current correlation function in terms of a quantized vector field is identical with the previous definition that employed a classical, external vector field.

The divergence relation

$$\partial_\mu K_{ab}{}^{\mu\nu}(x,x') = iC_{abc}j_c{}^\nu(x)\delta(x-x') + i(d_a(x)j_b{}^\nu(x'))_+ \quad (60)$$

follows directly from the current commutation relations (41) and (44). Its intimate connection with the fundamental divergence condition (12) can be seen from

$$K_{ab}{}^{\mu\nu}(x,x') = \lim_{g\to 0} \frac{1}{g} i(j_a{}^\mu(x)B_b{}^\kappa(x')) \times[(-\overleftarrow{\partial}'^2+\mu_0{}^2)\delta_\kappa{}^\nu+\overleftarrow{\partial}'_\kappa\overleftarrow{\partial}'^\nu], \quad (61)$$

for this gives[16]

$$\partial_\mu K_{ab}{}^{\mu\nu}(x,x') = \lim_{g\to 0}\left\{iC_{adc}j_{\mu d}(x)D_{db}{}^{\mu\kappa}(x,x') + i\frac{1}{g}(d_a(x)B_b{}^\kappa(x'))_+\right\}[(-\overleftarrow{\partial}'^2+\mu_0{}^2)\delta_\kappa{}^\nu+\overleftarrow{\partial}'_\kappa\overleftarrow{\partial}'^\nu], \quad (62)$$

from which (60) follows immediately.

We should perhaps note in conclusion that similar results are easily obtained for the simpler case of radiation-gauge quantum electrodynamics.

[16] We have omitted the appropriate time ordering of the current operator that occurs here, since it commutes with the vector field in the limit of vanishing coupling.

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Perturbation Theory and the Mass Sum Rule for the Baryon Octet

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Many dynamical derivations of the Gell-Mann–Okubo mass formula for the baryon octet assume that the symmetry-violating interactions can be treated to first order in perturbation theory. The validity of this assumption is tested by a detailed evaluation of second-order effects. A pole model with no free parameters contributes the major portion of the second-order effect; final-state scattering and other corrections are argued to be small. The second-order mass shifts are found to be 200 MeV or more. Since the second-order mass shifts are as large as the first-order shifts, the conclusion is that derivations of the mass splittings based on first-order perturbation theory must be considered as little more than restatements of the pure group-theoretic derivation.

I. INTRODUCTION

EVER since the development of unitary symmetry,[1] the Gell-Mann–Okubo (GMO) sum rule[1,2] for the masses of the baryon octet has been cited as a success of the theory. In fact, the sum rule is almost embarassingly successful. In the equation

$$m_N+m_\Xi=\tfrac{3}{2}m_\Lambda+\tfrac{1}{2}m_\Sigma,$$

the difference between the right- and left-hand sides is about 13 MeV when the average mass of each isotopic multiplet is used. This difference should be compared with the average octet mass of 1150 MeV and the average mass splitting within the multiplet of 173 MeV. Up to this time there has been no satisfactory *dynamical* derivation of this sum rule. The group-theoretic derivation simply involves taking the appropriate matrix elements of a mass operator containing both an invariant term and a term transforming as the $Y=0$, $T=0$ member of an octet. Why the mass operator should have this form is left unexplained.

There are three basically different ways of justifying the group-theoretic approach. The first argument states that the really strong interactions are invariant under $SU(3)$, but that there are a class of medium-strong interactions which are not invariant. In analogy with electrodynamics, the interactions are supposed to violate $SU(3)$ in a particular way corresponding to the $Y=0$, $T=0$ member of an octet. This component of the octet is uniquely determined by the requirement that the violation conserve hypercharge and isospin. The noninvariant part of the strong-interaction Hamiltonian will give the mass formula as a first-order effect in a perturbation expansion of the mass. This approach may involve tadpole or bubble-diagram contributions to self-energies, but it is basically an application of first-order perturbation theory.[3] The second approach

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[2] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[3] There are many models that fall into this category. Tadpole diagrams were used by S. Coleman and S. Glashow, [Phys. Rev. **134**, B671 (1964)]; while J. J. Sakuraii, [*ibid.* **132**, 434 (1963)] used bubble diagrams. Other perturbative calculations have been done by J. Arafune, Y. Iwasaki, K. Kikkawa, S. Matsuda, and K. Nakamura, *ibid.* **143**, 1220 (1966); Y. Ne'eman, *ibid.* **134**, B1355 (1964); and S. L. Cohen and C. R. Hagen, *ibid.* **149**, 1138 (1966).

regards the octet mass splitting as resulting from the sum total of all dynamics. Concepts such as a Hamiltonian or first-order effects are done away with. This approach, usually based on bootstrap methods, is favored by S-matrix theorists.[4] The third approach to the mass formula does not attempt to provide a dynamical basis for the observed mass splittings; rather the physical masses are taken as given, and the problem is one of finding corrections to the mass formula itself in order to bring it into agreement with the physical situation. In other words, the difference in the two sides of the GMO sum rule should not be zero, but rather it should be a small quantity which can be calculated. These corrections have been evaluated using current algebra ideas or field theory and have been found to be small.[5] Since it avoids all discussion of the dynamical origins of the mass splittings or symmetry violations, this approach is less ambitious than the other two; and its results have been more satisfactory for that reason.

In this paper the perturbative approach to the mass formula is examined in detail. In such a theory, the success obtained by treating to first order an interaction whose strength can be measured in terms of the average mass splitting, is explained by saying that, for some reason, second-order effects are small. However, no one has calculated these second-order contributions. Some work has been done on second-order mass corrections for resonances[6]; the assumption is made that the splittings are generated by mass differences among the particles interacting to form the resonance. The masses of the fundamental particles are treated as given, somewhat in the spirit of the S-matrix approach to octet enhancement. The second-order effects found are quite large. This result supports the conclusions reached in this paper, but the technique used is totally different and does not constitute a test of the conventional calculations of the octet mass formula. Within the framework of the strict perturbative approach to be tested here, if second-order corrections are found to be as large or larger than the first-order terms, derivations of the mass splittings based on first order alone must be considered as little more than restatements of the pure group-theoretic derivation.

