Structure of the $\eta \rightarrow 3\pi$ Amplitude*

CHING-HUNG WOO
Department of Physics and Astronomy, University of Maryland, College Park, Maryland
(Received 7 December 1966)

Difficulties with the usual phenomenological theory of the C-conserving part of $\eta \rightarrow 3\pi$, with $|\Delta I| = 1$ and dominance by the nearly constant symmetric amplitude, are summarized. It is discussed why the difficulties are indicative of the presence of a significant asymmetric part in the amplitude. An attempt is made to find the possible reductions in the theoretical branching ratio $R = \Gamma(\eta \rightarrow 3\pi^0)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ as a result of including some large asymmetric parts. We found that a branching ratio of the order $R \approx 1.1$ is the lower limit of what can be reasonably achieved in these theories. If the experimental value should turn out to be $R \simeq 0.5$, it will be extremely difficult to reconcile with the theory even when large energy variations in the decay amplitude are allowed for; and it is nearly certain that a part with $|\Delta I| > 2$ is present in $\eta \rightarrow 3\pi$ in that case.

THERE are growing indications that the usual picture of the $\eta \rightarrow 3\pi$ amplitude is inadequate. We mean the picture, suggested historically by the similarity in the $\tau \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ spectra and by centrifugal-barrier considerations, in which the decay is taken to be essentially into the totally symmetric $I = 1$ final state with a constant matrix element, plus a smaller "linear term" to account for the observed asymmetry in the spectrum. The evidences against such a picture include the following:

(1) The branching ratio $R = \Gamma(\eta \rightarrow 3\pi^0)/\Gamma(\eta \rightarrow \pi^+\pi^-\pi^0)$ is of the order 1.7 in this picture,[1] whereas experimentally the value of R has been consistently lower. At the present time there is a controversy[2,3] as to whether $R \approx 1$ or $R \approx 0.5$, but either case represents a significant reduction from $R = 1.7$.

(2) Sutherland has recently shown[4] that the use of current commutation relations in a manner analogous to their use in $K \rightarrow 3\pi$ would result in $\eta \rightarrow 3\pi$ being forbidden in the usual picture. Das *et al.*,[5] by a procedure different from the usual $K \rightarrow 3\pi$ computations in the treatment of the Schwinger terms, obtain a nonzero decay rate; but then the "successes" of the $K \rightarrow 3\pi$ calculations are hard to understand.

(3) Historically, the simplest dynamical model in support of this picture is the pion pole model.[6] In fact it has been argued that the pion pole model is necessary for the similarity between the $\tau \rightarrow 3\pi$ and the $\eta \rightarrow 3\pi$ spectrum in this picture, in addition to the general consideration that the 3π are in $I = 1$ for both decays.[7] However, Hori *et al.*[8] have pointed out that $\eta \rightarrow 3\pi$ in the pseudoscalar pole model actually vanishes in the $SU(3)$ limit for constant vertices. This consideration is of course distinct from the forbiddenness in (2), since no $SU(3)$ invariance is invoked in (2).

All these difficulties may be avoidable if one relaxes the $\Delta I \leq 2$ requirement for the processes $\eta \rightarrow 3\pi$. This would be a somewhat radical assumption insofar as the $\eta \rightarrow 3\pi$ is usually taken to be electromagnetic in nature (although there is at the present no clear objection to having also a $\Delta I > 1$ part in the electromagnetic interaction).[9] Alternatively, one may drop the "dominance by the nearly constant symmetric matrix element" assumption, and consider a decay amplitude which has considerable energy dependence over the Dalitz plot. Our main purpose here is to investigate what can reasonably be achieved in avoiding the difficulties mentioned above if a more strongly energy-dependent amplitude is considered. In other words, we would like to define more precisely the limit beyond which the $\Delta I > 2$ alternative becomes almost unavoidable.

A rapid energy dependence in the $\eta \rightarrow 3\pi$ amplitude can come about in at least two ways. It can arise from a resonant final-state interaction in the s-wave 2π system; or it can be "inherent" in the $\eta \rightarrow 3\pi$ matrix element. The former possibility has been considered

* Research sponsored, in part, by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under Contract No. AFOSR 500-66.

[1] K. C. Wali, Phys. Rev. Letters **9**, 120 (1962); M. A. B. Bég, *ibid.* **9**, 67 (1962).

[2] F. S. Crawford, C. J. Lloyd, and E. C. Fowler, Phys. Rev. Letters **16**, 907 (1966).

[3] The smaller value is based on the work of G. Di Giugno *et al.* [Phys. Rev. Letters **16**, 767 (1966)]; and the larger one on M. A. Wahlig, E. Shibata, and I. Mannelli, *ibid.*, **17**, 221 (1966).

[4] D. G. Sutherland, Phys. Letters **23**, 384 (1966).

[5] T. Das, M. Grynberg, and K. Kikkawa, Phys. Rev. (to be published). There are also other current commutation relation calculations which yield a nonzero decay rate [S. K. Bose and A. M. Zimerman, Nuovo Cimento **43A**, 1165 (1966); R. Ramachandran, University of California, Riverside, report, 1966 (unpublished); and Y. T. Chiu, J. Schechter, and Y. Ueda, University of Chicago report, 1966 (unpublished)]. These authors assume an effective local operator for the second-order electromagnetic transition, and assume its commutation relation with the other curents to be simply that of a component of a scalar octet density in the quark model. This is a stronger assumption, and the transition operator of Sutherland and Das *et al.* should have more relevance; unless, of course, the decay is not of the electromagnetic origin.

[6] S. Okubo and B. Sakita, Phys. Rev. Letters **11**, 50 (1963); where references to earlier work can also be found.

[7] C. Kacser, Phys. Rev. **130**, 355 (1963).

[8] S. Hori, S. Oneda, S. Chiba, and A. Wakasa, Phys. Letters **5**, 339 (1963).

[9] N. Dombay and P. K. Kabir, Phys. Rev. