Scattering of Polarized Neutrons by Spin Waves in Magnetite and Yttrium Iron Garnet

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We have employed the neutron diffraction technique to study the inelastic scattering of polarized 1.08 Å neutrons by magnetic spin waves in magnetite and in yttrium iron garnet (YIG). Measurements were made for the cases when the magnetite and YIG scatterers were oriented near the positions for (111) and (220) Bragg reflections, respectively. Two new and widely applicable methods are given for determining the parameter D appearing in the usual expression Dq^2 (q=magnon wave number) for the energies of long-wavelength acoustic spin waves in usual ferromagnets and ferrimagnets. These methods are based on a comparison between experimental and calculated intensity ratios of magnetic diffuse reflections measured by the polarized neutron technique. The value $D=0.49\pm0.04$ eV Å^2 was found for magnetite using one of these methods; for YIG, the respective values $D=0.26\pm0.01$ eV Å^2 and $D=0.26\pm0.02$ eV Å^2 were obtained by the aforementioned two methods.

I. INTRODUCTION

A GENERAL theoretical treatment of the one-magnon, zero-phonon scattering of polarized neutrons by a wide class of exchange-coupled spin structures, given by Sáenz,[1] led to the prediction of a useful new spin-wave effect. This effect consists of a sensitive dependence of the above scattering of polarized neutrons on the directions of incident neutron polarization and magnetization of the scattering crystal, and differs for magnon absorption and emission scattering. The measurements of Ferguson and Sáenz[2] on magnetite were the first to provide a qualitative experimental confirmation of the effect; later, a quantitative confirmation for this ferrimagnet was obtained by these authors[3] and by Samuelsen, Riste, and Steinsvoll.[4] Shirane *et al.*[5] have used the effect in the investigation of acoustic spin-wave spectra in iron, while Riste *et al.*[6] have applied it to a study of cobalt and nickel.

In the present investigation, we have studied the acoustic spin-wave spectra of magnetite and yttrium iron garnet (YIG) using the polarized neutron diffraction technique. Two new methods of determining the parameter D, occurring in the usual expression Dq^2 for the energies of acoustic magnons of sufficiently small wave number q in ferromagnets and ferrimagnets, are introduced in this paper and applied to these two ferrimagnets. These methods are based on the fact that comparisons between experimentally measured and theoretically calculated ratios of intensities of magnetic diffuse reflections allow one to infer accurate values of D in favorable cases. The chief advantage of these procedures over other methods in current use is that they only require that a few relative intensity measurements near the center of the diffuse reflections be made accurately. These intensities, when combined with a crude measurement of the widths of these reflections, then determine uniquely the value of D in the cases in question.

Section II of this paper summarizes the theoretical background of this work. In Sec. III, we give a description of the experimental apparatus and methods. An analysis of experimental results is carried out in Sec. IV.

II. THEORETICAL BACKGROUND

In this section, we confine our considerations to spin-wave scattering of polarized monochromatic neutrons by magnetic crystals satisfying the requirements stated in Ref. 1. In particular, the resultant electronic spins of the magnetic ions of these crystals are assumed to be oriented parallel or antiparallel to a given direction in each domain.

Assume, for simplicity, that in the long-wavelength limit the energy surfaces of the acoustic magnons are spherical in q space and that only this type of magnons with energies well approximated by Dq^2 contribute significantly to the spin-wave scattering. The methods presented in this paper may, however, be generalized to include the situation when higher powers of q need to be considered, as in the case of iron reported in Ref. 5.

