On forming a scalar product of Eq. (2) with **v**, it is found that

$$d\gamma/dt=\tfrac{1}{2}\gamma\beta^2[d(\ln H)/dt]-P\,, \quad (3)$$

where $\beta^2\equiv v^2/c^2=1-(1/\gamma^2)$. Subtraction of Eq. (3) from Eq. (1) leads to

$$d[W-\tfrac{1}{2}\gamma\beta^2]/dt=-\tfrac{1}{4}\gamma\beta^2 d(\ln W)/dt\,, \quad (4)$$

where $W=VH^2/8\pi m_0c^2$.

The integral of Eq. (4) is

$$\gamma\beta^2=4W_0[W/W_0-(1-\tfrac{1}{4}\gamma_0\beta_0{}^2/W_0)(W/W_0)^{1/2}], \quad (5)$$

where $\gamma_0$ and $W_0$ are, respectively, the initial electron and magnetic energies in units of the electron rest mass.

From Eq. (5) it follows that when the electron has lost its energy ($\gamma\rightarrow 1$, $\beta\rightarrow 0$), the change in the magnetic field $\Delta W\rightarrow\frac{1}{2}\gamma_0\beta_0{}^2$. As stated previously, this result is independent of the mechanism whereby the electron energy is dissipated. It is easily recognized that this change in the magnetic field energy $\Delta W$ is a simple consequence of Lenz's Law.

The considerations presented in this note affect our understanding of such phenomena as relaxation of magnetic fields in solar flares and elsewhere, the nature of such objects as the Crab Nebula, extars, quasars, as well as cosmic rays and stellar evolution. Also there arises the problem of magnetic field regeneration and its relation to particle acceleration in cosmic space. We hope to pursue these subjects in some detail in future publications. *Note added in proof.* A question has arisen as to when the irreversible change of $\Delta W$ occurs in the "lossless" system discussed above. It is our opinion that it takes place during the *sudden* insertion of the particle into the magnetic field.

The clarifying discussions with J. W. Carpenter and S. Frankenthal of American Science and Engineering, and S. Olbert of the Massachusetts Institute of Technology, are gratefully acknowledged. This work has been sponsored in part by NASA Contract No. NAS 5-9041.

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# Measurement of Pion Spectra in the Decay $K_2{}^0\rightarrow\pi^++\pi^-+\pi^0$†

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The pion spectra of the decay $K_2{}^0\rightarrow\pi^++\pi^-+\pi^0$ are presented. Results are based on 1198 $K_{\pi3}{}^0$ decays that were recorded with a spark chamber setup. $K_{\pi3}{}^0$ is well described by a linear matrix element of the form $M\propto 1+\sigma_0'[2T(\pi^0)-T(\pi^0)_{\max}]\,m_k/m_\pi{}^2$ with $\sigma_0'=-0.21\pm0.02$. There is no observable dependence of the matrix element on the $X$ variable, $X\propto[T(\pi^+)-T(\pi^-)]$. Our measurement is in agreement with the $|\Delta I|=\frac{1}{2}$ rule. Various models to explain the observed variation of the density of the Dalitz plot are discussed. The value for the slope parameter is in good agreement with certain speculative predictions based on equal-time current commutators. No evidence for $CP$ violation is found.

## I. INTRODUCTION

THE dominant decay modes of the neutral long lived $K$ meson, which we label $K_2{}^0$ meson, can be divided into two separate groups:

(a) $K_{\pi3}{}^0$ decay modes, that is to say, nonleptonic strangeness-nonconserving weak decays:

$$K_2{}^0\rightarrow\pi^++\pi^-+\pi^0\,,\quad[(K+-0),\,11.9\pm1\%] \quad (1)$$

$$K_2{}^0\rightarrow\pi^0+\pi^0+\pi^0.\quad[K(000),\quad 23.1\pm2\%] \quad (2)$$

(b) $K_{l3}$ decay modes, that is to say, leptonic strangeness-nonconserving weak decays:

$$K_2{}^0\rightarrow\pi^\pm+\mu^\mp+\nu_\mu\,,\quad(K_{\mu3},\,27.6\pm2\%) \quad (3)$$

$$K_2{}^0\rightarrow\pi^\pm+e^\mp+\nu_e.\quad(K_{e3},\,37.4\pm2\%) \quad (4)$$

The above branching ratios were obtained from the compilation by Trilling.[1]

The theoretical description of $K_{\pi3}{}^0$ decay is not very advanced. The reason is that little is known about a matrix element which involves only strongly interacting particles. In contrast to this, a useful theoretical framework exists for dealing with leptonic decays, group b; see for example, the survey by Jackson.[2]

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[1] G. H. Trilling, in Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory Report No. ANL-7130 (1965), p. 115 (unpublished).

[2] J. D. Jackson, *Elementary Particle Physics and Field Theory, Brandeis Summer Institute 1962* (W. A. Benjamin, Inc., New York, 1963), p. 263.

From a phenomenological point of view, an often used approach to $K_{\pi 3}$ decay, is the "linear matrix-element approximation."[3,4] We shall discuss it here because it provides a useful way for discussing the experimental data. The final state of three pions in $K_{\pi 3}{}^0$ is completely determined by two independent variables $X$ and $Y$. We define $X$ and $Y$ in terms of the Lorentz-invariant variables $S_i$ in such a way that they resemble the familiar Dalitz variables[5]:

$$X=-\sqrt{3}(S_1-S_2)/2m_KQ\,, \tag{5a}$$

$$Y=-3(S_3-S_0)/2m_KQ\,, \tag{5b}$$

with

$$Q=m_K-\sum m_i\,, \tag{5c}$$

$i=1, 2, 3$, representing $\pi^+$, $\pi^-$, and $\pi^0$, respectively,

$$S_i=(P_K-P_{\pi i})^2=(m_K-m_\pi)^2-2m_KT_i\,, \tag{5d}$$

$$S_0=\tfrac{1}{3}(S_1+S_2+S_3)=(m_K-m_\pi)^2-\tfrac{2}{3}m_KQ\,. \tag{5e}$$

$P$ is the four-momentum and $T_i$ is the kinetic energy of $\pi_i$ in the center-of-mass (c.m.) system of the $K_2{}^0$. A useful expansion of the matrix element $M(X,Y)$ for $K(+-0)$ decay is in powers of $X$ and $Y$ as follows:

$$M(X,Y)=a+bX+cY+\text{higher order terms}\,. \tag{6a}$$

This expansion is not quite valid[4] when there exists a strong $\pi$-$\pi$ final-state interaction, but we shall pursue the consequence of this expansion anyway. In the linear matrix-element approximation one ignores the higher-order terms; this reduces (6a) to

$$M(X,Y)=a+bX+cY\,. \tag{6b}$$

When $CP$ invariance holds it is easy to see that the matrix element for $K_2{}^0$ decay should not contain any odd powers of $X$, thus $b=0$ and Eq. (6b) becomes

$$M(X,Y)=a+cY\,. \tag{6c}$$

Substitution of Eqs. (5b)–(5e) into (6c) yields

$$M(X,Y)=a+c(3T_3-Q)/Q\,. \tag{7a}$$

This can be rewritten, using $T_{\max}\,\tfrac{2}{3}Q$, as follows:

$$M(X,Y)=a[1+\sigma_0(2T_3-T_{\max})m_K/m_\pi{}^2]\,, \tag{7b}$$

where $\sigma_0=3cm_\pi{}^2/2aQm_K$, which is called the slope parameter. The definition of $\sigma_0$ was chosen to make Eq. (7b) conform to the units used in Trilling's[1] compilation of the slope parameter in $K_{\pi 3}$ decay. The observed spectrum is given by $|M(X,Y)|^2/\phi$, where $\phi$ is the normalized invariant phase space. Thus,

$$|M(X,Y)|^2/\phi=1+2\sigma_0(2T_3-T_{\max})m_K/m_\pi{}^2\,; \tag{7c}$$

in this expression the quadratic term has been ignored, in the spirit of the linear matrix-element approximation. Expressions that are similar to Eqs. (6) and (7) also hold for the following decay modes of the charged $K$ meson:

$$K^\pm\to\pi^\pm+\pi^\pm+\pi^\mp\,,\quad [\tau \text{ or } K(++-)] \tag{8}$$

$$K^\pm\to\pi^\pm+\pi^0+\pi^0\,.\quad [\tau' \text{ or } K(+00)] \tag{9}$$

Note that the odd pion in $K^\pm$ decay has the same role as the $\pi^0$ in $K(+-0)$, and furthermore that Bose statistics for the two like pions in $K^\pm$ decay require that the expansion of the matrix element for $K^\pm$ decay does not have odd powers in $X$.

Gell-Mann and Pais[6] have suggested the existence of a $|\Delta I|=\frac{1}{2}$ rule for nonleptonic strangeness-changing weak decays. This rule allows only those transitions which involve a change $|\Delta I|=\frac{1}{2}$ in the total isotopic spin of the system. It leads to the following prediction[3,7] for the slope parameter $\sigma$ of the odd-pion spectrum in $K_{\pi 3}$:

$$\sigma_0(+-0)=\sigma_+(+00)=-2\sigma_-(++-)\,. \tag{10}$$

Another prediction[6,7] of the $|\Delta I|=\frac{1}{2}$ rule relates the decay rates $R$,

$$R(+-0)=2R(+00)=\tfrac{1}{2}R(++-)=\tfrac{2}{3}R(000)\,. \tag{11}$$

It is important to investigate to what extent the $|\Delta I|=\frac{1}{2}$ rule holds, because the nature of the rule is not clear. There are some interesting speculations based on the Cabibbo version of the current-times-current model for weak interactions. The hadronic weak current transforms like an octet and the matrix element for non-leptonic weak decay transforms like an octet and a 27-plet. To obtain the pure $|\Delta I|=\frac{1}{2}$ rule, one could eliminate the 27-plet by introducing a set of neutral currents, which have not been observed as yet. Another possibility is to postulate octet enhancement, which implies that $\Delta I=\frac{3}{2}$ currents will be suppressed and one has an approximate $|\Delta I|=\frac{1}{2}$ rule. Suggestive, but not conclusive of the last alternative, is the decay mode $K^+\to\pi^++\pi^0$ which is a well-known violation of the strict $|\Delta I|=\frac{1}{2}$ rule. The often performed test of the $|\Delta I|=\frac{1}{2}$ rule, through Eq. (11), by measuring the rates for the $\tau$ and $\tau'$ decay modes, yields only information about the presence of isotopic spin $I=1$ and $I=3$ final states, and is not a conclusive test of the $|\Delta I|=\frac{1}{2}$ rule. This is discussed in detail in Refs. 4 and 7. On the other hand, a comparison of the odd-pion slopes, Eq. (10), investigates the presence of an $I=2$ final state and thus gives information that is not otherwise available. We will elaborate on this statement in Sec. VI A.

