Asymmetries of the Critical Surface Current in Type-II Superconductors

P. S. SWARTZ AND H. R. HART, JR.
General Electric Research and Development Center, Schenectady, New York

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It is found that in two different and well-defined experimental situations the magnitude of the critical surface current changes when the direction of the transport current is reversed. This property is denoted as "partial rectification." In one case (single-surface rectifier), the asymmetry is measured when the external magnetic field ($H_{c1}<H<H_{c3}$) is directed parallel to a single, unpaired surface and perpendicular to the transport current directions. Here only a tentative explanation is offered. In the second case (asymmetric-field rectifier), the asymmetry can be understood with the assumptions: (1) that the critical surface state model applies, i.e., each region of the surface carries its maximum or critical current, and (2) that the local critical surface current decreases as the absolute magnitude of the local perpendicular field component is increased from zero. The asymmetry in the critical surface current occurs in foils when the perpendicular component of the applied magnetic field is zero along the center line of the foil ($x=0$), increases with distance from the center line, and has the symmetry that $H_{\perp a}(x)=-H_{\perp a}(-x)$.

A. INTRODUCTION

IT is usually found that the magnitude of the critical transport current of a superconductor does not change when direction of the transport current is reversed. In our earlier work,[1] however, we have shown that the magnitude of the critical surface current in a triangular prism of a type-II superconductor depends on the polarity of the transport current in the presence of a magnetic field directed perpendicular to the transport current directions and essentially parallel to one of the faces of the prism. In this paper we shall show that there are at least two separate and distinct experimental situations and, correspondingly, two separate and distinct underlying mechanisms by which the critical surface current of a type-II superconductor changes magnitude when the direction of the transport current is reversed.[2,3] This property is denoted as "partial rectification" in view of the observed asymmetric I–V characteristics as shown in Fig. 1. We distinguish between the two rectifiers by calling one a "single-surface rectifier" (Sec. B) and the second an "asymmetric-field rectifier" (Sec. C). In each case we can specify in some detail the experimental conditions that must be met for rectification to be observed.

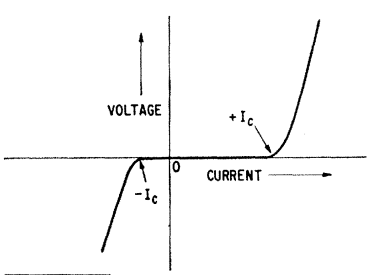

FIG. 1. A schematic of an asymmetric current-voltage characteristic. We denote this property as "partial rectification."

[1] P. S. Swartz and H. R. Hart, Jr., Phys. Rev. 137, A818 (1965).
[2] P. S. Swartz and H. R. Hart, Jr., Bull. Am. Phys. Soc. 11, 106 (1966).
[3] P. S. Swartz and H. R. Hart, Jr., Bull. Am. Phys. Soc. 10, 59 (1965).

However, our explanation for the operation of the single-surface rectifier is still only tentative. In certain experimental situations both mechanisms contribute to the observed rectification.

B. SINGLE-SURFACE RECTIFIER

The experimental conditions required for single-surface rectification[2] are the following:

(1) The type-II superconductor has at least one essentially planar surface.

(2) The transport current between H_{c1} and H_{c3} is associated primarily with the surfaces and not with the bulk. (Above H_{c2} this condition is always achieved; between H_{c1} and H_{c2} this condition can be achieved by annealing in vacuum for long times near the melting temperature if the alloy is a solid solution[1]).

(3) The applied magnetic field ($H_{c1}<H<H_{c3}$) is directed parallel (to within a few tenths of a degree) to a single, planar, current-carrying surface and essentially perpendicular (to within a few tens of degrees) to the transport current.

(4) If the geometry of the superconductor is such that there exist parallel surfaces (e.g., ribbon), then the current-carrying capacity of all but one of the surfaces must be essentially destroyed. (This condition can be met in a variety of ways. Perhaps the simplest method that we have employed is to copper plate one of the two surfaces of a ribbon).

When these four conditions are satisfied, the "diamagnetic" critical surface current (I_d) experimentally always exceeds the "paramagnetic" critical surface current (I_p). The identification of a diamagnetic and a paramagnetic transport current can be made when the current is carried by a single surface and the magnetic field is perpendicular to the transport current and parallel to the surface. A *diamagnetic* surface transport current is defined as a current whose self-field within the sample opposes the applied field. Conversely, the self-field of a *paramagnetic* surface transport current adds within the sample to the applied field. The "rectification ratio" is defined as $|I_d/I_p|$. The three experiments reported below demonstrate that rectification occurs when the stated conditions are satisfied.

Experiment I: A Diffused, Bimetallic Foil

A metal sandwich of Pb–$Pb_{0.90}Tl_{0.10}$ is formed by cold-rolling together equal thicknesses of the two metal sheets. At the final sandwich of 0.005 in., the two metal sheets are mechanically inseparable. The bimetallic strip is cut to dimension (test section 0.250 in×1 in.) and annealed for 60 h at 200°C and 1 h at 320°C in a vacuum of 10^{-6} to 10^{-7} mm Hg. The effect of this heat treatment is to cause a diffusion layer to form near the midplane. In this way the current-carrying capacity of the internal Pb–Tl surface is destroyed (as verified by Experiment II). This anneal is also sufficient to remove most of the bulk supercurrent. The sample is then chemically polished[1] and current and voltage leads are attached. The critical current (defined at a voltage level of 0.5 μV/cm) is measured at 4.2°K as a function of the magnetic field applied perpendicular to the transport current direction and parallel to the surfaces of the strip. The significant results are the following (see Figs. 2 and 3). (Qualitatively similar results were obtained with each of the ten or so different bimetallic strips tested).

