the loop. We calculate the probability of this occurring by using standard fluctuation theory. From this we are able to calculate the time-average resistance of the samples, and we find that while no infinitely sharp change of resistance occurs at any temperature, nevertheless, the resistance falls significantly below the normal resistance of the specimen as the temperature is lowered appreciably below the bulk T_c. A true phase transition to the superconducting state appears to be possible only in an infinite three-dimensional sample. In one dimension, if the range of the interaction force is finite, no phase transition is possible. The resistance of the one-dimensional system does approach zero, however, as $T \rightarrow 0°K$.

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Studies of Surface Transport Currents in Type-II Superconductors; a Surface-Flux-Pinning Model

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The critical surface transport current of type-II films is measured as a function of magnetic field ($H_{c1} < H < H_{c3}$), of the angle that the magnetic field makes with the surfaces, of surface condition, and of film thickness. The results show that the critical surface current (1) is much smaller than that predicted by the Abrikosov-Park model, (2) does not vary systematically with film thickness as predicted by certain recent theories, (3) *increases* as the surface is roughened, (4) decreases sharply as the perpendicular component of the applied magnetic field is increased, and (5) increases sharply as the applied magnetic field is lowered through H_{c2}. These results are interpreted as evidence for surface flux pinning, i.e. of a surface-critical-state model, rather than as evidence for any of the published theoretical models. In our model, quantized flux threads or spots intercepting the surface of the sample are pinned at surface pinning sites. When a transport current is applied, a Lorentz force is exerted on these surface flux threads or spots. At a transport current level below the intrinsic theoretical limit, the Lorentz force exceeds the pinning force; flux moves across the surface, a steady voltage is detected, and a critical surface current is thereby defined.

A. INTRODUCTION

THE large transport supercurrents supported in the mixed state ($H_{c1} < H < H_{c2}$) in hard superconductors like Nb_3Sn and Nb-Zr alloys flow predominantly through the bulk of the conductor.[1] These supercurrents exist by virtue of the interaction between the quantized magnetic flux threads that permeate the superconductor and some appropriate defect structure such as grain boundaries,[2] precipitate particles,[3] radiation damage,[4] etc.,—an interaction that inhibits the motion of the flux threads and the appearance of a voltage. It has recently been shown that the surfaces of a type-II superconductor can also support a transport supercurrent.[5–8] This effect has been demonstrated in at least two types of experiments. In the first type it has been shown that shielding surface transport supercurrents can be induced by a changing external magnetic field.[5,6] In the second type, that which we[7] and Bellau[8] perform, the ability of the surface to carry a transport supercurrent is demonstrated by applying transport currents directly to films and prisms. In both types of experiments it is found that a surface transport supercurrent will flow both in the mixed state and in the region of the Saint James and de Gennes surface film ($H_{c2} < H < H_{c3}$).[9] A

[1] For a general review, see J. D. Livingston and H. W. Schadler, Progr. Mat. Sci. **12**, 183 (1964).
[2] G. J. van Gurp and D. J. van Ooijen, J. Phys. Radium (to be published); J. J. Hanak and R. Enstrom, Air Force Material Laboratory, Wright-Patterson Air Force Base, Ohio, Technical Report No. AFML-TR-65-169, 1965, p. 86 (unpublished).
[3] J. D. Livingston, J. Appl. Phys. **34**, 3028 (1963).
[4] C. P. Bean, R. L. Fleischer, P. S. Swartz, and H. R. Hart, Jr., J. Appl. Phys. **37**, 2218 (1966); G. W. Cullen and R. L. Novak, Appl. Phys. Letters **4**, 147 (1964); S. H. Autler *et al.*, Bull. Am. Phys. Soc. **10**, 346 (1965).
[5] J. G. Park, Rev. Mod. Phys. **36**, 87 (1964); D. P. Jones and J. G. Park, Phys. Letters **20**, 111 (1966); L. J. Barnes and H. J. Fink, *ibid.* **20**, 583 (1966), also Phys. Rev. **149**, 186 (1966); H. A. Ullmaier and W. F. Gauster (to be published); M. Strongin and E. Maxwell, Phys. Letters **6**, 49 (1963); D. J. Sandiford and D. G. Schweitzer, *ibid.* **13**, 98 (1964); M Strongin, A. Paskin, D. G. Schweitzer, O. F. Kammerer, and P. P. Craig, Phys. Rev. Letters **12**, 442 (1964); M. Strongin, D. G. Schweitzer, A. Paskin, and P. P. Craig, Phys. Rev. **136**, A926 (1964); M. A. R. Leblanc, Phys. Letters **9**, 9 (1964); S. H. Goedemoed, A. Van der Giessen, D. De Klerk, and C. J. Gorter, *ibid.* **3**, 250 (1963); B. Bertman and M. Strongin, Phys. Rev. **147**, A268 (1966); R. W. Rollins and J. Silcox, Solid State Commun. **4**, 323 (1966).
[6] G. Bon Mardion, B. B. Goodman, and A. LaCaze, J. Phys. Chem. Solids **26**, 1143 (1965); L. J. Barnes and H. J. Fink (private communication); H. A. Ullmaier and W. F. Gauster (to be published).
[7] P. S. Swartz and H. R. Hart, Jr., Phys. Rev. **138**, A818 (1965).
[8] R. V. Bellau, Phys. Letters **21**, 13 (1966).
[9] D. Saint-James and P. G. de Gennes, Phys. Letters **7**, 306 (1964).

third experiment, the measurement of surface impedance at microwave frequencies,[10] can perhaps also be interpreted as evidence for a surface supercurrent.

