We have set $\delta\mu_2^2$ equal to zero. In the zero-scattering limit $X^{\rho}{}_{\lambda\tau}$ is

$$X^{\rho}{}_{\lambda\tau}=I\sum_{\sigma} C^{\rho}{}_{\lambda\sigma}C^{\rho}{}_{\tau\sigma}\,,$$

with I defined in (32). The $X^{\rho}{}_{\lambda\tau}$ are combined according to Eq. (31) to give Eq. (34).

APPENDIX B

Here we estimate the effect of a small, slowly varying phase shift on the α's. Let the phase shift δ be given by

$$\delta=\lambda[X^{3/2}/(X^2+\epsilon^2)]\,, \tag{B.1}$$

where $X=W-m-\mu$. Such a phase shift has correct threshold dependence and vanishes at infinite energy. The parameters λ and ϵ can be written in terms of δ_m, the maximum value of δ, and X_m, the position of that maximum.

$$\epsilon=X_m/\sqrt{3}\,,$$
$$\lambda=2\times3^{-3/4}\epsilon^{1/2}\delta_m. \tag{B.2}$$

Then, if $\Omega(X)$ is written as $\exp\Delta(X)$, we find that[17]

[17] *Tables of Integral Transforms*, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1954), Vol. II, p. 216.

$$\Delta(X)=\frac{\lambda(\epsilon/2)^{1/2}(X+\epsilon)}{X^2+\epsilon^2}+i\frac{\lambda X^{3/2}}{X^2+\epsilon^2}. \tag{B.3}$$

For simplicity we take X_m equal to $\sqrt{3}\mu$ or $\epsilon=\mu$. By a simple analytic continuation of (B.3) we obtain

$$\Delta(-\mu)=\lambda/2\mu^{1/2}. \tag{B.4}$$

Equations (B.3) and (B.4) are combined to give

$$|\Omega(X)/\Omega(-\mu)|^2=\exp\left[(\tfrac{4}{3})^{3/4}\delta_m\left(\frac{\mu(X+\mu)}{X^2+\mu^2}-\frac{1}{\sqrt{2}}\right)\right]. \tag{B.5}$$

The maximum magnitude of the exponent occurs for $X=\mu(\sqrt{2}-1)$. If in Eqs. (32) and (33) $|\Omega(X)/\Omega(\mu)|^2$ is replaced by its value at this point, an upper or lower bound α_m can be obtained for α depending on the sign of δ_m.

$$\alpha_m=-1+\exp[\tfrac{1}{2}(\tfrac{4}{3})^{3/4}\delta_m]. \tag{B.6}$$

The signs of α_m and δ_m are the same. In Table I we list some values of α_m and δ_m obtained from Eq. (II.6). The actual values of α may differ substantially from these bounds, though they will have the same sign.

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Some Consequences of $SU(3)$ and Charge-Conjugation Invariance for K-Meson Resonances*

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Implications of invariance of strong interactions under $SU(3)$ and charge conjugation are investigated for K-meson resonances (i.e., systems which are not eigenstates of C). It is pointed out that the charge-conjugation eigenvalue of the neutral nonstrange members of multiplets is determined by the decays (mesonic or electromagnetic) of the strange members. The $K'(1320)$, $K'(1420)$, and $K'(1800)$ are considered as examples. Possibilities for study of symmetry breaking and particle mixing in kaon resonance decays are mentioned; they may provide practical methods for studying symmetry breaking.

WE would like to point out some simple, but apparently not well-known, properties of mesons with nonzero strangeness. These properties follow whenever charge-conjugation invariance and $SU(3)$ invariance are supposed to hold. They may prove useful in the study of symmetry-breaking effects in $SU(3)$.

Assume that all the kaons we will consider are to be assigned to $SU(3)$ octets, and, throughout, let C be the charge-conjugation eigenvalue of the neutral, nonstrange (NNS) members of the octet. It is then possible to show quite simply that:

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(a) Two kaons belonging to multiplets whose NNS members have opposite C cannot mix in the limit of $SU(3)$ symmetry, but they can mix whenever symmetry breaking is present.