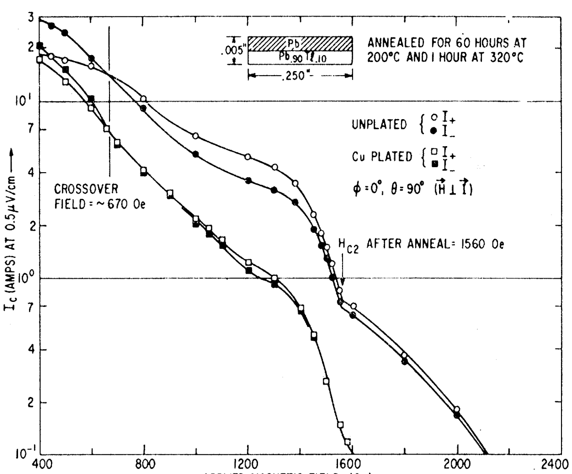

FIG. 2. The critical (surface) current at 4.2°K versus applied field for an annealed bimetallic strip of Pb–$Pb_{0.90}Tl_{0.10}$. The results show that the critical currents and the rectification are greatly diminished after copper plating.

(i) The critical currents of opposite polarity are *unequal* from the lowest field tested (400 Oe) to a field approaching 2400 Oe. (The maximum rectification ratio is about 1.35, occurring at ~1150 Oe).

(ii) The composition[4] of each of the two surfaces after the diffusion anneal can be inferred from the sharp change in critical current with field that occurs at 670 and 1560 Oe. From previous work[1,5,6] we are able to identify these changes in critical current as H_{c2} transitions for the two surfaces. From Fig. 4 we can associate a surface composition of $Pb_{0.984}Tl_{0.016}$ (H_{c2}=670 Oe) with the surface that was originally pure Pb and a composition of $Pb_{0.91}Tl_{0.09}$ (H_{c2}=1560 Oe) with the surface that was $Pb_{0.90}Tl_{0.10}$ before the annealing treatment.

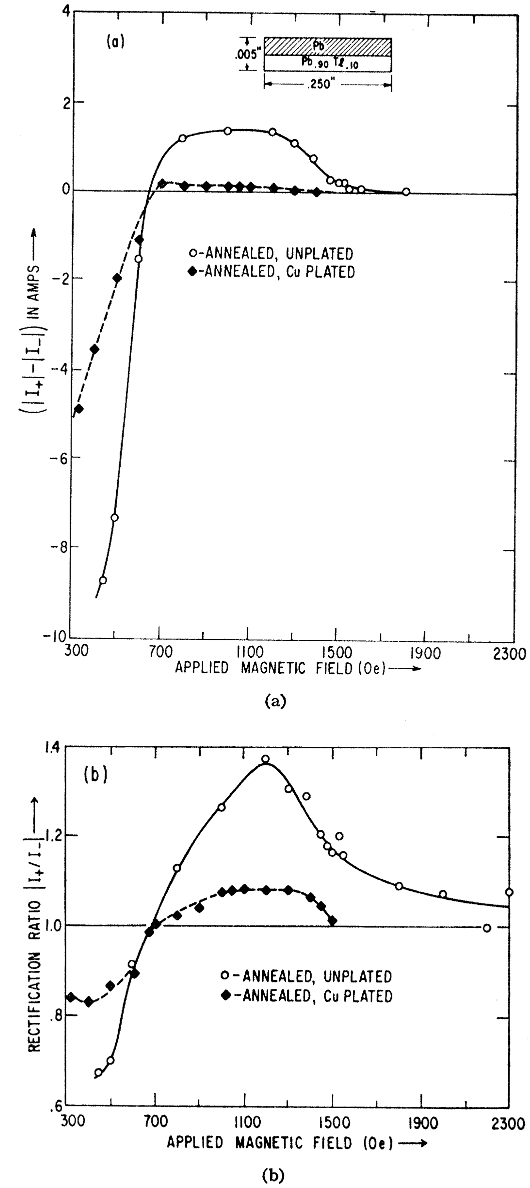

FIG. 3. The (a) absolute and (b) relative difference in the critical currents of opposite polarities of the sample of Fig. 2 before and after copper plating.

[4] H. R. Hart, Jr. and P. S. Swartz, Phys. Letters 10, 40 (1964).

[5] R. V. Bellau, Phys. Letters 21, 13 (1966).

[6] B. Bertman and M. Strongin, Phys. Rev. 147, 268 (1966).

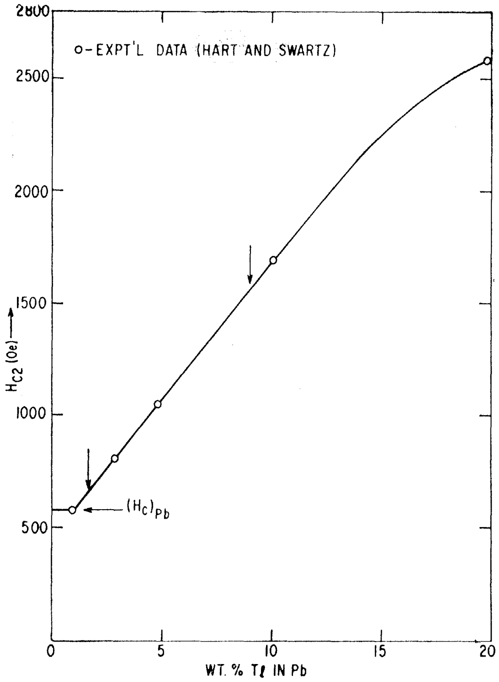

FIG. 4. A plot of H_{c2} at 4.2°K versus wt % Tl in Pb, taken from the experimental data of Hart and Swartz (Ref. 4).