The ability of the surfaces of a type-II superconductor to support a net superconducting transport current above H_{c1} cannot be explained in terms of the reversible, this effect has been demonstrated in at least two types nonzero magnetization that characterizes the ideal mixed state. For an illustration, consider a foil geometry with the applied magnetic field lying in the surface plane. The self-field of an impressed transport current that is orthogonal to the applied field and flows in the two foil surfaces will enhance the applied field at one surface and oppose the applied field at the opposite surface. If the sample is ideal (no bulk pinning, no surface pinning, no irreversible barrier to the passage of magnetic flux through the surface), there is only a single, unique value of the magnetic induction B for each value of the surface field $H_{c1}<H_a<H_{c2}$. Therefore, a superconducting state in which the magnetic induction B is uniform and the fields at the opposing surfaces are unequal is not an allowed state of the superconductor. No net transport supercurrent in the surfaces is possible unless some nonequilibrium state is allowed.

(These restrictions are not pertinent below H_{c1} or below the thermodynamic critical field H_c of a type-I superconductor. In these cases a magnetic induction equal to zero is allowed for all surface fields less than H_{c1} or H_c, respectively, and an impressed superconducting transport current can flow in the surfaces so long as the total field at the surface nowhere exceeds H_{c1} or H_c.)

A series of published theoretical models[11–14] predict that a particular nonequlibrium mechanism exists and that a net transport supercurrent can flow in the surfaces of type-II superconductors.[15] Our measurements (Sec. B) of surface transport supercurrents above H_{c1} in films and foils yield results not predicted by these models, results indicating that some other nonequilibrium mechanism is operative in our experiments.

Our experimental results show that the critical surface current measured in type-II films and foils: (1) is much smaller than that predicted by the Abrikosov[11]-Park[12] model; (2) does not vary with film thickness in a systematic way as predicted by Park[13]; (3) *increases* as the surface is roughened; (4) increases sharply as the magnetic field is lowered through H_{c2}; and (5) decreases as the perpendicular component of the applied magnetic field is increased.

[10] M. Cardona, J. Gittleman, and B. Rosenblum, Phys. Letters **17**, 92 (1965); G. Fischer, R. Klein, and J. McEnvoy, Solid State Commun. **4**, 361 (1966).

[11] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **47**, 720 (1964) [English transl.: Soviet Phys.—JETP **20**, 480 (1965)].

[12] J. G. Park, Phys. Rev. Letters **15**, 352 (1965).

[13] J. G. Park, Phys. Rev. Letters **16**, 1196 (1966).

[14] H. J. Fink and L. J. Barnes, Phys. Rev. Letters **15**, 792 (1965).

[15] These four theories of surface superconductivity are discussed briefly in Sec. C.

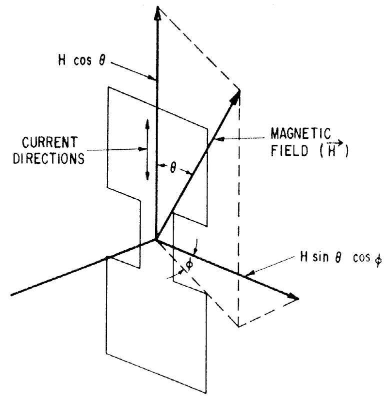

FIG. 1. The definition of the polar angle θ and the azimuthal angle φ in terms of the current **I**, the field **H**, and the sample geometry.

These experimental results have led us to introduce a surface-flux pinning model, or a surface critical state[16] model (Sec. C). In this model quantized flux threads or spots intercepting the surface of the sample are pinned at surface pinning sites. When a transport current is applied, a Lorentz force is exerted on these surface flux spots. At a transport-current level below the intrinsic theoretical limit,[11–14] the Lorentz force exceeds the pinning force; flux moves across the surface, a steady voltage is detected, and a critical surface current is thereby defined. In Sec. D we relate the experiments of Sec. B and of previous work[5–8] to our surface-pinning model.

B. EXPERIMENTS

1. Effect of Surface Roughness on Critical Current

A $Pb_{0.95}Tl_{0.05}$ ingot was rolled to 0.005 in., cut to a "dumbbell-shaped" (see Fig. 1) sample (test section 0.005 in.$\times\frac{1}{8}$ in.$\times\frac{3}{4}$ in.) and annealed *in vacuo* for 60 h at 320°C. This anneal is sufficient to remove most of the bulk current-carrying capacity.[7] The critical (surface) current was then measured three times, for three different surface preparations. Before the first measurements (A), the surface was given a partial chemical polish[7]; it reflected light well, but was slightly rough on the scale of tens of microns. Prior to the second testing (B), the sample was very briefly placed in a polishing solution to remove any possible surface contaminants, was then washed, dried, and placed in steam for about 30 sec. The steam gave the surface a uniformly "mat" appearance; the surfaces were densely pitted on a scale of a few tens of microns. The ribbon, after testing (B), was repolished to a high luster. The surfaces appeared much smoother under the microscope than prior to the first testing. The critical surface current was then measured a third time (C). In the interpretations of these measurements in Sec. D, we shall emphasize the observed changes in surface roughness rather than other possible, more subtle changes,

[16] C. P. Bean, Rev. Mod. Phys. **36**, 31 (1964).

such as local surface changes in the Ginsburg-Landau parameter κ.[1]

In each of the experiments the applied magnetic field is maintained perpendicular to the long axis of the ribbon ($\theta=90°$) and perpendicular to the direction of the transport current, while the azimuthal angle φ is varied. The critical current is defined at a voltage level of 0.5 μV/cm. The results are displayed in Figs. 2 and 3. The important features can be summarized as follows:

(i) Below H_{c2} the general effect of surface roughness is to *increase* the critical transport current (Fig. 2). At a field of 700 Oe (about halfway between H_{c1} and H_{c2}) the critical current of the very rough surface (B) is about 6 times as high as after polishing to a high luster [curve (C)]. Above H_{c2} the effect of surface roughening is more complicated, and will be discussed in Sec. D.

(ii) When the surface is roughened, the critical surface current is larger not only in a parallel magnetic field, but also in the presence of a perpendicular component of the applied magnetic field ($\varphi\neq0°$). (See Fig. 3.)

(iii) The critical current increases abruptly as the magnetic field is lowered through H_{c2} (Fig. 2). This is a rather general result that has been reported in earlier work (Refs. 7, 8, and Bertman and Strongin of Ref. 5).