Starting with conventional time-independent perturbation theory, we calculate the masses of an originally degenerate octet of baryon states. The total Hamiltonian contains two terms. The $SU(3)$-invariant piece $\bar{H}_0$, when acting on an unperturbed state at rest, gives the central octet mass. The noninvariant term $\bar{H}_I$ transforms as the $Y=0$, $T=0$ member of an octet. Corrections to the central mass are calculated to second order. (The procedure, with some minor revisions, could be applied to a calculation of self-energies.) Since the states which appear in all equations are always unperturbed, there is no ambiguity as to where the symmetry-breaking effects should be inserted. To first order, this approach yields the GMO mass formula. The parameters which appear in this formula are chosen to give the best fit to the experimental masses. The masses are then calculated to second order. Only two-particle states consisting of a baryon and a meson are retained. Other states will be higher in mass. Since all second-order contributions have the same sign, the restriction to two-particle states will underestimate the corrections. The problem thus reduces to one of calculating matrix elements of $\bar{H}_I$ between states of a single particle at rest and a two-particle state. For second-order contributions to be finite, this matrix element must vanish in the limit that the energy of the intermediate two-particle state becomes infinite. Conventional reduction techniques are used to determine the nearby singularities of the amplitude as a function of this energy. The result is a pole model modified by final-state scattering. The pole residues are given uniquely in terms of the first-order parameters. The integral equation so obtained is solved formally; in the resonance approximation for the final-state scattering amplitude, the equation is explicitly solved. The solution, in the limit of no scattering, depends only on the known meson-baryon coupling constants. In this limit, the second-order contributions to the masses are on the order of 200 MeV. The surprising first-order agreement with both the GMO mass formula and the Ξ-N and Σ-Λ mass differences is destroyed. Nonresonant scattering is shown to be incapable of improving the situation. A resonance in the scattering amplitude will only serve in general to make the effect larger; however, a delicate cancellation that almost completely suppresses the second-order corrections can occur for special values of the resonant parameters. We conclude that the medium-strong interactions are too strong; a perturbation expansion does not converge rapidly, if at all.

In the next section, the necessary perturbation formulas are stated; the $SU(3)$ and spin reduction of the appropriate amplitudes is treated. The integral equation for transition amplitude is derived and solved. Since the techniques used are quite standard, the development is only outlined. In Sec. III, the equations are evaluated numerically, and the results compared with the experimental situation. Scattering corrections are considered in detail. Various proposals for avoiding the obvious conclusion about the inadequacy of perturbation theory are discussed and rejected. Appendix A

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[4] The Dashen-Frautschi theory of octet enhancement [R. F. Dashen and S. C. Frautschi, Phys. Rev. **137**, B1331 (1965)] is in this category; however, its validity has been a matter of some controversy. Other bootstrap or dispersion relation calculations have been performed by Y. Hara, *ibid.* **144**, 1241 (1966); F. J. Gilman, *ibid.* **147**, 1094 (1966); and F. J. Ernst, R. L. Warnock, and K. C. Wali, *ibid.* **141**, 1354 (1966).

[5] The current algebra method of calculating corrections to the mass formula was developed by S. Fubini, G. Furlan, and C. Rossetti [Nuovo Cimento **40A**, 1171 (1965)]; it was first applied to the baryon octet by M. Boiti and C. Rebbi, *ibid.* **43A**, 214 (1966). An entirely different approach to the same problem was developed by J. W. Moffat, Phys. Rev. **145**, 1177 (1966). Both methods lead to corrections on the order of 10 MeV or less.

[6] F. J. Ernst (to be published).

gives a detailed expansion of the pole residues used in the text. Appendix B is used to develop a model to estimate the effect of a slowly varying scattering amplitude.

II. THEORY OF THE SECOND-ORDER MASS CORRECTION

If we assume the existence of a total Hamiltonian $\bar{H}$ which can be written in the form

$$\bar{H}=\bar{H}_0+\bar{H}_8{}^{\gamma}, \tag{1}$$

the application of time-independent perturbation theory to the calculation of the masses of a baryon octet is straightforward. The only difference between the formulas used here and those given in any introductory quantum mechanics book results from the use of covariant normalization for the states vectors,

$$\langle\alpha\mathbf{p}|\beta\mathbf{p}'\rangle=(2\pi)^3 2E\delta^3(\mathbf{p}'-\mathbf{p})\delta_{\alpha\beta}, \quad E=(p^2+m^2)^{1/2}. \tag{2}$$

The $SU(3)$ indices α, β label states of definite isospin and hypercharge; the perturbing Hamiltonian $H_8{}^{\gamma}$ is diagonal in these states for γ corresponding to $Y=0$, $T=0$. Spin indices are suppressed in (2). Along with the existence of $\bar{H}$, we assume the existence of a degenerate octet of baryons which are eigenstates of $\bar{H}_0$, the $SU(3)$ invariant portion of $\bar{H}$. We consider matrix elements of $\bar{H}$ between physical states at rest; the physical states are expanded in terms of the eigenstates of $\bar{H}_0$. The usual manipulations yield the following expression for the mass of particle α to second order in $H_8{}^{\gamma}$:

$$m_\alpha=m+\sum_{s=1}^{2}\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \gamma & \alpha\end{pmatrix}\delta m_s +\frac{1}{2m}\left\{\sum_s\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \gamma & \alpha\end{pmatrix}\delta m_s\right\}^2+X_\alpha. \tag{3}$$

The notation of de Swart[7] is used for the $SU(3)$ Clebsch-Gordan coefficients. The unperturbed mass is m. The two parameters δm_1, δm_2 in the second term on the right of (3) are defined by

$$\langle\alpha|H_8{}^{\gamma}(0)|\alpha'\rangle=2m\sum_s\begin{pmatrix}8 & 8 & 8_s\\ \alpha & \gamma & \alpha\end{pmatrix}\delta m_s, \tag{4}$$

where $H_8{}^{\gamma}(0)$ is the Hamiltonian density corresponding to $\bar{H}_8{}^{\gamma}$. The third and fourth terms on the right of (3) are second-order effects. If they are absent, the GMO mass formula follows for arbitrary $\delta m_{1,2}$. We shall determine δm_1 and δm_2 and m by requiring that the first-order expression give the best possible fit to the experimental masses. In other words, we assume at this stage that second-order effects are small. The third term in (3) is a kinematical correction which results from the covariant normalization; it is small and positive definite. The final term X_α contains the transitions to many-particle states, and it is negative definite. The purpose of this paper is to estimate its magnitude in order to judge whether or not it is permissible to limit (3) to first-order terms alone.

With the assumption that only two-particle states contribute to X_α, and that these states comprise an octet of pseudoscalar mesons of mass μ and the octet of baryons, X_α becomes

$$X_\alpha=-\frac{1}{2m}\sum_{\beta,\epsilon}\sum_{\text{spin}}\int\frac{d^3p\,d^3q}{(2\pi)^6 2E2\omega}\frac{(2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})}{\omega+E-m}\times|\langle\alpha|H_8{}^{\gamma}(0)|\beta\mathbf{p},\epsilon\mathbf{q}\rangle|^2. \tag{5}$$