Let neutrons of wave vector $\mathbf{k}$ fall on a ferromagnetic or ferrimagnetic crystal oriented near the position for Bragg reflection by a set of crystallographic planes normal to a given vector $\boldsymbol{\tau}$. We define the angle of misset $d\theta\equiv\theta-\theta_B$, where θ is the glancing angle between $\mathbf{k}$ and these planes and θ_B is the value of θ at this Bragg setting. The one-magnon zero-phonon scattering of such neutrons by spin waves obeying the Dq^2 dispersion relation is theoretically known[7] to be confined to the interior of a

[1] A. W. Sáenz, in *Proceedings of the Symposium on Pile Neutron Research in Physics, Vienna, 1960* (International Atomic Energy Agency, Vienna, 1962), p. 423; Phys. Rev. **125**, 1940 (1962).
[2] G. A. Ferguson, Jr., and A. W. Sáenz, J. Phys. Chem. Solids **23**, 117 (1962).
[3] G. A. Ferguson, Jr., and A. W. Sáenz, Bull. Am. Phys. Soc. **9**, 111 (1964).
[4] E. J. Samuelsen, T. Riste, and O. Steinsvoll, Phys. Letters **6**, 47 (1963).
[5] G. Shirane, R. Nathans, O. Steinsvoll, H. Alperin, and S. J. Pickart, Phys. Rev. Letters **15**, 146 (1965).
[6] T. Riste, G. Shirane, H. Alperin, and S. J. Pickart, J. Appl. Phys. **36**, 1076 (1965).
[7] R. J. Elliot and R. D. Lowde, Proc. Roy. Soc. (London) **A230**, 46 (1955).

circular cone whose axis is parallel to the vector $\mathbf{k}+2\pi\boldsymbol{\tau}$. This cone is of very narrow aperture when $|d\theta|$ is not too large and when, in addition, the dimensionless constant $\alpha\equiv(2m/\hbar^2)D$ obeys the typical inequality $\alpha\gg 1$. Theory[7] also predicts that the only magnon processes contributing to the last mentioned type of scattering by these spin waves are magnon-emission (absorption) processes when $d\theta<0(>0)$, provided $|d\theta|$ satisfies a condition which is expressed accurately enough for our purposes by the inequality

$$|d\theta|>(2\alpha)^{-1}\csc 2\theta_B\,, \tag{2.1}$$

if $\alpha\gg 1$. This inequality holds for all the values of $d\theta$ occurring in the measurements reported in this paper.

Figure 1 shows, schematically, the geometry of our magnon scattering experiment. Consider a sphere centered at a point A of the scattering crystal. The axis AA' of the cone of inelastically scattered neutrons intersects this sphere at the point O. The center O' of the neutron detector aperture is moved in a circular arc CC' on the surface of the above sphere, CC' being in the plane containing $\mathbf{k}$ and $\boldsymbol{\tau}$. The detector angle ψ (angle between AO and AO') is taken as $\geq 0(\leq 0)$ if O' is on $OC'(OC)$.

Let $I(\psi,d\theta,r)$ denote the intensity of neutrons scattered per unit time into a detector of radius r by one-magnon zero-phonon processes involving only acoustic magnons. Then, if the scattering crystal is oriented near enough to the position for Bragg reflection corresponding to a given $\boldsymbol{\tau}$, and if the energies of the magnons contributing to this scattering are sufficiently smaller than that of the incident neutron, this intensity is closely given theoretically[2] by

$$I(\psi,d\theta,r)=\tfrac{1}{2}[1+(\mathbf{e}\cdot\boldsymbol{\mu})^2\pm 2p(\mathbf{e}\cdot\boldsymbol{\lambda})(\mathbf{e}\cdot\boldsymbol{\mu})]I_0(\psi,d\theta,r)\,. \tag{2.2}$$

Here, $\mathbf{e}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$ are unit vectors defined as follows: $\mathbf{e}\equiv\boldsymbol{\tau}/|\boldsymbol{\tau}|$, and $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are unit vectors parallel to the direction of spin polarization of the incident neutron beam and to the direction of the net electronic spin polarization of the magnetic ions within the crystal, respectively. The factor p is the average value of the