(iv) The critical surface currents (at $\theta=90°$, $\varphi=0°$)

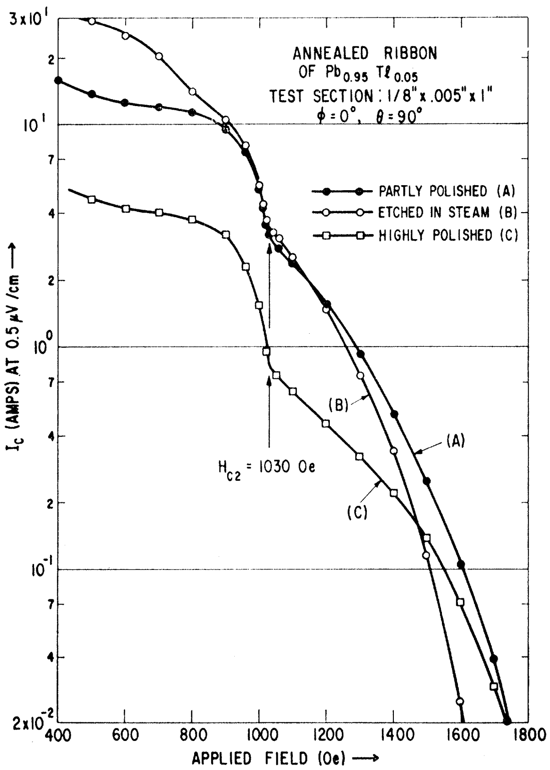

FIG. 2. The critical surface current at 4.2°K versus magnetic field for an annealed $Pb_{0.95}Tl_{0.05}$ ribbon ($\varphi=0°$, $\theta=90°$) after successive surface preparations. The results show that, except near H_{c3}, the highly polished sample carries the least current.

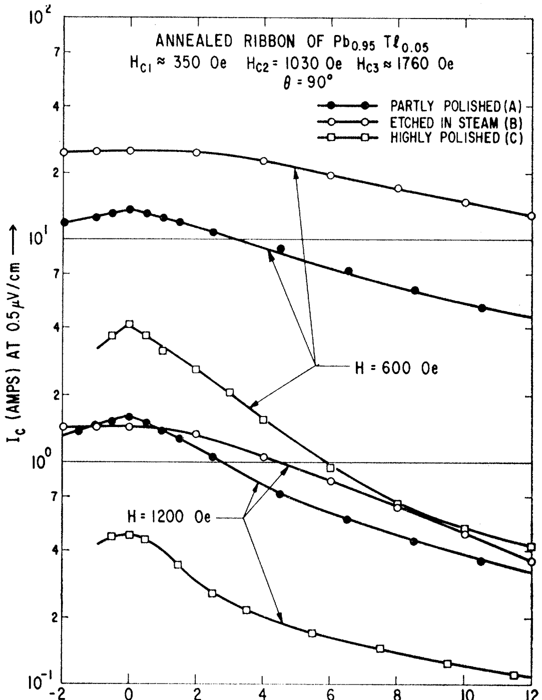

FIG. 3. The critical surface current at 4.2°K of an annealed $Pb_{0.95}Tl_{0.05}$ ribbon as a function of azimuthal angle φ. The results show that the highly polished sample is the most sensitive and the etched sample the least sensitive to the perpendicular component of the applied magnetic field.

both below and above H_{c2} are smaller by a factor of 3 to 10 than those calculated by either Abrikosov[11] or Park.[12]

2. Effect of the Perpendicular Component of the Applied Magnetic Field on Critical Current

Critical currents were measured in evaporated and annealed $Pb_{0.90}Tl_{0.10}$ films (test section 1 cm×1mm ×~1 μ) as a function of the angle that the magnetic field ($H_{c1}<H<H_{c3}$) makes with the surface plane (see Fig. 1). With evaporated films it is possible to produce surfaces that appear much smoother under a light microscope than those of rolled and polished ribbons. The critical currents that we measure in the annealed evaporated films are predominantly or wholly surface currents, for, when copper is flashed (~1000Å) on both surfaces, no supercurrent is carried above H_{c2} and the critical current between $\sim\frac{2}{3}H_{c2}$ and H_{c2} is significantly reduced. (See Sec. B of the following paper.[17]) Each of the experiments was performed with at least five separate films. The important results can be summarized as follows:

(i) The critical surface currents (Fig. 4) are much

[17] P. S. Swartz and H. R. Hart, Jr., following paper, Phys. Rev. **156**, 412 (1967).

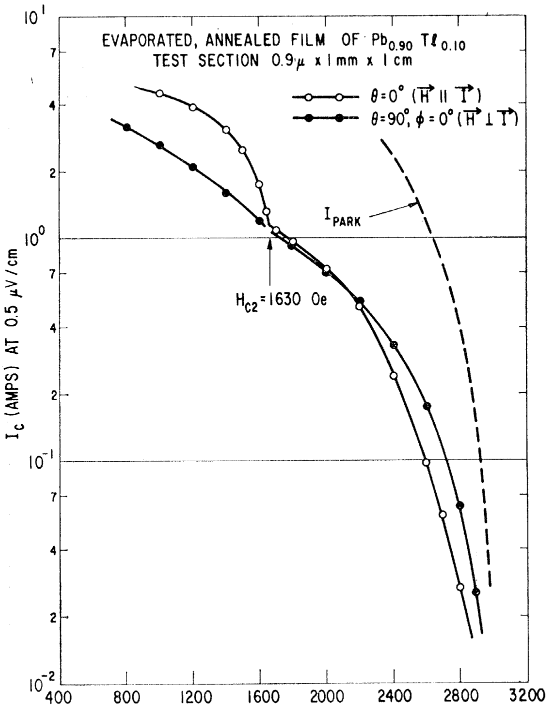

FIG. 4. The critical surface current at 4.2°K versus magnetic field for an evaporated and annealed film of $Pb_{0.90}Tl_{0.10}$. Below H_{c2} the critical current is larger when the field and current are parallel; above H_{c2}, when perpendicular. In both cases the critical surface current is much less than the calculated value (Ref. 12) (as shown).

smaller than the calculated Abrikosov[11]-Park[12] values (compared when $\theta=90°$, $\varphi=0°$).