(b) From the decay branching ratios of kaon resonances, it is possible to determine the C eigenvalue of the NNS members which can be placed in the same multiplet as the kaons. It should almost always be possible to do this in practice, with considerable restriction on one's freedom in making up meson multiplets.

(c) From the electromagnetic decays of kaon resonances it is also possible to determine the C eigenvalue of the NNS members which can be placed in the same multiplet as the kaons. In addition, the ratio of the neutral kaon to charged kaon electromagnetic decays is quite different for kaons from the same ratio for opposite C multiplets, and may provide a sensitive measure of $SU(3)$-breaking effects. If $C'=+1$, one expects $K'^0\to K^0+\gamma$ to be forbidden while $K'^+\to K^++\gamma$ is fully allowed; while if $C'=-1$, one expects a branching ratio $(K'^0\to K^0+\gamma)/(K'^+\to K^++\gamma)=4$.

In the following we will denote $K(495)$ by K, $K^*(890)$ by K^*, and other strange mesons by $K'(M)$. We will consider $K'(1320)$, $K'(1400)$, and $K'(1800)$ in some detail. An arbitrary strange meson will be denoted by k.

First we pose the question: Can two k's which belong to $SU(3)$ multiplets whose NNS members have opposite C eigenvalues show mixing effects? The answer is that they cannot in the limit of $SU(3)$ invariance, but that they can, in general, whenever $SU(3)$ is broken.

Questions such as these may be relevant in practice if a 16-plet of mesons of the same spin but consisting of two octets of opposite C is observed. The mixing will modify both the mass spectrum and the decays of the mesons involved. Even if no practical applications should arise, the study of possible particle-mixing effects is of some interest and provides a good example of the techniques we shall use.

We use a U-spin, V-spin formalism.[1] In terms of the I-spin singlet (I_0) and I-spin neutral vector member (I_1) we can write the U-spin and V-spin singlets and neutral vector members as

$$U_1^0=(\sqrt{3}I_0^0-I_1^0)/2\,,\quad U_0^0=(\sqrt{3}I_1^0+I_0^0)/2\,,\qquad(1)$$

$$V_1^0=(I_1^0+\sqrt{3}I_0^0)/2\,,\quad V_0^0=(-\sqrt{3}I_1^0+I_0^0)/2\,,\qquad(2)$$

and the V-spin and U-spin states are related by

$$V_1^0=(\sqrt{3}U_0^0+U_1^0)/2\,,\quad V_0^0=(\sqrt{3}U_1^0-U_0^0)/2\,.\qquad(3)$$

Finally, we take the photon to transform as

$$\gamma=U_0^0=(\sqrt{3}V_1^0-V_0^0)/2=(\sqrt{3}I_1^0+I_0^0)/2\,.\qquad(4)$$

Next, define the quantities analogous to G parity for U spin and V spin, G_U and G_V, respectively. The U-spin multiplets are $(k^0,U_1,\bar{k}^0)$ and U_0; the V-spin multiplets are (k^-,V_1,k^+) and V_0. Just as for G parity, we have $G_U=C(-1)^U$ and $G_V=C(-1)^V$, where C *is the charge-conjugation eigenvalue of the NNS member of the multiplet.*

To study mixing we consider $M=\langle k'|O|k\rangle$, where k' and k belong to two different octets and O is any operator. The mixing is of course zero unless k' and k have the same spin. Inserting $1=G_U^{-1}G_U$ for neutral k's and noting that $G_U|k\rangle=-C|k\rangle$, $G_U|k'\rangle=-C'|k'\rangle$, we have