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[5] R. H. Dalitz, Phil. Mag. **44**, 1068 (1953); Phys. Rev. **94**, 1046 (1954).
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[7] G. Barton, C. Kacser, and S. P. Rosen, Phys. Rev. **130**, 783 (1963).

Recently, the scheme of equal-time current commutators[8] has been applied successfully to a calculation of the ratio of axial vector to vector coupling constants in neutron $\beta$ decay,[9,10] to the $S$-wave part of nonleptonic hyperon decay[11,12] and to leptonic $K$-meson decay.[13–15] Callan and Treiman,[13] Suzuki,[16] and others[17–19] have extended this work and applied the current commutators to $K_{\pi 3}$ decay. These authors calculate the amplitude for $K(+-0)$ decay when the four-momentum $q(\pi)$ of any of the three pions is zero. In Sec. VI B we discuss how one can obtain the magnitude and the sign of the slopes of the pion spectra in $K(+-0)$ decay from the theory of equal-time current commutators. Furthermore, this theory of current commutators leads to a relation between the rates for $K(+-0)$ and $K_1{}^0 \to \pi^- + \pi^+$ decay.

In the following, we present the analysis of 1198 $K_2{}^0 \to \pi^+ + \pi^- + \pi^0$ decays, together with a brief discussion of the errors which might falsely indicate—or disguise—a possible structure in the $\pi^0$ spectrum. In Sec. VI A, our measured $\pi^0$, $\pi^-$, and $\pi^+$ spectra are analyzed in terms of a linear matrix element and the isotopic spin relations with $K(+00)$ and $K(++-)$ decay. In Sec. VI B, a comparison is made with the theory of equal-time current commutators. In Sec. VI C, there is a brief discussion of the models which have been proposed to explain the pion spectra in $K_{\pi 3}$ decay. Finally, in Sec. VI D we shall discuss the consequences of $CP$ noninvariance in $K(+-0)$ decay, in particular with respect to a charge asymmetry which is described by the coefficient $b \neq 0$ in Eq. (6b).

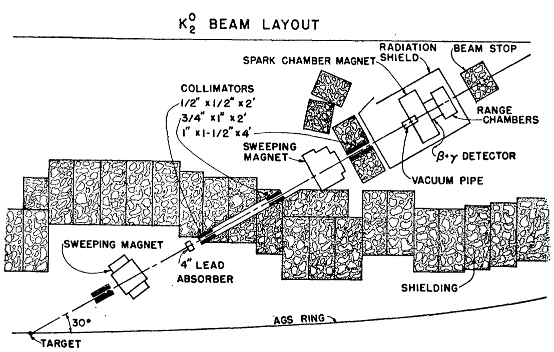


FIG. 1. Beam layout.

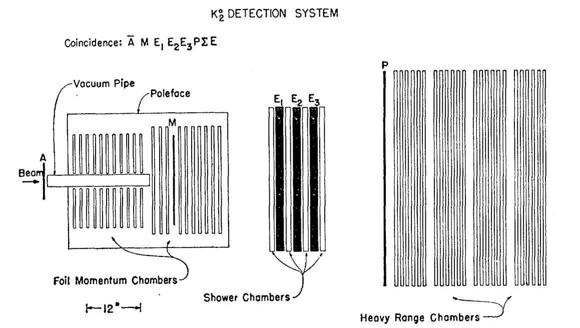


FIG. 2. Top view of detection apparatus.

## II. BEAM AND DETECTION APPARATUS

The experiment was performed in the 30° neutral beam of the AGS, the alternating gradient synchrotron at Brookhaven National Laboratory. The layout of the beam and the detection apparatus used have been described in detail elsewhere.[20–22] We shall limit this section to mentioning the characteristic features and essential parts of the setup.

The layout of the experiment is shown in Fig. 1. The neutral beam was suitably collimated to a size $\frac{3}{4}$ in. $\times 1$ in. by three brass collimators. It was swept clean of charged particles by two magnets, and filtered by four inches of lead to remove most of the $\gamma$ rays. The detection apparatus was located at 67 ft from the internal AGS target. At this place the beam consisted of neutrons, antineutrons, neutrinos, a few $\gamma$ rays, and $K_2{}^0$ mesons. All other known neutral particles had decayed to a completely negligible level. The top view of the detection system is shown in Fig. 2. The neutral beam passed through a 2-ft long evacuated pipe to eliminate confusing interactions from neutrons and $K_2{}^0$ mesons with air. The pipe had 3-mil Mylar windows on the sides and ends. It was surrounded by 21 thin-foil spark chambers, called momentum chambers, to measure the trajectory of the charged particles that emerged from the evacuated pipe. The foil spark chambers and pipe were placed between the pole faces of a large analyzing magnet (10 kG). A $\beta$-$\gamma$ detector followed by a range chamber was installed after the magnet, as shown in Fig. 1. The $\beta$-$\gamma$ detector consisted of three 12-in.$\times$32-in. shower counters, sandwiched between four thin spark chambers, each four gaps, to observe electron showers. The shower counters $E_1E_2E_3$ were made of four layers of $\frac{3}{32}$-in. lead sheet between five layers of $\frac{1}{8}$-in. scintillator. There was a hole in each counter to let the beam pass through. The range chamber consisted of 24 two-gap spark chambers, made of 48-in.$\times$36-in. aluminum plates, $\frac{1}{8}$ in. thick. In front of

[8] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[9] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965).
[10] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965).
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[14] M. Suzuki, Phys. Rev. Letters **16**, 212 (1966).
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[16] M. Suzuki, Phys. Rev. **144**, 1154 (1966).
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[18] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).
[19] B. M. K. Nefkens, Phys. Letters **22**, 94 (1966).

[20] D. W. Carpenter, A. Abashian, R. J. Abrams, G. P. Fisher, B. M. K. Nefkens, and J. H. Smith, Phys. Rev. **142**, 871 (1966).
[21] D. W. Carpenter, Ph.D. thesis, University of Illinois, 1965 (unpublished).
[22] G. P. Fisher, Ph.D. thesis, University of Illinois, 1964 (unpublished).

each of the first 20 chambers was a $\frac{3}{8}$-in. brass plate, and a $\frac{3}{4}$-in. brass plate in front of each of the others.

The top and side views of the momentum chambers in the magnet were photographed through a system of mirrors with a single camera. A 10° stereo pair picture of the top view of the shower and range chambers was taken with a second camera.

The spark chambers were fired by a coincidence $\bar{A}ME_1E_2E_3(\sum E)P$. $\bar{A}$ is an anticounter in the beam in front of the evacuated pipe that eliminated stray particles. $M$ is a counter placed after the pipe in the magnet, between the foil chambers, that signaled a $K_2^0$ decay. $P$ is one of six contiguous counters which indicated that a charge particle had entered the range chamber. $(\sum E)$ refers to the sum of the three undiscriminated pulses from the shower counters that were linearly added in a mixer. The discriminator of $(\sum E)$ was set "just above minimum ionizing" to prevent one straight through minimum ionizing particle from triggering the system. This requirement made the system trigger somewhat preferentially on measurable $K$ decays. About halfway in the experiment the direction of the magnetic field was inverted to minimize the effect of possible counter biases on a measurement of the $\pi^{\pm}$ asymmetry.

### III. SCANNING AND MEASURING

43 000 pictures were taken with the apparatus described in Sec. II. The photographs of the momentum chamber were scanned for measurable and analyzable $K_2^0$ decays, defined as a $V$ with two oppositely curved tracks, appearing to intersect in the evacuated pipe. Both tracks of the $V$ had to have a minimum of four sparks for a useful momentum determination. Double scanning of all pictures yielded 7400 $V$'s for measuring, or 18%. The rest were useless photographs which fall into two categories: (a) interactions of the beam in counter $M$ in the magnet which occurred in 58% of the pictures taken; (b) events either with one measurable prong, or with a single track from a stray particle, or with two tracks that curved in the same direction, a total of 24% of the pictures.

Measurements were done with a Hydel measuring machine and processed on an IBM 7094 computer. Each track was fitted to a helix and the distance of closest approach of the two helices of each $V$ was taken as the decay point of the $K_2^0$ meson. This decay point had to lie within the fiducial volume of the evacuated pipe. This criterion, plus requirements concerning the goodness of the track fitting (see Ref. 21, Sec. III, and Ref. 22, Sec. III), reduced the sample to 4124 useful $V$'s for a kinematical analysis. About 20% of the measured $V$'s turned out to be interactions of the beam in the counters $\bar{A}$ or $M$ which had not been rejected by the cautious scanner. Another 19% fell outside the fiducial volume, which was shorter than the length of the pipe, because the decay region was limited to the homogeneous part of the magnetic field. It excluded interactions of the beam in the front and end window of the pipe. Less than 5% of the measured $V$'s failed the criterion for track fitting or closest approach of the two helices.

The accuracy of measuring the momentum in the laboratory was typically about 3% for a track of 1 BeV/$c$.

### IV. TREATMENT OF EXPERIMENTAL DATA

All $V$'s that originated in the fiducial volume in the evacuated pipe were considered $K_2^0$ decays. This was a safe assumption since neutron decays could be ignored and the interactions of the neutral beam with the residual gas in the pipe were negligible.

The 4124 good $V$'s were followed through the $\beta$-$\gamma$ detector and 1079 events showed one track that showered, which was indicative of an electron track. These events were labeled $K_{e3}$ candidates. Details about the scanning for electrons are given in Ref. 22 which deals with the $K_{e3}$ spectrum. The remaining sample of 3050 events consisted of $K(+-0)$ decays and $K_{l3}$ decays—that is, $K_{\mu 3}$ and undetected $K_{e3}$—where the electron did not pass through the $\beta$-$\gamma$ detector. The rare decay modes, such as $K_2^0\to\pi^-+\pi^+$, $K_2^0\to\pi^-+\pi^++\gamma$, radiative $K$-meson decay, etc., can be ignored. In general, $K(000)$ did not trigger the system, except possibly via the Dalitz pair mode: $K_2^0\to 3\pi^0\to 5\gamma+e^++e^-$. This mode will also be ignored.