(ii) The critical surface current decreases much more rapidly with the perpendicular component of the magnetic field (angle θ or $\varphi \approx H_\perp/H_{\parallel}$) than with the rougher rolled and polished surfaces (Fig. 5 versus Fig. 3). For typical evaporated films, the critical surface current is decreased by a factor of 2 when the applied magnetic field is turned out of the plane of the surface by only a few tenths of one degree.

(iii) When the azimuthal angle φ is held constant at 90° and the polar angle θ is varied we observe that $1/I_c \propto (\sin\theta)^{+1.0}$ for $\theta > \sim\frac{1}{2}°$ (see Fig. 6). If this relationship is written as $I_c(H)=\gamma(H)/\sin\theta$, then $\gamma(H)$ follows the general shape of the type-II magnetization curve below H_{c2} (Fig. 7).

(iv) When the polar angle θ is held constant at 90° and the azimuthal angle φ is varied, we observe that $1/I_c$ varies roughly linearly with $\sin\varphi$:

$$1/I_c = 1/I_0 + b(\sin\varphi)^r,$$

where $\sim 0.9 < r < \sim 1.2$ (see Fig. 8).

(v) When the azimuthal angle is held constant at 0°(**H**‖ the surface plane) and the polar angle θ is varied, we find that the critical current does not vary in a systematic way with θ. In general, however, the critical current either *decreases* or is unchanged above H_{c2} as θ is decreased from 90°(**H**⊥**I**) to 0°(**H**‖**I**), while below H_{c2} the critical surface current is always larger when **H** and **I** are parallel (see Fig. 4).

C. SURFACE-FLUX-PINNING MODEL

Prior to introducing our surface-pinning model we might consider briefly the four published surface-supercurrent theories mentioned in the Introduction. Abrikosov[11] has calculated the critical surface supercurrent from fields near H_{c2} up to H_{c3}. He used as an approximate solution of the Ginsburg-Landau equations a Gaussian error function centered on the surface, obtaining that solution yielding the largest surface supercurrent. Park[12], extending the earlier work of Fink[18] to include the presence of a surface transport supercurrent, obtained numerical solutions of the Ginsburg-Landau equations yielding the maximum surface transport current. The results of these first two theories are in fairly good agreement.[12] They predict that in a parallel magnetic field some critical surface supercurrent J_c can flow in the surface or, equivalently, that no magnetic flux will cross the surface until some critical field difference $\Delta H_c = 4\pi J_c/10$ exists across the surface. (Below H_{c2} this critical field difference ΔH_c, or equivalent surface current J_c, is *in addition* to the field drop at the surfaces associated with the mixed state magnetization.)

The more recent theories of Fink and Barnes[14] and of Park[13] differ from the above theories in two major ways. They include the magnetic field energy term ignored in the earlier calculations and make the additional assumption that the transition to the normal state occurs when the Gibbs free energies of the normal

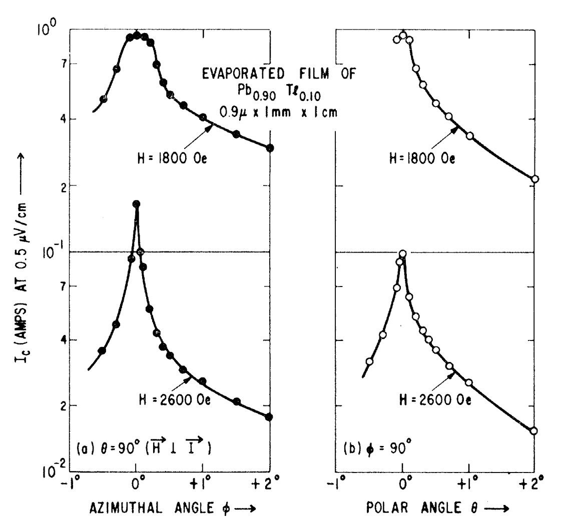

FIG. 5. The critical (surface) current at 4.2°K versus azimuthal angle φ and polar angle θ for an evaporated and annealed film of $Pb_{0.90}Tl_{0.10}$. The results show that the critical (surface) current is very sensitive to the perpendicular component of the magnetic field.

[18] H. J. Fink, Phys. Rev. Letters **14**, 309 (1965).

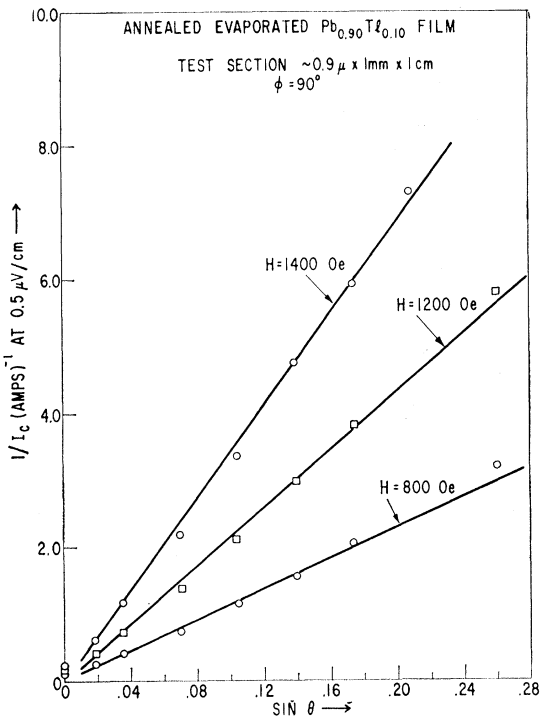

FIG. 6. A plot of the inverse of the critical (surface) current at 4.2°K versus the sine of the polar angle θ, for an evaporated and annealed $Pb_{0.90}Tl_{0.10}$ film. The results show that the inverse of the critical surface current is essentially linear with $\sin\theta$.

and superconducting states become equal. These theories do not, therefore, permit the metastable states observed with hollow type-I superconducting bodies[19] and with type-II superconductors that support large bulk supercurrents.[20] They further predict a surface-critical-current density that depends on the size of the sample. We shall return to this point later.