$$M=CC'\langle k'|G_UOG_U^{-1}|k\rangle\,.$$

Now, if O is an $SU(3)$-invariant operator, then $G_UOG_U^{-1}=O$ and we find $M=CC'M$ so that $M=0$ if $C\neq C'$. If O is not $SU(3)$ invariant, we can use the operator form $G_U=Ce^{-i\pi U_y}$ (analogous to $G=Ce^{-i\pi I_y}$ for G parity) to evaluate $G_UOG_U^{-1}$. For octet symmetry breaking, for example, $O\sim\lambda_8\sim I_0\sim U_0+\sqrt{3}U_1$. Since $e^{-i\pi U_y}U_0e^{i\pi U_y}=U_0$ while $e^{-i\pi U_y}U_1e^{+i\pi U_y}=-U_1$, $G_UOG_U^{-1}$ is not simply related to O and we cannot prevent mixing between k' and k to lowest order in the symmetry breaking. (An Ademello-Gatto–type theorem[2] which would make the mixing higher order in the symmetry breaking is apparently not possible, because the k's are in different multiplets and we cannot construct any conserved quantities from them.) The possibility that two meson octets with the same spin but opposite C will be observed, with kaon members whose masses and decay rates are modified by mixing effects, should be kept in mind when $SU(3)$ assignments are attempted for meson resonances. We will not consider such a situation any further in the following.

Next consider the problem: From the properties of a given k can we tell whether the NNS members which can be associated with it in an $SU(3)$ multiplet have even or odd eigenvalues under C. The answer is yes, and it appears that a study of this question leads to some useful comments on kaon resonances.

Consider the decays $K'^+\to K^++\pi^0$ and $K'^+\to K^++\eta$. The actual decay which occurs must [assuming $SU(3)$ invariance] be the decay $K'^+\to K^++V_n$, where $n=0$ if $C'=+1$ and $n=1$ if $C'=-1$. This is because K'^+ has $G_V=-C'$, K^+ has $G_V=-1$, so V_n must have $G_V=C'$. Since $G_V=-1$ for V_1, $G_V=+1$ for V_0, we have finally $C'=(-1)^n$. Thus from Eq. (2) we find that

$$C'=+1\Rightarrow(K'^+\to K^++\pi)/(K'^+\to K^++\eta)=3\,,$$
$$C'=-1\Rightarrow(K'^+\to K^++\pi)/(K'^+\to K^++\eta)=\tfrac{1}{3}\,.$$

If we consider decays $K'\to K^*+\pi$ we have $G_V{}^*=+1$, so the results are just reversed.

For the pseudoscalars we have ignored $\eta-\eta'$ mixing (see below). For the decays $K'\to K+\rho$, ω, we should include such effects. For the decay which requires V_1 this is straightforward if we ignore the fact that the mixing breaks $SU(3)$, as we simply put $V_1\simeq(\rho+\sqrt{3}\omega_8)/2=(\rho+\omega+\sqrt{2}\varphi)/2$, where we have used $\omega_8\simeq(\sqrt{\tfrac{1}{3}})\omega+(\sqrt{\tfrac{2}{3}})\varphi$. Thus we get

$$C'=+1\Rightarrow(K^{+\prime}\to K^++\rho)/(K^{+\prime}\to K^++\omega)=1\,.$$

But for $C'=-1$, the decay can either go to V_0 or to the unitary singlet combination; if we neglect the latter,[3]

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[1] S. Meshkov, C. A. Levinson, and H. J. Lipkin, Phys. Rev. Letters **10**, 361 (1963).

[2] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).

[3] See for example S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965); H. Sugawara and F. von Hippel, Phys. Rev. **141**, 1331 (1966); and V. Barger, M. Olsson, and K. V. L. Sarma, *ibid.* **147**, 1115 (1966).

we find a decay into $V_0=(-\sqrt{3}\rho+\omega/\sqrt{3}+2\varphi/\sqrt{3})/2$ so that

$$C'=-1 \Rightarrow (K^{+\prime}\to K^{+}+\rho)/(K^{+\prime}\to K^{+}+\omega)=9\,.$$

We summarize these results in Table I. The quantities given are the ratios of the squares of the matrix elements. The numbers in each column are to be compared to determine C_{K}'; there is not necessarily any relation among decays in different columns. We note that the $(K'\to K^*\pi)/(K'\to \rho K)$ ratio is one for either C assignment; this is more easily seen from the usual matrix approach than from the U-spin, V-spin analysis.