All parameters labeling the intermediate states are summed over. The baryon β and meson ϵ have four-momenta $(\mathbf{p},E)$ and $(\mathbf{q},\omega)$, respectively. The three-dimensional delta function indicates that $\bar{H}_8$ conserves three-momenta, but not energy. The matrix element has the kinematic structure of the scattering amplitude for the process $0_\gamma{}^+ + \frac{1}{2}{}_\alpha{}^+ \to 0_\epsilon{}^- + \frac{1}{2}{}_\beta{}^+$, with particle α and the 0^+ spurion both at rest. Therefore, in analogy with π-N scattering we can write

$$\langle\alpha|H_8{}^{\gamma}(0)|\beta,\epsilon\rangle=\sum_{\rho\sigma\lambda\tau}\begin{pmatrix}8 & 8 & \rho_\lambda\\ \alpha & \gamma & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_\tau\\ \beta & \epsilon & \sigma\end{pmatrix}\times 2m\bar{u}(0)\left[F_{\lambda\tau}{}^{\rho}-\frac{G_{\lambda\tau}{}^{\rho}}{m}\gamma\cdot q\right]\gamma_5 u(p),$$

$$=2m\bar{u}\gamma_5 u\sum\begin{pmatrix}8 & 8 & \rho_\lambda\\ \alpha & \gamma & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_\tau\\ \beta & \epsilon & \sigma\end{pmatrix}K^{\rho}{}_{\lambda\tau}, \tag{6}$$

where we have taken advantage of our particular Lorentz frame and momentum conservation to simplify the matrix element. The amplitude $K^{\rho}{}_{\lambda\tau}$ is given by

$$K^{\rho}{}_{\lambda\tau}=F^{\rho}{}_{\lambda\tau}+[(W+m)/m]G^{\rho}{}_{\lambda\tau}, \tag{7}$$

where $W=\omega+E$. Upon substituting (6) in (5) and carrying out the sums over spin and unitary spin, and integrating over the momenta, we obtain X_α in the form

$$X_\alpha=-\sum_{\rho,\sigma,\lambda,\lambda'}\begin{pmatrix}8 & 8 & \rho_\lambda\\ \alpha & \gamma & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_{\lambda'}\\ \alpha & \gamma & \sigma\end{pmatrix}\times\frac{1}{2(2\pi)^2}\int_{m+\mu}^{\infty}\frac{p(E-m)}{W(W-m)}dW\sum_\tau K^{\rho}{}_{\lambda\tau}K^{*\rho}{}_{\lambda'\tau}. \tag{8}$$

Conservation of angular momentum and parity restricts the intermediate states to those having $J=\frac{1}{2}$ and positive parity. Hence, K is independent of the direction of the internal momentum. The sum over ρ includes all multiplets formed by a product of two octets.

In order to calculate X_α we need to determine K. Since the amplitude of interest is a function of W, we

[7] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

assume it is an analytic function with no singularities other than the ones we determine directly. Moreover, K must vanish at infinite W in order for X_α to be finite.

As the first step in enumerating the singularities of K we contract out the pseudoscalar meson in the matrix element $\langle\alpha|H_8{}^\gamma(0)|\beta,\epsilon\rangle$, and obtain the following expression for the absorptive part of the matrix element:

$$\mathrm{Im}\langle\alpha|H_8{}^\gamma(0)|\beta,\epsilon\rangle=-(\pi/\sqrt{2})(2\pi)^3$$
$$+\{\sum_n \delta^4(p_0-q-n)\langle\alpha,p_0|j^+{}_\epsilon(0)|n\rangle\langle n|H_8{}^\gamma(0)|\beta,p\rangle$$
$$-\sum_n \delta^4(p+q-n)\langle\alpha,p_0|H_8{}^\gamma(0)|n\rangle\langle n|j^+{}_\epsilon(0)|\beta p\rangle\}\,. \quad (9)$$

In the first term on the right, the initial contribution in the sum over intermediate states will come from a single baryon state; it is given by

$$\mathrm{Im}\langle\alpha|H_8{}^\gamma(0)|\beta\epsilon\rangle_1=-\pi(2\pi)^3\int\frac{d^3n}{2n_0}\frac{1}{(2\pi)^3}\delta^3(\mathbf{q}-\mathbf{n})$$
$$\times\delta(n_0+\omega-m)(2m)^2\bar{u}(0)\gamma_5\frac{\gamma\cdot p+m}{2m}u$$
$$\times\sum_{s,t}\begin{pmatrix}8 & 8 & 8_s\\ n & \epsilon & \alpha\end{pmatrix}\begin{pmatrix}8 & 8 & 8_t\\ n & \gamma & \beta\end{pmatrix}g_s\delta m_t\,. \quad (10)$$

The momentum delta function constrains the state to have four-momentum $(\mathbf{p},E)$. The meson-baryon coupling constants g_s defined by

$$\frac{1}{\sqrt{2}}\langle\alpha|j^+{}_\epsilon(0)|n\rangle=2m\bar{u}\gamma_5u\sum_s\begin{pmatrix}8 & 8 & 8_s\\ n & \epsilon & \alpha\end{pmatrix}g_s \quad (11)$$

are known quantities. Symmetry violations in the coupling constants would first affect the masses in third order. When the integrations are performed, $\bar{u}\gamma_5u$ projected out, and the $SU(3)$ indices summed over, we find

$$\mathrm{Im}(K^\rho{}_{\lambda\tau})_1=-\frac{\pi\delta(W-m)}{1-\mu^2/2m^2}\sum_{\beta,\epsilon,\alpha,\gamma,n}\begin{pmatrix}8 & 8 & \rho_\lambda\\ \alpha & \gamma & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_\tau\\ \beta & \epsilon & \sigma\end{pmatrix}$$
$$\times\sum_{s,t}\begin{pmatrix}8 & 8 & 8_s\\ n & \epsilon & \alpha\end{pmatrix}\begin{pmatrix}8 & 8 & 8_t\\ n & \gamma & \beta\end{pmatrix}g_s\delta m_t$$
$$=-\frac{\pi\delta(W-m)}{1-\mu^2/2m^2}\sum_{s,t}G(\rho,\lambda,\tau)_{st}g_s\delta m_t\,. \quad (12)$$

Every parameter in (12) is known. The energy of the intermediate state is evaluated for $W=m$,

$$2E=2m(1-\mu^2/2m^2)\,.$$

In a similar fashion we obtain another one-particle contribution from the second term of (9).

$$\mathrm{Im}(K^8{}_{\lambda\tau})_2=\pi\delta(W-m)g_\tau\delta m_\lambda\,. \quad (13)$$

This term contributes only to the octet channels. There is a third one-particle absorptive part which can be obtained by contracting out the baryon in the initial state instead of the meson. It involves a one-meson intermediate state.

$$\mathrm{Im}(K^\rho{}_{\lambda\tau})_\mu=-\pi\delta(W-m)\frac{m}{2\mu^2}\sum_{st}F(\rho,\lambda,\tau)_{st}g_s\delta\mu^2{}_t\,. \quad (14)$$

The coefficient $F(\rho,\lambda,\tau)_{st}$ is given by

$$F(\rho,\lambda,\tau)_{st}=\sum_{\beta,\epsilon,\alpha,\gamma,n}\begin{pmatrix}8 & 8 & \rho_\lambda\\ \alpha & \gamma & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_\tau\\ \beta & \epsilon & \sigma\end{pmatrix}$$
$$\times\begin{pmatrix}8 & 8 & 8_s\\ \beta & n & \alpha\end{pmatrix}\begin{pmatrix}8 & 8 & 8_t\\ n & \gamma & \epsilon\end{pmatrix}. \quad (15)$$

The meson matrix element which enters (14) is

$$\langle n|H_8{}^\gamma(0)|\epsilon\rangle=\sum_t\begin{pmatrix}8 & 8 & 8_t\\ n & \gamma & \epsilon\end{pmatrix}\tfrac{1}{2}\delta\mu^2{}_t\,, \quad (16)$$

and the meson masses are

$$\mu_\epsilon=\mu+\tfrac{1}{2}\sum_t\begin{pmatrix}8 & 8 & 8_t\\ \epsilon & \gamma & \epsilon\end{pmatrix}\frac{\delta\mu^2{}_t}{\mu}\,. \quad (17)$$

If this last formula is squared and only the leading terms retained, the usual quadratic GMO mass formula for mesons[2] is obtained.