An assumption common to all four theories which does not apply in our experiments is the assumption

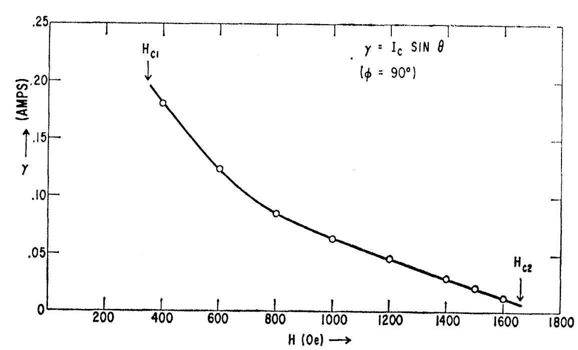

FIG. 7. A plot of $\gamma = I_c \sin\theta$ versus applied field for the film shown in Fig. 6. The result is that γ follows the general shape of the type-II magnetization curve.

[19] D. L. Coffey, W. F. Gauster, and H. E. Rorschach, Jr., Appl. Phys. Letters **3**, 75 (1963).

[20] C. P. Bean, Phys. Rev. Letters **8**, 250 (1962); Y. B. Kim, C. F. Hempstead, and A. R. Strnad, *ibid.* **9**, 309 (1962).

that the local magnetic field is everywhere parallel to the surface carrying the current; the inapplicability of this assumption has nontrivial consequences. In our experiments the local magnetic field has a nonzero component over most of the surface. This perpendicular field arises from several sources: the intentional misalignment of the applied magnetic field relative to the surfaces for many of our experimental measurements

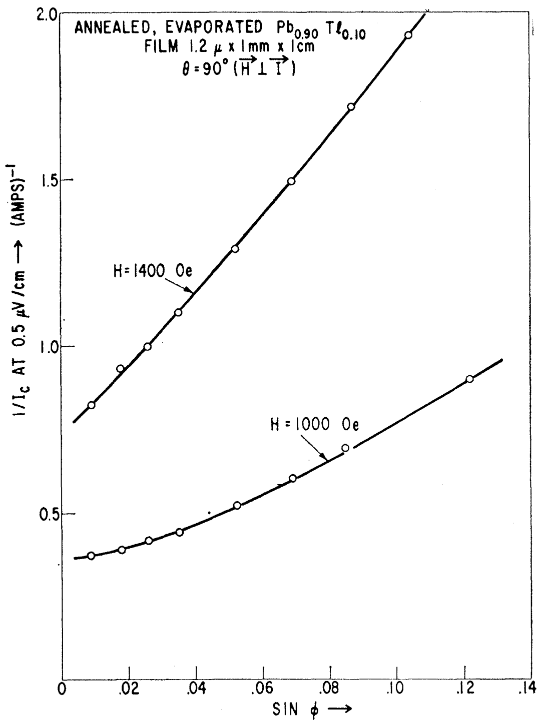

FIG. 8. A plot of the inverse of the critical (surface) current at 4.2°K versus the sine of the azimuthal angle φ, for an evaporated and annealed film of $Pb_{0.90}Tl_{0.10}$.

(i.e., for θ or φ not zero), the self-field of the applied transport current,[21–23] the demagnetizing factor of the

[21] In Sec. C of the following paper (Ref. 17) we describe the current distribution across the width of our surface sheath films, a current distribution which yields a perpendicular field component over most of the surface. It is shown that in the presence of a magnetic field ($H_{c1} < H < H_{c3}$) applied essentially parallel to the surfaces, the current is largest along the center line and smallest at the edges. This current distribution is radically different from that obtained for a thin, planar type-I superconducting film in zero magnetic field. For the type-I film the current is distributed across the film in a way which maintains a zero perpendicular magnetic field; the current is sharply peaked at the edges and small along the center line (Ref. 22).

The possibility of a similar edge-peaked current distribution for the surface sheath films in the presence of a magnetic field applied parallel to the surfaces can be ruled out in part for the following reason: We have calculated an upper limit for the critical surface current of an edge-peaked current distribution for our films using the theory of Bowers (Refs. 22, 23), together with the largest of the theoretical surface-sheath critical currents, that calculated by Abrikosov (Ref. 11). This calculated upper limit is smaller by at least a factor of 5 than the values measured in our experiments.

[22] E. H. Rhoderick and E. M. Wilson, Nature **194**, 1167 (1962).

[23] R. E. Glover and H. T. Coffey, Rev. Mod. Phys. **36**, 299 (1964).

sample, and local surface roughness. We can infer from the work of Tinkham[24] and of Giaever[25] with thin films that this perpendicular component intercepts the surface sheath as an array of quantized flux spots (or quantized flux threads, below H_{c2}).[26] The flux per spot is $\Phi_0 = 2\times10^{-7}$ G cm^2.

If flux is quantized in the surface sheath, then a surface-flux-pinning model can be developed in a straightforward way. Our model is a direct analogy to the flux-pinning model for quantized flux threads that permeate the body of type-II superconductors.[27,28] We propose that the free energy of individual flux spots or flux threads in the type-II surface sheath is sensitive to certain local properties of the sheath itself; it these local properties are spatially varying, then likewise the free energy of the flux spots will be spatially varying. The consequences of such a spatially varying free energy are several.[27] The individual flux spots will reside or be pinned at those locations where their free energy is a minimum. To set the flux-spot or flux-thread array into motion (and thus observe a steady voltage), the force by which each flux spot is pinned must be overcome by some driving force. A familiar example of such a driving force is the Lorentz force that results from the application of a transport current to a sample placed in a magnetic field. When surface pinning is absent, the critical surface current is zero; only when the surface flux spots or threads are pinned (or, in special cases, when a barrier to flux spot nucleation is effective; see Sec. D) is a nonzero critical transport current possible. This critical current, then, reflects the strength of surface-pinning sites and can be much smaller than that predicted by the parallel-field models discussed earlier.[11–14]

Giaever's results[25] on magnetic coupling between thin films support this line of reasoning. In his experiments, the Lorentz force associated with the transport current acts against a pinning force. At some current level the Lorentz force exceeds the pinning force, the quantized flux-spot array moves, and a voltage is detected in both the primary and the secondary.