To separate the $K(+-0)$ decays from $K_{l3}$ we exploited the small energy release in $K(+-0)$ in comparison with $K_{l3}$, as has been done in similar experiments.[23] One assumes that every $V$ is a $K(+-0)$ decay and calculates $P_0'^2$, where $P_0'^2$ is the square of the momentum of the $K_2^0$ meson in that reference frame in which the total longitudinal momentum of the two charged particles is set equal to zero:

$$P_0'^2=\frac{(m_k^2-m_0^2-m_c^2)^2-4m_0^2m_c^2-4m_k^2P_t^2}{4(P_t^2+m_c^2)}\,. \quad (12)$$

Here $m_k$ is the mass of the $K_2^0$, $m_0$ is the mass of the $\pi^0$, $m_c$ is the invariant mass of the two charged particles that one assumes to be pions, and $P_t$ is the sum of the transverse momenta of the two charged particles. The $P_0'^2$ distribution of our total sample of 4124 $K_2^0$ decays, before removal of the $K_{e3}$ decays, is shown in Fig. 3. Genuine $K(+-0)$ have $P_0'^2>0$, while $K_{l3}$ have a $P_0'^2$ distribution that is centered around $P_0'^2\sim-20\,000$ (MeV/$c$)$^2$ with a small tail extending into the region of $P_0'^2>0$. The finite resolution of our experiment causes the $P_0'^2$ distribution for $K(+-0)$ to have a short tail into the region of negative $P_0'^2$. The clear separation between the $K(+-0)$ and $K_{l3}$ peaks is evident in Fig. 3. The previously mentioned sample of $K_{e3}$ events

[23] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. **133**, B1276 (1964).

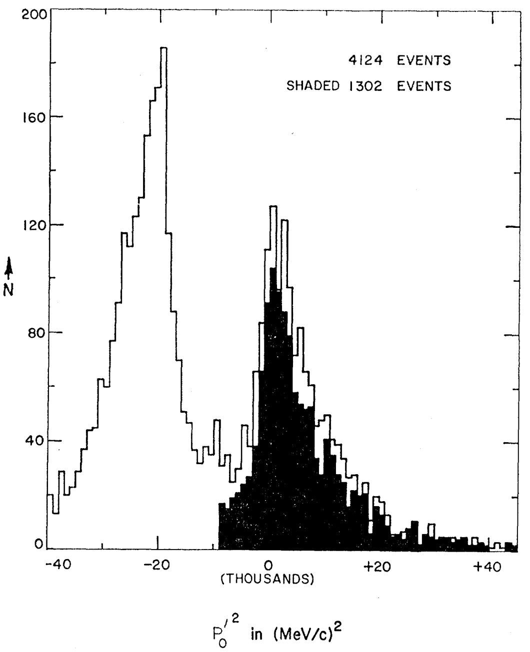


FIG. 3. $P_0'^2$ distribution for all $V$'s. The shaded part is the sample after the criteria for small measurement error have been applied. Furthermore, only events with $P_0'^2 > -9000$ $(MeV/c)^2$ were shaded.

was also used to obtain a $P_0'^2$ distribution. From a comparison of this $P_0'^2$ spectrum of the $K_{e3}$ candidates with a calculated $P_0'^2$ spectrum for a pure $K_{e3}$ sample,[24] we concluded that our sample of 1079 $K_{e3}$ candidates contained $60\pm30$ $K(+-0)$ events. The mislabeled events were due to pion charge exchange in the $\beta$-$\gamma$ detector: $\pi^{\pm}\to\pi^0\to 2\gamma$, where one of the photons converted promptly and simulated an electron shower. All $K_{e3}$ candidates were removed from our sample; this did not affect our measurement of the pion spectra of $K(+-0)$, because the few mislabeled $K(+-0)$ events were uniformly distributed over the Dalitz plot. All events with $P_0'^2 < -9000$ $(MeV/c)^2$ were considered $K_{l3}$ decays and not further used. This low cutoff in $P_0'^2$ ensured that all useful $K(+-0)$ were retained. Only a few grossly mismeasured $K(+-0)$ decays (we estimate less than 20) can have $P_0'^2 < -9000$ $(MeV/c)^2$. There remained 1574 events for further analysis, most of them $K(+-0)$.

A study of the measurement accuracy revealed that the curvature of a track of four sparks could not be determined well enough for our purpose, when the momentum was high. To improve our measurement resolution we removed all events which had a track with only four sparks. Furthermore, we required the calculated accuracy of each track measurement, defined by a goodness-of-fit criterion of the fitted helix through the measured points, to be better than a certain number, which corresponded to an uncertainty in the measured momentum of less than 4%. In this way we removed 272 bad events which could not be adequately measured. This purification did not affect the measurement of the pion spectra, because the bad events were more or less uniformly distributed over the Dalitz plot. (The subsequent analyses of the slope of the $\pi^0$ spectrum were also carried out for the case where the bad events had been retained. The resulting value for the slope parameter $\sigma_0$ was the same as for the purified sample but with a larger error.) The shaded spectrum in Fig. 3 is the $P_0'^2$ distribution of the purified sample; the improved resolution is apparent in the peak-to-valley ratio.

Assuming for a moment that the whole sample of 1302 events that remained were $K(+-0)$ decays, one can calculate the $\pi^0$ energy of each event in the $K_2{}^0$ c.m. system from the equation

$$E(\pi^0)=\frac{m_K{}^2+m_0{}^2-2m_\pi{}^2}{2m_K}-\frac{E_1\cdot E_2-\mathbf{P}_1\cdot\mathbf{P}_2}{m_K}, \quad (13)$$

where $E_1$, $E_2$ is the energy in the laboratory of $\pi^-$, $\pi^+$, and $\mathbf{P}_1$, $\mathbf{P}_2$ is the momentum in the laboratory. The result is plotted in Fig. 4(a). Note that in our calculation $E(\pi^0)$ can have a value smaller than 135 MeV. In this way we avoided deleting the spectrum due to small measurement errors, especially at $E(\pi^0)$ close to threshold. Finally, we required that a real $K(+-0)$ decay was consistent with the expression

$$[E(\pi^0)+5\text{ MeV}]^2\geq m_0{}^2+P_t{}^2, \quad (14)$$

where $E(\pi^0)$ was calculated from Eq. (13). The extra 5 MeV at the left-hand side of Eq. (14) was inserted to avoid deleting the $K(+-0)$ sample due to measurement errors, which in general caused an error in $E(\pi^0)$ of a few MeV. The "$K(+-0)$ consistency requirement" of Eq. (14) removed 104 $K_{l3}$ decays from our sample. We are left with the final sample of 1198 events for the study of the pion spectra in $K(+-0)$ which is shown in the "unshaded" distribution of Fig. 4(a). We have also shown in this figure the phase-space distribution,

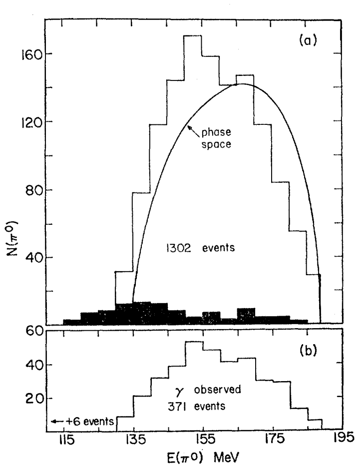


FIG. 4. (a) Calculated energy spectrum, using Eq. (13), of $\pi^0$ in $K_{\pi3}{}^0$ c.m. system for the sample of 1302 events. The shaded part consists of 104 events that do not fulfill the $K_{\pi3}{}^0$ consistency requirement of Eq. (14). The smooth curve is the invariant phase space, normalized to the "unshaded" part (1198 events). (b) Same as (a) for the subsample of "$\gamma$ observed" events.

[24] We assumed a vector interaction for $K_{e3}$. The number quoted does not change, within the stated error, when the $K_{e3}$ form factor is varied over a wide range.

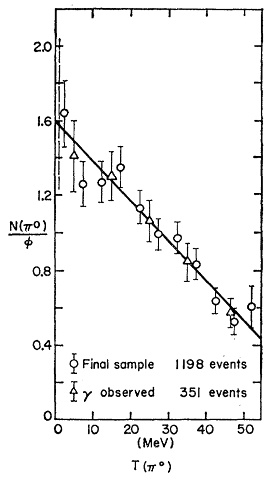


FIG. 5. $\pi^0$ spectrum divided by phase space $\phi$. The straight line is a least-squares fit to our final sample. It fits the expression $N(\pi^0)/\phi = 1+2\sigma_0[2T(\pi^0)-T_{\max}]m_K/m_\pi^2$, with $\sigma_0=-0.204\pm0.018$. The solid error bars indicate only statistical errors. The dashed line along the first point indicates the systematic uncertainty of this point.

normalized to the sample of 1198 events. We estimate that the final sample contained 30±18 undetected $K_{e3}$ and 68±20 $K_{\mu3}$ events. These numbers were obtained from a study of artificially generated $P_0'^2$ distributions for $K_{e3}$ and $K_{l3}$ which included the triggering efficiency. They do not change appreciably when the form factors for $K_{e3}$ and $K_{\mu3}$ are varied within the known range. We estimate a background of 72±36 events due to: unnoticed faulty measurements (~20 events), scattering of pions in the wall of the pipe and in the scintillator (~25 events), radiative and rare decay modes (~5 events), and $\pi$ decay in flight (~22 events). These events are more or less uniformly distributed over the $K(+-0)$ Dalitz plot.

The triggering efficiency of our apparatus for $K(+-0)$ decay as a function of the pion energy in the $K_2{}^0$ c.m. system was calculated with a Monte Carlo program, using the matrix method discussed in Ref. 20. The efficiency in various parts of the Dalitz plot was constant to ±10%. The average efficiency for bins of constant pion energy varied less than ±5%. The resolution function was obtained from the analyses of remeasurements and a study of the measurement accuracy. The rms error in the pion energy in the $K_2{}^0$ c.m. system was 2 MeV in the middle of the Dalitz plot. This value varied somewhat across the Dalitz plot. In Fig. 5 is shown the $\pi^0$ spectrum divided by phase space and corrected for efficiency and resolution. The straight line through the data points is discussed in Sec. VI.

We now discuss the charged-pion spectrum. There is a twofold ambiguity in the calculation of the laboratory energy of the $K_2{}^0$ from the measured momenta of the charged pions. Consequently, there is a related twofold ambiguity in the calculation of the energy of the charged pions $E(\pi^\pm)$, in the $K_2{}^0$ c.m. system. Monte Carlo calculations showed that the solution which gave the lower laboratory energy for the $K_2{}^0$ [called the low-$E(K_2{}^0)$ solution] was the correct one for 78% of the $K(+-0)$ decays registered by our system. All calculations concerning the charged pions were performed twice, once using the low-$E(K_2{}^0)$ solution for all events in the sample, and once using the high-$E(K_2{}^0)$ solution. Although the positions corresponding to the high- and low-energy solutions on the Dalitz plot may be quite different for a particular event, the changes seem to average out. As we shall see, both our high- and low-energy solutions give the same spectra within our experimental errors.