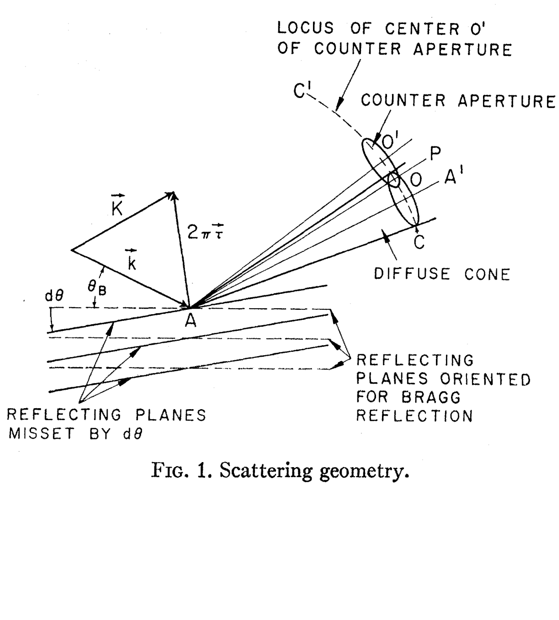

FIG. 1. Scattering geometry.

neutron polarization, in a direction parallel to $\boldsymbol{\lambda}$, of the transmitted neutron beam over the total path length traversed by this beam in the crystal. The $+(-)$ sign in (2.2) corresponds to magnon absorption (emission), and $I_0(\psi,d\theta,r)$, the value of $I(\psi,d\theta,r)$ when the incident neutron beam is unpolarized and $|(\mathbf{e}\cdot\boldsymbol{\mu})|=1$, is obtained from the equation

$$I_0(\psi,d\theta,r)=\int R\left[\frac{d\sigma_{01}}{d\Omega}\right]_0 d\Omega\,. \tag{2.3}$$

In this equation, $[d\sigma_{01}/d\Omega]_0$ represents the energy-integrated differential cross section for one-magnon zero-phonon emission or absorption scattering of unpolarized neutrons by acoustic spin waves when $\mathbf{e}$ and $\boldsymbol{\mu}$ obey the above condition, and R is the response function of the detector.[8]

When magnons of sufficiently small q are the only ones contributing significantly to the spin-wave scattering under discussion, a theorem of the second reference in Ref. 1 tells us that this scattering is well approximated by that from a suitable simple ferromagnet (ferromagnetic approximation). We evaluated I_0 on the NRL Electronic Computer (NAREC), using the latter approximation and the Dq^2 dispersion relation for the case $\psi=0$ which is of interest in this investigation. It is believed that the conditions of applicability of these two approximations were well met in our work.[9] The standard ferromagnetic scattering formulas used in

[8] In our calculations of I_0, the effect on R of the small divergence of the inelastically scattered neutrons in the present measurements was ignored. This should be an excellent approximation in this case.

[9] The satisfactory agreement between the experimental results in Ref. 17, on the total cross section for spin-wave scattering by magnetite oriented near the (111) position, with theoretical values calculated by employing both the ferromagnetic and Dq^2 approximations supports the applicability of these approximations for the conditions of interest here. In the case of YIG, a strong indirect argument for their applicability under these conditions is afforded by the fact that the values of D which we obtained for this substance by comparing the results of two different measurement procedures with calculated values based on the simultaneous use of the latter two approximations were closely equal. However, to our best knowledge, no quantitative results on the accuracy of these approximations are available. Of course, one can estimate roughly the error in I_0 due to the use of the Dq^2 dispersion relation in the few examples where complete analytical formulas for the acoustic branch are known, as is the case for magnetic but apparently not for YIG. With the aid of such a formula given by Kaplan [Phys. Rev. **109**, 782 (1958)] for a model of magnetite with only nearest-neighbor A-B interactions, we estimated that this use caused errors in I_0 of probably less than 5% for most of the values of d_a and α occurring in our numerical work. (The small corrections required in Kaplan's formulas by the spin-wave theory of Ref. 1 are irrelevant in the present connection.) In this estimate, we also took into account that, in the Dq^2 approximation, there are generally two acoustic magnons contributing to the relevant neutron scattering for every allowed scattering direction. Of these magnons, the one with the smallest q contributes most to I_0 in the ferromagnetic approximation for the parameter values of interest. This suggests that the Dq^2 dispersion relation need only be accurate for this spin wave. The error introduced by our employment of this relation in the value of D obtained by comparing theory and experiment is judged to be even smaller than the cited error in I_0, and to be negligible relative to the errors of our measurements.

these calculations are given in Ref. 10 in a form convenient for numerical purposes.