TABLE I. Squared matrix-element ratios.

C_{K}'	$\dfrac{(K'\to K+\pi)}{(K'\to K+\eta)}$	$\dfrac{(K'\to K^*+\pi)}{(K'\to K^*+\eta)}$	$\dfrac{(K'\to K+\rho)}{(K'\to K+\omega)}$
+	3	$\frac{1}{3}$	1
−	$\frac{1}{3}$	3	9

We might emphasize that the factor of 9 between C-even and C-odd results for a given decay will persist independent of mixing, so long as there is no coupling to the unitary singlet. This large factor should make it easy to decide in practice which decay is being observed. If the branching ratios could be accurately determined, they would allow good determinations of the mixing angles. It should, however, require considerably less data to decide on the value of C' than to determine the mixing angles, so we have not emphasized the latter. Note that for an even-C particle the $K\rho/K\omega$ ratio cannot be disturbed by coupling to a singlet, while for an odd-C particle the $K\pi/K\eta$ ratio cannot be disturbed by the singlet coupling (opposite for K^*'s). For the case where there is no coupling to the unitary singlet, any deviation from the given ratios must be due to $SU(3)$ breaking, unless, of course, the octet assignment is wrong. It is usually assumed that taking account of mass splittings in a phase-space factor and of particle mixing is sufficient to correct for symmetry breaking. Such notions should soon be testable in these decays. We can try to apply these results to K' (1420), $K'(1320)$, and $K'(1800)$. Some data are presented in Table II.[4]

$K'(1420)$. The customary assignment is to a $J=2^+$ nonet with $C=+1$. The decays involving π's and η's must be corrected for the mass differences; proceeding in the usual way with a phase-space factor p^{2l+1} we find the π decay enhanced over the η decay by an additional factor of about 4, giving altogether a ratio of 12. This is quite consistent with most experimental determinations which give the $K\eta/K\pi$ ratio to be less than about 0.1. However, the $K\rho/K\omega$ ratio, which should involve a phase-space correction factor to the ratio of unity of only about 1.4 (though $m_\rho \simeq m_\omega$, there is very little energy available for the decay), tends to be appreciably larger than that in most experiments, though 1.4 is within the quoted error. If the data continue to give too large a ratio, it implies appreciably greater symmetry breaking (this is the case where the unitary singlet cannot couple) present in these decays than we have grown accustomed to expecting.

[4] B. C. Shen, I. Butterworth, Chumin Fu, G. Goldhaber, S. Goldhaber, and G. H. Trilling, Phys. Rev. Letters **17**, 726 (1966).

TABLE II. Some data on Kaon resonance decays.

(a) $K'(1800)$[a,b]

$K\rho$	$K\omega$	$K\eta$	$K\pi$	$K^*\eta$	$K^*\pi$	$K\pi\pi$
7.5±5%	10±3%	0±3%	0±3%	2±2%	35±12%	40±15%

(b) $K'(1420)$[c]

$K^*\pi/K\pi$	$K\rho/K\pi$	$K\omega/K\pi$	$K\eta/K\pi$
0.87±0.22	0.34±0.25	0.09±0.09	0.06±0.06

(c) $K'(1320)$

$\omega K/(K^*\pi+\rho K)$	$K\pi\%$	$K\eta\%$	$K\rho\%$	$K\omega\%$	$K^*\pi\%$	Ref.
$(6_{-6}^{+4})/(80\pm 20)$	no decay	···	···	···	···	d
···	68±12	0±3	6±6	2±2	24±9	e

[a] G. Goldhaber, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (to be published).
[b] D. R. O. Morrison (private communication).
[c] In (a) results were given for these decays from six separate experiments, including Ref. e. The various results were consistent with one another, so we have averaged them, giving the numbers shown.
[d] Reference 4.
[e] J. Bishop *et al.*, Phys. Rev. Letters **16**, 1069 (1966).