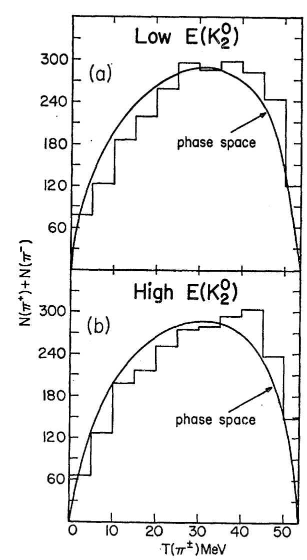


FIG. 7. Charged-pion spectrum as function of the kinetic energy in the $K_2{}^0$ c.m. system. The smooth curve shows the invariant phase space. (a) shows the low-$E(K_2{}^0)$ solution; and (b) shows the high-$E(K_2{}^0)$ solution.

First we show that the $\pi^+$ and $\pi^-$ spectra are the same, then it is permissible to add the two curves which reduces statistical fluctuations. A sensitive way to investigate how identical the two spectra are, is through the ratio of the number of $\pi^+$ mesons and the number of $\pi^-$ mesons as a function of the kinetic energy $T(\pi^\pm)$. This ratio is illustrated in Fig. 6(a) for the low-$E(K_2{}^0)$ solution. Though the points on the plot are somewhat scattered, there is no indication of a systematic deviation from unity, and we conclude that within the limited statistics the $\pi^+$ and $\pi^-$ spectra are identical. The obvious implications of this statement with respect to $CP$ invariance are discussed in Sec. VI D. In Fig. 6(b) the same ratio is shown as in Fig. 6(a), but now for the high-$E(K_2{}^0)$ solution. From this picture we conclude that the

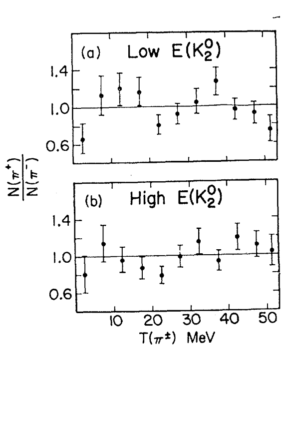


FIG. 6. (Number of $\pi^+$)/(number of $\pi^-$), as a function of their kinetic energy in the $K_2{}^0$ c.m. system. (a) is for the low-$E(K_2{}^0)$ solution; and (b) is for the high-$E(K_2{}^0)$ solution to the twofold kinematical ambiguity. The straight line shows the ratio for the case of no $CP$ violation.

twofold ambiguity does not introduce a charge asymmetry (as expected). In Figs. 7(a) and (b) is shown the charged-pion spectrum (sum of $\pi^+$ and $\pi^-$ spectra) for the low- and high-$E(K_2{}^0)$ solution, respectively. Also shown in Fig. 7 is the normalized phase space. As mentioned earlier, the ambiguity does not seem to have a noticeable effect on the shape of the pion spectrum. Finally, in Fig. 8(a) is shown the charged-pion spectrum, for the low-$E(K_2{}^0)$ solution, divided by phase space and corrected for the detection efficiency and measurement resolution. The corresponding plot for the high-$E(K_2{}^0)$ solution is shown in Fig. 8(b). The straight line through the data points is discussed in Sec. VI.

In view of the later discussion on $CP$ invariance and the linear matrix-element approximation to $K_{\pi 3}$ decay,

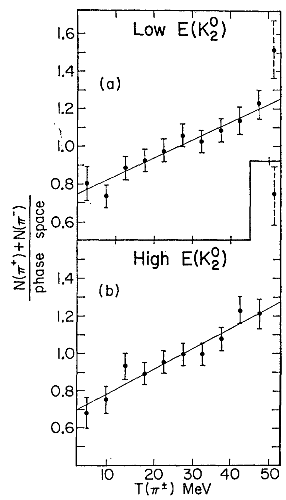


FIG. 8. Charged-pion spectrum divided by phase space. The straight line is a least-squares fit with the expression $N(\pi^\pm)/\phi=1+\sigma_\pm[2T(\pi^\pm)-T_{\max}]m_K/m_\pi{}^2$. (a) shows the low-$E(K_2{}^0)$ solution, where we found

$$\sigma_\pm=+0.096\pm0.09;$$

and (b) shows the high-$E(K_2{}^0)$ solution with

$$\sigma_\pm=+0.111\pm0.011.$$

we recast the $\pi^+$ and $\pi^-$ spectra in terms of the $X$ parameter, defined in the Introduction:

$$X=-\sqrt{3}(S_1-S_2)/2m_kQ=\sqrt{3}[T(\pi^+)-T(\pi^-)]/Q.$$

Figure 9 illustrates the Dalitz plot for $K(+-0)$ decay. First we show that our data are consistent with a Dalitz plot that is symmetric in the $X$ parameter. This is done through the ratio $R=N(\text{at } X_i)/N(\text{at } -X_i)$ ($R$ is the number of events in a vertical bin in Fig. 9 at $X_i$, divided by the number of events in a vertical bin at $-X_i$). The ratio $R$ as function of $X$ is shown in Figs. 10(a) and 10(b) for the low- and high-$E(K_2{}^0)$ solution, respectively. There is no deviation in $R$ from unity that is statistically significant; thus the $X$ parameter is symmetric with respect to the point $X=0$ and the left- and right-half of Fig. 9 can be added to reduce the statistical fluctuations. In Fig. 11 is plotted the spectral density as function of $|X|$. This is the number of events per vertical bin of Fig. 9, divided by phase space and corrected for efficiency and resolution. Figure 11(a) shows the low-$E(K_2{}^0)$ solution and Fig. 11(b) shows the high-$E(K_2{}^0)$ solution. The end point at $|X|=0.9$ is

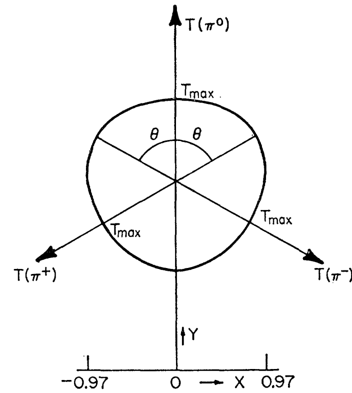


FIG. 9. Relativistic Dalitz contour for a $K$ decay into three pions. At the bottom of the figure are indicated the Dalitz axis $Y\propto T(\pi^0)$ and $X\propto[T(\pi^+)-T(\pi^-)]$.

very sensitive to the exact resolution of the experiment and it was considered unreliable; it has been ignored in further analysis. The straight line through the data points of Fig. 11 is discussed in Sec. VI.

There is no noticeable difference between the two solutions of the kinematic ambiguity as presented in Figs. 6–8 and in 10–11. In view of the considerable symmetry in $K(+-0)$ decay, together with an almost uniform detection efficiency across the Dalitz plot, this is not a surprising finding.

A special sample of $K(+-0)$ candidates was obtained by scanning all 4129 good $V$'s for a $\gamma$-ray in the $\beta$-$\gamma$ detector. After applying the criterion for the goodness of the track fit and after removing four- and five-spark events, there remained 372 events. The two charged tracks of one event looked like $e^-$ and $e^+$. This event was considered a $K_2{}^0\to 3\pi^0\to 5\gamma+e^++e^-$ and removed. The search for the decay mode $K_2{}^0\to\pi^++\pi^-+\gamma$ in our "$\gamma$-ray observed" sample has already been reported.[25] The upper limit, with 85% confidence, based on one

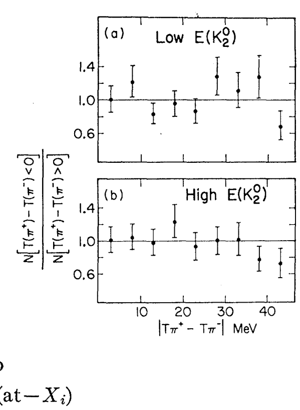


FIG. 10. Ratio $R=N(\text{at}X_i)/N(\text{at}-X_i)$
$=\{N[T(\pi^+)-T(\pi^-)]<0\}/\{N[T(\pi^+)-T(\pi^-)]>0\}$ as a function of $|T(\pi^+)-T(\pi^-)|$. Figure 10(a) shows the low-$E(K_2{}^0)$ solution; and (b) shows the high-$E(K_2{}^0)$ solution. The straight line is the expected ratio for no $CP$ violation.

[25] B. M. K. Nefkens, A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, and J. H. Smith, Phys. Letters 19, 706 (1966).

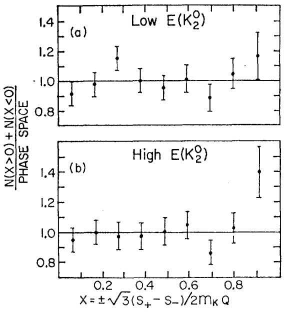


FIG. 11. Density of events as function of the Dalitz variable

$$|X|=\sqrt{3}\,|(T_+-T_-)|Q.$$

The straight line is the expected distribution for the linear-matrix-element approximation, Eq. (7c). (a) shows the low-$E(K_2^0)$ solution, and (b) shows the high-$E(K_2^0)$ solution.

possible event is

$$R=\frac{\text{rate}(K_2^0\rightarrow\pi^++\pi^-+\gamma)}{\text{rate}(K_2^0\rightarrow\text{all decay modes})}<3.0\times10^{-3}.$$

The $P_0'^2$ distribution of the 371 events is shown in Fig. 12. It indicates that the sample contains mainly $K(+-0)$. The $E(\pi^0)$ distribution in the $K_2^0$ c.m. system for these events is shown in Fig. 4(b), it was calculated using Eq. (13). To remove most of the faulty measurements due to $\pi$ decay in flight, interactions in the scintillator, mislabeled $\gamma$ events and radiative $K_{l3}$ decays, we further required that each event be consistent with a $K(+-0)$ decay, defined by Eq. (14). There remained 343 clean $K(+-0)$ events. The $\pi^0$ spectrum of this sample, divided by phase space, and corrected for efficiency and resolution, is shown in Fig. 5. It is in excellent agreement with the large sample.

