tion is smaller than 1/30 of the Weisskopf unit.[17] The $M1$ part of the 874-keV transition, though somewhat stronger than the $M1$ part of the 618-keV transition in Re^{187} (Ref. 18), is still hindered, being an order of magnitude slower than the 773-keV spin-flip transition[18] in Re^{187}. The small $E2$ strength and the hindrance of the $M1$ part are expected if the 874-keV level is the $\frac{3}{2}^+$ member of the rotational band based on the $\frac{1}{2}$ [411] Nilsson state and corresponds to the 618-keV level in Re^{187}. This again raises the question of whether the 879-keV member of the doublet, which might well correspond to the 625-keV level in Re^{187}, could be the $\frac{1}{2}^+$ band head.

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Vibrational Spectrum of Cd^{114}†

J. R. COMFORT AND C. K. BOCKELMAN
Nuclear Structure Laboratory, Yale University, New Haven, Connecticut

AND

P. D. BARNES*
Nuclear Structure Laboratory, Yale University, New Haven, Connecticut
and
The Niels Bohr Institute, University of Copenhagen, Copenhagen, Denmark

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The energies and angular distributions of proton groups produced in the $Cd^{113}(d,p)Cd^{114}$ reaction were measured with an over-all resolution of 12 keV. A distorted-wave analysis, employing experimentally determined optical-model parameters, was used to extract values of l_n and spectroscopic strengths for states up to 3.8-MeV excitation in Cd^{114}. The spectroscopic factors of the low-energy states are compared with those predicted by a microscopic theory of nuclear vibrations. Reasonable agreement is found for the ground and one-quadrupole-phonon states, but not for the two-phonon states.

I. INTRODUCTION

THE present report is the second of two papers describing measurements of the (d,p) spectroscopic factors in even cadmium isotopes. The previous paper[1] described the $Cd^{111}(d,p)Cd^{112}$ reaction at a deuteron bombarding energy of 7.7 MeV. The $Cd^{113}(d,p)Cd^{114}$ reaction observed at a bombarding energy of 7.5 MeV will be discussed here.

Cd^{114} has been extensively studied experimentally and much is known about the properties of levels below 2 MeV of excitation energy. Information has been obtained from studies[2,3] of positron decay and the γ cascade following K capture in In^{114}, Coulomb excitation measurements with α particles[4,5] and oxygen ions,[6,7] inelastic proton scattering,[8,9] deuteron stripping and pickup reactions,[10] and from γ-ray and internal conversion spectra[11–14] following slow neutron capture in Cd^{113}. A summary of the known information for levels below 2-MeV excitation is given in Fig. 1.

The strongly enhanced $B(E2)$ transition strengths shown in Fig. 1 are evidence for a collective nature of

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* NATO Fellow. Present address: Tandem Accelerator Laboratory, Physics Division, Los Alamos Scientific Laboratory, Los Alamos, New Mexico.
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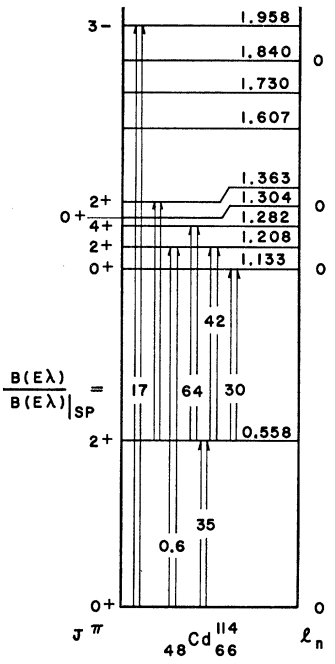

FIG. 1. The level scheme of Cd^{114} prior to the present experiment. The arrows indicate the transitions observed in the Coulomb-excitation work of Refs. 4 and 7; transition rates in single-particle units are indicated, as are the l_n values from Ref. 10, and the excitation energies of Ref. 12.

the low-lying levels in Cd^{114}. In particular, the existence of a triplet of states of spin and parity 0^+, 2^+, and 4^+ near 1.2 MeV, the strongly enhanced nature of the $E2$ transitions between each of these levels and the 2_1^+ level at 0.558 MeV and between the 2_1^+ level and the ground state, as well as the retardation of the 2_2^+ to 0^+ transition are evidence for a vibrational spectrum. The vibrational spectrum can in turn be interpreted by the microscopic model of harmonic nuclear vibrations,[15] which uses pairing and long-range quadrupole forces as shell-model residual interactions, treated in the random-phase approximation (RPA).[16–19]

To obtain anharmonic effects such as the splitting of the triplet two-phonon states, a higher random phase approximation (HRPA) can be employed; approximate calculations[20,21] of the $E2$ strengths have produced agreement to within a factor of 2. The model also predicts a three-phonon quintet of states with spin and parity of 0^+, 2^+, 3^+, 4^+, and 6^+ at approximately three times the excitation of the one-phonon first excited 2^+ state. Assignments for the known states in Cd^{114} between 1.6 and 2.2 MeV have been suggested.[22] However, these microscopic models of collective vibrations have been challenged by recent measurements[23,24] of the static quadrupole moment of the 2_1^+ state of Cd^{114}; the measurements indicate that this state has a static quadrupole moment of magnitude nine times that predicted by the RPA calculations.[25] In order to provide additional information on the wave functions of the Cd^{114} nucleus, the present paper reports measurements of the (d,p) spectroscopic factors for states in Cd^{114} and compares them with predictions for the zero-, one-, and two-phonon final states.

II. EXPERIMENTAL PROCEDURES

The data were obtained from a target produced by evaporation of Cd metal enriched in Cd^{113} onto a Formvar backing which had been flashed with a thin layer of gold. The isotopic constitution of the 35-$\mu g/cm^2$ Cd layer is given in Table I. The impurities observed in the spectra were $C^{12,13}$, N^{14}, O^{16}, Na^{23}, Al^{27}, Si^{28}, S^{32}, and Cl^{35}. The target was exposed to 7.5-MeV deuterons from the MIT-ONR Van de Graaff accelerator. The reaction protons were momentum analyzed in the MIT 24-gap spectrograph[26] and detected with 50-μ NTB nuclear emulsions. Tantalum and aluminum foils covered the emulsions to prevent particles other than protons from being recorded.

The developed emulsions were scanned for proton tracks in 0.5-mm strips, i.e., about 5-keV intervals. A typical spectrum is shown in Fig. 2 where the observed track density is graphed as a function of proton magnetic rigidity and proton energy. The quality of the data is somewhat superior to the Cd^{112} data[1] in that the effective linewidth was reduced to about 12 keV and the interference from $Au(d,p)$ was strongly reduced. As a consequence, it was possible to identify Cd^{114} levels to 4.2-MeV excitation, although an examination of the spectrum will show that weak peaks may have been missed at the higher excitations. Angular distributions were obtained for all states below 3.35 MeV. Above 3.35 MeV, the decreasing dispersion of the

TABLE I. Mass-spectroscopic analysis of the enriched Cd^{113} target. The enriched material was obtained from the Isotope Sales Division of the Oak Ridge National Laboratory.

