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de Haas–van Alphen Effect in Arsenic by the Torque Method*

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The de Haas–van Alphen (dHvA) effect in arsenic has been studied using a torque magnetometer. Techniques were used employing detection of torque, first or second derivative of torque, and torque with magnetic field modulation. An effect that showed hysteresis and a discontinuity in each dHvA oscillation was observed. The effect is explained in terms of the finite angular compliance of the magnetometer coupled with the periodicity of the curve of torque versus reciprocal magnetic field. The dHvA periods are reported for the magnetic field in the bisectrix-trigonal, binary-trigonal, and binary-bisectrix planes. Long periods are from necks of hyperboloidal shape tipped $-10°\pm1°$ from the trigonal direction. Short periods are from carriers of two sets of pockets with tilt angles of minimum area from the trigonal direction of $+86°\pm1°$ for β carriers and $+38°\pm1°$ for α carriers. These data support Lin and Falicov's model of the Fermi surface of arsenic.

I. INTRODUCTION

THE de Haas–van Alphen (hereafter called dHvA) effect of arsenic has been under study for some time by several investigators. The first investigation was carried out by Berlincourt[1] with a torque balance. He observed two sections of the Fermi surface. One section was analyzed as three or six equivalent ellipsoids with a carrier density of about 6×10^{19} carriers/cm^3 per ellipsoid. The other segment was of negligible size compared to the first, and there was evidence that it could consist of a set of segments that would be related by the symmetry of the crystal. It was evident, however, that the work of Berlincourt was not complete since there was no carrier compensation. Shapira and Williamson[2] observed a new and large portion of the Fermi surface, which could give the carrier compensation of a semimetal. This portion as well as that observed by Berlincourt has also been observed by several other investigators.[3,4] More recent Fermi-surface studies of arsenic have been greatly aided by the recent pseudopotential band-structure calculation of Lin and Falicov.[5] They proposed a model of the Fermi surface of arsenic which was consistent with data obtained by Priestley *et al.*[4] and by the authors of the present paper.[6] In this paper, we report our investigation of the dHvA effect of arsenic. The data were taken with a self-balancing torque magnetometer.

The theory of the dHvA effect is outlined in Sec. II. The self-balancing torque magnetometer and the various techniques used to obtain data are described in Sec. III. These techniques employ dc detection of torque, detection of the first or second derivative of torque, and detection with magnetic field modulation. An effect which showed hysteresis and a discontinuity in high-field dHvA oscillations is reported. The effect is explained in terms of the finite angular compliance of the magnetometer coupled with the sinusoidal nature of the torque versus reciprocal magnetic field curve. The experimental results are presented in Sec. IV. These data are discussed in Sec. V in terms of the model of Lin and Falicov and are compared with results of other experiments on arsenic.

II. THEORY OF THE DE HAAS–VAN ALPHEN EFFECT

The dHvA effect is the oscillatory magnetization in reciprocal magnetic field of a system of degenerate holes

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[1] T. G. Berlincourt, Phys. Rev. 99, 1716 (1955).
[2] Y. Shapira and S. J. Williamson, Phys. Letters 14, 73 (1965).
[3] J. B. Ketterson and Y. Eckstein, Phys. Rev. 140, A1355 (1965).
[4] M. G. Priestley, L. R. Windmiller, J. B. Ketterson, and Y. Eckstein, Bull. Am. Phys. Soc. 10, 1089 (1965); Phys. Rev. 154, 671 (1967).
[5] P. J. Lin and L. M. Falicov, Phys. Rev. 142, 441 (1966).
[6] J. Vanderkooy and W. R. Datars, Phys. Can. 22, 32 (1966).

or electrons at low temperatures. Onsager[7] showed that the frequency of an oscillation is directly proportional to the extremal cross-sectional area of the Fermi surface perpendicular to the magnetic field. Lifshitz and Kosevich[8] calculated the dHvA effect for an arbitrary Fermi surface. The oscillatory portion of the electronic free energy which gives rise to the dHvA effect is[8]

$$(F)_{\rm osc}=2VkT(eH/hc)^{3/2}|\partial^2A/\partial p_z^2|^{-1/2}{}_{E_0,p_m}$$
$$\times\left[\sum_{j=1}^{\infty}\frac{\exp(-2\pi^2jkT_D/\beta^*H)}{j^{3/2}\sinh(2\pi^2jkT/\beta^*H)}\right.$$
$$\left.\times\cos\left(\frac{2\pi jF}{H}-2\pi j\gamma\mp\frac{\pi}{4}\right)\cos(j\pi gm^*/2m_0)\right], \quad (1)$$