(iii) We can infer from the abrupt change in the slope of the critical current versus field curve at 670 Oe that below 670 Oe most of the current is being carried on the Tl-poor surface, while above 670 Oe the Tl-rich surface carries essentially all of the current. Using this inference, we can identify the larger critical current below 670 Oe as a diamagnetic current flowing on the Tl-poor surface and the larger critical current above 670 Oe as a diamagnetic transport current on the Tl-rich surface. Thus, we conclude that the diamagnetic surface transport current is larger than the paramagnetic critical current for each surface.

Experiment II: A Copper-Plated, Diffused Bimetallic Ribbon

The *same* ribbon as tested in Experiment I is electroplated with a few microns of copper and retested. The significant results (see Figs. 2 and 3) are:

(i) The magnitude of the critical currents of both polarities is significantly reduced at all fields. This observation reveals that the transport currents measured in Experiment I are predominantly or wholly surface currents.

(ii) Both the absolute [Fig. 3(a)] and relative [Fig. 3(b)] difference between the critical currents of opposite polarity is significantly reduced. This result indicates that it is the surface current that is being rectified.

(iii) The field at which the sense of the rectification switches (670 Oe) remains essentially unchanged after copper plating, suggesting that the two surfaces are about equally affected by the copper.

(iv) The critical current of both polarities becomes too small to detect (less than a few milliamperes) above about 1700 Oe.

Experiments I and II seem to confirm the conditions listed at the beginning of this section. However, in polishing the samples before testing, the cross section is brought from a rectangular to a slightly ellipsoidal shape. It will be demonstrated in Sec. C that a curved surface can bring about rectification for entirely different reasons. Although the amount of surface curvature introduced during the polishing operations was probably too small to account for the measured rectification, the following experiment was performed to eliminate surface curvature as a possible origin of the effect.

Experiment III: An Evaporated Film with a Flash of Copper on its Surface

$Pb_{0.90}Tl_{0.10}$ films are evaporated (test sections are $\sim 1\,\mu \times 1$ mm$\times 1$ cm). A flash ($\sim$1000 Å) of copper is evaporated either just before or just after the evaporation of the superconductor itself. In this way the current-carrying capacity of one of the two surfaces is destroyed or significantly reduced. (We performed experiments to

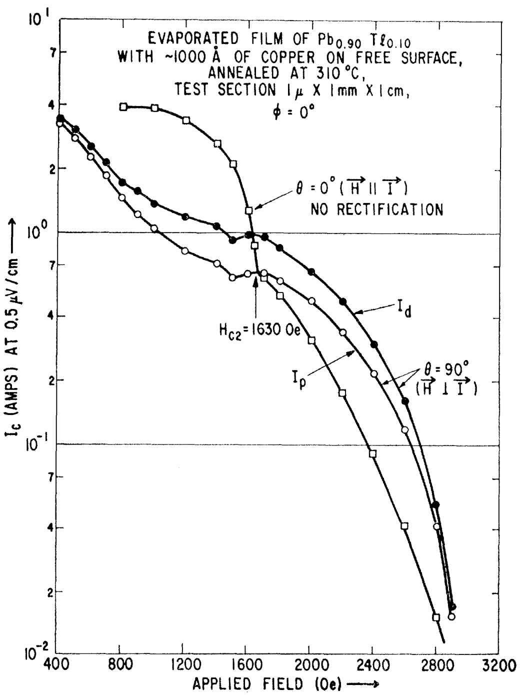

FIG. 5. The critical (surface) current at 4.2°K versus applied field of an evaporated, annealed film of $Pb_{0.90}Tl_{0.10}$ with a flash of copper on the free surface. When the magnetic field is parallel to the surfaces and perpendicular to the current direction, partial rectification is observed; when the magnetic field and current directions are coincident, no rectification is observed.

assure ourselves that this is the case; when copper is flashed on *both* surfaces, *no* supercurrent is measured above H_{c2} while the current-carrying capacity in fields between $\sim\frac{2}{3}H_{c2}$ and H_{c2} is significantly reduced). The evaporated films are then annealed for a few hours at 310°C in a vacuum of 10^{-6} to 10^{-7} mm Hg. This treatment is sufficient to level any compositional gradients in the superconductor and to reduce the bulk current close to zero. There is no evidence (such as unusual H_{c2} or H_{c3} transitions) that the copper and the Pb–Tl alloy interdiffused.

Rectification was found in all but 1 out of about 20 experiments where copper was flashed on either of the two surfaces (see Fig. 5). The significant results are summarized as follows:

(i) The sense of the rectification is always such that the diamagnetic current associated with the current-carrying surface is larger than the paramagnetic current.

(ii) The magnitude of the rectification ratio is generally found to be 1.3–1.7, of the same magnitude as that observed in the bimetallic ribbons that had been rolled from the bulk.

(iii) The rectification disappears quickly at all applied fields as the magnetic field vector is turned out of the plane of the surface (see Fig. 6).

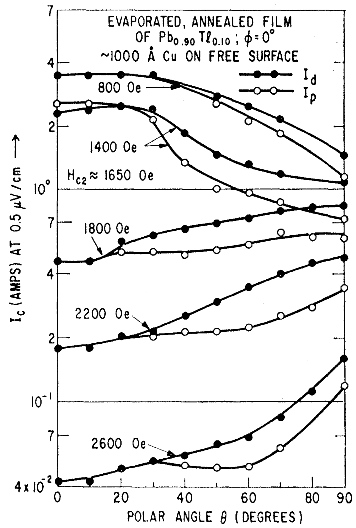

FIG. 7. The critical (surface) current at 4.2°K versus polar angle θ for an evaporated, annealed film of $Pb_{0.90}Tl_{0.10}$ with copper on the free surface. The magnetic field is in the plane of the surface ($\varphi=0°$). The results show that the critical currents and the rectification ratio do not vary in a systematic way as a function of θ.

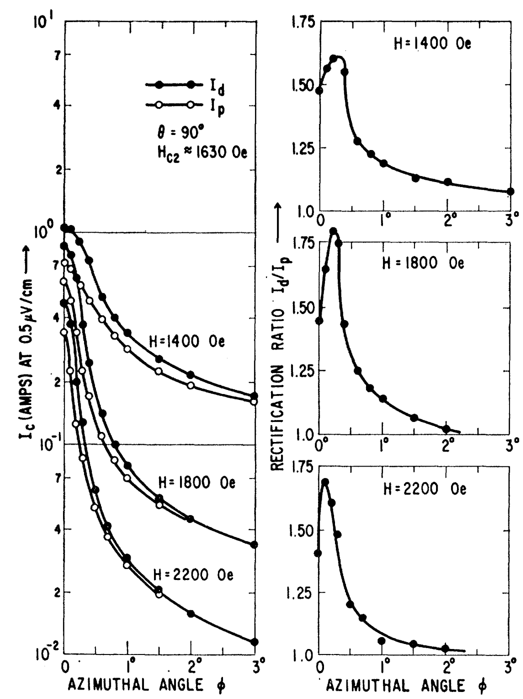

FIG. 6. The critical (surface) currents and rectification ratio at 4.2°K as a function of azimuthal angle φ for the sample of Fig. 5. The results show that the rectification ratio approaches unity as the perpendicular component of the magnetic field ($\propto\varphi$) is increased.

(iv) The rectification does not seem to be associated with the details of the edge structure of the film. On two samples the edges were trimmed and the magnitude of the critical currents of both polarities was found to decrease linearly with the width of the film while the rectification ratio was essentially unchanged (results not displayed).

(v) The rectification ratio goes to unity as the magnetic field vector, while kept in the plane of the film, is brought from $\mathbf{H}\perp\mathbf{I}$ to $\mathbf{H}\|\mathbf{I}$ (Figs. 5 and 7).

At present we can offer only a tentative explanation for the single-surface rectifier. Park[7] has calculated that the paramagnetic surface transport currents should be larger than the diamagnetic surface transport currents. We observe just the opposite. As discussed in the preceding paper,[8] however, a surface pinning model, rather than the Abrikosov[9]–Park[7] model, provides a better explanation for the surface transport currents that are measured with planar geometries. Consequently, we should not expect to confirm predictions made from the Abrikosov–Park model.

The following is offered as a tentative explanation for the rectification that we measure. It is assumed that one of Park's results[7] is pertinent to our experimental situation; namely, that the depth below the surface to which the surface sheath extends is increased by a net diamagnetic surface transport current and is decreased by a net paramagnetic surface transport current. Then the depth below the surface to which an intercepting quantized flux thread persists and the length of the flux line that is pinned is greater in the presence of a diamagnetic surface transport current than for a paramagnetic surface transport current. The Lorentz driving force acting on a single flux spot,[8]

[7] J. G. Park, Phys. Rev. Letters **15**, 352 (1965).
[8] H. R. Hart, Jr. and P. S. Swartz, preceding paper, Phys. Rev. **156**, 403 (1967).
[9] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **47**, 720 (1964) [English transl.: Soviet Phys.—JETP **20**, 480 (1965)].

$J\Phi_0$ (J is a *surface* current density or current per unit width), does not depend on the depth to which the flux spot extends below the surface. Since the force required to unpin a flux line increases with the length that is pinned, the diamagnetic transport current should be the larger of the two.

C. ASYMMETRIC FIELD RECTIFIERS

1. Principles of Operation

In this section we shall show a second means by which partial rectification of the surface transport current ($H_{c1}<H<H_{c3}$) is possible.[2,3] We make two assumptions regarding the physical properties of the superconducting surface: (1) The surface critical state model applies, i.e., each region of the surface carries its maximum or critical current (see accompanying paper[8]); and (2) the critical current density $J_c(H_\perp)$, of each surface region decreases as the absolute magnitude of the perpendicular component of the local magnetic field $H_\perp$ is increased from zero. The maximum value of the local surface critical-current density (measured in A/cm), that achieved when the local $H_\perp=0$, is designated as J_0 (Fig. 8). The second assumption is consistent with the experimental result that the critical surface transport current decreases as the perpendicular component of the applied field is increased from zero.[1,5,8] The second assumption follows from the first on simple theoretical grounds.[8]

To show how these two assumptions can lead to partial rectification in certain experimental situations, we shall concern ourselves with the distribution of current across the width of planar type-II films ($H_{c1}<H<H_{c3}$) when the maximum or critical current is reached. The assumption of a surface critical state model rules out the current distribution of a type-I film, a distribution in which only the very edges of the film support the critical current density and in which the local perpendicular field is zero across a major portion of the film (see accompanying paper[8]). With the surface-critical-state model, the maximum or critical-current density in each region of the surface is determined by the perpendicular component of the local magnetic field; in turn, the local magnetic field is in part determined by the distribution of the current. Thus, the actual current and field distributions must be determined in a self-consistent way. The perpendicular component of the local magnetic field can be written as

$$H_\perp(x)=H_{\perp a}(x)+H_{\perp d}(x)+H_{\perp i}(x), \tag{1}$$

where $H_{\perp a}(x)$ is the perpendicular field applied externally, $H_{\perp d}(x)$ is that resulting from induced diamagnetic currents, and $H_{\perp i}(x)$ is that due to the applied transport current. The diamagnetic contribution of the equilibrium magnetization of a type-II superconductor can be neglected for the present thin films and foils, though not for bulk samples (e.g., the triangular prism of Ref. 1) at low fields. When the surface critical state is reached the local perpendicular field is determined from Ampere's law using the following integral equation:

$$H_\perp(x)=H_{\perp a}(x)+P\int_{-w/2}^{+w/2}\frac{J_c[H_\perp(x')]dx'}{5(x-x')}, \tag{2}$$

where $J_c[H_\perp(x')]$ is the total self-consistent current density at x', w is the width of the film and P indicates that the principal value of the integral is to be taken. For any dependence of the local critical-current density on local perpendicular field, this integral equation can be solved numerically to yield $H_\perp(x)$, $J_c(x)$, and thus the sample critical current as a function of the applied perpendicular field distribution, $H_{\perp a}(x)$.[10]