The concept of a surface critical state, analogous to the bulk critical state,[15] follows if the surface transport supercurrent is determined by flux pinning. A local surface region is said to be in the critical state when it carries its maximum or critical-surface-current density. A *sample* critical surface current is reached only when the entire flux-spot array can move as an entity—only when the local surface transport supercurrent is everywhere equal to the local critical supercurrent, i.e., when every surface region is in the critical state.

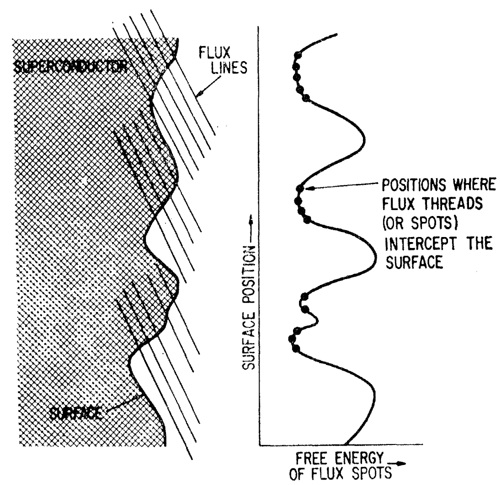

FIG. 9. On the left is a schematic showing magetic flux intercepting a rough superconducting surface. The corresponding schematic on the right shows the free energy of the flux spots as a function of their surface position.

Surface roughness on a scale comparable to or larger than the spacing between flux spots ($d \approx [\Phi_0/(B\sin\theta\sin\varphi)]^{1/2}$) is one possible source of the spatial variation of the free energy of flux spots that is required for pinning. Consider a flux spot moving along such a surface. It if is assumed that the flux spot, or the vortex current associated with the flux spot, can be treated as a magnetic dipole moment $\boldsymbol{\mu}$ normal to the local surface plane, there will be a spatially varying term in the free energy, $-\boldsymbol{\mu}\cdot\mathbf{H}$. The flux spot (or thread, below H_{c2}) will be pinned at those locations where $-\boldsymbol{\mu}\cdot\mathbf{H}$ is a minimum (see Fig. 9). As the experiments in Sec. B have shown, surface roughness can lead to an enhanced critical surface current.

It is quite difficult to develop a microscopic model of the dependence of the critical current on the perpendicular component of the applied magnetic field; however, some simple first steps can be taken. The force per unit area acting on a surface carrying a current in a field is the Lorentz force $\mathbf{J}\times\mathbf{B}$, where J is a *surface* current density (A/cm). This Lorentz force acts on the flux-spot array just as the Lorentz force acts on the flux threads that permeate the bulk in the mixed state. If we assume initially that the perpendicular component of the applied field over the surface is large compared to the perpendicular component of the self-field associated with the surface transport current, then the number of flux spots per unit surface area N is given by

$$N = (B/\Phi_0)\sin\theta\sin\varphi, \tag{1}$$

and the Lorentz force per flux spot is

$$F_d = \frac{\mathbf{J}\times\mathbf{B}}{N} = \frac{\mathbf{J}\times\mathbf{B}\Phi_0}{B\sin\theta\sin\varphi} = \frac{J\Phi_0}{\sin\varphi}. \tag{2}$$

[24] M. Tinkham, Phys. Rev. **129**, 2413 (1963); Rev. Mod. Phys. **36**, 268 (1964).
[25] I. Giaever, Phys. Rev. Letters **15**, 825 (1965); I. Giaever, *ibid.* **16**, 460 (1966).
[26] Between H_{c1} and H_{c2} magnetic flux is quantized through the bulk as well as in the surfaces; the flux spots in the surface sheath extend throughout the bulk as flux threads or current vortices.
[27] P. W. Anderson, Phys. Rev. Letters **9**, 309 (1962); P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39 (1964).
[28] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).

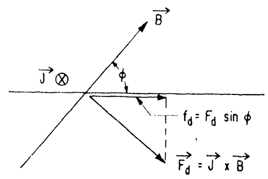

FIG. 10. A schematic showing the direction and the magnitude of the force exerted on a surface flux spot in the presence of a transport current.

The component of this force that drives a flux spot across the surface is given as (see Fig. 10)

$$f_d = F_d \sin\varphi = J\Phi_0. \tag{3}$$

The force that opposes this driving force is the pinning or restoring force f_p. In order that we may write an expression for this pinning force we assume that there are m pinning centers per unit area, each of strength $A(H,T,\theta,\varphi)$. Then if $N \gg m$, so that each pinning center is shared among many flux spots (see Anderson[27] and Friedel, De Gennes, and Marticon[29] for analogous arguments for the bulk pinning of flux threads), then we can write for the pinning force per flux spot

$$f_p = \frac{m}{N} A(H,T,\theta,\varphi) = \frac{mA(H,T,\theta,\varphi)}{B\sin\theta\sin\varphi}\Phi_0. \tag{4}$$

