean energy is calculated by

$$\bar{E}(\Theta(n))=2M\bar{\gamma}_{\Theta(n)}^2, \quad (3.24)$$

where $M$ is the nucleon mass.

Then we introduce the following quantity:

$$\bar{P}_{t(n)}\equiv\bar{E}(\Theta(n))\,\Theta(n). \quad (3.25)$$

As long as

$$\bar{P}_{t(n)}\lesssim 200\ \mathrm{MeV}/c \quad (3.26)$$

is satisfied for a certain value of $\Theta(n)$, the interactions contained in the cone of half-angle $\Theta(n)$ are considered

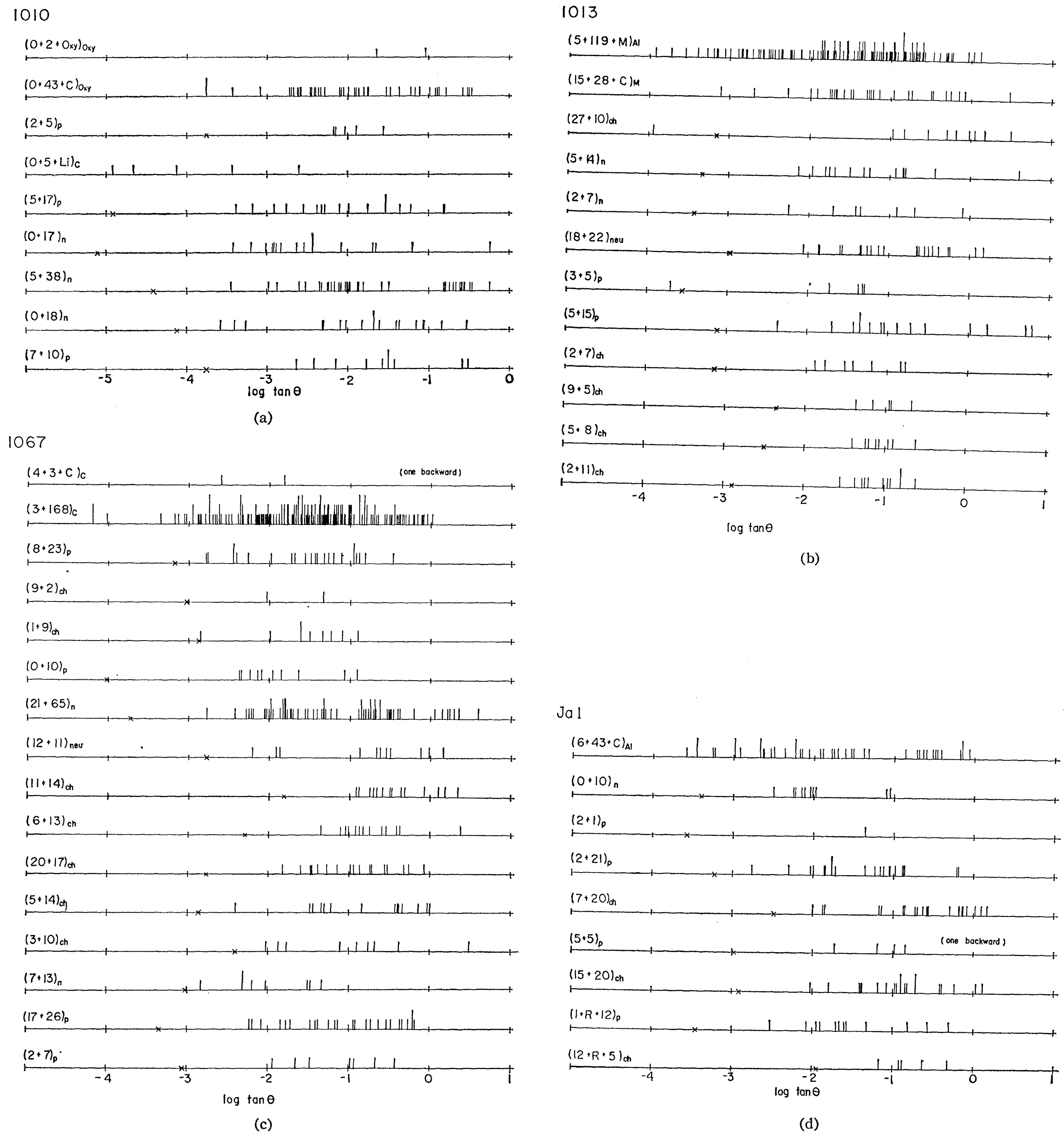


FIG. 2. log tan$\theta$ distributions of the analyzed families of jets are shown. The emission angles $\theta$ are measured with respect to the directions of the primary particles of the individual interactions by using the surviving fragments. The accuracy is about $10^{-5}$ rad for the small emission angles. In the case of a secondary interaction the emission angles of the primary particle in the parent interaction are shown by the symbol ×. The interactions with subscripts "ch" and "neu" are not included in the present analysis. For the selection of nucleon interactions, see text.