where the frequency of the oscillation is

$$F=cA/eh. \quad (2)$$

Equation (1) includes the effects of electron spin[9] and Landau level broadening.[10] The notation is similar to that used by Williamson *et al.*[10]: A is the extremal cross-sectional area of the Fermi surface in momentum space; E_0 is the Fermi energy; T_D is the Dingle temperature; $\beta^*=eh/m^*c$ where m^* is the cyclotron effective mass; g is the effective spectroscopic splitting factor, γ is a constant; and m_0 is the free-electron mass. In the phase of Eq. (2), the upper sign is used when A is a maximum, the lower for a minimum.

Oscillations in the magnetization $\mathbf{M}$ and the torque $\mathbf{T}$ are predicted by Eq. (1) since $\mathbf{M}=-\partial F/\partial\mathbf{H}$, $\mathbf{T}=\mathbf{M}\times\mathbf{H}$, and $T_\varphi=-\partial F/\partial\varphi$. Also, since $F/H\gg1$, only the first few terms in the summation need be considered and the fundamental oscillation is dominant. Usually, the Fermi surface has several extremal areas for a direction of magnetic field. Each contributes an oscillation with little interaction and mixing except at very high fields. Extremal cross-sectional areas are determined from Eq. (2). The cyclotron effective mass may be determined from the temperature dependence of the oscillations.

III. EXPERIMENTAL TECHNIQUES

Single crystals of As were grown from high-purity As by a modified Bridgman technique. These crystals were prepared by Dr. J. B. Taylor of the National Research Council. Analysis showed that Si was one of the chief impurities of the crystal (up to 200 ppm). The residual resistivity ratio of each crystal was approximately 1000. Rectangular samples were cut from a single crystal by cleaving it and cutting it with a Servomet spark cutter. Each sample was oriented to within 1° of a desired crystallographic plane by the back-reflection Laué method. The sample was attached to a mount at the end of the quartz rod of the magnetometer and was immersed in liquid helium which was reduced to a temperature of 1.2° by pumping.

The magnetic field was provided by a 12-in. electromagnet. Magnet gaps of 2 and 1 in. yielded maximum magnetic fields of 24 and 33 kOe, respectively. Both magnet configurations were used in the experiment. The magnetic field intensity was measured to an accuracy of 0.1% with a Rawson-Lush rotating-coil gaussmeter that was calibrated by NMR and by ESR of diphenyl picryl hydrazl (DPPH) at a frequency of 35 GHz.

The torque was measured with a self-balancing torque magnetometer that was similar to the one described by Condon and Marcus.[11] It used a Weston inductronic amplifier.[12] A quartz rod which supported the sample was suspended from the moving coil of the galvanometer in the inductronic amplifier by a copper strip. The torque on the sample was proportional to the current in the feedback loop of the amplifier. The first and second derivative of torque was measured by the method of Higgins *et al.*[13]

Most data were obtained by observing dHvA oscillations as a function of magnetic field strength. However, some data were taken by observing oscillations at a fixed magnetic field and as a function of the orientation of the magnetic field with respect to the crystal axes. The electromagnet was rotated slowly by a motor and chain drive. In this way, variations in extremal area of the Fermi surface were mapped out directly. One dHvA frequency, which is a reference frequency, is determined at a specific angle by sweeping magnetic field. The change in the dHvA frequency from the reference frequency at a fixed magnetic field is the product of the magnetic field and the number of dHvA oscillations between the magnetic field direction and the reference direction. This follows from Eq. (1) since the phase $2\pi F/H$ must change by 2π for one oscillation to occur.