The three one-particle contributions to the imaginary part of K are just the result of inserting the mass-splitting operator $\bar{H}_8{}^\gamma$ into each leg of the basic strong interaction vertex. When used in a dispersion relation for K, each of these leads to a pole at $W=m$.

Next we consider two-particle intermediate states. Since only two-particle states are retained in (5), it is consistent to retain only those singularities in K which correspond to the two-particle intermediate states alone. There is only one such contribution. This arises from the second term on the right of (9). A two-particle state in the first term would amount to including a baryon–two-meson intermediate state in the total amplitude as would the two-particle part of the baryon-contracted representation of ImK. The two-particle contribution which we shall retain leads to the following imaginary part in K:

$$(\mathrm{Im}K^\rho{}_{\lambda\tau})^s=-\frac{1}{4\pi}\sum_\tau\frac{P(E-m)}{4Wm}K^\rho{}_{\lambda\sigma}H^{*\rho}{}_{\sigma\tau}\,. \quad (18)$$

The $J=\frac{1}{2}^+$ portion of the total scattering amplitude is

$H^{\rho}{}_{\lambda\tau}$. If the matrix element for meson-baryon scattering is

$$\langle\beta'\mathbf{p}',\epsilon'\mathbf{q}'|\beta\mathbf{p},\epsilon\mathbf{q}\rangle=\sum_{\rho,\lambda,\sigma,\tau}\begin{pmatrix}8 & 8 & \rho_\lambda\\ \beta & \epsilon & \sigma\end{pmatrix}\begin{pmatrix}8 & 8 & \rho_\tau\\ \beta & \epsilon & \sigma\end{pmatrix}$$

$$\times\bar{u}(p')\left[A^{\rho}{}_{\lambda\tau}-\frac{1}{2}\frac{(\gamma\cdot(q+q'))}{m}B^{\rho}{}_{\lambda\tau}\right]u(p),\quad(19)$$

then

$$H=\left(A+\frac{W+m}{m}B\right)_{J=1/2}.\quad(20)$$

The meson momenta are q and q'.

When Eqs. (12), (13), (14), and (18) are used to write a dispersion relation for K, the following expression results:

$$K^{\rho}{}_{\lambda\tau}=\frac{C^{\rho}{}_{\lambda\tau}}{W-m}+\frac{1}{\pi}$$

$$\times\int_{m+\mu}^{\infty}\frac{dW'p'(E'-m)}{4W'm(W'-W)}\frac{1}{4\pi}\sum_{\sigma}K^{\rho}{}_{\lambda\sigma}H^{*\rho}{}_{\sigma\tau}.\quad(21)$$

The residue at the pole $C^{\rho}{}_{\lambda\tau}$ is equal to

$$C^{\rho}{}_{\lambda\tau}=(g_\tau\delta m_\lambda)_{\rho=8}-\frac{1}{1-\mu^2/2m^2}\sum_{st}G(\rho,\lambda,\tau)g_s\delta m_t$$

$$-\frac{m}{2\mu^2}\sum_{st}F(\rho,\lambda,\tau)_{st}g_s\delta\mu^2{}_t.\quad(22)$$

$C^{\rho}{}_{\lambda\tau}$ is given for each of the channels in Appendix A. Equation (21) is a singular integral equation for $K^{\rho}{}_{\lambda\tau}$. For $\rho\neq8$, the equations are uncoupled, while for $\rho=8$ there are two sets of two coupled equations. If $H^{\rho}{}_{\sigma\tau}$ is replaced by

$$H^{\rho}{}_{\lambda\tau}=-\frac{16Wm}{p(E-m)}(e^{\delta}\sin\delta)^{\rho}{}_{\lambda\tau},\quad(23)$$

the equations take on a more familiar appearance. For the octet channels we make the simplifying assumption that there is only one octet phase shift and the various octet couplings can be described by a parameter equivalent to an F/D ratio. Specifically we write the octet scattering amplitude as a 2×2 matrix times an octet phase shift; in order to preserve unitarity the matrix is restricted so that

$$(e^{i\delta}\sin\delta)^{8}{}_{\lambda\tau}=\begin{pmatrix}\sin^2\theta & \sin\theta\cos\theta\\ \sin\theta\cos\theta & \cos^2\theta\end{pmatrix}e^{i\delta_8}\sin\delta_8.\quad(24)$$

The angle θ constitutes an unknown parameter of the theory. If a resonance is present in one of the channels, it is possible to estimate its value; however, the results are insensitive to θ.

When (23) is used in (21), we find the integral equation for the nonoctet amplitudes to be a standard type.[8]

$$K^{\rho}=\frac{C^{\rho}}{W-m}+\frac{1}{\pi}\int_{m+\mu}^{\infty}\frac{dW'}{W'-W}K^{\rho}e^{-i\delta_\rho}\sin\delta_\rho.\quad(25)$$

The coupled equations in the octet channel, by virtue of Eq. (24), may be decoupled by choosing a particular linear combination of the amplitudes. Specifically we find that

$$(K^{8}{}_{\lambda1}\sin\theta+K_{\lambda2}{}^{8}\cos\theta)=\frac{C^{8}{}_{\lambda1}\sin\theta+C^{8}{}_{\lambda2}\cos\theta}{W-m}$$

$$+\frac{1}{\pi}\int_{m+\mu}^{\infty}\frac{dW}{W-W}(K^{8}{}_{\lambda1}\sin\theta+K^{8}{}_{\lambda2}\cos\theta)e^{-i}{}_{8}\sin\delta_8\quad(26)$$

and

$$K^{8}{}_{\lambda1}\cos\theta-K^{8}{}_{\lambda2}\sin\theta=\frac{C^{8}{}_{\lambda1}\cos\theta-C^{8}{}_{\lambda2}\sin\theta}{W-m}.\quad(27)$$

Equation (26) is identical in type to Eq. (25). For real values of the phase shift, these equations can be formally solved.[8]

$$K=\frac{C}{W-m}\frac{\Omega(W)}{\Omega(m)},\quad(28)$$

$$\Omega(W)=\exp\left(\frac{1}{\pi}\int_{m+\mu}^{\infty}\frac{\delta(W')}{W'-W}dW'\right).\quad(29)$$

This solution has both the pole and the correct phase along the cut. A possible polynomial in the numerator of the solution has been set equal to a constant by the requirements that K vanish as W becomes infinite and that the solution become just the pole term in the limit of zero scattering, or $\Omega=1$. We assume that the phase shift vanishes at infinity. All that remains is to evaluate K for various values of δ and substitute it into the expression for X_α. This is done in the next section.