Cadmium isotope	Percent abundance
106	<0.02
108	<0.02
110	0.28
111	0.55
112	2.27
113	91.07
114	5.57
116	0.27

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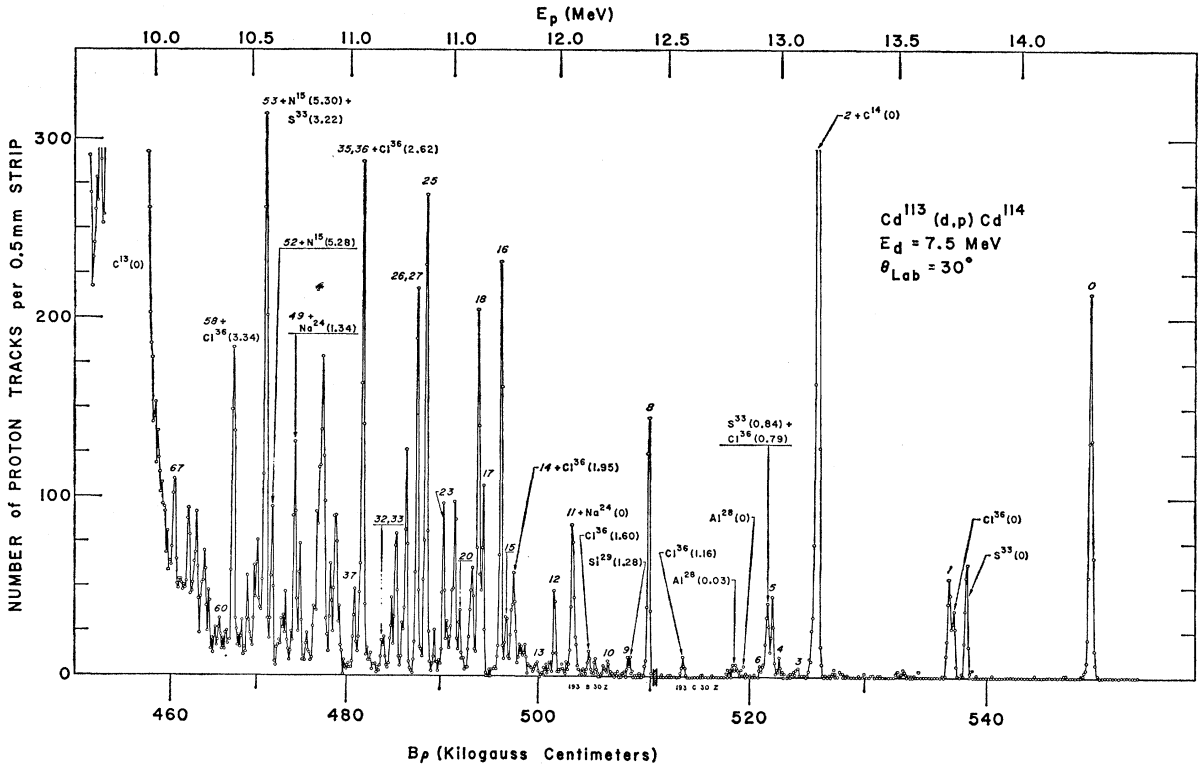

FIG. 2. Proton spectrum observed at a laboratory angle of 30°. The number of proton tracks in a 0.5-mm strip across the exposed zone is graphed as a function of proton momentum (kG-cm) and energy (MeV). The peaks corresponding to levels in Cd^{114} are numbered as in Table V. Prominent peaks associated with contaminants in the target are also identified.

spectrograph and the increasing number of contaminant groups precluded the determination of angular distributions for all except the strongest Cd peaks.

As was also the case in the $Cd^{111}(d,p)Cd^{112}$ experiment,[1] differential hysteresis in the spectrograph required that a set of effective magnetic fields be used to establish a Q-value scale. These were determined by requiring that 256 observed impurity groups have Q values consistent with those given in the literature. With these effective fields, the Q value for the Cd^{114} ground-state transition was measured as 6.822 ± 0.008 MeV. It is believed that the excitation energies of the levels in Cd^{114} are known to ±6 keV.

At each angle, some of the cadmium proton groups in the spectra were partially obscured by contaminant groups. By normalizing the measured (d,p) angular distributions for the contaminant reactions, as reported in the literature, to isolated contaminant groups observed here, estimates of the contribution of each contaminant to peaks at other angles were made. In cases where the contribution of known impurity groups could not be ascertained, the observed yield is taken as an upper limit.

Immediately following the (d,p) exposure, a series of exposures of the elastic deuteron scattering at 7.5 and 2.5 MeV were made. The 2.5-MeV elastic scattering measurement, assumed to follow the Rutherford law, was used to establish both the 7.5-MeV elastic scattering and the (d,p) cross-section scales. These measurements were greatly facilitated by the use of "Rutherford slits," which adjusted the spectrograph apertures in the forward quadrant in accordance with the Rutherford law, so that the entire angular distribution could be obtained in a single exposure. The 7.5-MeV scattering results are shown in Fig. 3. The normalization error assigned to the absolute cross sections for both the scattering and the (d,p) reaction is estimated as $\pm10\%$, in addition to the statistical errors given by the individual data points.

III. DISTORTED-WAVE ANALYSIS

The differential cross section representing the observed proton angular distributions is given as

$$\frac{d\sigma}{d\Omega}=\frac{3}{2}\frac{2J_f+1}{2J_i+1}\sum_{l,j}S_{lj}\sigma_{lj}(\theta), \qquad (1)$$

where J_i and J_f are the total angular momenta of the target and residual nuclei and the factor $\frac{3}{2}$ is associated with the use of a Hulthén wave function for the deuteron. The spectroscopic factor S_{lj} is discussed in Sec. IV. The factor $\sigma_{lj}(\theta)$ is a theoretical estimate for the probability that there will be a transition from the incident deuteron

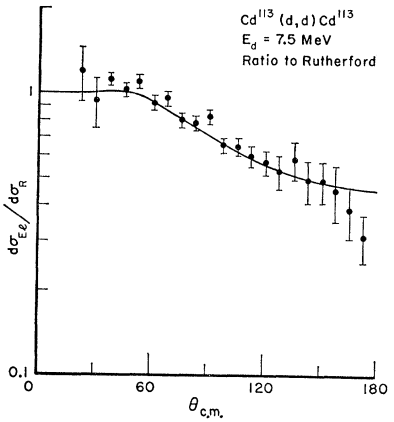

FIG. 3. The differential cross section for elastic scattering of 7.5-MeV deuterons from Cd^{113} expressed in ratio to the calculated Rutherford cross section. The ordinates are based on the assumption that the elastic scattering of 2.5-MeV deuterons on Cd^{113} follows the Rutherford law. The vertical bars indicate statistical errors only. The solid line represents the cross section calculated from the deuteron optical potential described by the parameters listed in Table II.

channel to an outgoing proton channel with the neutron capture into a single-particle orbit labeled by the orbital and total angular momenta l and j, respectively. In this experiment, all calculations have been done in the distorted-wave (DW) Born approximation through the use of the computer code JULIE written by Drisko.[27] The calculations are zero range and use local potentials.

Wave functions for the captured neutron were obtained by using a Woods-Saxon potential

$$V_n(r)=-V_n\left[1+\exp\left(\frac{r-r_0A^{1/3}}{a}\right)\right]^{-1}$$
$$+\text{spin-orbit term}. \quad (2)$$

The parameters used were those of Bjorklund and Fernbach[28]; these are listed in Table II. The well depth V_n was adjusted by the program to yield the observed neutron binding energy for each state.