Below we shall consider the distribution of current across the width of a superconducting ribbon for transport currents of opposite polarities and for two applied perpendicular field distributions. In the first case, the applied magnetic field is parallel to the surface(s) and $H_{\perp a}(x)=0$. In the second, $H_{\perp a}(x)$ is an odd function of x, $+H_{\perp a}(x)=-H_{\perp a}(-x)$, where $x=0$ along the center of the strip. These distributions of surface current density are calculated from the present model. In the experiments described subsequently (Sec. C 2) the integrated or total critical currents are measured, not the critical current density distributions.

Case I: $H_{\perp a}(x)=0$. The element along the center line of the two surfaces ($x=0$) always sees a local magnetic field that is parallel to the surface; consequently, the maximum current density J_0 will flow there [Fig. 9(a)]. This current produces a magnetic field that is perpendicular to all other surface elements. Because J_c decreases with increasing perpendicular field component, the current-carrying capacity of the two elements adjacent to the center line element will be less than the current J_0 that flows in the center line element.

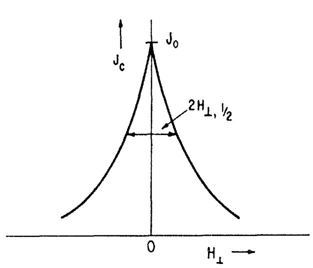

FIG. 8. A schematic of the microscopic critical surface current as a function of the local perpendicular magnetic field, as assumed in the present model.

[10] If $H_{\perp a}$ is independent of x, Eq. (2) can be written as

$$H_\perp(\eta)=H_{\perp a}+P\int_{-1}^{+1}\frac{J_c[H_\perp(\eta')]d\eta'}{5(\eta-\eta')},$$

where $\eta\equiv x/(\frac{1}{2}w)$. This equation indicates that $H_\perp(\eta)$ and thus $J_c(\eta)$ are independent of the film width w. Thus, for a planar surface exposed to a field making an arbitrary angle with the surface, as in the experiments of Refs. 1 and 5, the sample critical current should vary linearly with w. We have observed this linear variation in two experiments in which the widths of deposited $Pb_{0.90}Tl_{0.10}$ films were repeatedly reduced by trimming (results not displayed).

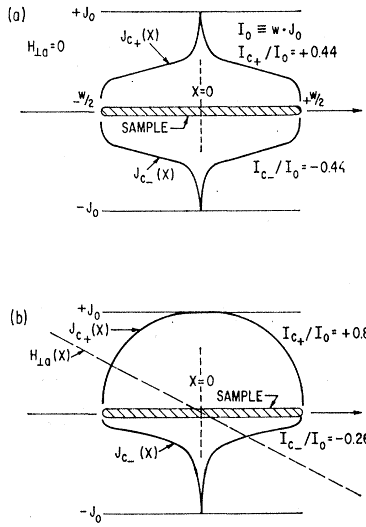

FIG. 9. (a) The calculated distribution of surface transport supercurrents in the presence of a field applied parallel to the surface when the critical condition is reached (a voltage first detected). (b) The calculated distribution of surface transport currents of opposite polarities, in the presence of a perpendicular field as shown, when the critical condition is reached. The drawing indicates that partial rectification will occur. The curves displayed are for a computer solution of Eq. (2) using $J_c(H_\perp) = J_0/(1+|H_\perp/H_{\perp,1/2}|)$ where $H_\perp$ is the local perpendicular field. The parameters J_0 and $H_{\perp,1/2}$ are appropriate for a $Pb_{0.95}Tl_{0.05}$ ribbon for $H_{||,a}=400$ Oe. For 9(a), $H_{\perp a}(x)=0$; for 9(b), $H_{\perp a}(x) = -56(x/w)$ Oe. The rectification ratio in 9(b) is 3.3:1. For more details, see Sec. C 1.

When these arguments are carried out for all elements across the surface plane, it can be seen that the maximum current flows in the center line element and that the current in all other surface elements decreases with distance from the center line element. The total transport current that a macroscopic surface will carry is the integral of $J_c(x)$ [Fig. 9(a)]. The case illustrated is a numerical solution of Eq. (2) in which the decrease in local critical current density with local perpendicular magnetic field $H_\perp$ is taken as

$$J_c(H_\perp)=J_0/(1+|H_\perp/H_{\perp,1/2}|).$$

This expression is equivalent to Eq. (7) of the preceding paper[8] for the small azimuthal angles φ of interest. The use of this expression is justified by the experimental results in Sec. C 2. In this expression $H_{\perp,1/2}$ is defined as the local perpendicular field reducing the local critical current density by one-half (Fig. 8) and is taken as 6 Oe here. The second microscopic parameter, J_0, is taken as 50 A/cm for the two surfaces combined. For the case illustrated, the macroscopic critical current density is considerably smaller than the intrinsic critical current $I_0 \equiv wJ_0$. This relative reduction of the macroscopic critical current caused by the self-field of the transport current becomes more important as the ratio $J_0/H_{\perp,1/2}$ increases. Our analysis shows that this reduction is significant for our ribbons in low magnetic fields, near H_{c1}. This same self-field effect also tends to blunt the macroscopic angular dependence at low fields (Fig. 3 of Ref. 1). The parameters chosen for the curves of 9(a) yield values for the magnitude and angular dependence of the macroscopic critical current similar to those measured experimentally on a $Pb_{0.95}Tl_{0.05}$ ribbon near H_{c1} (Fig. 3 of Ref. 1).