III. NUMERICAL EVALUATION

In order to evaluate the second-order masses it is necessary to assign values to all the parameters appearing in Eqs. (8), (22), (27), and (28). The first-order splittings are determined by fitting the experimental masses to the first-order mass formulas. The results are

$$m=1150,\quad\delta m_1=39\sqrt{5},\quad\delta m_2=-379;$$
$$\mu=413,\quad\delta\mu_1{}^2=-(\sqrt{5})14.4\times10^4,\quad\delta\mu_2{}^2=0.$$

All masses in this section are expressed in MeV. The meson-baryon coupling constants g_1 and g_2 are discussed by de Swart,[7] who writes them in terms of the

[8] G. Barton, *Dispersion Techniques in Field Theory* (W. A. Benjamin, Inc., New York, 1965). Chaps. 9 and 10.

renormalized pion-nucleon coupling constant g, and f. The relative strengths of the antisymmetric and symmetric $SU(3)$ couplings are given by f and $1-f$.

$$g_1=2(5/3)^{1/2}g(1-f),$$
$$g_2=2\sqrt{3}gf. \tag{30}$$

We set $g^2/4\pi$ equal to 14.6. Various theories predict an f in the range between 0.25 and 0.40.[9] Our solution will be written as a polynomial in f to see if there is a best value in that range. The coefficients $G(\rho,\lambda,\tau)_{st}$ and $F(\rho,\lambda,\tau)_{st}$ are elements of the crossing matrix from 8_{st} in the u or t channels to $\delta_{\lambda\tau}$ in the s channel; they are available in the literature.[10] In Appendix A we use these coefficients to explicitly evaluate the pole residues $C^{\rho}{}_{\lambda\tau}$.

For each of the physical particles we evaluate the first three terms on the right of (3) and write X_α as a sum of contributions from the various channels:

$$\begin{aligned}
m_N&=961.6-\left\{\tfrac{1}{4}X^{10*}+(9/20)X^{27}+(1/20)X^8{}_{11}+\tfrac{1}{4}X^8{}_{22}-\frac{\sqrt{5}}{20}(X^8{}_{12}+X^8{}_{21})\right\},\\
m_\Lambda&=1111.7-\{\tfrac{1}{8}X^1+(27/40)X^{27}+\tfrac{1}{5}X^8{}_{11}\},\\
m_\Sigma&=1189.7-\{\tfrac{1}{4}X^{10}+\tfrac{1}{4}X^{10*}+\tfrac{3}{10}X^{27}+\tfrac{1}{5}X^8{}_{11}\},\\
m_\Xi&=1334.9-\left\{\tfrac{1}{4}X^{10}+(9/20)X^{27}+(1/20)X^8{}_{11}+\tfrac{1}{4}X^8{}_{22}+\frac{\sqrt{5}}{20}(X^8{}_{12}+X^8{}_{21})\right\}.
\end{aligned}\tag{31}$$

We have used $X^{\rho}{}_{\lambda\tau}$ to represent the coefficients of the two Clebsch-Gordan coefficients in X_α. For the octet $X^{\rho}{}_{\lambda\tau}$ there is a sum over intermediate channels involved in the definition. In the limit of zero scattering in the $\frac{1}{2}^+$ channel $\Omega(W)/\Omega(m)=1$, and $X^{\rho}{}_{\lambda\tau}$ will involve only the pole portion of $K^{\rho}{}_{\lambda\tau}$. If the integral with K given by just a pole is denoted by I,

$$I=\frac{1}{2}\frac{1}{(2\pi)^2}\int_{m+\mu}^{\infty}\frac{\rho(E-m)}{W(W-m)^3}dW, \tag{32}$$

then the integrand of $I^{\rho}{}_{\lambda\tau}$ contains an additional factor $|\Omega(W)/\Omega(m)|^2$, and $I^{\rho}{}_{\lambda\tau}$ can be written as

$$I^{\rho}{}_{\lambda\tau}=(1+\alpha^{\rho})I. \tag{33}$$

In the limit of no scattering α is equal to zero; in general it can range from -1 to infinity. The integral I is equal to 2.127×10^{-6} MeV^{-1}, when the aforementioned values for m and μ are used.

[9] In the work of A. W. Martin and K. C. Wali [Phys. Rev. **130**, 2455 (1963)], it was found that $f=0.25$ would explain the observed properties of the decuplet resonances. $SU(6)$ symmetry predicts an $f=0.40$. C. J. Goebel [Phys. Rev. Letters **16**, 1130 (1966)] finds f to be 0.36 in strong-coupling theory, and he gives references to other values of f between 0.29 and 0.39.

[10] M. M. Nieto, Phys. Rev. **140**, B434 (1965).

We first evaluate Eq. (31) in the approximation of zero scattering and then look at the scattering corrections. The masses in this pole-model limit are second-order polynomials in the coupling parameter f. Upon inserting the residues from Appendix A into Eq. (28), we obtain the following expressions:

$$\begin{aligned}
m_N&=333.4+1283.4f-1689.2f^2,\\
m_\Lambda&=678.0+1197.3f-1274.4f^2,\\
m_\Sigma&=782.8+582.6f-1019.5f^2,\\
m_\Xi&=1024.6+923.2f-1181.5f^2.
\end{aligned}\tag{34}$$

For f equal to 0.35, the resulting values are

$$m_N=575.7,\quad m_\Lambda=941.0,\quad m_\Sigma=861.8,\quad m_\Xi=1203.0.$$

Not only are the magnitudes of the calculated masses wrong, but also the mass differences $m_\Sigma-m_\Lambda$ and $m_\Xi-m_N$ are wrong. The Σ-Λ mass difference even has the wrong sign. In addition, we find that the error in the GMO mass formula is equal to -63.7 MeV, compared with the experimental value of -13 MeV. By varying f we find that the minimum error of -42.4 MeV occurs for f equal to 0.13. This error is still smaller than that which would be expected in first order on the basis of a realistic estimate of the strength of the medium-strong interactions. As a function of f, the maximum value of m_N is 577.2 MeV and that of m_Σ is 866.6 MeV. The maximum values for all the masses occur for f's between 0.24 and 0.45, and these values are between 100 and 375 MeV below the physical values. The masses are quite insensitive to the value of f in this region.

If the unphysical assumption is made that the mesons are degenerate in mass ($\delta\mu_1{}^2=0$), the situation is altered. For an f of 0.35, we would find the pole approximation masses to be

$$m_N=930.0,\quad m_\Lambda=1081.3,\quad m_\Sigma=1150.0,\quad m_\Xi=1313.4.$$

The second-order mass shifts are small compared with first order but the error in the GMO formula is $+46.4$ MeV. If the first-order baryon mass shifts are set equal to zero, and the second-order effect is generated by the meson splitting alone, we find mass shifts ranging from 250 to 400 MeV for the same value of f. The error in the GMO formula is 118 MeV. These unphysical choices of the parameters may lead to smaller or larger second-order corrections, but the agreement with the GMO mass formula is not improved. These choices do show, however, that the dominant contribution to the second-order baryon mass shifts comes from the large first order meson mass splitting. Obviously a real understanding of the baryon masses will involve solving the problem of the origin of the meson mass splitting. The simple pole model with no free parameters leads to very large second-order corrections to the octet masses. The corrections are sufficiently large to render perturbation theory meaningless. The question is whether this conclusion can be avoided either by invoking scattering effects to suppress the pole model results or by objecting to some fundamental aspect of the whole model.

Before considering scattering effects, we note that in the pole model, the error in the GMO mass formula is substantially less than the error in the magnitudes of either the individual masses or the mass differences between pairs of particles. This indicates that the GMO formula is not a sensitive test of the correctness of a dynamical calculation of mass shifts. Most of the perturbative[3] and bootstrap[4] calculations of mass splittings concentrate on satisfying this sum rule; the actual magnitudes of the masses are not calculated, and often they are infinite. One or two parameters are adjusted to make the "calculated" masses fit the physical ones in some approximation. A successful dynamical calculation of mass shifts must predict the correct magnitudes of the individual masses and mass differences as well as satisfy the weak requirements of the GMO sum rule.

The pole model is the result of solving (25) in the limit that the phase shifts are identically zero for each channel. For a nonzero phase shift, the problem is more complicated. There are the correction factors proportional to the α^ρ. For f equal to 0.35 the masses, including these factors, are

$$\begin{aligned}
m_N &= 575.7-43.3\alpha^{27}-260.2\alpha^{10*}\\
&\quad -\alpha^8(45.9\sin^2\theta+36.4\cos^2\theta+81.8\sin\theta\cos\theta)\,,\\
m_\Lambda &= 941.0-65.0\alpha^{27}-67.2\alpha^1\\
&\quad -\alpha^8(28.0\sin^2\theta+10.5\cos^2\theta-34.3\sin\theta\cos\theta)\,,\\
m_\Sigma &= 861.8-28.9\alpha^{27}-0.3\alpha^{10}-260.2\alpha^{10*}\\
&\quad -\alpha^8(28.0\sin^2\theta+10.5\cos^2\theta-34.3\sin\theta\cos\theta)\,,\\
m_\Xi &= 1203.0-43.3\alpha^{27}-0.3\alpha^{10}\\
&\quad -\alpha^8(2.2\sin^2\theta+86.1\cos^2\theta+27.6\sin\theta\cos\theta)\,. \qquad (35)
\end{aligned}$$

The angle θ was introduced in Eq. (26). Clearly values of α^ρ near -1, or $|\Omega(W)/\Omega(m)|^2$ near zero, are needed to suppress the second-order effects. (The second-order corrections will not vanish even for all the α's equal to -1 because of the parametrization of the octet channel in terms of θ.) Moreover, such values of α are needed in the 1, 8, 10*, and 27 channels simultaneously. The 10 channel can be neglected for this value of f. If the phase shifts are small and slowly varying it seems obvious that the α^ρ's will be near zero. In Appendix B we check this explicitly by using a simple parametrization of the phase shift that has the correct threshold and asymptotic behavior. For small positive (negative) phase shifts the α^ρ's are positive (negative). In Table I we list bounds on α for various values of the maximum value of the phase shift. For a phase shift which has a maximum magnitude of $\pi/4$, α would be bounded by $+0.63$ or -0.39 depending on the sign of the phase shift. Even if the phase shifts reached $\pi/2$, α would be 1.65 or -0.62. If all the α's were equal to -0.39 and the most advantageous value of θ were chosen, the nucleon mass would still be 200 MeV too low. Also it is unlikely that all the channels would have the same behavior. Hence, a slowly varying phase shift will not substantially reduce the second-order effect.

TABLE I. The parametrization of a slowly varying phase shift discussed in Appendix B is used to generate bounds on α for various values of the maximum phase shift δ_m. These bounds are $\alpha_m^\pm$, where the superscript indicates the sign of δ_m.

$\|\delta_m\|$	α_m^+	α_m^-
$\pi/32$	0.063	-0.059
$\pi/16$	0.13	-0.115
$\pi/8$	0.28	-0.22
$\pi/4$	0.63	-0.39
$3\pi/8$	1.08	-0.52
$\pi/2$	1.65	-0.62

The only experimentally known $\frac{1}{2}^+$ phase shift comes from pion-nucleon scattering where there is evidence for a highly inelastic $T=\frac{1}{2}$ resonance at about 1480 MeV center-of-mass energy.[11] Such a resonance could be a member of a 27, 10*, or 8, and arguments have been put forth to favor either the 10* [12] or 8 [13] assignments. The inelasticity could be due to other $SU(3)$ allowed channels. Since the actual resonance occurs below threshold in terms of the unperturbed baryon and the meson masses, there is some ambiguity in assigning a position and width to the resonance multiplet. Neglecting this problem for the moment, we examine the effect of a resonance on the α. For a P-wave resonance with a width factor $\Gamma(W)$ and position a_r, $\Omega(W)$ is given by[14]

$$\Omega(W)=\frac{\epsilon}{[a_r-W-i\Gamma(W)/2]}\,, \qquad (36)$$

where

$$\frac{\Gamma(W)}{2}=\frac{(p\beta/\mu)^3}{1+(p\beta/\mu)^2}\gamma^2\,. \qquad (37)$$

In Eq. (36) and (37) ϵ is an arbitrary constant, β/μ is the range of interaction for mesons and baryons, γ^2 is the reduced width, and p is the c.m. momentum. The arbitrary constant cancels out of the ratio $\Omega(W)/\Omega(m)$.

[11] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964).

[12] J. J. Brehm and G. L. Kane, Phys. Rev. Letters **17**, 764 (1966).

[13] V. Barger (private communication) argued that the resonance must be an octet since it is strongly produced in proton-proton scattering, and the exchange of an $SU(3)$ singlet Pomeranchuk Regge trajectory is expected to dominate the production process.

[14] The functional form of $\Omega(W)$ for a resonance is derived in Ref. 8. The parametrization of the resonance width is suggested by K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1963), pp. 100, 101.