III. EXPERIMENTAL APPARATUS AND METHODS

Figure 2 is a schematic diagram of our polarized-neutron diffractometer, the details of which are described elsewhere.[11] To achieve the maximum effect predicted by Eq. (2.2), our apparatus was arranged to make **e**, **λ**, and **μ** collinear. Magnetic resonance techniques were employed to reverse the spins of the incident neutrons in order that **λ** could be made either parallel or antiparallel to **μ**. We shall refer to these two respective cases as the parallel and antiparallel cases.

It can be shown that the relative spin-wave intensities calculated from Eq. (2.3) can be strongly influenced by the form of the response function of the detector. For this reason, a careful measurement of R was made by scanning the 1 in. diameter of our detector with a small neutron beam of circular cross section. The observed response was compared with that calculated for a wide variety of cylindrically symmetric functions R. From this comparison, it was concluded that, of all these functions, the one which best fitted the experimental data was one of trapezoidal form. More precisely, this last function was constant over 95% of the detector diameter and fell linearly to zero at the counter's edge.

In order to evaluate the average polarization p entering Eq. (2.2), we measured the relative depolarization experienced by the neutron beam in traversing the magnetite crystal (in which depolarization effects were most severe). This depolarization was determined by measuring the double reflection effect for the case when this crystal was first in, then out, of the polarized neutron beam which traveled between two Co-Fe crystals situated at positions M_p and A of Fig. 2. From these measurements and the measured value of the polarization of the incident neutrons, we concluded that $p \geq 0.92$ for our experiments.

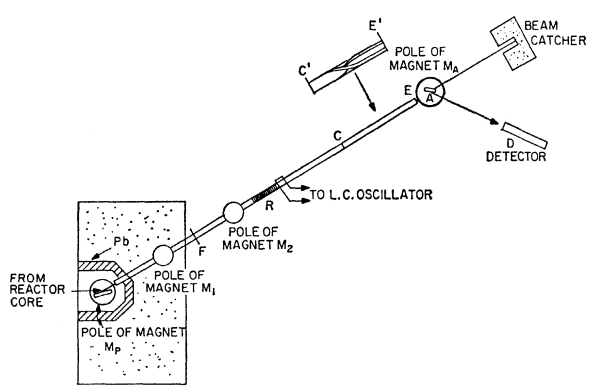

FIG. 2. Schematic diagram of polarized neutron diffractometer.

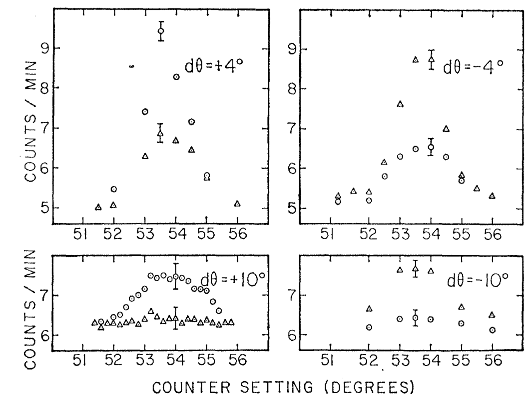

FIG. 3. Scattering of polarized 1.08 Å neutrons by spin waves in a magnetite crystal (293°K) misset by $d\theta = \pm 4°$, $\pm 10°$ with respect to the (111) Bragg position, for $r = \frac{1}{2}$ in. The triangles represent scattered neutron intensities for the parallel case, while the circles denote these intensities for the antiparallel case.

IV. DISCUSSION OF SPIN-WAVE RESULTS

A. Magnetite (Fe_3O_4)

In the measurements on magnetite discussed in detail below, the magnetite crystal was oriented in the vicinity of the (111) Bragg position, and the distance from the crystal to the detector was kept fixed. The angular aperture of the counter was 1.94° in these measurements.