The magnetometer was calibrated by applying a specified torque and measuring the output voltage across the galvanometer current coil. The torque was applied by a double-beam assembly. One light balsa wood beam was glued to the quartz rod in place of a sample and a second beam of equal length was suspended from it by two equal lengths of silk thread attached to the ends of the beams. The lower beam was of known mass and was twisted relative to the upper beam by forces in the horizontal plane. These forces were applied by two silk threads at right angles

[7] L. Onsager, Phil. Mag. **43**, 1006 (1952).
[8] I. M. Lifshitz and A. M. Kosevich, Zh. Eksperim. i Teor. Fiz. **29**, 730 (1955) [English transl.: Soviet Phys.—JETP **2**, 636 (1956)].
[9] M. H. Cohen and E. I. Blount, Phil. Mag. **5**, 115 (1960).
[10] S. J. Williamson, S. Foner, and R. A. Smith, Phys. Rev. **136**, A1065 (1964).
[11] J. H. Condon and J. A. Marcus, Phys. Rev. **134**, A446 (1964).
[12] R. W. Gilbert, AIEE Trans. **70**, 1121 (1951). Model 1411, Weston Instrument Division, Newark, New Jersey.
[13] R. J. Higgins, J. A. Marcus, and D. H. Whitmore, Phys. Rev. **137**, A1172 (1964).

to the ends of the beam. From a force analysis, the torque on the upper beam was determined to be

$$T=Mg\frac{d^2\sin\phi}{l[1-4d^2\sin^2(\phi/2)/l^2]^{1/2}}, \qquad (3)$$

where g is the acceleration due to gravity, d is the half-length of the balsa wood beam, l is the length of the silk thread in the suspension, ϕ is the angular displacement of the lower beam relative to the upper beam, and M is the mass of the lower beam. The sensitivity at the input to the galvanometer coil was determined to be 0.33 mV/dyn cm.

The angular compliance of the magnetometer is defined by

$$\eta=\partial\varphi/\partial T, \qquad (4)$$

where φ is the angular rotation of the sample and T is the applied torque. It was measured by determining the angular displacement with torque of a small pencil beam spotlight reflected from a mirror that was attached to the end of the quartz rod. This measurement, made at room temperature, was taken to be the compliance of the magnetometer during the experiment. The compliance was 11.4×10^{-5} rad/dyn cm with the design of Condon and Marcus[11] and 1.42×10^{-5} rad/dyn cm using a 2-mm quartz rod for sample suspension and an additional amplifier to provide current gain in the feedback loop. The latter was used in the final experiments.

During the course of experiments, large crystals were employed in an attempt to increase the signal-to-noise ratio. It was observed that beyond a certain magnetic field intensity oscillations were skewed and, at a point in each oscillation, the signal displayed a step discontinuity limited in speed by the torque magnetometer response. Upon reversal of the magnetic field sweep, the signal retraced most of the monotonic sections but the step discontinuity occurred at a displaced magnetic field. The first oscillation exhibiting this effect is shown in Fig. 1 for increasing and decreasing magnetic field. The dashed lines indicate the discontinuity of the torque.

This effect is to be understood in terms of the compliance of the torque magnetometer in the following way. With finite compliance, the specimen oscillates with increasing or decreasing magnetic field due to the oscillatory torque of the dHvA effect. A small rotation changes the value of the dHvA frequency which depends on the orientation of the sample. We shall assume that the torque can be expressed using only one fundamental dHvA oscillation and is of the form

$$T=T_0(H)\sin(2\pi F/H), \qquad (5)$$

where $T_0(H)$ is a torque which varies only slowly with magnetic field. Using Eq. (5), the change in torque for an angular rotation $\Delta\theta$ may be expressed to first order

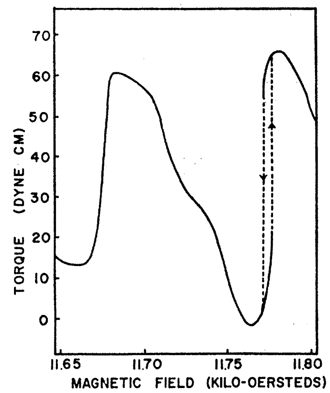


FIG. 1. A de Haas–van Alphen oscillation in a large crystal of arsenic for **H** 20° from the trigonal direction in the binary-trigonal plane. The dashed lines are step discontinuities for increasing and decreasing magnetic field.

by[14]

$$y=\kappa\sin(x-y), \qquad (6)$$

where $y=(2\pi/H)(\partial F/\partial\theta)\eta T$, $x=2\pi F/H$, and $\kappa=(2\pi\eta/H)\times(\partial F/\partial\theta)T_0(H)$. θ is the angle between a crystal axis and $\mathbf{H}$ thus $\Delta\theta=-\eta T$. If $|\kappa|>1$ there is a multivalued solution for Eq. (6). This is the condition for the onset of the effect that we observed. We see physically that when $|\kappa|>1$, a small angular displacement is unstable since the change in phase of Eq. (5) causes a greater change in torque than the restoring force of the magnetometer.