Rectification is not expected for the case that $H_{\perp a}=0$, as the distribution of the surface transport current across the width of the film is independent of the polarity of the transport current.

Case II: $H_{\perp a}(x)=-H_{\perp a}(-x)$ and $|H_{\perp a}(x)|$ increasing continuously with x. The surface element along the center line always sees only a magnetic field parallel to the surface, independent of the polarity of the transport current. Consequently, the maximum current density J_0 flows there. The current that flows in all other surface elements, however, is polarity-dependent. Currents in the center line element of one polarity produce a perpendicular magnetic field in all other surface elements that aids the applied perpendicular magnetic field. Currents of the opposite polarity produce a perpendicular magnetic field that opposes the applied perpendicular field in all other surface elements. The critical transport current of the polarity that reduces the magnitude of the perpendicular magnetic field will be larger than the transport current of the polarity that enhances the perpendicular magnetic field because of the dependence of the critical current on the perpendicular component of the magnetic field. The distribution of the critical current of each polarity over the width of the surface will be qualitatively similar to that shown in Fig. 9(b) and partial rectification should be observed. The curves of Fig. 9(b) represent numerical solutions of Eq. (2). The only assumption that is different between the curves of (9a) and 9(b) is the assumption in 9(b) that $H_{\perp a}(x) = (2x/w)\cdot H_{\perp a}(w/2)$, where the maximum perpendicular applied field, $H_{\perp a}(w/2)$, is 28 Oe. For these parameters a rectification ratio of 3.3:1 is predicted.

The assumed value of 28 Oe for $H_{\perp a}(w/2)$ yields an essentially maximum rectification ratio for $J_0=50$ A/cm and $H_{\perp,1/2}=6$ Oe. A smaller $H_{\perp a}(w/2)$ yields current distributions for both polarities closer to those resulting for $H_{\perp a}=0$ [Fig. 9(a)] and a smaller rectification ratio. If larger values of $H_{\perp a}(w/2)$ are chosen, the currents of both polarities are decreased near the edges of the film; the rectification ratio decreases relatively slowly with increasing $H_{\perp a}(w/2)$. For values of $H_{\perp a}(w/2)$ much larger than the optimum value of 28 Oe, the portion of the sample carrying currents approaching J_0 is reduced to a narrow strip centered at $x=0$.

The calculated maximum rectification ratio increases as the ratio $J_0/H_{\perp,1/2}$ increases. Thus, significant rectification should occur by this mechanism in the same field regions in which the self-field of the transport current reduces the macroscopic critical current relative to I_0 in the absence of an asymmetric perpendicular field.

2. Experiments

The foregoing arguments suggest a number of experiments in which partial rectification should be observed in type-II superconductors. We require only that the critical current density decrease sharply with the perpendicular field component and that there be means available for externally applying a small perpendicular field that varies across the width of the plane in the manner indicated. Two simple ways for satisfying these requirements are illustrated in Fig. 10. In both cases the bulk currents of a type-II ribbon, which tend to be insensitive to the angle between the applied field and the surface, are decreased essentially to zero by annealing. The remaining current-carrying capacity is that associated with the surfaces and is highly sensitive to the perpendicular field. In the first example the ribbon is planar, and a steady applied magnetic field is directed parallel to the surface and perpendicular to the long axis and the transport current direction. In addition, a small perpendicular magnetic field is produced by two wires laid parallel to the ribbon at its edges through which the applied control current I_{con} is equally divided. The magnitude of the perpendicular field is controlled by the magnitude of the control current. This experiment was performed with a well-annealed $Pb_{0.95}Tl_{0.05}$ ribbon ($H_{c1}\approx 350$ Oe; $H_{c2}=1030$ Oe) 0.250 in. wide and copper control wires. As I_{con} is increased, the critical transport current through the ribbon of the same polarity as the control is increased, while the critical current of the opposite polarity is decreased. The results are shown in Fig. 11 for a steady field of 400 Oe. Edwards and Newhouse[11] have previously used this experimental technique to produce rectification in evaporated type-I films. In their experiment, too, rectification comes about by the cancellation of the applied perpendicular field by self-field of the transport current. With their type-I films, however, the current distribution is edge-peaked while with type-II films ($H_{c1}<H<H_{c3}$) the current distribution is centerpeaked (Fig. 9).

We also performed an experiment in which the control current was applied through an insulated normal strip placed directly on the superconducting strip. Here, again, an appropriate perpendicular field distribution is provided by the control current and partial rectification is observed (results not displayed).