to be due to the quasimonoenergetic fragment nucleons. Denoting the largest angle which satisfies the condition (3.26) by $\Theta(m)$, $\bar{E}(\Theta(m))$ is regarded as the incident energy per nucleon of the family.

The correction factor $C$ in Eqs. (3.20) and (3.21) does not cover the fluctuation in the energy determination due to asymmetric emission of the secondary particles in the nucleon-nucleon c.m. system. The situation is improved, however, by taking the mean energy of several interactions, since predominantly forward or predominantly backward emissions can be expected to occur with the same probability.[17]

The observed log tan$\theta$ distribution for each interaction is shown in Fig. 2 and the primary energy as-

TABLE II. Comparison of $\langle K_{ch}\rangle$ with previous experiments. The various data from the nuclear emulsion experiments are normalized by using $\langle P_{t,\pi,K}\rangle=0.4$ GeV/$c$. $\langle K\rangle$ is estimated from $\langle K\rangle=1.6\langle K_{ch}\rangle$ except in the case of machine experiments.

| Author | Primary energy | $\langle K_{ch}\rangle$ in lab system | $\langle K\rangle$ in lab system | Target | Remarks |
|---|---|---|---|---|---|
| Lohrmann *et al.* (Ref. 8) | ~250 GeV | 0.40 | 0.56 | Nucl. emul. | Fragment-nucleon beam is used. Deuteron contamination is corrected. |
| Dobrotin *et al.* (Ref. 17) | ~300 GeV | 0.20 | 0.32 | LiH | The momenta of the secondary particles are measured in a magnetic field. Primary energy is estimated by a calorimeter. |
| Guseva *et al.* (Ref. 6) | ~300 GeV | 0.20 | 0.32 | LiH | |
| Hansen *et al.* (Ref. 5) | 100–500 GeV | 0.18 | 0.29 | C | The mean $K_{ch}$ is calculated from their data by using their results on the $K$-$\pi$ ratio. The momenta of the secondary particles are measured in a magnetic field. The primary energy is estimated by the momentum balance of the particles produced. The low value of $\langle K_{ch}\rangle$ may be due to the fact that the authors could not detect some particles with large emission angles. |
| Koshiba *et al.* (Ref. 18) | ~3 TeV | 0.25 | 0.4 | Nucl. emul. | Primary energy is estimated by using $\langle P_t\rangle_{pn}=0.4$ GeV/$c$. |
| Barkow *et al.* (Ref. 7) | ~3.5 TeV | 0.38 | 0.60 | Nucl. emul. | |
| Edwards *et al.* (Ref. 2) | 1–100 TeV | 0.22 | 0.35 | Nucl. emul. | |
| This exp. | 1–20 TeV | 0.28 | 0.45 | Nucl. emul. | |
| | Machine energy | | 0.4–0.5 | | |

signed to each family is listed in Table I. A total of 9 events are thus selected out of 33 to have been initiated by fragment nucleons.

## IV. INELASTICITY DISTRIBUTION

For the 9 interactions selected in Sec. III the fractional energies carried away by charged secondaries are obtained. Denoting this quantity by $K_{ch}$, we have

$$K_{ch}=\frac{E_{ch}}{E_0}=\frac{E_{ch}}{\bar{E}(\Theta(m))}. \quad (4.1)$$

Since we cannot tell in which of the two charge states the interacting nucleon emerges, there still remains an ambiguity in estimating $E_{ch}$. The following two alternatives are used to evaluate $E_{ch}$ for each interaction:

$$E_{ch,0}=\sum_{j=1}^{n_s}\frac{0.4}{\theta_j} \quad \text{(in GeV)}, \quad (4.2)$$

and

$$E_{ch,1}=\sum_{j=2}^{n_s}\frac{0.4}{\theta_j} \quad \text{(in GeV)}, \quad (4.3)$$

where $\theta_1$ is the smallest emission angle in the jet and this track may in fact be the interacting nucleon in the proton state. This procedure of eliminating the nucleon contribution to $E_{ch}$ can be neglected for the recoil nucleon since the error caused thereby is very small.