Figure 3 shows experimental results on the angular distribution of neutrons scattered by magnons in magnetite for $d\theta = \pm 4°$, $\pm 10°$. The data in Fig. 3 fully confirm the correctness of the $\pm$ signs in Eq. (2.2).[12] From this equation, the diffuse peaks in Fig. 3 corresponding to the parallel (antiparallel) case should be quite small when $d\theta = +4°$, $+10°$ ($-4°$, $-10°$) for the present situation where $p \approx 1$. This figure makes it clear that this theoretical prediction holds for $d\theta = \pm 10°$, but that for $d\theta = +4°$ and $d\theta = -4°$ there are quite sizeable "residual" peaks in the parallel and antiparallel cases, respectively. The most straightforward explanation of these residual peaks is that they are due to the scattering of neutrons by phonons, or to dipole-dipole interactions among the magnetic ions, or to both of these causes.[13]

[10] G. A. Ferguson, Jr., A. W. Sáenz, and A. D. Anderson, Report of NRL Progress, February 1965, p. 10 (unpublished). In Eqs. (A5a) and (A5b) in the appendix of this reference, replace h by $\hbar$ and replace the exponent $\frac{1}{2}$ nearest to the end of the second line of Eq. (A5a) by the exponent 2.

[11] G. A. Ferguson, Jr., Report of NRL Progress, November 1962, p. 10 (unpublished).

[12] Our measurements for magnetite in the vicinity of the (220) Bragg setting are also in complete accord with these $\pm$ signs.

[13] The modification of (2.2) caused by the presence of dipole-dipole interactions is readily calculated for simple ferromagnets, and this modification predicts the existence of residual peaks of the type mentioned in the text. This result has been obtained independently by Dr. L. Dobrzynski (private communication). Since **e** and **μ** were essentially collinear in our measurements, one does not expect any significant contributions to the observed residual peaks from magnetovibrational scattering of the usual variety. Estimates based on data in Ref. 17 confirm this expectation. The measurements of Dr. H. A. Alperin (private communication) on magnetite show that these peaks persist below a temperature of 120°K. This result shows that the spin disorder existing at the crystallographic B-sites above this temperature makes no contribution to the peaks in question.

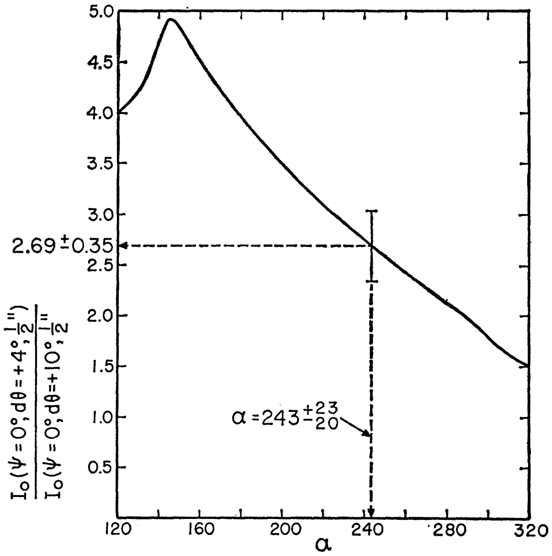

FIG. 4. This figure refers to magnetite, for the same values of temperature and neutron wavelength as in the caption of Fig. 3. Calculations are depicted of

$$I_0(\psi=0°, d\theta=+4°, r=\tfrac{1}{2}\text{ in.})/I_0(\psi=0°, d\theta=+10°, r=\tfrac{1}{2}\text{ in.}),$$

the experimental value 2.69 ± 0.35 of this ratio, and the inferred value $243(+23, -20)$ of α.