Wave functions for the deuteron and proton channels were determined from an optical potential of the form

$$U(r)=-Vf(r,r_0,a)+i4a'W'\frac{d}{dr}f(r,r_0',a')+V_c(r,r_c), \quad (3)$$

where

$$f(r,r_0,a)=\left[1+\exp\left(\frac{r-r_0A^{1/3}}{a}\right)\right]^{-1}.$$

V and W' designate the well depths of the real and surface absorption potentials, respectively. V_c is the Coulomb potential resulting from a uniformly charged sphere of radius $R_c=r_cA^{1/3}$. A spin-orbit term may also be added to the potential $U(r)$. The effects of a spin-orbit potential, 6 MeV for protons and 8 MeV for deuterons, were found to be negligible and were not included thereafter.

Great care has geen exercised in the selection of the parameters of the optical potentials which describe the proton and deuteron channels. Perey[29] has studied the elastic scattering of protons from various targets at several energies between 9 and 22 MeV and has found a single set of parameters which reproduce the data well. These parameters were used during most of the search for the best deuteron parameters, described below. However, recent measurements of the elastic scattering of 13-MeV protons from Cd^{112} by Ford *et al.* have resulted in two further sets of parameters.[30,31] One of these sets gives the "best fit" to the Cd^{112} proton scattering, while the other is suggested as an average set for fitting elastic proton scattering in the mass region of palladium and cadmium. Each set differs significantly from the Perey set.[29] The "best-fit" set has been adopted in the present analysis and is given in Table II. The real-well depth V_p was varied according to the relation

$$V_p=60.1-0.32E_p \text{ MeV}, \quad (4)$$

thus using the energy dependence of Perey[29] adjusted to give the potential of Ford *et al.*

It is interesting to point out that a change from the Perey proton parameters to the Ford *et al.* "best-fit" parameters, with only small changes in the "best-fit" deuteron parameter set, resulted in identical (d,p) angular distributions, except for a 30% decrease in the theoretical cross-section scale. On the other hand, the "average set" and the "best-fit" proton parameter set yielded essentially identical results.

The deuteron optical-model parameters were more difficult to select. Perey and Perey[32] have examined the data on the elastic scattering of deuterons from various targets at several energies between 11 and 27 MeV and have found six families of parameters which reproduce

TABLE II. Parameters used in the distorted-wave calculations. The real well depths V and imaginary well depths W' are given in MeV, and the lengths in fermis. The real well depth of the proton optical potential was varied with excitation energy as specified by Eq. (4) of the text.

Particle	V	r_0	a	W'	r_0'	a'	r_c
n	~48	1.25	0.65				
p	≥55.5	1.20	0.68	9.6	1.21	0.73	1.25
d	85	1.24	0.78	19.9	1.316	0.68	1.24

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the scattering data reasonably well. More recently Dickens and Perey[33,34] have measured the elastic and inelastic scattering of deuterons from Cd^{114} at eight energies between 8 and 15 MeV and produced another parameter set which reproduced all these data very well. The parameter sets B, C, D, and F of Perey and Perey[32] have been tested as well as the Dickens and Perey set,[34] and each of them was found inadequate for the (d,p) data as will be explained below.

Using the experimental angular distributions for the ground- and first excited states of Cd^{114}, a search was carried out for a better set of deuteron parameters. It was found that for the $l_n=0$ ground-state distribution, the ratios of the cross sections at the first and second maxima, and at the second and third maxima were sensitive to the choice of r_0 (always adjusting the real well depth V_d to place the maxima and minima at the observed angles) and were very insensitive to all other parameter variations. This criterion led to the rejection of the parameter sets cited above and produced the selection of $r_0=1.24$ F and $V_d=85$ MeV. For all reasonable values of r_0, V_d is near 85–90 MeV, thus excluding $V_d\simeq 70$ MeV as used by Dickens and Perey.[34] A value near 20 MeV was found necessary for the imaginary well depth W_d'.

The parameters a, r_0', and a' were less sensitive to the (d,p) data. Taking the deuteron elastic scattering data of Dickens and Perey[33] for the energies 12.0 to 15.0 MeV and using the ABACUS-2 search program[35] a search for a minimum χ^2 was performed allowing these parameters to vary. The angular distributions at energies between 8 and 11 MeV were insensitive to most variations in these parameters. The best set of deuteron parameters obtained from the search with both the (d,p) and (d,d) data is given in Table II. It is important to point out that this parameter set will not only produce fits to the (d,p) data, but also fits all the deuteron elastic scattering data up to 14 MeV. Figure 3 shows the fit to the 7.5-MeV elastic scattering data of the present work. Table III presents a list of χ^2 values for all the scattering data obtained with the parameters of Dickens and Perey and with the parameters of Table II. Limits of 0.6 to 1.3 for χ^2 were accepted as indicating statistically reliable fits. For the deuteron parameters of Table II, the χ^2 values at 7.5 and 7.7 MeV are dominated by a few data points, and the values at 10 and 14 MeV can be rapidly reduced if V is increased to 86 or 87 MeV. The value at 15 MeV was significantly lowered only by introducing a spin-orbit potential with a well depth of about 8 MeV. It may be additionally noted that the deuteron parameters given in Table II are similar to those used[36] for fitting the 15 MeV (d,p) and (d,t) reaction angular distributions on targets of Cd^{110}, Cd^{112}, Cd^{114}, and Cd^{116}. The differences between the parameter sets are made insignificant by the use in Ref. 36 of a lower radial cutoff of 6.7 F. The parameters will also fit the 12-MeV $Cd^{114}(d,p)Cd^{115}$ and $Cd^{116}(d,p)Cd^{117}$ angular distributions[37] with no change in spectroscopic factors provided a cutoff of 6.5 F is used.

TABLE III. Elastic scattering parameters for deuterons on Cd. The data of Refs. 33 and 34 for 7.96- to 15.00-MeV deuterons scattered from Cd^{114}, the elastic scattering data for 7.7-MeV deuterons on Cd^{111} from Ref. 1, and the present elastic scattering data on Cd^{113} were used as input to the optical-model code ABACUS (Ref. 35) for calculating the goodness-of-fit parameter χ^2. For N data points in an angular distribution,

$$\chi^2=(1/N)\textstyle\sum_i[(\sigma_{i,\,\mathrm{th}}-\sigma_{i,\,\mathrm{exp}})^2/(\Delta\sigma_{i,\,\mathrm{exp}})^2].$$

Columns 3 to 5 list the well depths given by Dickens and Perey (Ref. 34) and the χ^2 values obtained from their parameters ($r_0=1.18$, $a=0.866$, $r'=1.30$, and $a'=0.746$). Column 6 lists χ^2 values obtained with the parameters in Table II, assuming no energy dependence.

		Reference 34			
E_d (MeV)	No. of points	V (MeV)	W' (MeV)	χ^2	Present work χ^2
7.50	21				(Cd^{113}) 1.33
7.70	18				(Cd^{111}) 2.66
7.96	27	70.1	14.0	0.19	0.90
8.97	28	67.8	16.0	0.36	0.70
9.98	27	67.3	16.1	0.48	1.44
10.99	29	65.0	17.5	0.38	0.64
12.00	31	63.6	17.9	0.41	0.75
13.00	29	63.7	17.5	0.32	0.91
14.00	29	63.7	17.2	0.63	1.43
15.00	28	63.0	16.9	1.05	2.58

The use of a lower cutoff on the radial integrals in code JULIE was investigated at values of 5.5 and 6.5 F. It was found that the fits to the present data were very poor even when the deuteron parameters were varied widely over the parameter space. Cutoffs were not used in the final calculations. Their use would increase the S_{lj} by approximately a factor of 2.