By inserting (4.2) and (4.3) into (4.1), two different values of $K_{ch}$ are obtained, though only one of them is correct. We weigh the two values equally to get the $K_{ch}$ distribution shown in Fig. 3. The error of $\pm 20\%$ is taken into account considering also the uncertainty of the primary energy. The arithmetic mean of $K_{ch}$ is 0.28.

In order to make a correction for the deuteron contamination we assume that the mean $E_{ch}$ for a deuteron interaction is larger than that of a nucleon interaction by a factor 1.5 at the same incident energy per nucleon.[8] If 30% of the 9 interactions are due to deuterons, the mean $K_{ch}$ reduces to 0.24. If the composition of the secondary particles, i.e., 70% pions and 30% kaons, is taken into account and charge-independent production of these particles is assumed, the total inelasticity $K$ is obtained from

$$K=1.6K_{ch}. \quad (4.4)$$

Our mean $K_{ch}$ gives 0.45 for the mean total inelasticity. The result is compared with other experiments in Table II.

## V. MEAN TRANSVERSE MOMENTUM OF THE DEGRADED NUCLEONS

In this section we study the mean transverse momentum of the nucleons which can be regarded to have actually participated in the multiple meson production in the nucleus-nucleus interaction. The nucleons whose emission angles $\Theta$ fall in the region $\Theta(m)<\Theta\leqslant\Theta(N_p)$

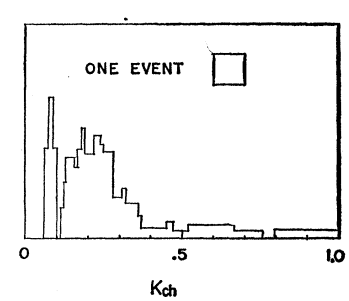


FIG. 3. The $K_{ch}$ distribution obtained from the interactions of quasi-monoenergetic nucleons.

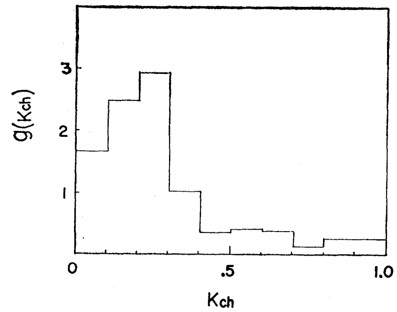


FIG. 4. Normalized distribution $g(K_{\mathrm{ch}})$ of $K_{\mathrm{ch}}$, obtained by taking the arithmetic mean in each 0.1 interval of $K_{\mathrm{ch}}$ of the distribution presented in Fig. 3. This function $g(K_{\mathrm{ch}})$ is used for calculating the distribution $H(\eta)$ shown in Fig. 5.

are considered. When a nucleon of emission angle $\Theta$ makes a secondary interaction, the following approximate equation follows from energy conservation:

$$E_{\mathrm{ch}}=K_{\mathrm{ch}}P_{t,N}/\Theta\,, \tag{5.1}$$

where $E_{\mathrm{ch}}$ is the energy of the shower particles produced by the secondary interaction and $P_{t,N}$ is the transverse momentum of the nucleon emerging from the primary interaction.

We define the quantity

$$\eta=\Theta E_{\mathrm{ch}}=P_{t,N}K_{\mathrm{ch}}\,, \tag{5.2}$$

where both $E_{\mathrm{ch}}$ and $\Theta$ are measurable quantities. From the $K_{\mathrm{ch}}$ distribution and the $\eta$ distribution, the mean $P_{t,N}$ value can be obtained by assuming some functional form for the distribution of $P_{t,N}$. Let us represent the differential distribution functions for $\eta$, $K_{\mathrm{ch}}$, and $P_{t,N}$ by $h(\eta)$, $g(K_{\mathrm{ch}})$, and $f(P_{t,N})$, respectively. Then, by using Eq. (5.2), $h(\eta)$ can be calculated from $f(P_{t,N})$ and $g(K_{\mathrm{ch}})$ as

$$h(\eta)=\int_0^\infty dp_{t,N}\int_0^1 dK_{\mathrm{ch}}f(p_{t,N})g(K_{\mathrm{ch}})\delta(\eta-p_{t,N}K_{\mathrm{ch}})$$
$$=\int_\eta^\infty dp_{t,N}\frac{1}{p_{t,N}}f(p_{t,N})g\left(\frac{\eta}{p_{t,N}}\right). \tag{5.3}$$