TABLE II. The parameter α is displayed for various values of the resonance position a_r and observed width Γ. Two values of the range parameter β are tested. The position, width, and reduced width γ^2 are all given in MeV.

a_r	Γ	β	γ^2	α
400	200	0.88	139	1.23
400	200	0.50	442	5.25
400	390	0.88	236	−0.77
400	100	0.50	236	12.84
200	200	0.88	255	−0.97
200	200	0.50	942	1.31
200	75	0.88	117	1.33
200	75	0.50	464	6.33
100	50	0.88	184	−0.97
100	50	0.50	828	2.98
800	300	0.88	106.5	4.55
800	300	0.50	274	7.91

Equation (37) is analytically continued in W to give $\Omega(m)$ a real quantity. The two-meson state is the lowest mass system that can be exchanged in meson-baryon scattering; this would indicate that β is 0.5. On the other hand, when a similar resonance formula is fitted to the $N^*(1238)$, β is found to be 0.88.[12] Since Ω depends sensitively on β, both values will be tested.

The integral in Eq. (32) with the additional factor of $|\Omega(W)/\Omega(m)|^2$ is evaluated numerically for a variety of resonance positions and widths, as well as for the two values of β. In Table II we present some typical results. We give what would be the observed width and position of the resonance. The reduced width γ^2 is calculated from the observed width. For $\beta=0.5$ the resonance always acts to give positive α's, with narrow resonances giving larger α's. However, we find that for $\beta=0.88$, the experimentally favored value, it is possible to obtain negative values as well as positive ones. Negative α's result from cancellations occurring when $\Omega(W)$ is continued to $\Omega(m)$. The position and reduced width of the resonance must be properly matched for this cancellation to occur; it is necessary that the resonance have a width comparable to its distance above threshold. Although this cancellation depends on continuing the resonance approximation away from the region where it is most valid, similar extrapolations of the resonance formula have been found to be valid in other processes.[15] Hence, we find that although a resonance is most likely to enhance the second-order effects in the mass formulas, the opposite situation is possible. We would need either several multiplets of such resonances, or a single 10* resonance with the rest of the phase shifts moderately negative. It might then be possible to reduce the second-order mass shifts to 50 MeV or less, but only at the expense of invoking unknown dynamics and an improbable concellation.

There is one parameter in Eq. (35) which has yet to be discussed. The octet mixing angle θ has heretofore been arbitrary. However, if the $\frac{1}{2}^+$ resonance is assigned to a particular multiplet and its observed inelasticity is caused by coupling to the other $SU(3)$ allowed channels, an estimate of $\sin\theta$ can be made in the limit of all other phase shifts vanishing at the resonance energy.[16] For π-N scattering the $T=\frac{1}{2}$ S matrix is

$$\eta e^{2i\delta}=\frac{1}{20}e^{2i\delta^{27}}+\tfrac{1}{4}\exp 2i\delta^{10*}+\left(\frac{3\sqrt{5}}{10}\sin\theta+\frac{1}{2}\cos\theta\right)^2 e^{2i\delta^{8}}. \quad (38)$$

Experimentally η is about 0.2.[11] If the resonance belongs to a decuplet, we have in the limit of degenerate thresholds

$$-\eta=\frac{1}{20}-\frac{1}{4}+\left(\frac{3\sqrt{5}}{10}\sin\theta+\frac{1}{2}\cos\theta\right)^2=-0.2\,,$$

or $\sin\theta=-3/(14)^{1/2}$. If the resonance belongs to an octet, we would have $\sin\theta=0.21$ or 0.997; there is no solution if the resonance belongs to the 27. In any case, our parametrization of the octet channel is consistent with the assignment of the resonance to either 8 or 10* multiplets, and with the observed inelasticity. There is no point in inserting these values into Eq. (35); the second-order mass shift is large in any case.

There is one further dynamical possibility that should be mentioned. It may be that in the symmetry limit the $\frac{1}{2}^+$ resonance should be treated as a bound state. This will mean there is another pole in $K^{\rho}{}_{\lambda\tau}$, and interference effects between the two poles could produce almost any result. Since the sign of the meson-baryon coupling is unknown, constructive or destructive interference would be possible. However, again this would occur in only one channel and would, therefore, have the same effect as a resonance.

IV. SUMMARY

We have calculated the mass of the baryon octet to second order in the symmetry-breaking Hamiltonian by a straightforward application of perturbation theory. With the restriction to two-particle intermediate states, the second-order mass shifts are as large or larger than the first-order shifts. In the limit of no scattering in the $J=\frac{1}{2}^+$ channel, the predictions are unambiguous. We have also discussed the scattering corrections to the pole model. Except under very special circumstances, these scattering corrections will not reduce the magnitudes of the second-order mass shifts obtained from the pure pole model, but are more likely to lead to larger effects. Although a mechanism has been found whereby a resonance can strongly suppress the second-order corrections, delicate cancellations are involved, and any arguments based on this mechanism are suspect. Several multiplets of $\frac{1}{2}^+$ resonances would be needed to reduce

[15] L. Durand, III (private communication).

[16] This determination of $\sin\theta$ is suggested by some of the discussion in D. Atkinson and M. B. Halpern, Phys. Rev. **150**, 1377 (1966).

the second-order effects sufficiently to make a first-order derivation of the GMO mass formula believable. These resonances would necessarily be very broad if well above threshold; there is no evidence for the existence of more than one $\frac{1}{2}^+$ resonance.

Since dynamics seems to be incapable of explaining the success of the first-order mass formulas, perhaps the manner in which perturbation theory has been applied can be questioned. For one thing, perhaps the parameters δm_1 and δm_2 should not have been determined in the way they were. (The GMO mass formula is satisfied for arbitrary values of these parameters.) However, this is tantamount to saying that first-order perturbation theory is inadequate to explain the mass splittings even approximately and *a fortiori* to assuming that perturbation theory is meaningless from the start. A similar objection could be raised to the choice of the unperturbed mass, and the same argument made against the objection. It may be that the pole approximation to $K^{\rho}{}_{\lambda\tau}$ has the wrong asymptotic behavior, leading to a large high-energy contribution to the integral in Eq. (32). However, looking at the integral equation for K, we see that, for a scattering amplitude that vanishes at infinite W, K indeed goes asymptotically as W^{-1}. Possibly perturbation theory is all right, but first- and second-order effects ought to be calculated together and the parameters adjusted in some self-consistent fashion. However, if second-order effects are so large as to necessitate consideration in this fashion, third- and higher-order terms generated from the first- and second-order terms should also be considered. On a different level, it is conceivable that the inclusion of multiparticle intermediate states in the determination of K would reduce its magnitude. Consistency then requires that multiparticle states be included in X_α as well. If these states strongly affect K, then they should also contribute strongly to X_α. These contributions would all have the same sign. Hence, we believe that multiparticle effects will increase rather than decrease the magnitude of the second-order masses. We therefore state our final conclusion: The reason for the success of the GMO mass formula when applied to the octet of baryons does not lie in the relative weakness of the symmetry-breaking interaction and the consequent applicability of perturbation theory. The explanation must come from a nonperturbative S-matrix approach.