It was found that the parameter κ had the value 0.90 at the onset of the effect. This is rather good agreement with the predicted value of one if we realize that several effects not yet considered could have reduced the effective value of κ. These effects are the presence of additional oscillations, angular displacements about a second axis, and the variation of $T_0(H)$ with magnetic field.

It is to be noted that the expression given by Eq. (6) has the same form as that given by Plummer and Gordon[15] for the magnetic interaction of the dHvA effect. The magnetic interaction has been proposed by Shoenberg[16] and discussed by Pippard.[17] The torque can change in step-like fashion because of either effect. Also, hysteresis effects, harmonic content, and mixing of the dHvA oscillations can occur from either effect. However, the effects of magnetic interaction were known to be small in the present experiment because a reduction of the magnetometer compliance resulted in dHvA oscillations of low harmonic content.

Torque experiments were also done using modulation of the magnetic field and synchronous demodulation of the torque signal. For these experiments, an ac magnetic field, $\mathbf{h}_m=\mathbf{h}_m(0)\sin\omega t$ is applied at an angle α from $\mathbf{H}$. The ac output of the magnetometer at frequency ω is

[14] J. Vanderkooy, Ph.D. dissertation, McMaster University, 1967 (unpublished).
[15] R. D. Plummer and W. L. Gordon, Phys. Rev. Letters **13**, 432 (1964).
[16] D. Shoenberg, Phil. Trans. Roy. Soc. London **A255**, 85 (1962).
[17] A. B. Pippard, Proc. Roy. Soc. (London) **A272**, 192 (1963).

proportional to $\partial T/\partial h_m$. To first order from Eq. (6)[14]

$$\frac{\partial T}{\partial h_m}=T_0(H)\cos\left(\frac{2\pi F}{H_0}\right)\frac{2\pi}{H_0^2}\left[\left(\frac{\partial F}{\partial\theta}\right)\sin\alpha-F\cos\alpha\right]. \quad (7)$$

Since the condition for an extremum of signal is

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial T}{\partial h_m}\right)=0, \quad (8)$$

the condition for maximum signal is

$$\frac{1}{F}\left(\frac{\partial F}{\partial\theta}\right)=\cot(\alpha_\pm\pm\pi/2) \quad (9a)$$

for $\alpha=\alpha_\pm$. From Eq. (7) there is no signal when

$$\frac{1}{F}\left(\frac{\partial F}{\partial\theta}\right)=\cot\alpha_0 \quad (9b)$$

for $\alpha=\alpha_0$. The amplitude of modulation must be chosen so that the phase $2\pi F/H$ does not change by more than $\pi/2$. This condition is satisfied when

$$\frac{h_m(0)}{H_0}<\frac{H_0}{4[(\partial F/\partial\theta)^2+F^2]^{1/2}}. \quad (10)$$

The mathematics can be generalized to give the output amplitudes of all the harmonics in terms of Bessel functions.[18] This is hardly necessary in the analysis of an experiment with a torque magnetometer for which harmonics can be influenced by mechanical resonances of the system.

Modulation experiments were done using a two-channel amplifier to drive two sets of coils which were mutually perpendicular in the horizontal plane. Equation (10) was satisfied for $h_m(0)=5$ Oe, $H_0>10$ kOe, and $F\sim5\times10^6$ G. This provided a method of discriminating against the oscillations of a period.

IV. EXPERIMENTAL RESULTS

The dHvA data of arsenic are divisible into sets of short-period oscillations and long-period oscillations. The short periods that were measured for magnetic field directions in bisectrix-trigonal, binary-trigonal, and binary-bisectrix planes are shown in Figs. 2, 3, and 4, respectively. These periods are analyzed in terms of carriers of two bands that are called α and β carriers Lin and Falicov's model of the Fermi surface of arsenic.[5] provides a basis for the interpretation of the experimental results. This model consists of three electron pockets and a single, multiply connected hole surface. Each electron pocket resembles a distorted ellipsoid. The hole surface consists of six pockets which are connected by six thin cylindrical sections. For each band of carriers in the bisectrix-trigonal plane, there is a principal branch of periods and a nonprincipal branch derived from pockets that are related to the principal one by $\pm120°$ rotations.