In a number of cases of experimental interest, our computations showed that the ratio $I_0(\psi=0°, d\theta_1, r)/I_0(\psi=0°, d\theta_2, r)$ was quite sensitive to variations of α when $|d\theta_1|$ and $|d\theta_2|$ were sufficiently different. This and the other intensity ratios mentioned in this paper were conveniently measured by the polarized neutron technique.[14] Our measured value of $I_0(\psi=0°, d\theta=+4°, r=\frac{1}{2}\text{ in.})/I_0(\psi=0°, d\theta=+10°, r=\frac{1}{2}\text{ in.})$ for magnetite was 2.69 ± 0.35. A crude analysis of the magnon peaks in Fig. 3 showed that the inequality $\alpha\geq120$ held for this substance. Theoretical results for this ratio are exhibited in Fig. 4 (full curve) together with the pertinent experimental value.[15] Comparing these theoretical and experimental findings and employing the last inequality, we obtain the unique value $\alpha=243(+23, -20)$ for magnetite. This is in very satisfactory agreement with those obtained by earlier experimenters[16,17] using other neutron scattering methods.

[14] Let $\Delta(\psi,d\theta,r)$ denote the intensity of neutrons scattered in the parallel case by one-magnon zero-phonon acoustic spin-wave processes minus the value of this intensity in the antiparallel case, for given ψ, $d\theta$, and r. To measure the ratio of two quantities I_0 of interest, we measured the corresponding quantities Δ and employed the equation

$$I_0(\psi_1,d\theta_1,r_1)/I_0(\psi_2,d\theta_2,r_2)=|\Delta(\psi_1,d\theta_1,r_1)/\Delta(\psi_2,d\theta_2,r_2)|,$$

which follows from (2.2) and is applicable when $d\theta_1$ and $d\theta_2$ obey (2.1).

[15] The ratio in Fig. 4 decreases monotonically with α to the value unity for values of α greater than that corresponding to the peak in this figure. This behavior also holds for the intensity ratios for YIG discussed subsequently in the text.

[16] B. N. Brockhouse, Phys. Rev. **106**, 859 (1957).

[17] T. Riste, K. Blinowski, and J. Janik, J. Phys. Chem. Solids **9**, 153 (1959).

B. Yttrium Iron Garnet

Conflicting experimental determinations of the parameter D for YIG have appeared in the literature,[18] and for this reason we have employed the polarized neutron diffraction method to shed further light on its correct value.

In the experiments reported below, a YIG crystal was oriented near the position for (220) Bragg reflection and maintained at a fixed distance from the detector. The angular aperture of the counter was 2.02° for the measurements on this crystal for which no collimation was used to narrow its $\frac{1}{2}$ in. radius. For this latter aperture value, spin-wave intensities were measured in the manner previously described for magnetite. Figure 5 illustrates these intensities for $d\theta=\pm2°$, $\pm5°$. This figure shows that the intensities for the parallel (antiparallel) case essentially vanish for $d\theta=+2°$, $+5°$ ($d\theta=-2°$, $-5°$), in accordance with Eq. (2.2), for the present case $p\approx1$.

The parameter D was determined for YIG by two methods, one of which was identical to that employed for magnetite.

The other method is of comparable sensitivity and is somewhat more convenient experimentally for YIG. In this procedure, one measures the ratio $I_0(\psi=0°, d\theta, r_1)/I_0(\psi=0°, d\theta, r_2)$ for a given $d\theta$ and for two different counter radii, r_1 and r_2. The detector radius was varied by means of cadmium-shielded plugs inserted at the entrance of the counter.

A rough analysis of the magnon peaks in Fig. 5 yielded the fact that $\alpha\geq60$ for YIG. In Fig. 6(a) (curves I, II, III), we depict calculated values of the last cited intensity ratio when $d\theta=+2°$ for several choices of r_1 and r_2. The experimental values of this ratio are also shown in this figure. For the respective