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APPENDIX A

When Eq. (22) is evaluated for the various choices of the following expressions for $C^{\rho}{}_{\lambda\tau}$ are obtained:

$$C^{27}=-\frac{1}{1-\mu^2/2m^2}\frac{1}{5}(g_1\delta m_1+g_2\delta m_2)-\frac{m}{2\mu^2}\frac{1}{5}(g_1\delta\mu_1^2)\,,$$

$$C^{10}=-\frac{1}{1-\mu^2/2m^2}\frac{1}{5}(2g_1\delta m_1+(\sqrt{5})g_1\delta m_2+(\sqrt{5})g_2\delta m_1)-\frac{m}{2\mu^2}\frac{1}{5}(-2g_1\delta\mu_1^2+(\sqrt{5})g_2\delta\mu_1^2)\,,$$

$$C^{10*}=-\frac{1}{1-\mu^2/2m^2}\frac{1}{5}(2g_1\delta m_1-(\sqrt{5})g_1\delta m_2-(\sqrt{5})g_2\delta m_1)-\frac{m}{2\mu^2}\frac{1}{5}(-2g_1\delta\mu_1^2-(\sqrt{5})g_2\delta\mu_1^2)\,,$$

$$C^{1}=-\frac{1}{1-\mu^2/2m^2}(g_1\delta m_1-g_2\delta m_2)-\frac{m^2}{2\mu^2}(g_1\delta\mu_1^2)\,,$$

$$C^{8}{}_{11}=g_1\delta m_1-\frac{1}{1-\mu^2/2m^2}\frac{1}{10}(-3g_1\delta m_1-5g_2\delta m_2)-\frac{m}{2\mu^2}(-\tfrac{3}{10}g_1\delta\mu_1$$

$$C^{8}{}_{12}=g_2\delta m_1-\frac{1}{1-\mu^2/2m^2}(-g_1\delta m_2+g_2\delta m_1)-\frac{m}{2\mu^2}\frac{1}{2}(g_2\delta\mu_1^2)\,,$$

$$C^{8}{}_{21}=g_1\delta m_1-\frac{1}{1-\mu^2/2m^2}\frac{1}{2}(g_1\delta m_2-g_2\delta m_1)-\frac{m}{2\mu^2}\frac{1}{2}(g_2\delta\mu_1^2)\,,$$

$$C^{8}{}_{22}=g_2\delta m_1-\frac{1}{1-\mu^2/2m^2}\frac{1}{2}(-g_1\delta m_1+g_2\delta m_2)-\frac{m}{2\mu^2}\frac{1}{2}(g_1\delta\mu_1^2)\,.$$

We have set $\delta\mu_2^2$ equal to zero. In the zero-scattering limit $X^{\rho}{}_{\lambda\tau}$ is

$$X^{\rho}{}_{\lambda\tau}=I\sum_{\sigma} C^{\rho}{}_{\lambda\sigma}C^{\rho}{}_{\tau\sigma}\,,$$

with I defined in (32). The $X^{\rho}{}_{\lambda\tau}$ are combined according to Eq. (31) to give Eq. (34).

APPENDIX B

Here we estimate the effect of a small, slowly varying phase shift on the α's. Let the phase shift δ be given by

$$\delta=\lambda[X^{3/2}/(X^2+\epsilon^2)]\,, \tag{B.1}$$

where $X=W-m-\mu$. Such a phase shift has correct threshold dependence and vanishes at infinite energy. The parameters λ and ϵ can be written in terms of δ_m, the maximum value of δ, and X_m, the position of that maximum.

$$\epsilon=X_m/\sqrt{3}\,,$$
$$\lambda=2\times3^{-3/4}\epsilon^{1/2}\delta_m. \tag{B.2}$$

Then, if $\Omega(X)$ is written as $\exp\Delta(X)$, we find that[17]

[17] *Tables of Integral Transforms*, edited by A. Erdelyi (McGraw-Hill Book Company, Inc., New York, 1954), Vol. II, p. 216.

$$\Delta(X)=\frac{\lambda(\epsilon/2)^{1/2}(X+\epsilon)}{X^2+\epsilon^2}+i\frac{\lambda X^{3/2}}{X^2+\epsilon^2}. \tag{B.3}$$

For simplicity we take X_m equal to $\sqrt{3}\mu$ or $\epsilon=\mu$. By a simple analytic continuation of (B.3) we obtain

$$\Delta(-\mu)=\lambda/2\mu^{1/2}. \tag{B.4}$$

Equations (B.3) and (B.4) are combined to give

$$|\Omega(X)/\Omega(-\mu)|^2=\exp\left[\left(\tfrac{4}{3}\right)^{3/4}\delta_m\left(\frac{\mu(X+\mu)}{X^2+\mu^2}-\frac{1}{\sqrt{2}}\right)\right]. \tag{B.5}$$

The maximum magnitude of the exponent occurs for $X=\mu(\sqrt{2}-1)$. If in Eqs. (32) and (33) $|\Omega(X)/\Omega(\mu)|^2$ is replaced by its value at this point, an upper or lower bound α_m can be obtained for α depending on the sign of δ_m.

$$\alpha_m=-1+\exp[\tfrac{1}{2}(\tfrac{4}{3})^{3/4}\delta_m]. \tag{B.6}$$

The signs of α_m and δ_m are the same. In Table I we list some values of α_m and δ_m obtained from Eq. (II.6). The actual values of α may differ substantially from these bounds, though they will have the same sign.

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Some Consequences of $SU(3)$ and Charge-Conjugation Invariance for K-Meson Resonances*

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Implications of invariance of strong interactions under $SU(3)$ and charge conjugation are investigated for K-meson resonances (i.e., systems which are not eigenstates of C). It is pointed out that the charge-conjugation eigenvalue of the neutral nonstrange members of multiplets is determined by the decays (mesonic or electromagnetic) of the strange members. The $K'(1320)$, $K'(1420)$, and $K'(1800)$ are considered as examples. Possibilities for study of symmetry breaking and particle mixing in kaon resonance decays are mentioned; they may provide practical methods for studying symmetry breaking.

WE would like to point out some simple, but apparently not well-known, properties of mesons with nonzero strangeness. These properties follow whenever charge-conjugation invariance and $SU(3)$ invariance are supposed to hold. They may prove useful in the study of symmetry-breaking effects in $SU(3)$.

Assume that all the kaons we will consider are to be assigned to $SU(3)$ octets, and, throughout, let C be the charge-conjugation eigenvalue of the neutral, nonstrange (NNS) members of the octet. It is then possible to show quite simply that:

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(a) Two kaons belonging to multiplets whose NNS members have opposite C cannot mix in the limit of $SU(3)$ symmetry, but they can mix whenever symmetry breaking is present.

(b) From the decay branching ratios of kaon resonances, it is possible to determine the C eigenvalue of the NNS members which can be placed in the same multiplet as the kaons. It should almost always be possible to do this in practice, with considerable restriction on one's freedom in making up meson multiplets.

(c) From the electromagnetic decays of kaon resonances it is also possible to determine the C eigenvalue of the NNS members which can be placed in the same