Considering the quality of the fits obtained using the parameters of Table II for all the data cited, together with the sensitivity of S_{lj} to the value of the proton well parameters noted above, the possible error in S_{lj} resulting from ambiguity in the choice of parameters is believed to be of the order of 30%. An additional uncertainty arises from the decision not to use a spin-orbit term in the neutron potential because of the inability to divide the observed angular distributions into classes with $j_n=l_n\pm\frac{1}{2}$. The effects on S_{lj} if a spin-orbit

TABLE IV. Changes in S_{lj} from the neutron spin-orbit force. Columns 2 and 3 list the calculated changes in S_{lj} when a spin-orbit potential of 8-MeV depth is inserted in Code JULIE (Ref. 27). The calculations were made for an excitation energy of 2.5 MeV.

l_n	$j_n=l_n-\frac{1}{2}$	$j_n=l_n+\frac{1}{2}$
1	+5%	−2%
2	+9%	−5%
3	+16%	−11%
4	+27%	−22%

[33] J. K. Dickens and F. G. Perey, Oak Ridge National Laboratory Report No. ORNL-3727, 1965 (unpublished).
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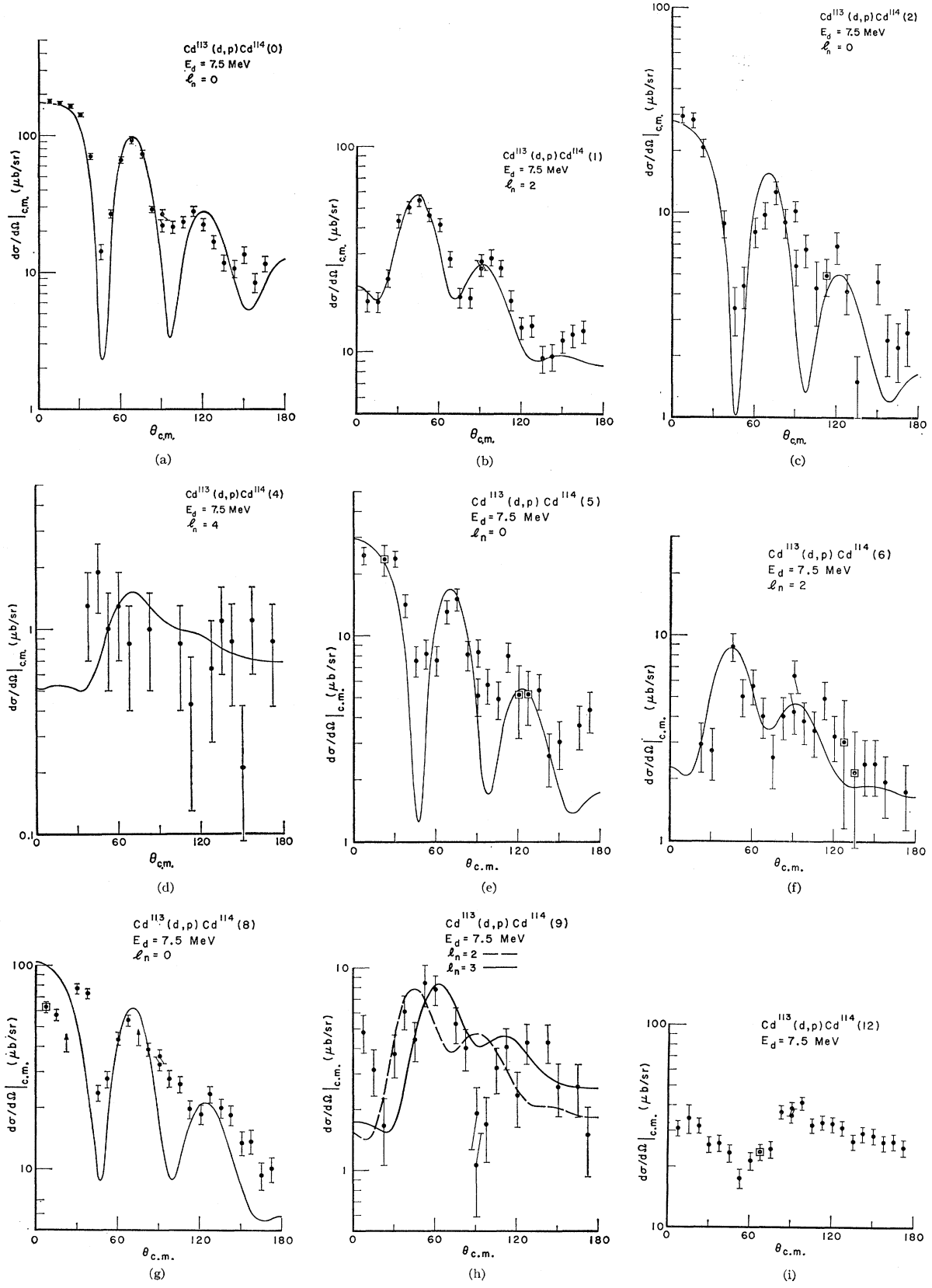
Cd113 (d,p)Cd114 (0)
Ed = 7.5 MeV
ℓn = 0
Cd113 (d,p)Cd114 (1)
Ed = 7.5 MeV
ℓn = 2
Cd113 (d,p)Cd114 (2)
Ed = 7.5 MeV
ℓn = 0
Cd113 (d,p)Cd114 (4)
Ed = 7.5 MeV
ℓn = 4
Cd113 (d,p) Cd114 (5)
Ed = 7.5 MeV
ℓn = 0
Cd113 (d,p) Cd114 (6)
Ed = 7.5 MeV
ℓn = 2
Cd113 (d,p) Cd114 (8)
Ed = 7.5 MeV
ℓn = 0
Cd113 (d,p) Cd114 (9)
Ed = 7.5 MeV
ℓn = 2
ℓn = 3
Cd113 (d,p)Cd114 (12)
Ed = 7.5 MeV
dσ/dΩ|c.m. (μb/sr)
θc.m.
(a)
(b)
(c)
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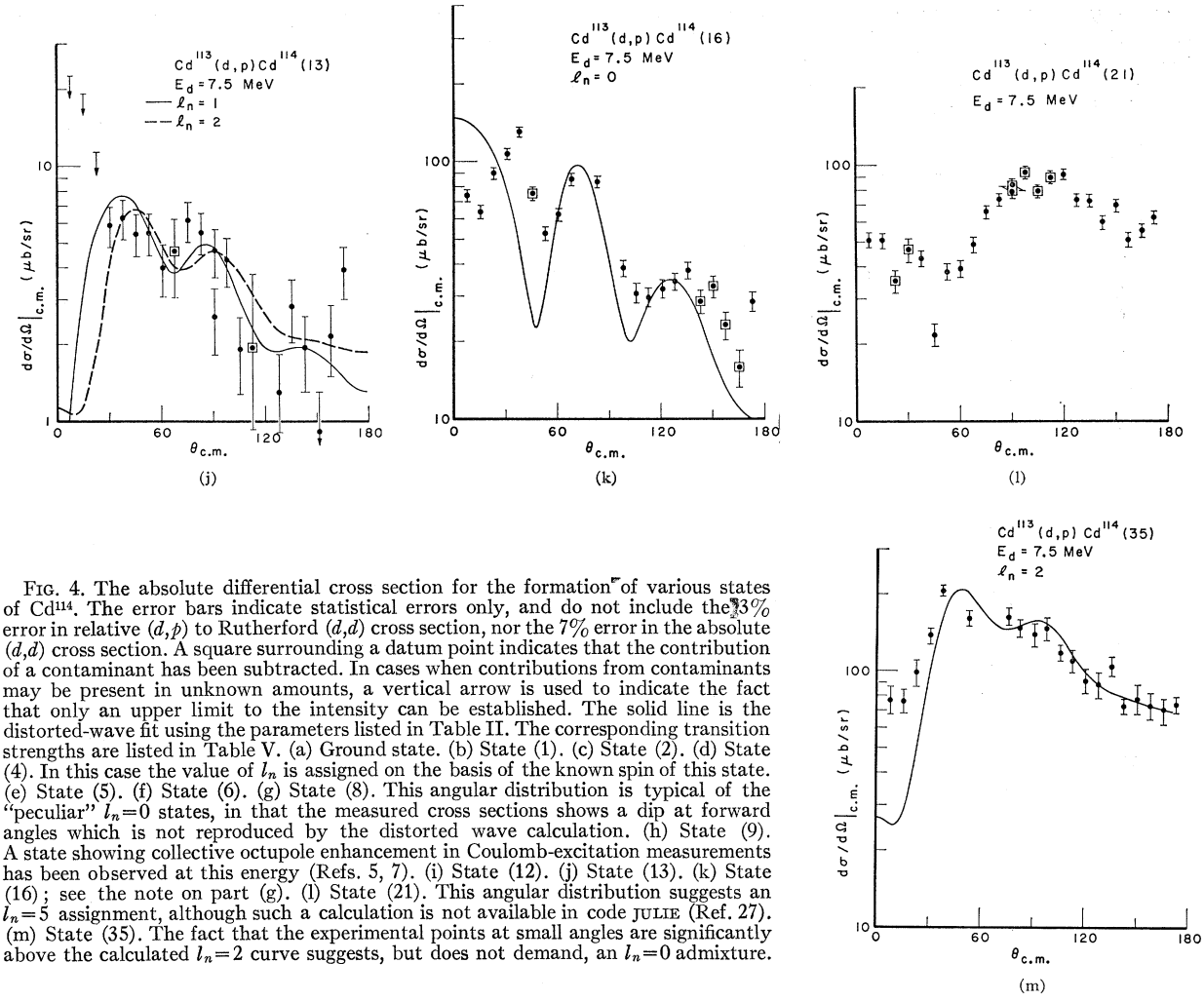