Flux will move and a voltage will be measured when $f_d > f_p$. In this way we obtain a surface critical current density J_c;

$$J_c = mA(H,T,\theta,\varphi)/B\sin\theta\sin\varphi\,. \tag{5}$$

If the pinning strength A is only weakly dependent on the angles φ and θ (i.e., the perpendicular field component), we can write for fixed field and temperature

$$1/J_c \propto \sin\theta\sin\varphi. \tag{6}$$

Certain of the assumptions used in deriving expression (6) break down when either θ or φ approaches zero. At very small θ or φ the number N of flux spots (or threads) intercepting the surface may depend either on the flux trapped by surface pinning sites during the previous magnetic history of the sample or on the self-field of the transport current itself. In either event Eq. (1) is incomplete. Also, if the surface is very rough, flux will intercept local elements of the surface even when the applied magnetic field is parallel to the gross surface place ($\theta=0$ or $\varphi=0$); Eq. (1) again fails, the sample critical current will saturate as θ or φ approaches zero at a value J_0 determined by the ratio of mA/B to the average spread in local angle. Equation (6) can fail at very small θ or φ if the local magnetic field is so nearly parallel to the surface that N is small compared to m.[27] Here Eq. (4) is not valid and the critical surface current will again saturate as θ or φ approaches zero. This saturation of J_c at very small θ or φ suggests that the sample critical current will vary roughly as

$$1/I_c = 1/I_0 + b\sin\theta\sin\varphi. \tag{7}$$

D. DISCUSSION

1. Magnitude of the Critical Surface Current

The critical surface current ($H_{c1} < H < H_{c3}$) that we (Sec. B and Ref. 7) and others[5–8] measure is lower than that predicted by the Abrikosov[11]-Park[12] model by a factor of 3–10. This result is consistent with a surface-flux-pinning model. If the surface pinning sites are few and weak, the critical surface current is very small; only if surface pinning is very strong will the critical surface current approach its theoretical limit.[11,12]

2. Size Dependence of the Critical Current

As mentioned in Sec. C, Fink and Barnes,[14] and more recently Park,[13] have introduced new models that yield critical surface supercurrents which depend on the sample size. We have already emphasized that neither these theories nor the earlier theories[11,12] pertain to our experimental situation because of the assumption in the theories that the magnetic field is everywhere parallel to the current-carrying surface. There is, however, an additional reason for ruling out the two recent theories[13,14] in our experimental situation. These theories calculate the critical transport current with the assumption that a superconducting-to-normal transition takes place when the maximum surface supercurrent is reached. In our experiments no superconducting-to-normal transition takes place when the critical surface transport current is reached; instead, the sample goes from a superconducting state in which no flux moves to a superconducting state in which flux flows and a steady voltage is observed (see Figs. 10 and 12 of Ref. 7, and Refs. 25 and 28).

Disregarding for the moment the general objections to the applicability of these recent theories to the present experiments, let us note the prediction[13] that the critical surface current should vary with sample thickness as $t^{-1/2}$. We have measured the critical current of evaporated and annealed $Pb_{0.90}Tl_{0.10}$ films over a thickness range of 2×10^{-5} cm to 3×10^{-4} cm. We have also measured the critical current of rolled and annealed $Pb_{0.90}Tl_{0.10}$ ribbons in the thickness range of 5×10^{-5} cm to 2×10^{-2} cm. In these experiments encompassing a thickness range of $\sim10^3$: 1, no systematic varia ion in the critical surface transport current was observed. Park's calculations[13] implicitly assume the presence of a ground plane since the samples are of finite width; ground planes were not used in most of our experiments. In one set of measurements in which a (niobium) ground plane was used (sample thickness 0.005 in.), the critical surface current was enhanced by $\sim$20–50%. This enhancement is in agreement with the surface critical

[29] J. Friedel, P. G. de Gennes, and J. Matricon, Appl. Phys. Letters **2**, 119 (1963).

state model. (See the analysis given in Sec. C of the following paper.[17])

3. Effect of Surface Roughness

In Sec. B (Figs. 2 and 3) we have shown that in our foils the effect of surface roughness on a scale greater than the spacing between surface flux spots is to *increase* the critical surface current below H_{c2}. Above H_{c2} the effect of surface roughness is more complicated. As the surface is roughened slightly [curve (A) versus curve (C) of Fig. 2], the strength of surface pinning and the critical currents increase at all fields between H_{c2} and H_{c3}. If the surface is made *very* rough [curve (B)] the apparent H_{c3} is decreased and the critical current near H_{c3} is likewise decreased. This decreased critical current, or apparent shifting of H_{c3}, is probably due to the fact that with the much rougher surface there are virtually no continuous paths sufficiently parallel to the applied field to be capable of supporting a transport supercurrent up to the limiting field, $H_{c3}=1.69\ H_{c2}$.[9] Our results also show that when the surface is roughened the critical surface current is larger not only in a parallel magnetic field, but also in the presence of a perpendicular component of the applied magnetic field ($\varphi\neq 0°$). (See Fig. 3.)

Bellau[8] has made similar measurements of the critical surface current with rectangular samples of $Nb_{0.55}Ta_{0.45}$. He also finds that below H_{c2} the critical current *increases* as the surface is roughened. Above H_{c2} he finds an apparent downward shift of H_{c3} and a decreased critical current as the sample is roughened.

Bellau also reports that the critical surface current is increased both below and above H_{c2} as the samples are allowed to sit in the cryostat. We have observed the same effect with our polished Pb-Tl ribbons as they are allowed to tarnish (results not displayed). These results suggest that surface contaminants can act as effective flux pinning sites.

Our observation of an increase in critical current with roughness agrees with the recent measurements of Jones and Rose-Innes,[30] who found that if the surface of a cylindrical wire is roughened on two opposite sides, then the critical current is a maximum when the applied field is parallel to these two areas. The finding of Niessen *et al.*,[31] that magnetic flux tends to move in directions parallel to surface furrows, can also be interpreted in terms of surface pinning.

These various experimental results regarding the effects of surface roughness support the arguments in Sec. C that surface roughness can be one source of the spatial variation in the free energy of flux spots or threads required for flux pinning.