In the bisectrix-trigonal plane, we shall adopt the convention that angles be measured positive in the sense of rotation from ΓT towards ΓX.[19] The ΓX and ΓL directions were determined using back-reflection Laué techniques and were related directly to the dHvA data. In Fig. 2, the maximum period of the principal branch of the α carrier is $6.75\pm0.05\times10^{-7}$ G^{-1} at $+38°\pm1°$. The maximum period of the nonprincipal branch of the α carrier is $5.80\pm0.05\times10^{-7}$ G^{-1} at $-22°\pm1°$. Only one period branch of the β carrier was observed. This is the principal branch with maximum period of $4.70\pm0.05\times10^{-7}$ G^{-1} at $+86°\pm1°$. The lower periods in Fig. 2 were obtained by using magnetic field modulation and derivative techniques.

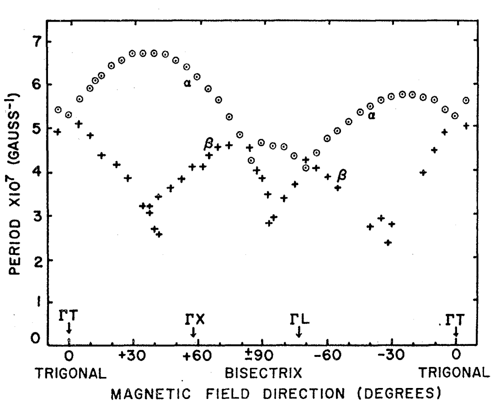


FIG. 2. The observed short periods with **H** in the bisectrix-trigonal plane of arsenic. ⊙ periods of fundamental oscillations; + periods of beat oscillations. ΓT, ΓX, ΓL are directed line segments in the Brillouin zone.

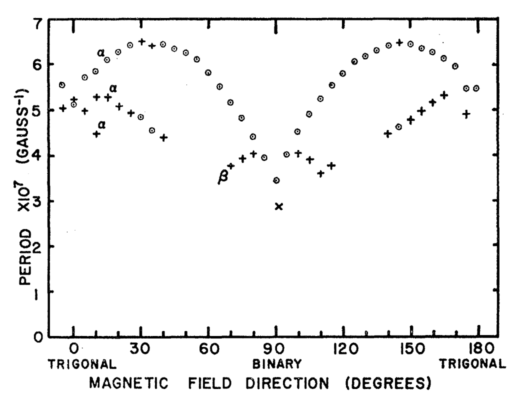


FIG. 3. The observed short periods in the binary-trigonal plane of arsenic. ⊙ periods of fundamental oscillations; + periods of beat oscillation.

[18] S. J. Williamson, Ph.D. dissertation, Massachusetts Institute of Technology, 1965 (unpublished).

[19] L. R. Windmiller, Phys. Rev. **149**, 472 (1966).

Three period branches of the α carriers and one branch of the β carrier are evident in Fig. 3. This assignment of period branches to carriers is made from the tilt angles of the pockets determined from Fig. 2. At first glance, the period of 2.9×10^{-7} G^{-1} belongs to the β-barrier branch that is symmetrical about the binary axis, but an ellipsoidal fit indicates that this branch has a lower period. However, the period 2.9×10^{-7} G^{-1} is assigned to the hole orbit which is not in the mirror plane and arises from a skew orbit associated with the junction of the necks and the α pockets.[4]

The data in the binary-bisectrix plane were taken by the rotation method and there was a large oscillation from only one of the carriers. The three period branches in Fig. 4 are assigned to the β carriers. The period 0.97×10^{-7} G^{-1} 2° from the binary axis was determined at a beat of two longer-period oscillations. It is 25% lower than the lowest period of the β carrier.[4] It is close to the fourth harmonic of the longer-period oscillations. However, the fourth harmonic is expected to be of very low amplitude. Thus, the period 0.97×10^{-7} G^{-1} could be from the hole orbit which is in the mirror plane and in the central section of the α pocket for **H** parallel to the binary axis.[20] The period of this orbit is small since the orbit encloses a large area. The orbit was not measured by Priestley *et al.*[4] but was detected in a cyclotron-resonance experiment.[21]