FIG. 4. The absolute differential cross section for the formation of various states of Cd^{114}. The error bars indicate statistical errors only, and do not include the 3% error in relative (d,p) to Rutherford (d,d) cross section, nor the 7% error in the absolute (d,d) cross section. A square surrounding a datum point indicates that the contribution of a contaminant has been subtracted. In cases when contributions from contaminants may be present in unknown amounts, a vertical arrow is used to indicate the fact that only an upper limit to the intensity can be established. The solid line is the distorted-wave fit using the parameters listed in Table II. The corresponding transition strengths are listed in Table V. (a) Ground state. (b) State (1). (c) State (2). (d) State (4). In this case the value of l_n is assigned on the basis of the known spin of this state. (e) State (5). (f) State (6). (g) State (8). This angular distribution is typical of the "peculiar" $l_n=0$ states, in that the measured cross sections shows a dip at forward angles which is not reproduced by the distorted wave calculation. (h) State (9). A state showing collective octupole enhancement in Coulomb-excitation measurements has been observed at this energy (Refs. 5, 7). (i) State (12). (j) State (13). (k) State (16); see the note on part (g). (l) State (21). This angular distribution suggests an $l_n=5$ assignment, although such a calculation is not available in code JULIE (Ref. 27). (m) State (35). The fact that the experimental points at small angles are significantly above the calculated $l_n=2$ curve suggests, but does not demand, an $l_n=0$ admixture.

force of 25 times the Thomas term (corresponding to to about 8 MeV well depth) is included are given in Table IV. There may be additional errors in S_{lj} resulting from the use of the distorted-wave approximation itself, or from the finite-range and local-potential approximations. No quantitative estimates for these errors are available; yet, judging from empirical experience in this type of analysis it is suggested that an over-all uncertainty of a factor of 2 is a reasonable statement of the present accuracy in S_{lj}.

IV. EXPERIMENTAL RESULTS

Typical proton angular distributions are shown in Figs. 4(a)–4(m) along with results of appropriate DW calculations. A complete set of angular distributions may be obtained from the authors. A list of all the states observed in Cd^{114} is given in Table V. Column 1 gives a code number for identifying the states. Column 2 gives the excitation energy of each level based on the effective-field calibration of the spectrograph; the ground-state Q value was measured to be 6.822 ± 0.008 MeV. Where possible, th angular distributions have been fitted with DW curves for the single values of the transferred neutron orbital angular momentum listed in column 3, thus yielding $(2J_f+1)$ times the spectroscopic factor (termed the strength of the transition) given in column 5. If the final state total angular momentum J_f was known from the literature, the spectroscopic factor S_{lj} was calculated and listed in column 7. The l_n's could not be uniquely determined in some cases, and more than one value has been suggested for these. A plot of the measured strength function $(2J+1)S_{lj}$ against excitation energy for various values of l_n is given in Fig. 5. The lack of a value for $(d\sigma/d\Omega)_{\max}$ in column 8 signifies that although the existence of the state is clear from the data, the angular distribution obtained is not reliable.

Neglecting possible second-order processes, such as target excitation, the $\frac{1}{2}^+$ character of the target nucleus and conservation of angular momentum and parity

TABLE V. Levels observed in Cd^{114}. Column (1) is a code number identifying each observed level. Column (2) gives the excitation energies in Cd^{114}. Columns (3)–(5) give the values of orbital angular-momentum transfer l_n, parity, and transition strengths $(2J_f+1)S_{lj}$ as obtained from analysis of the observed differential cross sections. Column (7) is the value of the spectroscopic factor S_{lj} obtained from column (5) using the final-state spins in column (6). Column (8) lists the maximum observed differential cross sections.