A second example is also illustrated in Fig. 10. In this case, the ribbon is curled about its long axis. The applied magnetic field is rotated about this axis. When $\varphi=0°$ (applied magnetic field parallel to the chord of the arc), the applied magnetic field can be divided into parallel and perpendicular components that satisfy the criteria set down for partial rectification. The applied

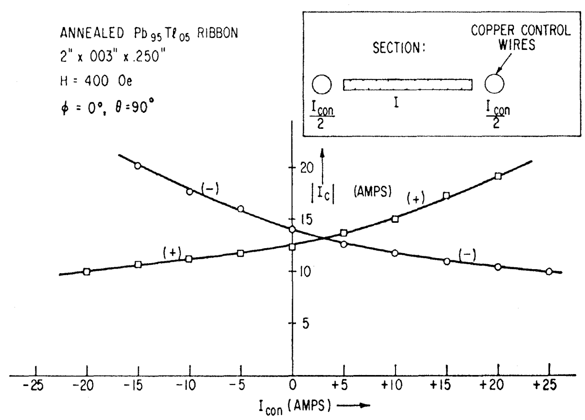

FIG. 11. The critical transport currents of opposite polarities in an annealed $Pb_{0.95}Tl_{0.05}$ ribbon versus the current in the control wires. The results show that partial rectification occurs in the presence of a perpendicular field distributed as shown in Fig. 10(c).

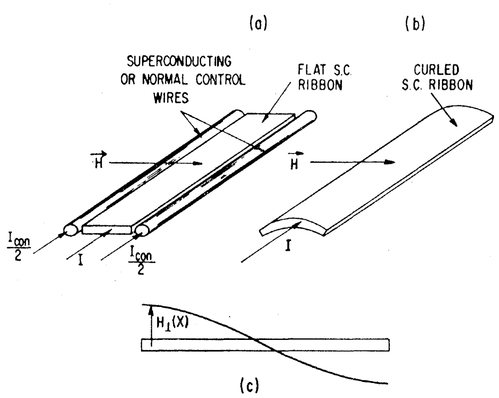

FIG. 10. A schematic showing experimental techniques for achieving a distribution of the perpendicular field as shown in (c).

perpendicular field distribution is given by $H_{\perp a}(x) \cong H_{\parallel a}x/R$. The experiment was carried out using a well-annealed $Pb_{0.95}Tl_{0.05}$ ribbon 0.240 in. wide and wrapped on an insulating cylinder of 0.39-in. radius. Between the angles $\varphi=-18°$ and $\varphi=+18°$, the applied magnetic field is tangent to some portion of the major surfaces of the ribbon. Some of the experimental results are shown in Fig. 12 for applied fields of 400 and 900 Oe. In both fields rectification begins at $\varphi=\pm 18°$, when the applied magnetic field first becomes parallel to some portion of the major surfaces of the ribbon. The rectification ratio increases until $\varphi=\pm 12°$ is reached in a field of 400 Oe and $\varphi=\pm 16°$ in a field of 900 Oe. Between these limits the rectification ratio is essentially constant. These results are significant because they show that the surface transport current flows in a well-defined band that is moved across the surface as φ is changed. At 400 Oe the angular width of the current-carrying band is $\sim 12°$, while at 900 Oe

[11] H. H. Edwards and V. L. Newhouse, J. Appl. Phys. 33, 868 (1962).

the band is reduced to $\sim 4°$. This experiment lends further support to the local surface critical-state model; if the current distribution were edge-peaked, the plateau region would not occur.

From the discussion of Sec. C 1, the truncated shape of the critical current curves in Fig. 12 can be interpreted as showing that the applied perpendicular field distribution is stronger than that required for a maximum sample critical current; the radius of curvature is too small. The experimental results of Fig. 12 are to be compared with a numerical solution [Eq. (2)] of the surface critical-state model (shown in Fig.

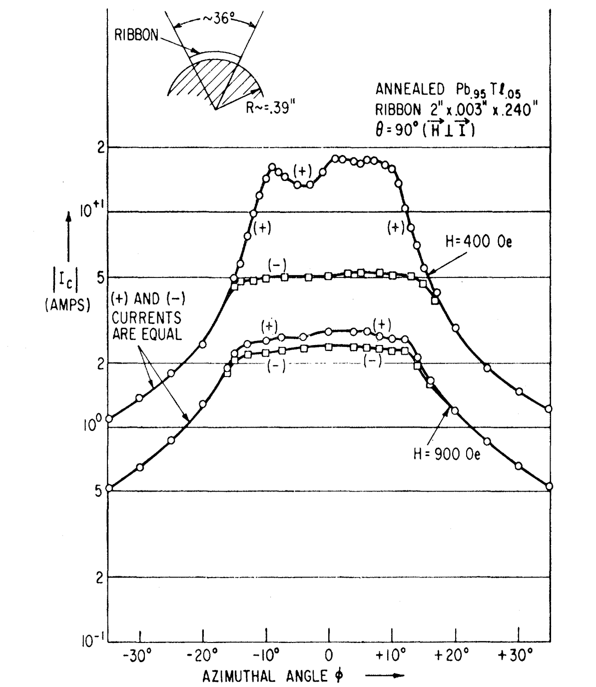

FIG. 12. The critical transport currents of opposite polarities for an annealed and curled $Pb_{0.95}Tl_{0.05}$ ribbon at 4.2°K versus the angle that steady fields of 400 and 900 Oe make with the chord of the arc. The results demonstrate that partial rectification occurs when the magnetic field becomes tangent to some portion of the major surfaces and a perpendicular field distribution as shown in Fig. 10(c) is obtained.