The long periods for the bisectrix-trigonal and binary-trigonal planes are shown in Figs. 5 and 6, respectively. Long periods in the bisectrix-trigonal plane show two branches having periods which are quite low near the bisectrix axis. The solid lines in Fig. 5 are plots

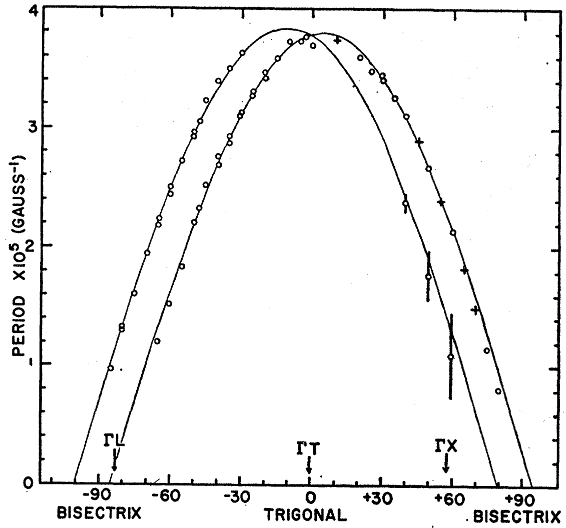


FIG. 5. The observed long periods in the bisectrix-trigonal plane of arsenic. The solid curves are for infinite cylinders.

of $P=P_{\max}\cos\theta$. This is the plot for an infinite cylinder. The fit to the data is good except near the bisectrix direction where the period is smaller than that for a cylinder. This indicates that this section of the Fermi surface may have the general shape of a hyperboloid of revolution. The tilt angle ϕ from the trigonal direction of the period maximum of the nonprincipal branch is related to the tilt angle δ of the section by the relation $\tan\delta=-\tan\phi\cos60°$. Since $\phi=+5°\pm\frac{1}{2}°$, the tilt angle $\delta=-10°\pm1°$. Then, for a hyperboloid of revolution $(x^2+y^2)/a^2-z^2/b^2=1$

$$P(\theta)=P_{\max}\cos\theta[1-(a/b)^2(\sec^2\delta\sec^2\theta-1)]^{1/2} \quad (11)$$

FIG. 4. The observed short periods in the binary-bisectrix plane of arsenic. Because of crystal symmetry all information is contained in a 30° interval in this plane.

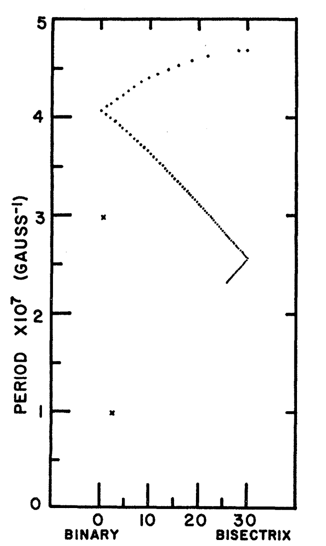
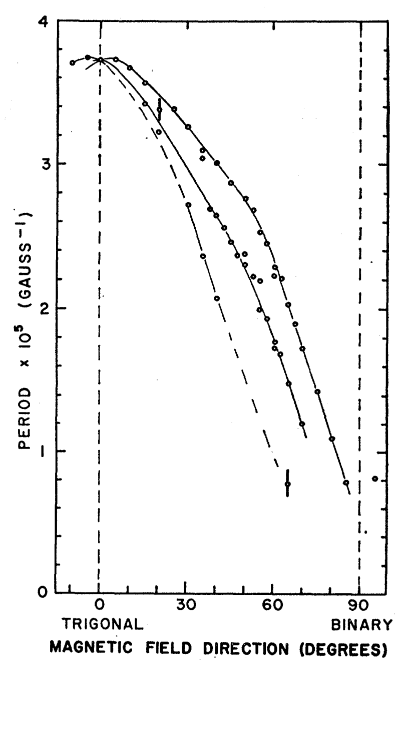


FIG. 6. The observed long periods in the binary-trigonal plane of arsenic. The curves are drawn through the experimental points.



[20] A period for the α carrier of $\sim2\times10^{-7}$ G^{-1} satisfies carrier compensation and the period 0.97×10^{-7} G^{-1} could be the second harmonic.