No.	E_x (MeV)	l_n	π	$(2J+1)S_{lj}$	J_f	S_{lj}	$(d\sigma/d\Omega)_{max}$ (μb/sr)
0	0.000	0	+	0.42	0	0.42	177.0
1	0.560	2	+	0.44	2	0.089	54.3
2	1.137	0	+	0.051	0	0.051	29.8
3	1.206	(2)	+	$\leq$0.045	2	$\leq$0.009	<5.0
4	1.289	(4)	+	$\leq$0.095	4	$\leq$0.001	<2.0
5	1.310	0	+	0.053	0	0.053	24.5
6	1.369	2	+	0.061	2	0.012	8.8
7	1.840						2.0
8	1.862	0	+	0.17			76.9
9	1.959	(3)	(−)	(0.089)	(3)	(0.013)	8.4
10	2.049	2	+	0.049			7.8
11	2.219	2	+	0.18			24.7
12	2.300						41.1
13	2.386	(1, 2)		(0.018, 0.04)			6.3
14	2.505	0	+	0.030			14.6
15	2.527	2	+	0.24			31.7
16	2.554	0	+	0.22			130.0
17	2.638	0	+	0.13			62.0
18	2.659	2	+	1.4			241.0
19	2.701	3	−	0.34			39.6
20	2.751	2	+	0.43			75.4
21	2.772	$>$4					92.9
22	2.810	(2, 3)		(0.20, 0.30)			35.8
23	2.829	0	+	0.092			51.1
24	2.874	3	−	0.35			35.8
25	2.913	0	+	0.33			212.0
26	2.938	$\geq$2					27.0
27	2.956	2	+	1.5			210.0
28	3.003						31.7
29	3.026	2	+	0.17			28.8
30	3.054	0	+	0.030			17.7
31	3.080	2	+	0.16			34.5
32	3.119	(2, 3)		(0.15, 0.21)			26.0
33	3.130	(2, 3)		(0.18, 0.28)			32.2
34	3.192	3	−	0.17			27.3
35	3.221	2	+	1.1			208.0
36	3.236	2	+	0.26			51.8
37	3.262	2	+	0.22			42.6
38	3.285	$\geq$3					14.5
39	3.331	0	+	0.064			58.4
40	3.348						61.3
41	3.365						
42	3.383						
43	3.410	2	+	0.34			68.6
44	3.448	$\geq$3					45.8
45	3.461	2	+	0.26			66.9
46	3.480						
47	3.499						
48	3.521						
49	3.557						
50	3.582						
51	3.613						60.7
52	3.670						
53	3.712	2	+	0.28			55.6
54	3.747						
55	3.789	2	+	0.20			48.8
56	3.823						
57	3.853						
58	3.876						
59	3.902						
60	3.940						
61	3.962						
62	3.996						
63	4.017						
64	4.075						
65	4.099						
66	4.142						
67	4.177						
68	4.211						

require that only a single l_n can contribute to the excitation of a natural-parity final state, i.e., a state for which the parity $\pi=(-1)^J$. This fact suggests that unnatural parity states might be identified by angular distributions which indicate contributions from more than one l_n. However, in the present experiment, the presence of a small $l_n=2$ component in an $l_n=0$ angular distribution, and vice versa, is very difficult to detect. In several cases, angular distributions for such mixtures were calculated, but the fits were not materially improved. Therefore, only single values of l_n have been assigned. Only transitions characterized by l_n values between 0 and 4 could be calculated with the available code. According to the shell model, the $h_{11/2}$ ($l_n=5$) state is available for population in the low-energy spectrum. State 21 at excitation 2.772 MeV, Fig. 4(l), is a good candidate for such a transition. The weak level 13 at 2.386 MeV, Fig. 4(j), is best fit by an $l_n=1$ angular distribution, although an $l_n=2$ assignment is also acceptable. The binding-energy systematics of single-particle states[38] suggests that the main $3p$ strength should be observed at excitations greater than 4 MeV.

In addition to the zero- and one-phonon ground- and first excited states of Cd^{114}, all three components of the two-phonon triplet near 1.2-MeV excitation were observed in this experiment. However, because of its low yield, the presence of the 2^+ state at 1.206 MeV could only be affirmed at five forward angles. The 4^+ state at 1.289 MeV, Fig. 4(d), was identified at 14 angles. The yield was too small to determine its angular distribution well enough to specify the l_n as being 4; this particular assignment is made only on the basis of its measured spin and parity.[2,3] The 0^+ and 2^+ states at 1.310 and 1.369 MeV, Figs. 4(e) and 4(f), have transition strengths very nearly equal to the two-phonon 0^+ and 2^+ states, respectively.

Although the calculated DW fits to the data are in general satisfactory, several systematic difficulties have been encountered. All the calculated $l_n=0$ angular distributions have minima which fall well below the experimental data points. This feature is not substantially modified by changes in the optical-model parameters. In addition, many of the $l_n=2$ states at high excitation have data points at the forward angles which lie approximately a factor of 2 above the theoretical curves. An example is level 35 at 3.221 MeV, as shown in Fig. 4(m). While this might be taken to indicate the presence of an $l_n=0$ component, the backgrounds are rather high in this region, and the over-all evidence for the presence of such admixtures is not convincing.

Another difficulty with the proposed $l_n=0$ states involves a group which cannot be fit very well with the theoretical curves at the forward angles, where the data show pronounced dips. None of these angular distributions can be fit with any other single value of l_n, nor has any mixture of l_n values been found satisfactory.

[38] B. L. Cohen, Phys. Rev. **130**, 227 (1963).

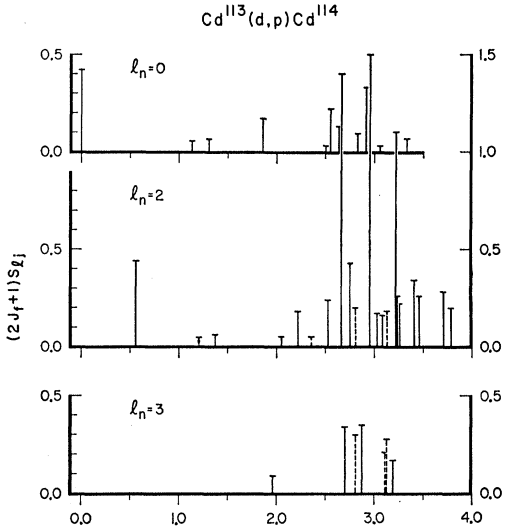

FIG. 5. The observed strength function $(2J_f+1)S_{lj}$ for Cd^{114}. This is a graph of columns (2) and (5) of Table V.

Examples of these angular distributions are shown in Figs. 4(g) and 4(k). Six states show the anomalous behavior, namely those at 1.862-, 2.505-, 2.554-, 2.638-, 2.829-, and 2.913-MeV excitation. Five other $l_n=0$ states, both above and below these in excitation, do not exhibit the effect. It is interesting to point out that Groshev *et al.*[13] observed primary γ rays leading to the five "normal" states following thermal neutron capture in Cd^{113}, but did not identify any of the six peculiar states. No explanation of the phenomenon is offered.

V. COMPARISON WITH OTHER MEASUREMENTS

The determination of an l_n value does not uniquely determine the spin and parity of the final state, as may be seen in Table VI. As discussed in the previous section, there are no discrepancies between the l_n values determined for states below 1.4 MeV and the spins and parities given in Fig. 1. Above this excitation energy, some differences between the present and previous work can be noted. The 1.607-MeV level tentatively introduced by Smither,[12] but not identified by Groshev *et al.*[13] or Bäcklin *et al.*,[14] is not seen here, nor is the 1.730-MeV level identified by these three authors.

TABLE VI. Final-state spin and parity values. For each value of l_n the possible final-state spins and parities produced from a $\frac{1}{2}^+$ target are listed.

l_n	J^π
0	0^+, 1^+
1	0^-, 1^-, 2^-
2	1^+, 2^+, 3^+
3	2^-, 3^-, 4^-
4	3^+, 4^+, 5^+

Two states have been identified in the region of 1.84-MeV excitation. The first, at 1.840 MeV, was detected at 17 of the 24 angles, but no l_n could be assigned to the transition because of its very low strength. This state has been seen strongly in the (n,γ) reaction[11–14] and assigned a J^π of $(1,2)^+$. The second state, at 1.862 MeV, was observed as a strong $l_n=0$ transition. It was previously seen by Cohen and Price,[10] but not in the (n,γ) experiments.