## V. DISCUSSION OF ERRORS

The experiment described in the previous section required a "zero constraint fit" to obtain the pion spectra. We investigate here the effect of important systematic errors on the results. We mainly discuss the $\pi^0$ spectrum divided by phase space, Fig. 5. The $\pi^0$ energy of each event was calculated using Eq. (13) and the accuracy is determined by the uncertainty of the $\pi^\pm$ momentum measurement and the errors in the masses of the mesons. The obvious effect of a systematic error in mass or momentum is to shrink or expand the area of the Dalitz plot, that is to say, events are pushed in or out of the theoretical boundary of the Dalitz plot. Therefore, the first and last bin of Fig. 5 are the ones to be affected. The masses of the pions are known accurately enough; however the mass of $K_2^0$ is not sufficiently known for the purpose. In our calculation we have used $m_K=497.8$ MeV. A recent compilation[26] gave $m_K=497.7\pm0.3$ MeV, and we will use an uncertainty $\delta m_K=0.3$ MeV. The uncertainty in the magnetic field in our experiment is $\frac{1}{2}\%$. Table I shows the effect of these uncertainties on the population of the first, second, and last data point of Fig. 5. The total

TABLE I. Systematic errors and their effect on the $\pi^0$ spectrum. $\delta E(\pi^0)$ is the shift or change in the calculated $\pi^0$ energy, using Eq. (13), due to the uncertainty in $m_K$ or in $P(\pi^\pm)$.

| Error | $\delta E(\pi^0)$ due to error | $\delta N(\pi^0)/\phi$ | Data point in Fig. 5. | Comment |
|---|---|---|---|---|
| $\delta P=\frac{1}{2}\%$ | $\frac{1}{2}$ MeV | 14% | First | $\mathbf{P}(\pi^\pm)\simeq\mathbf{P}(\pi^\mp)$ |
| $\frac{1}{2}\%$ | 1 MeV | 28% | First | $\mathbf{P}(\pi^\pm)\gg\mathbf{P}(\pi^\mp)$ |
| $\frac{1}{2}\%$ | $\frac{1}{2}$ MeV | 2% | Second | $\mathbf{P}(\pi^\pm)\simeq\mathbf{P}(\pi^\mp)$ |
| $\frac{1}{2}\%$ | 1 MeV | 4% | Second | $\mathbf{P}(\pi^\pm)\gg\mathbf{P}(\pi^\mp)$ |
| $\frac{1}{2}\%$ | $\sim$0 | $\sim$0 | Last | |
| $\delta m_K=0.3$ MeV | 0.2 MeV | 6.5% | First | |
| $\delta m_K=0.3$ MeV | 0.2 MeV | 1% | Second | |
| $\delta m_K=0.3$ MeV | 0.2 MeV | $\sim$0 | Last | |

systematic error of the first point of Fig. 5 amounts to about 26%, for other points it can be ignored. It is clear that no statements can be made concerning a possible depletion of the first bin, as has been suggested by other measurements.[27–29] Because the (solid) error bars in Fig. 5 indicate the statistical uncertainty only, we have indicated the systematic errors in the first point by a dashed line.

Similar arguments as above concerning the effect of the systematic errors hold for the charged-pion spectrum of Fig. 8, but only the last bin is affected here and it has been omitted for the numerical analysis.

As stated earlier, the unnoticed $K_{l3}$ decays, mismeasurements in our sample, etc., are more or less uniformly distributed over the Dalitz plot. Their

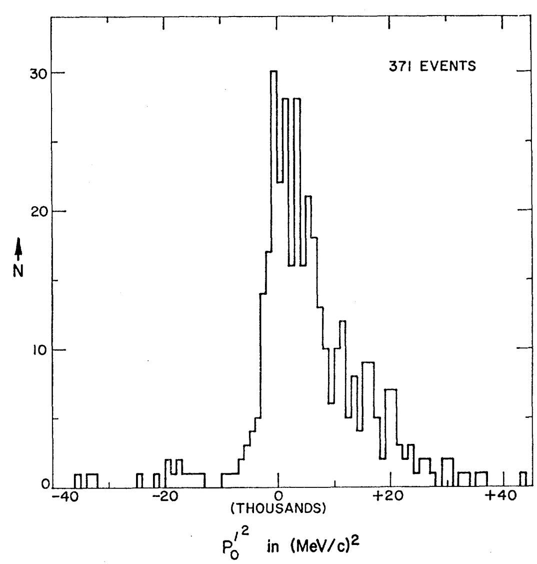


FIG. 12. $P_0'^2$ distribution for all events which showed a $\gamma$ ray in the $\beta$-$\gamma$ detector.

[26] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

[27] P. Basile, J. Bingert, J. W. Cronin, B. Thevenet, R. Turlay, S. Zylberach, and Z. Zylberstejn, in Ref. 1, p. 77.

[28] P. Astbury, G. Finocchiaro, R. D. Fortune, A. Michelini, C. Verkerk, C. H. West, W. Beusch, M. Pepin, and M. A. Pouchon, Phys. Letters **16**, 80 (1965).

[29] P. Astbury, A. Michelini, C. Verkerk, F. Verkerk, W. Beusch, M. Pepin, and M. A. Pouchon, Phys. Letters **18**, 175 (1965).

number is small and the uncertainty in this contamination does not effect the slope of the pion spectra beyond the statistical error. (We estimate that this background has less than 6% effect on the slope.)

Finally, we mention the finite resolution function of our system which tends to "smear" some events outside the boundary of the Dalitz plot. We have calculated and measured the resolution function. It has been taken into account in obtaining Figs. 5 and 8. Again, the first and last bin of Fig. 5 and the last bin of Fig. 8 are most sensitive. We have checked the shape of our resolution function by comparing the observed $P_0'^2$ distribution with a calculated distribution in which the resolution function was folded in. It is worthwhile to remark that a good check is provided by the position of the peak in the $P_0'^2$ distribution. This peak lies between 0 and 1000 $(\mathrm{MeV}/c)^2$ and depends sensitively on the value of the magnetic field and on the direction of the $K_2^0$ beam.

## VI. ANALYSIS OF RESULTS

Traditionally[23,27–35] the $\pi^0$ spectrum in $K(+-0)$ decay [indicated as $N(T_3)$] is fitted with the expression

$$N(T_3)/\phi=1+\alpha T_3/m_K, \qquad (15)$$

where $\alpha$ is the slope parameter. We have compared our data on the $\pi^0$ spectrum to this formula and obtained from a least-squares fit the value $\alpha=-(6.7_{-0.5}^{+0.4})$. Our result is in satisfactory agreement with other experiments, as shown in Table II.

We prefer to fit our $\pi^0$ spectrum to the symmetric first-order linear matrix-element approximation (symmetric with respect to the center of the Dalitz plot), Eq. (7c), which was introduced in Sec. I:

$$N(T_3)/\phi=1+2\sigma_0(2T_3-T_{\max})m_K/m_\pi^2, \qquad (7c)$$

where $m_K=497.7$ MeV, $m_\pi=139.6$ MeV, and $T_{\max}=53.8$ MeV. A least-squares fit to the data—indicated by the straight line in Fig. 5—was used to obtain the slope parameter: $\sigma_0=-0.204\pm0.018$; the error is the statistical one only. When we include an estimate of other experimental errors, the final value is $\sigma_0=-0.204\pm0.025$.

TABLE II. Published measurements of the $\pi^0$ slope parameter $\alpha$ in $K(+-0)$ decay.

| Reference | Events | $\alpha$ |
|---|---|---|
| 23 | 83 | $-7.3\pm1.6$ |
| 30 | 79 | $-7.3\pm1.7$ |
| 28 and 29 | 66 | $-5.5\pm1.5$ |
| 29 | 310 | $-(7.3_{-0.8}^{+0.6})$ |
| 31 | 1729 | $-7.9\pm0.9$ |
| 32 | 566 | $-5.8\pm1.4$ |
| 33 | 130 | $-(7.65_{-1.25}^{+0.95})$ |
| 34 | 126 | $-8.6\pm0.7$ |
| 35 | 280 | $-(8.2_{-1.3}^{+0.9})$ |
| This expt. | 1198 | $-(6.7_{-0.5}^{+0.4})$ |

The charged pion spectrum, shown in Fig. 8, was fitted with an equation which is similar to Eq. (7c), namely,

$$N(T_\pm)/\phi=1+2\sigma_\pm[2T_\pm-T_{\max}(\pi^\pm)]m_K/m_\pi^2. \qquad (7d)$$

Note that $T_{\max}(\pi^\pm)=53.1$ MeV. We ignored the last bin of Fig. 10, as discussed in Sec. IV. A least-squares fit, indicated by the straight line in Fig. 8(a), resulted in the value $\sigma_\pm=+0.096\pm0.009$ for the low-$E(K_2^0)$ solution. The quoted error is the statistical one. The high-$E(K_2^0)$ solution for the charged-pion spectrum, Fig. 8(b), was treated similar as the low-$E(K_2^0)$ solution. This resulted in $\sigma_\pm=+0.111\pm0.012$. We consider the agreement between the two values of $\sigma_\pm$ a sufficient reason for neglecting any effect of the twofold ambiguity.

Since $\sigma_0$ and $\sigma_\pm$ are different projections of the same density variation of the Dalitz plot of $K(+-0)$ decay, they are related; in the linear matrix-element approximation to $K_{\pi3}$ decay, and assuming $CP$ invariance, this relation is a very simple one: Consider the Dalitz plot for $K_{\pi3}$ decay of Fig. 9; assume that the three pion masses are equal and that the density along the $X$ axis is uniform. From Fig. 9 one concludes immediately that $\sigma_+=-\sigma_0\cos\theta=-\sigma_0\cos60°=-\frac{1}{2}\sigma_0$. The mass difference between $\pi^0$ and $\pi^\pm$ introduces a correction to this relation of the order of $T_{\max}(\pi^0)/T_{\max}(\pi^+)=1.01$, which is negligible. From the measurement of the neutral pion spectrum we obtained $\sigma_\pm=+\frac{1}{2}\times0.204=+0.102\pm0.009$, in good agreement with the direct measurement of $\sigma_\pm$. We interpret this close agreement between the two determinations of $\sigma_\pm$ as evidence that the linear matrix-element approximation is valid for $K(+-0)$ decay. Figure 11 shows that there is no structure in the spectral density as function of $X$, as expected from the discussion of the charged pion spectrum. Somewhat surprising perhaps is the fact that no $X^2$ dependence of the matrix element is apparent in Fig. 11, because an $X^2$ term is the simplest observable correction to the linear matrix-element approximation.

For completeness we also made a least-squares fit of our data with a linear matrix element, Eq. (7b) [not to be confused with Eq. (7c)]. We obtained $\sigma_0'=-0.21\pm0.02$ and $\sigma_+'=0.102\pm0.012$. We conclude that the linear matrix-element approximation as expressed either in Eq. (7b) or in Eq. (7c) gives an adequate description of $K(+-0)$ decay, and that any meaningful distinction between them is beyond the sensitivity of this experiment. (To avoid confusion we have labeled the slope parameter as $\sigma'$ when the data were fitted with the linear matrix element, and as $\sigma$ when the fit was made to a linear spectrum.)