We assume the following form for $f(P_{t,N})$:

$$f(P_{t,N})=\frac{P_{t,N}}{p_0^2}e^{-P_{t,N}/p_0}\,, \tag{5.4}$$

where $p_0$ is the parameter to be determined. The mean transverse momentum is

$$\langle P_{t,N}\rangle=2p_0. \tag{5.5}$$

For $g(K_{\mathrm{ch}})$ we used the $K_{\mathrm{ch}}$ distribution obtained in the previous section, replacing the ideogram of Fig. 3 by the histogram of Fig. 4. $h(\eta)$ was calculated with these forms of $f(P_{t,N})$ and $g(K_{\mathrm{ch}})$, and was then converted to its integral form $H(\eta)$. In Fig. 5, curves of $H(\eta)$, calculated for several values of $p_0$, are compared with the $\Theta E_{\mathrm{ch}}$ distribution obtained from the degraded nucleons in the present experiment; the best fit is obtained for $p_0=(0.26\pm0.10)\mathrm{GeV}/c$. The mean value of $P_{t,N}$ thus becomes

$$\langle P_{t,N}\rangle=(0.5_2\pm0.2)\mathrm{GeV}/c.$$

This value of $\langle P_{t,N}\rangle$ is obtained from the nucleus-nucleus interactions. The correction for Fermi motion in the interacting nuclei amounts to a few tens of MeV/$c$ and can be neglected since the errors from other sources are considerably larger.

Dekkers *et al.*[16] measured secondary-proton momentum and angular distribution using 23.1 GeV/$c$ $p$-$p$ and $p$-nucleus interactions. For the 23.1-GeV/$c$ $p$-Be interaction, their result gives $\langle P_{t,p}\rangle=0.42$ GeV/$c$. Therefore, there does not seem to be a significant difference between the mean $P_{t,p}$ at 23 GeV/$c$ and the one in the TeV/$c$ region.

Kim[9] identified the recoil protons in cosmic-ray jets in emulsion and obtained the mean $P_t$ value of these protons. His result is $\langle P_{t,p}\rangle=(0.35\pm0.11)$ GeV/$c$.

## VI. MULTIPLICITY OF SHOWER PARTICLES

The multiplicities of the shower particles of the secondary interactions are plotted versus energy in Fig. 6. We took only those jets with $N_H\leqslant5$. For the incident energy of the interactions initiated by the fragment nucleons we used the primary energy per nucleon of the family, and for that of the interactions initiated by the degraded nucleons we used the following relation:

$$E=\frac{(0.5)_2}{\Theta}\quad(\text{in GeV})\,, \tag{6.1}$$

where $(0.5)_2$ GeV/$c$ is the mean transverse momentum of the degraded nucleons evaluated in the preceding section and $\Theta$ is the emission angle of the nucleon in the primary interaction. Results from previous experiments are also shown in Fig. 6.

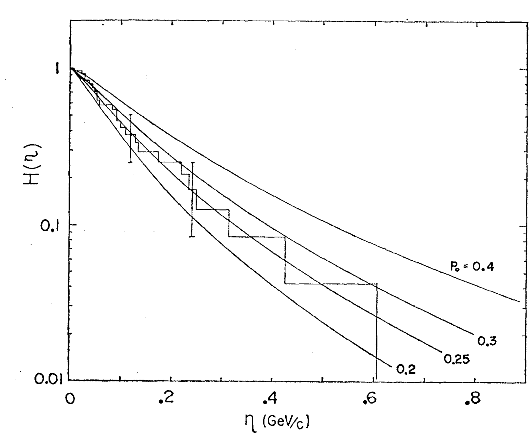


FIG. 5. The calculated and the measured $H(\eta)$ are compared. The histogram is the observed $\Theta E_{\mathrm{ch}}$ distribution of the degraded nucleons and the four curves were calculated from $f(P_t)$ and $g(K_{\mathrm{ch}})$ for different values of $p_0$.