$K'(1320)$. The data are not consistent in this case, in that Shen *et al.* see no $K\pi$ decay mode while Bishop *et al.* find that this mode dominates. It is of course possible that this particle is not real, and the two experiments are seeing different sorts of kinematic effects. Both experiments do suggest a $K\rho/K\omega$ ratio rather larger than 1, but not enough larger to be conclusive as the small available phase space favors the ρ, and for the odd-C case possible singlet mixing can affect the result. Consequently, we only remark that if $C=-1$, as is slightly suggested by both sets of data by the ρ/ω ratios,[5] the $K'(1320)$ could only be assigned to a multiplet whose NNS members were odd under C. This would rule out the speculative assignment of Shen *et al.*[4] to a multiplet containing the $D(1286)$ (if the latter has positive C as is usually assumed) and the A_1. The only known NNS meson in this mass region with odd C are the $B(1220)$ and the $H(980)$; the K' associated with these in an octet would then decay mainly into $K\eta$, $K\rho$, and $K^*\pi$.

$K'(1800)$. Here the $K^*\pi/K^*\eta$ decay strongly suggests $C=-1$. If the $K\rho/K\omega$ ratio continued to be of order unity, it would imply strong cancellation by coupling to the unitary singlet.

Now consider electromagnetic decays. The photon is a U-spin singlet with odd C so it has $G_U=-1$. Thus the decay $K'^0\to K^0+\gamma$ is forbidden if K' has G_U odd, or C even. But the decay $K'^+\to K^++\gamma$ can occur, as

[5] The data are suggestive though obviously not conclusive. This is most easily seen by assuming $C'=+1$. Then we expect a ρ/ω ratio of unity, which would not be affected by coupling to a singlet. The phase-space correction for a 1^+ (for example) particle decaying by s wave into $K\rho$ and $K\omega$ would favor the ρ decay by a factor of about 1.25. A large ρ/ω ratio could only be due to a $C'=-1$ assignment (with coupling to the singlet interfering perhaps).

$\gamma\sim(\sqrt{3}V_1-V_0)/2$, so it corresponds to a transition $(V=1,V_3=1)\rightarrow(V=1, V_3=1)+\gamma$ and the transition will go to whichever part of the photon has the correct G_V, V_1 in this case. Thus we find

$$C'=+1: \qquad \underset{\text{allowed}}{K'^+\rightarrow K^++\gamma;} \qquad \underset{\text{forbidden}}{K'^0\rightarrow K^0+\gamma.}$$

On the other hand, for C' odd both decays are allowed, and the K'^+ decay is to the V_0 state, which can itself be written $V_0=-(U_0+\sqrt{3}U_2)/2$. Since $\gamma=U_0$, we then find a branching ratio

$$C'=-1\Rightarrow\frac{K'^+\rightarrow K^++\gamma}{K'^0\rightarrow K^0+\gamma}=\frac{1}{4}.$$

For a final K^* with C opposite to the K, the results would just invert. These results will eventually be useful for two purposes. First, as with the other decays above, they will allow us to determine the C eigenvalue of the NNS member of the $SU(3)$ multiplet for K'. Second, they will allow relatively sensitive study of symmetry-breaking effects, and perhaps a good test of the $SU(3)$ character of the photon, because they involve no corrections for mass differences.

It would be useful if symmetry breaking did not affect these numbers in lowest order. As above, however, this is not the case. We can show this by an explicit counter example. If such a theorem is possible, it would certainly apply in the case where $K'^0=K^0$, i.e., for the kaon electromagnetic form factor. It is well known that this is zero in the $SU(3)$ limit (because the K^0 is in a U-spin multiplet with the π^0, whose electromagnetic form factor is zero by charge-conjugation invariance). If we compute it with vector-meson dominance we get an answer proportional to $m_\rho^2-m_{\omega 8}^2$. This is indeed zero in the $SU(3)$ limit (equal masses), but it is nonzero in lowest order in the mass differences (i.e., to lowest order in the symmetry breaking).