[30] R. Adair and L. Leipuner, Phys. Letters **12**, 67 (1964).
[31] H. W. K. Hopkins, T. C. Bacon, and F. R. Eisler, in Ref. 1, p. 67.
[32] P. Guidoni *et al.*, in Ref. 1, p. 49.
[33] L. Behr *et al.*, in Ref. 1, p. 59.
[34] C. J. B. Hawkins, Phys. Letters **21**, 238 (1966).
[35] M. Anikina *et al.*, Soviet J. Nucl. Phys. **2**, 339 (1966).

### A. Isotopic-Spin Analysis

We compare here the slope parameter $\sigma$ of the *linear spectrum* [Eq. (7c)] in $K(+-0)$ decay with $K(+00)$ and $K(++-)$. We use

$$\sigma_0(+-0)=-0.204\pm0.025\,,$$
$$\sigma_+(+00)=-0.25\pm0.02\,,^{36-38}$$
$$\sigma_-(++-)=+0.093\pm0.011\,.^{38}$$

The ratios of the slopes are therefore

$$\sigma(+-0)/\sigma(+00)=0.82\pm0.12\,, \quad (16a)$$
$$\sigma(+-0)/[-2\sigma(++-)]=1.10\pm0.19\,, \quad (16b)$$
$$\sigma(+00)/[-2\sigma(++-)]=1.34\pm0.19\,. \quad (16c)$$

To investigate the relevance of the observed ratios of the slopes, we express the slopes in terms of the various amplitudes to symmetric and nonsymmetric final states of three pions. We assume that the $3\pi$ final state with isotopic spin $I=3$ is negligible. This is consistent with the observed decay rates for $K(+00)$ and $K(++-)$, as discussed in Ref. 4. The slopes are given by

$$\sigma(+-0)\propto A^0(1,L)/A^0(1,S)\,, \quad (17a)$$
$$\sigma(+00)\propto[A^+(1,L)-A^+(2,L)]/A^+(1,S)\,, \quad (17b)$$
$$2\sigma(++-)\propto[A^+(1,L)+A^+(2,L)]/A^+(1,S)\,. \quad (17c)$$

We have used the notation and definition of Ref. 7. Thus, $A^0(1,L)$ is the transition amplitude in $K_2^0$ decay to the $I=1$ nonsymmetric final state, and $A^0(1,S)$ is the same to the symmetric final state.[39] $A^+(1,L)$ is the transition amplitude in $K^+$ decay to the $I=1$ nonsymmetric final state, etc. In this analysis we assume *CP* variance in $K_2^0$ decay. The ratios of the odd-pion slopes are

$$\frac{\sigma(+-0)}{\sigma(+00)}=\frac{A^0(1,L)}{A^+(1,L)-A^+(2,L)}\,\frac{A^+(1,S)}{A^0(1,S)}\,, \quad (18a)$$

$$\frac{\sigma(+-0)}{-2\sigma(++-)}=\frac{A^0(1,L)}{A^+(1,L)+A^+(2,L)}\,\frac{A^+(1,S)}{A^0(1,S)}\,, \quad (18b)$$

$$\frac{\sigma(+00)}{-2\sigma(++-)}=\frac{A^+(1,L)-A^+(2,L)}{A^+(1,L)+A^+(2,L)}\,. \quad (18c)$$

If the $|\Delta I|=\frac{1}{2}$ rule were strictly valid, the ratios (18a)–(18c) should all be unity.[7] Because of discrepancies between the various experiments on the slopes of the odd-pion spectra in $K_{\pi 3}$ decay as apparent from Trilling's compilation,[1] it is safe to say that our experiment, as expressed in Eqs. (16a) and (16b), is in agreement with the $|\Delta I|=\frac{1}{2}$ rule. However, the data do not exclude an admixture of $\sim15\%$ of $A^+(2,L)$ compared to $A^+(1,L)$. Therefore, we cannot rule out the existence of a small $|\Delta I|=\frac{3}{2}$, $\frac{5}{2}$ transition to the $I=2$ nonsymmetric final state in $K^+$ decay. In view of the approximations that were used in the linear matrix-element expansion and the aforementioned discrepancies in the experimental data, this percentage of $I=2$ admixture should not be taken too seriously.

Note that a comparison of the odd-pion slope in $K(+-0)$, $K(+00)$, and $K(++-)$ decay is not an exhaustive test of the validity of the $|\Delta I|=\frac{1}{2}$ rule. Even when there is no transition to an $I=2$ state, the transition to the $I=1$ final state can occur via a $\Delta I=\frac{1}{2}$ and via a $\Delta I=\frac{3}{2}$ transition. A possible way to investigate this problem is to compare the rates for $K(+-0)$ and $K(+00)$ decay.[4]

### B. Equal-Time Current Commutators

It was pointed out by Callan and Treiman[13] that, on the basis of equal-time current commutators, $A(+-0)$ vanishes when $q(\pi^+)=0$ or when $q(\pi^-)=0$. $A(+-0)$ is the amplitude for $K(+-0)$ decay and $q(\pi^\pm)$ is the four-momentum of $\pi^\pm$. To obtain this result, Callan and Treiman assumed that $H$, the Hamiltonian for $K_{\pi 3}$ decay, consists of two parts: $H^+$ and $H^-$, a parity conserving and a parity nonconserving part, respectively. They generalized the "quark symbolism" for the currents, used by Gell-Mann, to a similar structure for $H$, namely, $H=H^++H^-\sim\bar{\psi}(1+\gamma_5)\psi$ [the $SU(3)$ indices have been omitted]. Furthermore, they assumed that $H$ transforms like a member of an $SU(3)$ octet, which implies the validity of the $|\Delta I|=\frac{1}{2}$ rule. The commutation relations in the notation of Callan and Treiman are

$$[A^{(-)}(x),H(\Delta S=1)]_{x_0=0}=[A^{(+)}(x),H(\Delta S=-1)]_{x_0=0}=0\,, \quad (19a)$$

$$[A_0{}^3(x),H^\pm(0)]_{x_0=0}=\tfrac{1}{2}\delta(\mathbf{x})H^\pm(0)\,. \quad (19b)$$

From Eq. (19a) one obtains directly that $A(+-0)=0$ at $q(\pi^+)=0$ or at $q(\pi^-)=0$. To compare these predictions with experiments, one has to extrapolate the measured amplitude for $K(+-0)$ decay in the physical region to $q(\pi)=0$, which seems an impossible enterprise. It was argued in Ref. 19 that a logical place to which to extrapolate $A(+-0)$ is $E(\pi)=0$, where $E$ is the total energy of the pion, $(E^2=m_\pi{}^2+\mathbf{p}^2)$. One can then compare the predictions of Callan and Treiman at $E(\pi)=0$.

In Fig. 13 is shown our measurement of $A(+-0)$, plotted as a function of $E(\pi^\pm)$. Notice that in the

[36] G. E. Kalmus, A. Kernan, R. T. Pu, W. M. Powell, and R. Dowd, Phys. Rev. Letters **13**, 99 (1964).

[37] V. Bisi *et al.*, Nuovo Cimento **35**, 768 (1965).

[38] Weighted average quoted in Ref. 1. Note that the experiment of Kalmus *et al.* [Ref. 36] is quoted by Trilling to yield $\sigma(+00)=-0.24$. This is not correct, since Kalmus *et al.*, used a linear matrix element [Eq. (7b)] while Trilling used a linear spectrum [Eq. (7c)] to fit the experimental data. These different fits could give different values for the slope parameter, as shown for instance in Ref. 37.

[39] Note that $A^0(1,S)$ and $A^+(1,L)$, etc., are different linear combinations of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ transitions, for instance $A^0(1,S)=A(\Delta I=\frac{1}{2})+A(\Delta I=\frac{3}{2})$ and $A^+(1,S)=A(\Delta I=\frac{1}{2})-\frac{1}{2}A(\Delta I=\frac{3}{2})$.

physical region $A(+-0)$ is a linear function of $E(\pi^\pm)$. This is no great surprise, since we have seen earlier that $K(+-0)$ decay is described very well by a linear matrix element. It is reasonable, therefore, to use a linear extrapolation for $A(+-0)$. In this way we obtained $A(+-0)=0.13\pm0.10$ at $E(\pi^\pm)=0$, which is in agreement with the prediction by Callan and Treiman. Because of this success, we venture that the slope of the $\pi^\pm$ spectrum is actually determined by the fact that $A(+-0)=0$ at $E(\pi^\pm)=0$. If the matrix element for $K(+-0)$ decay is linear in $E(\pi)$, then the value for the slope parameter is $\sigma_+'=(T_{\max}+2m_\pi)m_\pi^2/m_K=+0.118$. Both the sign and the order of magnitude of the computed slope agree with our measurement $\sigma_+'=+0.102\pm0.012$.

From Eq. (19b), the following interesting relation, which holds at $q(\pi^0)=0$, was obtained by Callan and Treiman:

$$A(K_1^0\rightarrow\pi^-+\pi^+)=A(+-0)2g_Am_N/g_r. \quad (20)$$

Here $g_A$ is the axial-vector renormalization constant $g_A=1.18$, $g_r$ is the strong pion-nucleon coupling constant ($g_r\sim13.5$), and $m_N$ is nucleon mass. Thus, $g_Am_N/g_r\simeq0.32\pm0.02$. For the left-hand side of Eq. (20), we obtained[1]

$$A(K_1^0\rightarrow\pi^-+\pi^+)=(7.9\pm0.2)\times10^{-7}.$$

From the decay rate for $K(+-0)$,[1] we calculated that $A(+-0)=(8.3\pm0.3)\times10^{-7}$ at $T(\pi^0)=\frac{1}{2}T_{\max}(\pi^0)$, and thus that $0.32A(+-0)=(2.7\pm0.2)\times10^{-7}$. To obtain the value at $E(\pi^0)=0$, we multiplied by the extrapolation factor

$$f=1-\sigma_0'(T_{\max}+2m_\pi)m_K/m_\pi^2=2.74\pm0.16,$$

where $\sigma_0'$ is our experimental value: $\sigma_0'=-0.21$. Thus the right-hand side of Eq. (18) is $(7.4\pm0.7)\times10^{-7}$, in agreement with the left-hand side (see also Fig. 14). A relation very similar to Eq. (20) was obtained by Hara and Nambu[18] [and independently by Suzuki,[16] who treated the case of $K(++-)$ decay]. The main difference is that these authors symmetrized the $3\pi$ final state, while Callan and Treiman did not. This results in an extra factor of three on the right-hand side of Eq. (20), but no extrapolation of $A(+-0)$ is required. The numerical results obtained with the "symmetrized calculation" are identical to those of Callan and Treiman when the extrapolation factor $f=3$ [corresponding to a slope for the $\pi^\pm$ spectrum which gives $A(+-0)=0$ at $E(\pi^\pm)=0$]. When $f=3$, the right-hand side of Eq. (20) becomes $(8.1\pm0.7)\times10^{-7}$, which is also in accord with the left-hand side.