A weakly populated state has been identified at 1.959-MeV excitation and the data, Fig. 4(h), can be fit with an $l_n=3$ angular distribution, although $l_n=2$ is not excluded. This state may be the octupole 3^- vibrational state observed by Hansen and Nathan[5] at 1.94 MeV, by McGowan *et al.*[7] at 1.945±0.016 MeV, and by Sakai *et al.*[8] at 1.95 MeV. A state at this excitation is also seen in the (n,γ) reaction,[12–14] but the spin and parity assignment is ambiguous. Identification with the 3^- state is assumed to be correct in Table V and Fig. 5. [*Note added in proof.* R. K. Smither (private communication) has pointed out that this 1.959-MeV state contains a significant contribution from a strong $l_n=0$ state at 2.302 MeV in Cd^{112}. We are not able to make a reliable subtraction of this contribution.]

The agreement between the present and earlier experiments is rather good above 2 MeV of excitation. The existence of a 2.049-MeV state, level 10, is confirmed, while the $l_n=2$ character contradicts Smither's[12] $J^\pi=0^+$ suggestion and confirms the choice of $(1,2)^+$ by Groshev *et al.*[13] and Bäcklin *et al.*[14] The 2.202-MeV state[12–14] was not seen in the (d,p) reaction. Level 11 has $l_n=2$ and is consistent with the 2^+ assignment in Ref. 14 for a possible level at 2.219 MeV and with Smither's assignment of $J=(1,2)$ for his 2.225-MeV state. Level 13 for which $l_n=1$ is favored, Fig. 4(j), could be the same as the 2.393-MeV state of Smither which has a spin of 1 or 2. The level at 2.573, seen in all three (n,γ) experiments, was not identified, but the remaining levels of Smither and Bäcklin *et al.* are in agreement with the present measurements. The levels of Groshev *et al.*[13] at 2.438, 2.457, 2.580, 2.784, and 3.162 have no correspondence with the levels listed in Table V. There is a discrepancy for level 35 at 3.221 MeV. The primary γ ray populating a level at 3.218 MeV in the (n,γ) reaction is very strong and would imply an $M1$ transition matrix element[13] more than 20 times the average for transitions to lower-lying states. If the transition is $E1$, then the state has $J^\pi=(1,2)^-$. An $l_n=2$ transition is observed in the (d,p) reaction, Fig. 4(m), implying positive parity for the state. The closest odd l_n transition observed is for an $l_n=3$ state at 3.192 MeV. Level 35 was not entirely resolved from the weaker level 36 in the spectra. Nevertheless, a very reasonable separation could be made during analysis. The possibility of an additional level being present cannot be excluded.

General agreement may be found between the present (d,p) experiment and the work of Cohen and Price.[10] All of their levels can be associated with the present ones, although the association for the 3.64-MeV level is ambiguous. The $J^\pi=0^+$ assignment for the 3.23-MeV state contradicts the present measurement of $l_n=2$ for the 3.236-MeV state, level 36, or the $l_n=2$ assignment for the 3.221-MeV state, level 35.

Some general similarities have been observed between the results of the present experiment and the $Cd^{111}(d,p)$-Cd^{112} experiment of Ref. 1. In agreement with expectations of the pairing-plus-quadrupole model, both spectra are characterized by the weak excitation of a few number of levels between 2 MeV and the rapid increase in the number of levels and their cross sections above 2.3 MeV. The distribution of the transition strengths (the strength functions) are very similar, although not as many states were identified in Cd^{112}. The Cd^{114} level at 2.300 MeV, Fig. 4(i), has an angular distribution very similar to the Cd^{112} level 12 at 2.374 MeV, and neither can be fit with the present DW calculations. The failure of the experimental $l_n=0$ angular distributions to dip as much as the calculations and the difficulties at forward angles for some of the $l_n=2$ levels in both reactions is suggestive of a common feature of the reaction mechanism not included in the calculations. The peculiar set of $l_n=0$ levels is present in both experiments; the Cd^{112} levels suggested for this set are those at 1.876 and 2.302 MeV.

VI. DISCUSSION

In Ref. 1, the results of a model calculation of the spectroscopic factors for zero-, one-, and two-phonon states in Cd^{112} formed in the (d,p) reaction were presented. This microscopic description of spherical nuclei in which the residual shell-model interactions are approximated by a pairing plus a quadrupole interaction treated in the random-phase approximation has also been applied to Cd^{114}. The shell-model orbital energies used were the same as those used in Ref. 1, except for slight adjustments following the extrapolation in mass suggested by Kisslinger and Sorensen (KS).[18] The quadrupole coupling strength χ was chosen so that the

TABLE VII. Parameters used in the calculation of the wave functions for Cd^{113}. The units for Δ, χ, G, $\hbar\omega$, ϵ_j, and the quasiparticle energies E_j are MeV. V_j^2 is the occupation probability of the jth orbital.

	Neutron	Proton
Δ	1.247	0.615
χ	1.986	2.939
G	0.204	0.230
$\hbar\omega$	0.62	

Orbital	ϵ_j	E_j	V_j^2	Orbital	ϵ_j	E_j	V_j^2
$2d_{5/2}$	0.153	2.216	0.92	$1f_{5/2}$	−0.094	3.095	0.99
$1g_{7/2}$	1.386	1.383	0.72	$2p_{3/2}$	0.583	2.435	0.98
$3s_{1/2}$	2.286	1.282	0.38	$2p_{1/2}$	1.614	1.461	0.95
$1h_{11/2}$	2.764	1.470	0.23	$1g_{9/2}$	2.422	0.803	0.82
$2d_{3/2}$	4.572	2.871	0.05				

RPA description of the 2_1^+ state in Cd^{112} yields the correct excitation energy. The parameters of the present calculation are given in Table VII.

The spectrum of the positive-parity states of Cd^{113} calculated with the parameters of Table VII is presented in Fig. 6 in comparison with the experimental spectrum.[36] Agreement within 150 keV is obtained for the excitations of the lowest $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ states. Above 0.5-MeV excitation, the discrepancies are considerably larger and the identification of states is ambiguous. The energy fit is comparable to that obtained by KS[18] with a slightly different set of single-particle energies.

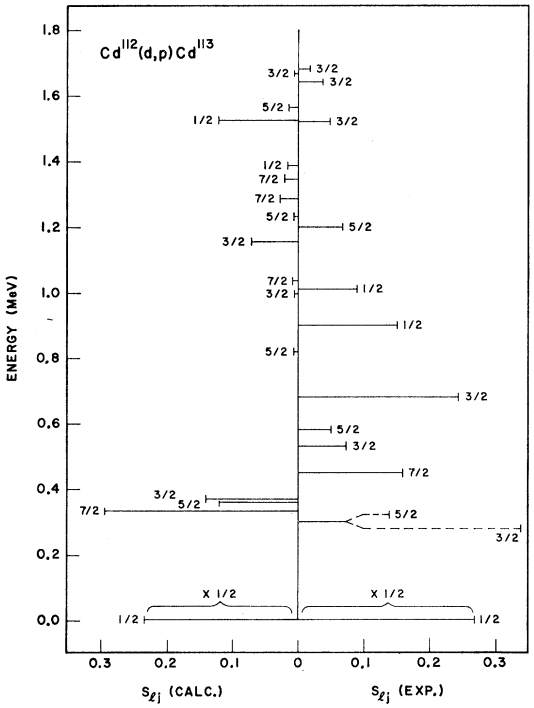

FIG. 6. The observed and calculated strength function for the positive-parity states of Cd^{113}. The dashed lines for the proposed $\frac{3}{2}^+$, $\frac{5}{2}^+$ doublet at 300 keV (Ref. 36) are calculated using the same cross-section ratio for these two d states in Cd^{113} as found for the lowest $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states in Cd^{111}.