The decay $K^{0*}\rightarrow K^0\gamma$ should, from $SU(3)$, be 4/9 the decay $\omega\rightarrow\pi^0\gamma$, if $\varphi\rightarrow\pi^0\gamma$ is negligible. (The same mechanism which inhibits $\varphi\rightarrow\rho\pi$ may apply to the $\varphi\rightarrow\gamma\pi$ decay.) The $\omega\rightarrow\pi\gamma$ width is known to be about 1 MeV, so the K^0 decay will have a branching ratio of about 1%.

It is, of course, possible to verify all of the above statements by constructing C and $SU(3)$-invariant couplings in the usual manner. We have presented the derivations in the manner above to illustrate the simple and useful application of the U-spin, V-spin techniques to problems involving strange mesons, and to emphasize the utility of defining the analog of G parity for the U-spin, V-spin subgroups of $SU(3)$, so that the kaons are eigenstates of a conserved operator. Applications to isospin—$\frac{1}{2}$ kaons in other $SU(3)$ multiplets and to higher-isospin kaons can be carried out by similar techniques.

In the Appendix we give the various branching ratios including an arbitrary mixing angle (even though probably only ω-φ and η-η' mixing will enter in applications, the situation is still not entirely clear in these cases) and allowing coupling to a unitary singlet where possible. When decay branching ratios become quite well known it may be possible to determine mixing angles and the size of coupling to singlets by using these decays.

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APPENDIX

We include mixing by putting $\omega_8=\omega\sin\theta+\varphi\cos\theta$, $\omega_1=\omega\cos\theta-\varphi\sin\theta$, with similar formulas for (η',η). Consider first the decay into NNS vector mesons.

Then for the case where $C'=+1$ the decay proceeds by emitting $V_1=[\rho+\sqrt{3}(\omega\sin\theta+\varphi\cos\theta)]/2$ so that

$$(K'\rightarrow K+\rho)/(K'\rightarrow K+\omega)=(1/\sqrt{3}\sin\theta)^2.$$

For $\sin\theta=1/\sqrt{3}$ we get the result in the text. When decays $K'\rightarrow K+\varphi$ are observed it is clear how to modify the formulas to obtain their branching ratios.

For $C'=-1$ the decay is $\alpha V_0+\beta S$, where S is a unitary singlet, $S=\omega\cos\theta-\varphi\sin\theta$, $V_0=(-\sqrt{3}\rho+\omega\sin\theta+\varphi\cos\theta)/2$, and β measures the amount of coupling to the singlet. Then we have

$$(K'\rightarrow K\rho)/(K'\rightarrow K\omega)=3/(\sin\theta+2\beta\cos\theta/\alpha)^2.$$

For $\beta=0$ and $\theta=1/\sqrt{3}$ we recover the result in the text.

To obtain the equivalent formulas for the pseudoscalars or tensors we exchange the $C'=+1$ and $C'=-1$ cases and replace (ω,φ) by (η',η) or (f,f'). Then, for example, for C' odd we obtain

$$(K'\rightarrow K+\pi)/(K'\rightarrow K+\eta)=(1/\sqrt{3}\cos\theta)^3.$$

Note that we have taken φ, η, and f' to belong to the octet in the limit $\theta=0$. For the pseudoscalars, the branching ratio for $(K+\pi)/(K+\eta')$ will be proportional to $(\sin\theta)^{-2}$, and will be quite sensitive to the difference between a mixing angle[6] for a linear mass formula $(\theta\sim 20^0)$ and that for a quadratic mass formula $(\theta\sim 10^0)$.

[6] A. J. MacFarlane and R. H. Socolow, Phys. Rev. **144**, 1194 (1966).