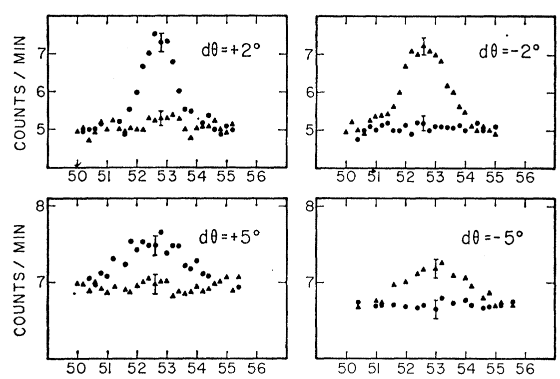

FIG. 5. Scattering of polarized 1.08 Å neutrons by spin waves in a YIG crystal (293°K) misset by $d\theta=\pm2°$, $\pm5°$ from the (220) Bragg position, for $r=\frac{1}{2}$ in. The triangles and circles have the same significance as in Fig. 3.

[18] H. Meyer and A. B. Harris, J. Appl. Phys. **31**, 49 (1960); D. T. Edmonds and R. G. Peterson, Phys. Rev. Letters **2**, 499 (1959); J. E. Kunzler, L. R. Walker, and J. K. Galt, Phys. Rev. **119**, 1609 (1960); E. H. Turner, Phys. Rev. Letters **5**, 101 (1960); S. S. Shinozaki, Phys. Rev. **122**, 388 (1961).

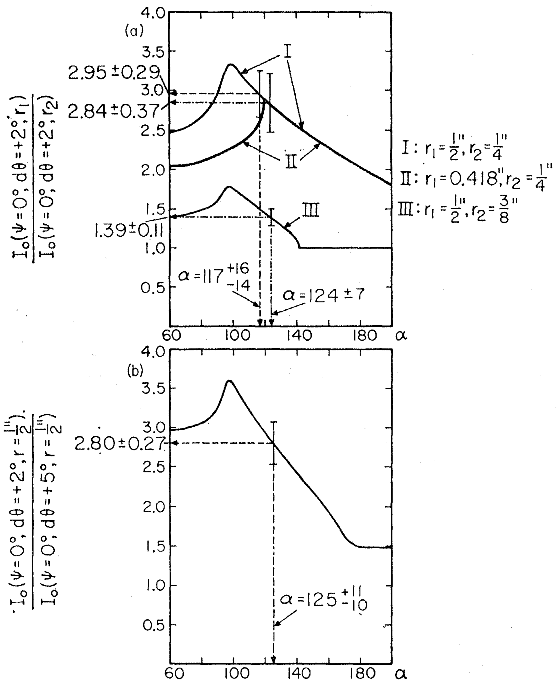

FIG. 6. Parts (a) and (b) of this figure refer to the vicinity of the (220) Bragg setting of YIG and to the same incident neutron wavelength and temperature as for Fig. 5. Figure 6(a) shows computations of the ratio

$$I_0(\psi=0°, d\theta=+2°, r_1)/I_0(\psi=0°, d\theta=+2°, r_2)$$

for various combinations of the values $\frac{1}{2}$, 0.418, $\frac{3}{8}$, $\frac{1}{4}$ in. of r_1 and r_2, together with the corresponding experimental values of this ratio and the deduced values of α. Figure 6(b) exhibits calculations of the ratio

$$I_0(\psi=0°, d\theta=+2°, r=\tfrac{1}{2}\text{ in.})/I_0(\psi=0°, d\theta=+5°, r=\tfrac{1}{2}\text{ in.}),$$

the experimental value of this ratio, and the corresponding value of α.

cases $r_1=\frac{1}{2}$, $r_2=\frac{1}{4}$ in. and $r_1=\frac{1}{2}$, $r_2=\frac{3}{8}$ in., this ratio was measured to be 2.95 ± 0.29 and 1.39 ± 0.11. On comparing these results with the theoretical ones and using the more refined lower bound on α for YIG given by the inequality $\alpha\geq100$, which holds with high probability from the measurements reported below, one finds that a unique value of α corresponds to each of the last two experimental values, namely,

$$\alpha=117(+16, -14)(D=0.24\pm0.03 \text{ eV Å}^2)$$

and

$$\alpha=124\pm7(D=0.26\pm0.01 \text{ eV Å}^2),$$

respectively. The need to supplement these two experimental results by such a more accurate lower bound on α in order to determine this parameter uniquely should be obvious from curves I and III.