13) for a sample of nominally the same geometry. In this calculation it is again assumed that $J_c(H_\perp) = J_0/(1+|H_\perp/H_{\perp,1/2}|)$, where J_0 and $H_{\perp,1/2}$ are taken to be 85 A/cm (for the two surfaces combined) and 4.8 Oe, respectively. This comparison of an experimental vs. a calculated angular dependence, together with comparisons for flat films (not shown), support the surface critical-state model as characterized by the local properties shown schematically in Fig. 8. Our analysis indicates that for a flat sample in large magnetic fields, where the critical currents and self-field contributions are small, the measured macroscopic angular dependence of the critical current approaches

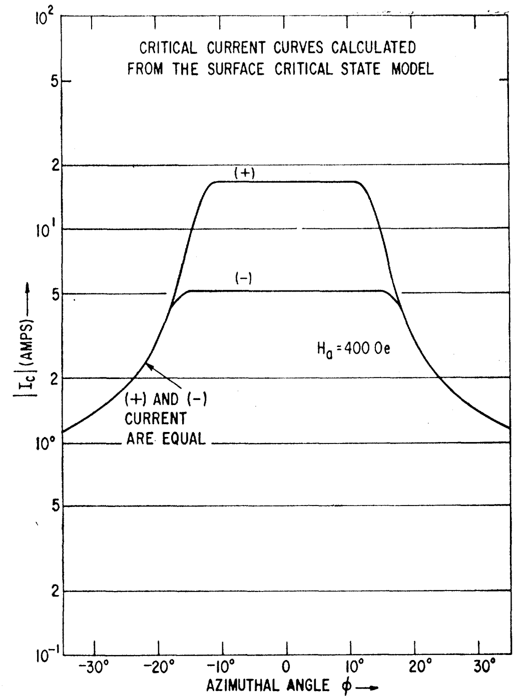

FIG. 13. Critical current curves calculated from the critical state model [Eq. (2)] for the geometric arrangement of Fig. 12 and the assumption that $J_c(H_\perp) = J_0/(1+|H_\perp/H_{\perp,1/2}|)$ where J_0 and $H_{\perp,1/2}$ are taken to be 85 A/cm (for the two surfaces combined) and 4.8 Oe, respectively.

the microscopic or local angular dependence. However, at low fields and small angles where the critical currents are large, self-field contributions lead to appreciable differences between macroscopic and microscopic angular dependences.

With the use of the techniques described in this section, we have been able to obtain rectification ratios as high as ~ 5. In contrast, the largest rectification ratios measured with the single-surface rectifiers (Sec. B) are generally ~ 1.7.

In Fig. 14 we reproduce a result from our earlier work[1] of rectification in a well-annealed triangular prism of $Pb_{0.95}Tl_{0.05}$. The current-carrying capacity fo two of the surfaces is reduced essentially to zero when the magnetic field is aligned parallel to the third surface (because of the dependence of critical current on perpendicular field). In this experimental situation rectification ratios of $\sim 3:1$ were measured. This rectification is almost certainly due to the slight curvature of the three surfaces introduced during the chemical polishing operation. Rectification comes about in the same way as it does in the curled ribbon. We have repeated the experiments with other prisms, carefully avoiding any significant curvature of the surfaces. We are then able to examine the properties of an unpaired planar surface and we find that the prism behaves as a single-surface rectifier only; above H_{c2},

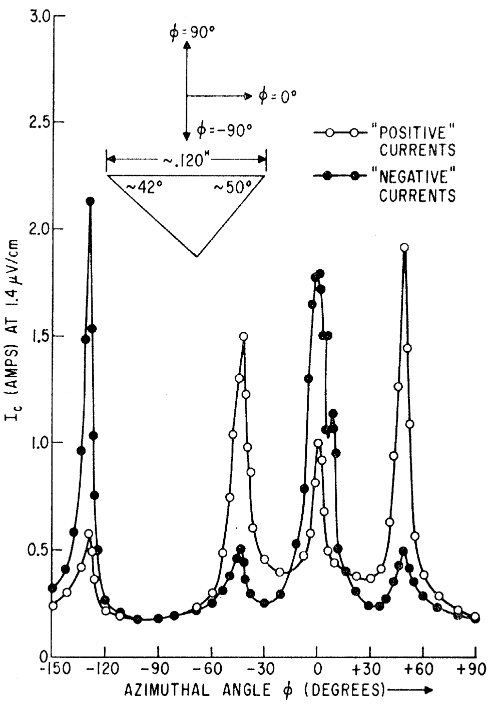

FIG. 14. Reproduced from Ref. 1. The critical current at 4.2°K of a well-annealed and polished prism of $Pb_{0.95}Tl_{0.05}$ as a function of azimuthal angle $\varphi(\mathbf{H}\perp\mathbf{I})$ at a magnetic field of 700 Oe. The results show that the critical (surface) currents of opposite polarities go through maxima of significantly different magnitudes when the magnetic field vector is aligned parallel to a surface.

where $H_{\perp d}=0$, rectification ratios in the range of 1.2:1 to 1.5:1 (results not displayed) are typically observed.

D. SUMMARY

We have described two different experimental situations in which the critical surface transport current changes in magnitude when the direction of the transport current is reversed. In Sec. B we have shown that rectification occurs in a single, unpaired superconducting surface when the applied magnetic field is directed parallel to the surface and perpendicular to the directions of the transport current. Maximum rectification ratios of $\sim 1.7:1$ are measured with single-surface rectifiers. The tentative explanation that we have given is consistent with the surface flux pinning model proposed in the accompanying paper.

In Sec. C we have shown that rectification of the surface supercurrents can also come about when the perpendicular component of the applied magnetic field is distributed across the surface in a particular way. Such rectification can be understood in terms of a surface critical-state model in which the critical current density of each local surface region decreases sharply as the local perpendicular field component is increased. With asymmetric-field rectifiers we have produced rectification ratios as large as $\sim 5:1$.

In our analysis of the asymmetric-field rectifier we have emphasized the transport supercurrent that flows on the surface. However, the analysis should also apply to bulk supercurrents, and rectification is possible if these currents are sufficiently anisotropic with field direction. Thus, rectification can perhaps be produced with rolled ribbons of hard superconductors like Nb-Zr in low applied fields since anisotropy with field direction is commonly observed.[12]

ACKNOWLEDGMENTS

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[12] E. D. Hoag, J. Appl. Phys. 36, 1183 (1965).