The effect of surface roughness has also been studied below H_{c2} in the type of experiment in which surface supercurrents are induced by a changing external magnetic field[6]; here it is found that the critical surface transport current below H_{c2} *decreases* as the surface is roughened. Though it is difficult to reconcile these apparently conflicting observations, it may be appropriate to note that the critical surface current (or magnetic hysteresis associated with the surface currents) can be controlled not only by surface flux-spot pinning, but also by barriers to the nucleation of new flux spots. An extreme example of a nucleation barrier is the Bean-Livingston surface barrier[32] for the nucleation of the first flux thread that enters the bulk of a type-II superconductor (above H_{c1}). The same barrier can also inhibit the exit of a flux thread from the sample. With such a surface barrier, the smoother the surface the harder the nucleation of the flux thread or spot and thus the larger the surface critical current.[33] A nucleation barrier (rather than surface pinning) may control the critical-surface-current density or hysteresis for cylindrical samples exposed to changing external magnetic fields. The planar geometry of our films or foils ensures easy flux-spot nucleation at the edges; surface flux-spot pinning rather than the nucleation of flux spots controls the surface-critical-current density. The resolution of the contrasting effect of surface roughness in the two types of experiments may require further careful experiments in which the same surface treatment is studied for several geometries and for both experimental techniques.

4. Change in Critical Surface Current at H_{c2}

A quite general experimental result (Fig. 2, Refs. 7, 8, and Bertman and Strongin of Ref. 5) is the abrupt increase in critical surface current with field as the field is lowered through H_{c2}. The effects of surface smoothness (Fig. 2) and of copper plating (Fig. 2 of the following paper[17]) on the abrupt change in critical current at H_{c2} show that this jump is predominantly a surface property. Note that in these figures the current scale is logarithmic. None of the four theories of critical surface current[11–14] predicts this result. This abrupt increase in critical surface current can perhaps be understood in terms of the surface-pinning model. As the field is lowered through H_{c2}, the bulk becomes superconducting and the surface flux spot or surface vortex current is extended throughout the bulk of the superconductor as a flux thread or tube of vortex current. In this way a greater length of flux line can interact with surface pinning sites, leading to an enhanced pinning force. Consequently, a larger driving force, or a larger surface transport current, is needed to set the flux-spot array into motion. As an

[30] R. G. Jones and A. C. Rose Innes, Phys. Letters **22**, 271 (1966).
[31] A. K. Niessen, J. van Suchtelen, F. A. Staas, and W. F. Druyvesteyn, Philips Res. Rept. **20**, 226 (1965).
[32] C. P. Bean and J. L. Livingston, Phys. Rev. Letters **12**, 14 (1964).
[33] R. W. DeBlois and W. DeSorbo, Phys. Rev. Letters **12**, 499 (1964); A. S. Joseph and W. J. Tomasch, *ibid.* **12**, 219 (1964).

example, the dipole moment associated with the vortex current of a flux thread is larger than the dipole moment associated with a flux spot. Consequently, the spatial variation across a rough surface of the free energy of the flux thread below H_{c2} will be larger than the spatial variation of the free energy of the flux spots above H_{c2}. A larger Lorentz force is thereby required to move flux below H_{c2} and a larger critical surface current is predicted.

5. Effect of the Perpendicular Component of the Applied Magnetic Field

The results of Sec. B show that $1/I_c \propto (\sin\theta)^{+1.0}$ for $\theta > \sim\frac{1}{2}°$ and $\varphi = 90°$, and that $1/I_c = 1/I_0 + b(\sin\varphi)^{+r}$, where $\sim 0.9 < r < \sim 1.2$ for $\theta = 90°$. These results are in satisfactory agreement with Eq. (7).

E. SUMMARY

In this paper we have introduced a surface-flux-pinning, or surface-critical-state, model to interpret our experimental results. The argument is made that the critical surface transport current is reached in a planar type-II superconducting foil ($H_{c1} < H < H_{c3}$) when the Lorentz force exceeds the force by which quantized flux spots (or threads, below H_{c2}) are pinned at surface pinning sites; thus, the maximum theoretical limiting surface transport current is not attained.

The published surface models are inadequate in at least two important respects. The critical currents that we and others have measured are much smaller than those predicted by the parallel field models.[11,12] Also, we observe no systematic dependence of the critical surface current on foil thickness, whereas the prediction is made[13] that the critical surface current should vary with thickness as $t^{-1/2}$.

A number of arguments and experimental results support a surface-flux-pinning model. In our experiments there is always a quantized flux-spot or flux-thread array in the surfaces. Consequently, in the absence of surface pinning, flux will move across the surface and a voltage will be measured as soon as the transport current is made nonzero. We presented arguments that surface roughness is one source of the spatial variation in the free energy of flux spots (threads) that is required in any flux-pinning model. Our experimental findings show that the critical surface transport current increases as the surface is roughened. Also, the fundamental dependence of the critical surface transport current on the perpendicular component of the applied magnetic field is found to be close to that predicted. Finally, the experimental result that the critical surface transport current increases sharply as the magnetic field is lowered through H_{c2}, though not explained by the parallel field models, is consistent with the surface-pinning model.

Experiments performed by others show that the critical surface transport current induced by a changing external magnetic field decreases as the surface is roughened. We have suggested that the contrasting effects of surface roughness in these two experiments may be explained through the different geometries. The limiting surface transport current can be determined either by the nucleation of quantized flux spots or threads at the surface or by the pinning of the flux spots or threads that intercept the surface. In the cylindrical geometries where the surface currents are induced by a changing magnetic field, nucleation at the surface may be limiting. In our planar film experiments, nucleation at the film edges should be easy, and surface pinning is limiting.

Note added in proof: H. J. Fink has recently presented a theory for the critical surface current of a foil of finite width placed in a magnetic field parallel to the major foil surface [Phys. Rev. Letters **17**, 696 (1966)]. In this theory, he uses the physical principles introduced by Fink and Barnes.[14] Fink predicts, as we observe (Sec. D), that the surface critical current does not depend on sample thickness. He furthermore predicts magnitudes of critical currents reasonably close to those we observe, in marked contrast to the earlier theories.[11,12] However, he calculates that the critical current is proportional to the square root of the width of the foil; our limited experiments (data not shown) indicate that except very near H_{c3} the critical current is linearly proportional to the width of the foil. Very near H_{c3} we find that the critical current increases less rapidly than linearly with foil width. Our criticisms (Sec. C) of the applicability of the Fink and Barnes[14] and Park[13] theories are also pertinent to this recent paper of Fink.

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