Also shown in Fig. 6 are the calculated spectroscopic factors for the $Cd^{112}(d,p)Cd^{113}$ reaction and the values suggested by Rosner's data.[36] There is good agreement for the lowest $\frac{1}{2}^+$ state. Because the proposed[36] $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states at 300 keV were not resolved in the (d,p) measurement, no unambiguous comparison can be made. However, the results are consistent with the conclusion of Ref. 1 that the $J^\pi=\frac{5}{2}^+$ strength is in agreement with the calculations while the calculated $J^\pi=\frac{3}{2}^+$ strength is a factor of 2 too small. The lowest $\frac{7}{2}^+$ state is overestimated by a factor of 2. Similar calculations using KS wave functions[18] yielded a strength for the $\frac{1}{2}^+$ state a factor of 2 small. It is believed that the parameters of Table VII produce a more satisfactory wave function for the ground state of Cd^{113}.

TABLE VIII. Sum-rule analysis of the spectroscopic factors in Cd^{113}. The values in column 4 equal $(2j+1)(1-V_j^2)$, where V_j^2 for Cd^{112} is given in Table IV of Ref. 36.

j	KS parameters	Table VII parameters	Observed strength $\sum(2j+1)S_{lj}$
$\frac{1}{2}$	1.07	1.40	1.6
$\frac{3}{2}+\frac{5}{2}$	4.58	4.71	4.5
$\frac{7}{2}$	3.44	2.93	1.3

A sum-rule analysis of the spectroscopic strengths for $Cd^{112}(d,p)Cd^{113}$ (using the data in Table IV of Ref. 36) is given in Table VIII. Not all of the $j=\frac{7}{2}$ strength has been observed. However, the total observed $l_n=2$ strength is consistent with calculations for both the present set and the KS set of parameters. It is to be noted that the parameters of Table VII produce a sum for the $j=\frac{1}{2}$ orbit much closer to the observed value, although calculations with both sets of parameters underestimate the total strength.

The calculated spectroscopic factors for the zero-, one-, and two-phonon states in Cd^{114} are given in Table IX. The differences between the tabulated values and those calculated by Sorenson, Lin, and Cohen[39] are associated with a different choice of parameters as well as with different approximations used in the calculations. A more detailed discussion is given in Ref. 1. As in the Cd^{112} calculations,[1] the parameters used here give agreement for the zero- and the one-phonon states well within the uncertainty of the present distorted wave calculations. The present experiment is not sufficiently sensitive to test the 2^+ member of the two-phonon triplet; for the 0^+ and 4^+ members, the calculations are unsatisfactory. The discrepancy for the 4^+ component may be related to the poor agreement for $l_n=4$ states in the odd isotopes, suggesting difficulty in the choice of parameters. In the case of the 0_2^+ state, an understanding of the discrepancy is presumably dependent on the understanding of the character of the 0^+ and 2^+ states at 1.310 and 1.369 MeV, respectively.

In summary, the energy spectra and the electromagnetic transition probabilities of the Cd isotopes have suggested the application of a harmonic vibrator model. In the present work, the observed spectra of Cd^{112} and

TABLE IX. Comparison of spectroscopic factors for states in Cd^{114} with N_f phonons. The calculated values in Column 3 were obtained from the parameters of Table VII. The ratios to the experimental values of column 4 are given in column 5.

N_f	J_f^π	S_{lj}(calc.)	S_{lj}(exp.)	S_{lj}(calc.)/S_{lj}(exp.)
0	0^+	0.45	0.42	1.1
1	2^+	0.075	0.089	0.85
2	0^+	0.010	0.051	0.20
2	2^+	0.007	$\leq$0.009	$\geq$0.8
2	4^+	0.007	$\leq$0.001	$\geq$7.0

[39] R. A. Sorenson, E. K. Lin, and B. L. Cohen, Phys. Rev. **142**, 729 (1966).

Cd^{114}, and the general behavior of the strength function, are in good qualitative agreement with the predictions of such a model. Detailed calculations of the (d,p) spectroscopic factors have produced very good quantitative agreement with experimental values for the zero- and one-phonon states, but poor agreement for the two-phonon states. It must be emphasized that there are uncertainties in the DW treatment of the reaction mechanism which affect the experimental spectroscopic factors as well as uncertainties in the selection of model parameters for the odd isotopes so that the observed agreement for the lowest two states in Cd^{112} and Cd^{114} need not be conclusive evidence for the model. Nevertheless, modification of the model in order to account for the static quadrupole moment of the one-phonon states must be constrained to preserve agreement with the measurements reported here.

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^{252}Cf Fission Neutron Spectrum from 0.003 to 15.0 MeV*

J. W. MEADOWS
Argonne National Laboratory, Argonne, Illinois
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The spontaneous-fission neutron spectrum of ^{252}Cf has been measured from 0.003 to 15.0 MeV by time-of-flight techniques. A hydrogenous liquid scintillator was used as a detector at the higher energies, while a ^{6}Li-loaded glass scintillator was used at the lower energies. The measured spectrum has an average energy of 2.348 MeV. A Maxwellian distribution, $N(E)\sim E^{1/2}\exp(-E/1.565)$, fits the data well for $0.5<E<10.0$ MeV. Below 0.1 MeV, $N(E)$ has a $\sqrt{E}$ dependence but with values $\sim$25% larger than those predicted by the extended Maxwellian spectrum. The results are interpreted in terms of a simplified evaporation model.

I. INTRODUCTION

RECENT measurements[1–3] of the energy and angular distributions of ^{235}U and ^{252}Cf fission neutrons have established that, for energies greater than 0.3–0.5 MeV, the majority are emitted isotropically from the fully accelerated fragments while 10 to 15% may be emitted by a source stationary in the laboratory system. The neutron energy distribution for isotropic emission from a moving fragment is given by[4]

$$N(E)=\int_{(\sqrt{E}-\sqrt{E_F})^2}^{(\sqrt{E}+\sqrt{E_F})^2}\frac{\phi(E_{\text{c.m.}})dE_{\text{c.m.}}}{4(E_{\text{c.m.}}E_F)^{1/2}}, \quad (1)$$

where E is the energy in the laboratory system, $E_{\text{c.m.}}$ is the center-of-mass energy, E_F is the energy per nucleon of the fragment, and $\phi(E_{\text{c.m.}})$ is the center-of-mass energy distribution. Those neutrons with small E are the ones emitted backwards and nearly parallel to the fragment direction. As E approaches zero, $E_{\text{c.m.}}$ becomes restricted to a small range of values around E_F and

$$\lim_{E\to 0}N(E)=E^{1/2}\phi(E_F)/E_F^{1/2}. \quad (2)$$

Thus, regardless of the form of $\phi(E_{\text{c.m.}})$, this part of the fission neutron spectrum should have an $E^{1/2}$ dependence at low energies. If $\phi(E_{\text{c.m.}})$ is slowly varying near $E_{\text{c.m.}}=E_F$, then $N(E)$ will retain the dependence $E^{1/2}$ to quite large values of E.

The inclusion of the neutrons not emitted from the moving fragments does not greatly affect this conclusion. They appear to have an evaporation type of spectrum,[1,2]

$$P(E)\propto\sigma_c(E)E\exp(-E/\Theta), \quad (3)$$

where σ_c is the cross section for compound-nucleus formulation and Θ, the nuclear temperature, is approximately 1 MeV. The high average energy, coupled with their low abundance, implies a small effect at low energies. Secondly, at very low energies $\sigma_c(E)$

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