[21] W. R. Datars and J. Vanderkooy, J. Phys. Soc. Japan Suppl. **21**, 657 (1966).

for the nonprincipal branch where θ is the direction of $\mathbf{H}$ in the bisectrix-trigonal plane. A fit of Eq. (11) to the data of Fig. 5 yields a value for a/b of 0.12 ± 0.04. This means that the small sections have a gentle flare at each end. The principal branch is not as clearly defined as the nonprincipal branch in Fig. 5 but shows directly that the tilt angle of the hyperboloidal sections is $-10°\pm2°$ from the trigonal axis.

Figure 6 shows the results of data which were in general difficult to analyze. The kink in one branch at 55° from the trigonal axis may be partially due to these difficulties. However, the shape of the branch in this region is consistent with the idea that the hyperboloidal sections join on to other parts of the Fermi surface. The three branches in Fig. 6 show directly that there are three or six hyperboloidal sections.

Several experiments with large crystals were carried out with $\mathbf{H}$ in the binary-bisectrix plane. Little or no long-period oscillations were observed. This is consistent with a tilt angle of $-10°$ for which the orbits are nearly cut off with the magnetic field in the binary-bisectrix plane.

The temperature dependence of the dHvA amplitude was measured to determine the cyclotron mass of carriers and to determine the Fermi degeneracy energy using an ellipsoidal approximation. A plot of $\ln\{a/T[1-\exp(4\pi^2kT/\beta^*H)]\}$ versus T was a straight line for the β carriers. The Fermi degeneracy energy was found to be $(31.0\pm0.3)\times10^{-14}$ erg. The plot for the α carriers had a positive curvature so that the cyclotron mass was poorly defined. The Fermi energy of the α carriers was approximately 28×10^{-14} erg. This value may be in considerable error if the Fermi surface is highly nonellipsoidal. The Fermi energy of carriers observed in the binary-bisectrix plane is 30.5×10^{-14} erg. This confirms that the β carriers were observed in this plane.

V. DISCUSSION

A comparison of our results with the Fermi-surface model of Lin and Falicov allows us to assign the β carriers to the electron pockets and the α carriers to the pockets of the hole crown. With this assignment, there is reasonable agreement between the theoretical and experimental values of the tilt of the minimum areas in the bisectrix-trigonal plane. The carriers of the long periods may be assigned to necks from the evidence given in the previous section. These necks join the pockets of Lin and Falicov's hole surface. The decision as to whether three or six hole pockets exist can be made by considering the symmetry of the complete hole Fermi surface. If there were three pockets, there would be three interconnecting necks whose axes are perpendicular to the direction ΓT so that the $\bar{3}\ m$ symmetry is preserved. With a tilt angle of $-10°$, the symmetry $\bar{3}\ m$ makes the requirement six pockets joined by six necks. This is satisfied by Lin and Falicov's multiply connected surface situated around T of the Brillouin zone.

The carrier concentration of each pocket of electron and holes is estimated to be 6.0×10^{-19} cm^{-3} and 4×10^{-19} cm^{-3}, respectively, by using an ellipsoidal approximation to fit the data. The largest error of these values is due to uncertainties in the minimum periods. With these values, six hole pockets and three electron pockets provide satisfactory carrier compensation of arsenic. The electron pockets are situated at point L of the Brillouin zone. The present results are in agreement with those of Priestley *et al.*[4] The latter determine the shortest periods by their modulation techniques and show clearly where orbits on the hole pockets are cut off by adjoining necks. The present data do not show this cutoff but it is evident from the comparison of the two experiments that the α carrier was observed in most of the possible directions in the bisectrix-trigonal plane by using the torque method. In this experiment, additional information to that of Priestley *et al.* is provided with the observation of both principal hole orbits for the binary magnetic field direction and by the study of the long periods in the binary-trigonal plane. The data of both experiments provide good evidence for the correctness of the topology of Lin and Falicov's model at the Fermi level.

However, Lin and Falicov predict an electron Fermi energy, i.e., the energy between the Fermi level and the bottom of the L_4 band to be 58.7×10^{-14} erg. This value is 90% larger than our value of 31.0×10^{-14} erg. Also cyclotron effective masses of the theory are a factor of 2 smaller than those measured by cyclotron resonance[20] and with the dHvA effect. This discrepancy indicates that the Lin and Falicov theory may not be adequate away from the Fermi level.

ACKNOWLEDGMENTS

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