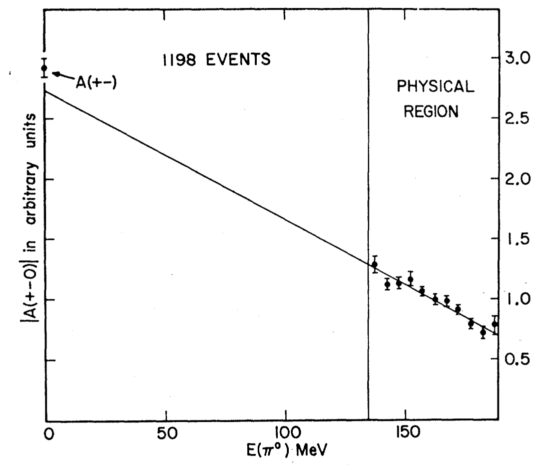


FIG. 14. Amplitude for $K(+-0)$ decay as a function of $E(\pi^0)$ in arbitrary units. The line shows a least-squares fit to the data points, assuming that the amplitude is linear. The point on the $x$ axis indicates the amplitude for $K_1^0\rightarrow\pi^++\pi^-$.

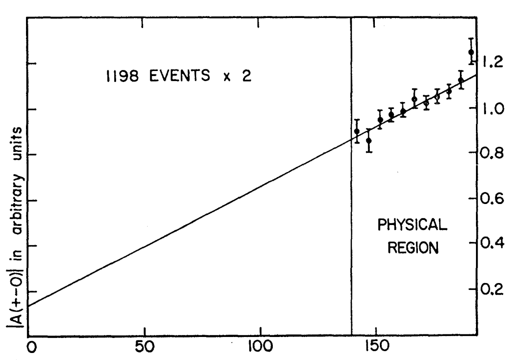


FIG. 13. Amplitude in arbitrary units for $K(+-0)$ decay as a function of $E(\pi^\pm)$. The events are plotted twice, once as a function of $E(\pi^+)$ and once as a function of $E(\pi^-)$. The line shows a least-squares fit to the data points (ignoring the dashed point), assuming that the amplitude is a linear function of $E(\pi^\pm)$.

Bose and Biswas[17] included in their calculation the case that the weak Hamiltonian for $K_{\pi3}$ decay contains 27-plet components $H_{27}$, besides the octet $H_8$. Specifically, they compared the various $K_{\pi3}$ and $K_{\pi2}$ decay rates. We can add to this that an admixture of $H_{27}$ results in $A(+-0)\neq0$ when $q(\pi^\pm)=0$ and also that Eq. (20) does not hold. Very recently, some aspect of this was discussed by Elias and Taylor.[40]

In conclusion, we can say that our data are in agreement with the theory of equal-time current commutators as proposed by Callan and Treiman,[13] in the manner suggested by Nefkens.[19] Our $\pi$ spectra indicate that the contribution of the 27-plet to the Hamiltonian for $K(+-0)$ decay is insignificant. We note that our measurement of the $\pi^\pm$ slope in $K(+-0)$, without reference to any other slope measurement, indicates—in the spirit of the equal-time current algebra and the above assumptions about extrapolation—the approximate validity of the $|\Delta I|=\frac{1}{2}$ rule [and octet dominance in $K(+-0)$ decay].

### C. $K_{\pi3}$ Decay Models

The pion spectra in $K(+-0)$ decay (see Fig. 5) show that the population of the Dalitz plot deviates

[40] D. K. Elias and J. C. Taylor, Nuovo Cimento 44, 518 (1966).

markedly from a pure phase-space distribution. The decay is well described by a linear matrix element. A similar situation is found in $K(+00)$ and $K(++-)$ decay.[1] Several authors[41–43] have attempted to explain the observed $K_{\pi 3}$ spectra as a result of $\pi$-$\pi$ final-state interactions. It is assumed that the decay proceeds initially to a totally symmetric $I=1$ state, which is then modified by final-state $S$-wave $\pi$-$\pi$ interaction, while $p$ waves are ignored. This model predicts[44] that the $S$-wave $\pi$-$\pi$ interaction is attractive in the $I=2$ state and repulsive or weak in the $I=0$ state, which is at variance with results obtained from crossing relations.[45,46] A more sophisticated calculation of the $\pi$-$\pi$ final-state interaction that includes rescattering of pions has been made by Gribov[47] and by Anisovitch.[48] The Feynman diagrams that illustrate these models are shown in Fig. 15.

A closer look at the $\pi$-$\pi$ interaction models reveals that they lead in many cases to a nonlinear matrix element. To search for possible evidence for this, we fitted our measured $\pi^0$ spectrum to the expression

$$N(T_3)/\phi = a + b(2T_3 - T_{\max}) + c(2T_3 - T_{\max})^2.$$

We found that the error in the coefficient $c$ was larger than the coefficient itself and thus conclude that there is no detectable quadratic term in the matrix element. This is not surprising, since we could not distinguish between a linear matrix element, Eq. (7b), and a linear spectrum, Eq. (7c), as mentioned in the beginning of this section. The absence of a quadratic term, together with the absence of any $X$ dependence, as shown in Fig. 11—which is easier to detect experimentally—imposes strong limitations on the models that try to explain the observed deviation of the Dalitz plot from

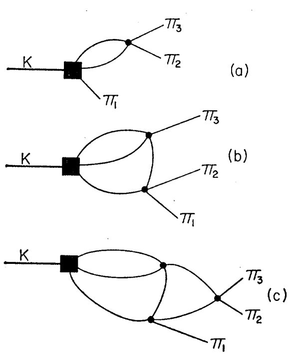


FIG. 15. Feynman diagrams for $K_{\pi 3}$ decay indicating possible final-state $\pi$-$\pi$ interactions that might account for the observed deviations from phase space. (a) single $\pi$-$\pi$ interaction; (b) and (c), rescattering.

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[42] N. N. Khuri and S. B. Treiman, Phys. Rev. **119**, 1115 (1960).
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[48] V. V. Anisovich, Zh. Eksperim. i Teor. Fiz. **47**, 240 (1964) [English transl.: Soviet Phys.—JETP **20**, 161 (1965)].

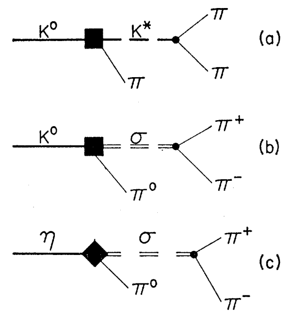


FIG. 16. Models for $K\rightarrow 3\pi$ and $\eta\rightarrow 3\pi$ decay. (a), $K^*$ intermediate state for $K_{\pi 3}$ decay; (b), $\sigma$ intermediate state for $K_{\pi 3}$ decay; (c), $\sigma$ intermediate state for $\eta$ decay.

phase space by a strong $S$-wave $\pi$-$\pi$ interaction. The inclusion of a $P$-wave $\pi$-$\pi$ interaction,[49] the $\rho$ meson, markedly influences the theoretical prediction of the pion spectra. Our experiment is not accurate enough to draw useful conclusions concerning this theory.

It has been suggested[50] that $K_{\pi 3}$ decay proceeds via the $K^*(891)$ intermediate state as indicated in Fig. 16(a). This is an interesting speculation in view of attempts to calculate the form factor in $K_{e3}$ and $K_{\mu 3}$ using this resonance.[2,51] The $K^*$ model predicts an almost-linear matrix element for $K_{\pi 3}$ decay,[50] in reasonable agreement with our experiment.

Brown and Singer[52] have suggested the existence of a $0^+$ dipion resonance named $\sigma$, which acts as an intermediate state in $K$ and $\eta$ decay, as shown in Figs. 16(b) and 16(c). The $\pi^0$ spectrum should have the form

$$\frac{N(T_3)}{\phi} = \frac{C}{(A-T_3)^2+B^2},$$

where $C$ is a normalization constant, $A=[(m_K-m_\pi)^2-m_\sigma^2]/2m_K$, $B=m_\sigma\Gamma_\sigma/2m_K$, $m_\sigma$ is the mass of the $\sigma$ meson, and $\Gamma_\sigma$ is the width. The present status of the $\sigma$ model may be evaluated from Table III, where the $K$ and $\eta$ experiments have been tablulated, for which a comparison with the $\sigma$ model has been made. This table indicates that there are two preferred values for the mass of the $\sigma$ meson: $m_\sigma \simeq 345$ MeV and $m_0 \simeq 400$ MeV. In Fig. 17 is shown our $\pi^0$ spectrum compared with the $\sigma$ hypothesis for $m_\sigma = 345$ and also for $m_\sigma = 400$ MeV. This figure shows that our $\pi^0$ spectrum, which suffers from a large systematic error in the first bin, as discussed in Sec. V, does not warrant drawing a definite conclusion concerning the $\sigma$ hypothesis. Our experiment in its entirety (in particular the relation between the slopes of the charged and neutral pion spectra) as stated earlier favors a simple linear matrix element for $K(+-0)$ decay. Evidence against the existence of the $\sigma$ meson with $m_\sigma \simeq 400$ MeV is the observed branching ratio

[49] G. Barton and C. Kacser, Phys. Rev. Letters **8**, 226 (1962); **8**, 353(E) (1962).
[50] Riazuddin and Fayyazuddin, Phys. Rev. Letters **7**, 464 (1961).
[51] V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters **16**, 947 (1966).
[52] L. M. Brown and P. Singer, Phys. Rev. **133**, B812 (1964).