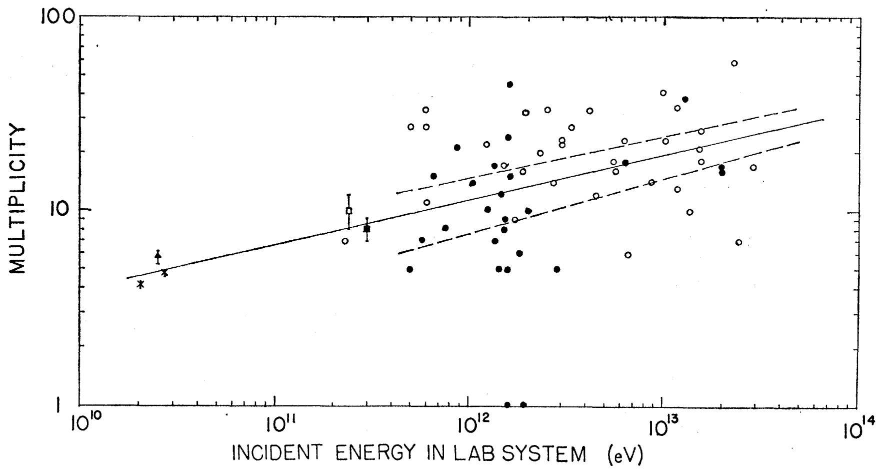


FIG. 6. The multiplicities of the events are plotted versus incident energy from the present experiment (●) and from Barkow *et al.*, (Ref. 7) (○). The mean multiplicities obtained by Meyer *et al.* (Ref. 19) (×), Marzari-Chiesa *et al.* (Ref. 12) (▲), Lohrmann *et al.* (Ref. 8) (□), and Dobrotin *et al.* (Ref. 17) (■) are also shown. The solid line is $\langle n_s\rangle = 2.8E^{1/4}$ ($E$ in GeV) and the broken lines show one standard deviation from the solid line expected from the simple Poisson distribution of the multiplicity.

The average multiplicities can be fitted by either of the following equations:

$$\langle n_s\rangle = aE^{1/4}, \quad \text{with} \quad a = 2.8\ (\text{GeV})^{-1/4}, \qquad (6.2)$$

or

$$\langle n_s\rangle = b\log(E/\epsilon), \quad b = 4.4, \quad \epsilon = 3\ \text{GeV}, \qquad (6.3)$$

where $E$ is measured in GeV.

The mean multiplicity and the variance of the multiplicity in the energy range of 0.5 to 2 TeV are

$$\langle n_s\rangle = 11.2 \pm 2.1,$$
$$\sigma = 9.7.$$

A correction for the effect of surviving protons was applied.

## VII. CONCLUSIONS

(1) The $K_{\text{ch}}$ distribution is obtained from the interactions initiated by the quasimonoenergetic fragment nucleons. The distribution has a broad peak at about $K_{\text{ch}} = 0.2$. The arithmetic mean of $K_{\text{ch}}$ is 0.28. The general shape of the $K_{\text{ch}}$ distribution does not seem to change in the energy range from 100 GeV up to 10 TeV.[6,17]

(2) The mean transverse momentum of the degraded nucleons, i.e., nucleons which interacted in the primary collision, is $(0.5_2 \pm 0.20)$ GeV/$c$; this value does not differ from the result of accelerator experiments. It seems that the mean transverse momentum of the degraded nucleons is insensitive to the incident energy, as is also the case for the transverse momentum of the produced mesons.

The observed small value of the mean transverse momentum of the degraded nucleons indicates that it cannot be regarded as the superposition of the recoils due to the statistically independent emission of secondary particles. If uncorrelated production took place, the mean transverse momentum of the nucleons would be expressed approximately by

$$\langle P_{t,N}\rangle = (1.6\langle n_s\rangle)^{1/2}\langle P_{t,s}\rangle, \qquad (7.1)$$

where $\langle P_{t,s}\rangle$ is the mean $P_t$ of the secondary particles and the factor 1.6 accounts for the recoil due to neutral secondaries. When the experimentally observed values of $\langle n_s\rangle$, i.e., $\sim 12$, and of $\langle P_{t,s}\rangle$, i.e., $\sim 0.4$ GeV/$c$, are employed, this equation gives $\langle P_{t,N}\rangle \simeq 1.8$ GeV/$c$, which is much larger than observed.

(3) The multiplicity distribution has a large variance and the energy dependence of the average multiplicity can be expressed either by $aE^{1/4}$ or $b\log(E/\epsilon)$. The variance is larger than expected from the simple Poisson distribution by a factor of 2 to 3. This also suggests that the secondary particles are created with certain dynamical correlations.

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