Our procedure for showing that $\alpha\geq100$ for YIG is simple and appears to be widely applicable to find the fairly good lower bounds on this parameter which may be needed in many other cases to secure such a unique determination when one uses the intensity methods of the present paper. Hence it is of interest to outline the simple theoretical conclusions on which this procedure is based. These conclusions are valid when one makes the ferromagnetic and Dq^2 approximations and assumes, in addition (for example), that the detector has a flat response over its accessible cross section. Consider the cone of neutrons scattered by pure magnon-emission or magnon-absorption processes of the type of interest here. Denote by $\rho(\alpha,d\theta)$ the radius of the cross section of this cone at the position of the detector aperture. For each $d\theta$ and each r_1 and r_2 such that $r_1>r_2$, $I_0(\psi=0°, d\theta, r_1)/I_0(\psi=0°, d\theta, r_2)$ has a continuous derivative with respect to α, except at those unique values of α at which $\rho(\alpha,d\theta)$ equals r_1 or r_2. For example, at the value α_1 of α at which $\rho(\alpha,d\theta)=r_1$, this ratio has a cusp such that the said derivative approaches $+\infty$ (a finite negative value) as α tends to α_1 from the left (right).[19]

Our previously cited measurements on YIG are compatible with an α close to 120. Now, $\rho(\alpha=120, d\theta=+2°)=0.418$ in. for the (220) reflection of interest. Hence, because of the stated flatness property of our detector's response function R, $I_0(\psi=0°, d\theta=+2°, r_1=0.418\text{ in.})/I_0(\psi=0°, d\theta=+2°, r_2=\frac{1}{4}\text{ in.})$ has a cusp at $\alpha=120$ for this R, as shown by curve II in Fig. 6(a). Our hope was that the measurement of this ratio would yield a value of the latter which would intersect curve II near this cusp, and hence would "trap" α in the neighborhood of 120. This ratio was found to be 2.84 ± 0.37 experimentally, a number which lies 1.54 standard deviations above its theoretical value for the case $\alpha=100$. From these facts, the shape of curve II, and the properties of the normal distribution, one easily shows that $\alpha\geq100$ for YIG with a probability of at least 94%.

The ratio $I_0(\psi=0°, d\theta=+2°, r=\frac{1}{2}\text{ in.})/I_0(\psi=0°, d\theta=+5°, r=\frac{1}{2}\text{ in.})$ was also measured for YIG, its experimental value being 2.80 ± 0.27. Theoretically, one obtains the dependence of this ratio on α shown in Fig. 6(b). Invoking again the inequality $\alpha\geq100$, and comparing calculated and measured values of the latter ratio, it is found that $\alpha=125(+11, -10)$ ($D=0.26\pm0.02$ eV Å^2), which is in excellent agreement with the most accurate value obtained for YIG by our other method.

ACKNOWLEDGMENTS

The valuable aid of S. Podgor in all of the numerical aspects of this paper is gratefully acknowledged. We are also grateful to L. A. Harris, who assisted in collecting the data.

[19] As is well known, in the joint ferromagnetic-Dq^2 approximation, the cross section $(d\sigma_{01}/d\Omega)_0$ for one-magnon zero-phonon scattering becomes infinite for scattered neutron momenta tangent to the scattering surface, i.e., momenta lying on the surface of the diffuse cone. When the detector has a flat response, this behavior of the cross section leads to the infinite discontinuities in slope of $I_0(\psi=0°, d\theta, r_1)/I_0(\psi=0°, d\theta, r_2)$ stated in the text. The fact that this response was not perfectly flat when the counter radius was $\frac{1}{2}$ in. caused the peaks of curves I and III in Fig. 6(a) to be smooth instead of cusp-like. A similar remark applies to the peaks in Figs. 4 and 6(b).