TABLE III. Compilation of published determinations of the mass $m_\sigma$, and the width $\Gamma_\sigma$, of the $\sigma$ dipion resonance of Brown and Singer (Ref. 52) based on $K_{\pi 3}$ and $\eta(+-0)$ decay spectra.

| Reference | Decay | Events | $m_\sigma$ (MeV) | $\Gamma_\sigma$ (MeV) |
|---|---|---|---|---|
| a | $K^+ \to \pi^+ + \pi^0 + \pi^0$ | 1792 | $337\pm4$ | $87\pm9$ |
| b | $K^+ \to \pi^+ + \pi^0 + \pi^0$ | 2027 | $\sim350$ | $\sim80$ |
| c | $K^+ \to \pi^+ + \pi^+ + \pi^-$ | 3587 | $\sim340$ | $\sim90$ |
| d | $K_2^0 \to \pi^+ + \pi^- + \pi^0$ | 310 | $346_{-7}^{+14}$ | $75_{-16}^{+27}$ |
| e | $K_2^0 \to \pi^+ + \pi^- + \pi^0$ | 229 | $350\pm10$ | $75\pm15$ |
| f | $\eta \to \pi^+ + \pi^- + \pi^0$ | 109 | $392\pm9$ | $88\pm15$ |
| g | $\eta \to \pi^+ + \pi^- + \pi^0$ | 274 | $407_{-12}^{+25}$ | $117\pm15$ |

[a] Reference 36.
[b] Reference 37.
[c] J. Huetter, S. Taylor, E. L. Koller, P. Stamer, and J. Grauman, Phys. Rev. **140**, B655 (1965).
[d] Reference 29.
[e] Reference 35.
[f] Reference 61.
[g] Reference 62.

$\eta(+-0)/\eta(000)$ and the data on $K_{e4}$ decay.[53] In a very recent paper. Lovelace *et al.*[54] concluded, from a study of the dispersion relations for backward $\pi$-$p$ scattering, that there exists a $\sigma$ meson with a very large width, $\Gamma_\sigma \simeq 400$ MeV, or that there is more than one $\pi$-$\pi$ resonance in the $\sigma$ region. Our experiment is not sensitive enough to reveal the existence of such a resonance. Our measurement of the pion spectra does not show evidence for the repulsive $S$ wave $\pi$-$\pi$ interaction theory of Mitra and Ray,[55] or for the ABC anomaly[56] (strong $\pi$-$\pi$ interaction at $\sim$320 MeV).

Finally, we mention the one-pion-pole model[49,57,58] in which $\eta(+-0)$ and $K_{\pi 3}$ both proceed predominantly through a one-pion intermediate state, as illustrated in Fig. 18. This model predicts that $\sigma(+-0) = \sigma(\eta \to \pi^+\pi^-\pi^0)$. Experimentally, $\sigma(\eta) = -0.20\pm0.03$, in good agreement with our measurement. However, this is not a very convincing test in favor of the one-pion model,[59,60] since the equality of the slopes follows from any model in which the $K$ and $\eta$ have the same $I=1$ final state and the slope is determined by $\pi$-$\pi$ final-state interactions. A serious difficulty of the one-pion-pole model is the fact that it does not explain the observed branching ratio for $\eta(+-0)/\eta(000)$.[61,62]

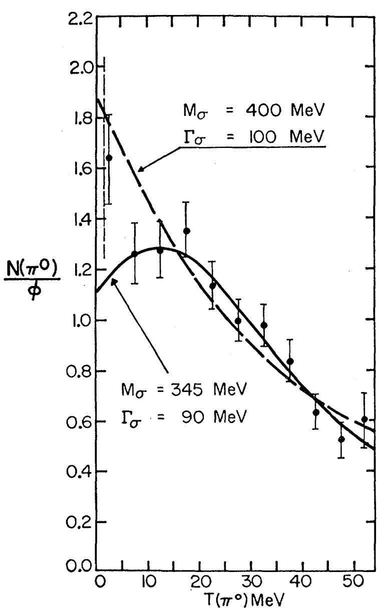


FIG. 17. A fit of our data with the $\sigma$ hypothesis of Brown and Singer. Dashed line along the first point indicates the uncertainty in this point due to systematic errors.

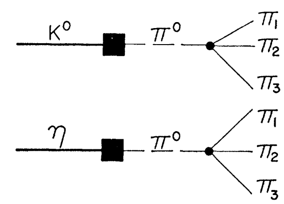


FIG. 18. Feynman diagrams for the one-pion-pole model for $K \to 3\pi$ and $\eta \to 3\pi$ decay.

### D. *CP* Conservation

Since the discovery of the decay mode $K_2^0 \to \pi^- + \pi^+$,[63,64] one cannot properly assume $CP$ conservation. In this section we shall use the symbol $K_L$ for the long-lived neutral $K$ meson and keep the symbols $K_2^0$ and $K_1^0$ for the $CP$-odd and $CP$-even eigenstates. Then $|K_L^0\rangle = (1+|r|^2)^{-1/2}(|K^0\rangle - r|\bar{K}^0\rangle)$. In the linear approximation the matrix element for $K^0 \to \pi^+ + \pi^- + \pi^0$ decay is

$$M(K^0) = a + bX + cY\,,$$

where $X \propto [T(\pi^+) - T(\pi^-)]$ and $Y \propto T(\pi^0)$. For $\bar{K}^0 \to \pi^- + \pi^+ + \pi^0$ decay we find, after applying the $CPT$ theorem,

$$M(\bar{K}^0) = -a + bX - cY\,;$$

thus,

$$M(K_L) = (1+r)/(1+|r|^2)^{1/2}[a+cY] + (1-r)/(1+|r|^2)^{1/2}bX\,.$$

We therefore expect that $CP$ violation will manifest itself as a charge asymmetry in $K(+-0)$ decay through the $X$ dependence of the matrix element.[65] The magni-

[53] R. W. Birge *et al.*, Phys. Rev. **139**, B1600 (1965).
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[62] F. S. Crawford, Jr., R. Grossman, L. J. Lloyd, L. R. Price, and E. C. Fowler, Phys. Rev. Letters **11**, 564 (1963); **13**, 421(E) (1964).
[63] J. H. Cristenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters **13**, 138 (1964).
[64] A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, B. M. K. Nefkens, and J. H. Smith, Phys. Rev. Letters **13**, 243 1(964).
[65] Using the terminology of Sec. VI A we can formulate the $CP$ violation in the following way: The $K_2^0$ meson may decay to $I=1$ and $I=3$ final states of the three pions and the $K_1^0$ meson decays to $I=0$ and $I=2$ final states. Thus $CP$ violation in $K_L$ decay, where $K_L$ is now written as

$$|K_L\rangle = \tfrac{1}{2}\sqrt{2}(1+|r|^2)^{-1/2}[(1+r)|K_2^0\rangle + (1-r)|K_1^0\rangle],$$

implies a small admixture of $I=0$ and $I=2$ final states to the predominant $I=1$ (and $I=3$) final state.

TABLE IV. Measurement of charge asymmetry.

| Solution of ambiguity | Comment | Number of events with $T(\pi^+)-T(\pi^-)>0$ | Number of events with $T(\pi^+)-T(\pi^-)<0$ | Charge asymmetry $[T(\pi^+)-T(\pi^-)<0]/[T(\pi^+)-T(\pi^-)>0]$ |
|---|---|---|---|---|
| Low $E(K_2^0)$ | Total sample | 593 | 605 | $1.02\pm0.04$ |
| Low $E(K_2^0)$ | Only events with $\|T(\pi^+)-T(\pi^-)\|\geq 21$ MeV | 259 | 273 | $1.05\pm0.06$ |
| High $E(K_2^0)$ | Total sample | 607 | 591 | $0.96\pm0.04$ |
| High $E(K_2^0)$ | Only events with $\|T(\pi^+)-T(\pi^-)\|\geq 21$ MeV | 293 | 257 | $0.88\pm0.06$ |

tude of this effect depends on $r$, which is unknown, but probably close to unity. We have searched for a (linear) $X$ dependence of the matrix element in $K(+-0)$ decay through the ratio $R=N(\text{at }X_i)/N(\text{at }-X_i)$, which is shown in detail in Fig. 10. As remarked earlier, the data points are consistent with no slope in the ratio $R$, thus no $X$ dependence and no $CP$ violation. For completeness we have included the high-$E(K_2^0)$ solution in Fig. 10. A quantitative statement concerning $CP$ conservation is contained in Table IV. We have included the ratio of a subsample (large $X$ only) to enhance a possible effect, but no effect was seen. Of course, there are many ways in which to look for a charge asymmetry (which is not simply related to $X$). Supplementary to Fig. 10, therefore, we mention the comparison of the $\pi^+$ and $\pi^-$ spectra, illustrated in Fig. 6, which does not indicate any charge asymmetry either. Both tests are consistent with no asymmetry and thus a value for $r$ that is not too different from unity. A particular model in which the $CP$ violation is confined to $\Delta I\geq\frac{3}{2}$ transition has been proposed by Truong.[66] He predicts a charge asymmetry of the order of a few percent, which is beyond the scope of this experiment. Other models, such as those of Sachs[67] and Lee,[68] predict even smaller numbers. It is no surprise, therefore, that our measurements do not indicate any $CP$ violation in $K(+-0)$ decay.

## VII. CONCLUSIONS

The pion spectra in the decay $K_2^0\rightarrow\pi^++\pi^-+\pi^0$ are well described by the linear matrix-element approximation formulated in Eq. (7). The $\pi^0$ slope parameter, defined by Eq. (7b), is $\sigma_0'=-0.21\pm0.02$, while $\sigma_\pm'(+-0)=+0.10\pm0.01$ is obtained for the charged pions. There is no experimental evidence for an $X$ dependence of the decay. Our value for $\sigma_0(+-0)$, when compared with $\sigma_+(+00)$ and $\sigma_-(++-)$, indicates that the final state of these pions is predominantly the $I=1$ state, in agreement with the $|\Delta I|=\frac{1}{2}$ rule. Taking the data on the $K_{\pi 3}{}^+$ spectra at face value and assuming that the linear approximation is still valid, the slopes do not exclude an admixture of $\sim$15% of $I=2$ final state in $K^+$ decay.

The fact that no deviation was found from the linear matrix-element approximation to $K(+-0)$ decay imposes definite limits to the models that explain the observed pion spectra in $K_{\pi 3}$ decay as a result of $\pi$-$\pi$ final-state interactions. Our data are not inconsistent with the $\rho$ or $K^*$ intermediate-state model or with the pion-pole model.

The $\pi^\pm$ and $\pi^0$ spectra are in excellent accord with calculations based on the algebra of equal-time current commutators. No effects indicating $CP$ noninvariance were observed, and none was expected.

## ACKNOWLEDGMENTS

We wish to thank Dr. A. Wattenberg for his crucial role in the planning of this experiment and his contributions to its analysis. We thank the staff at Brookhaven National Laboratory for their help and hospitality during the run. We appreciate the many clarifying discussions with Dr. R. L. Schult concerning the theoretical aspects, in particular the current commutators. We are grateful to our scanning and measuring staff for their patient work during the data analysis.

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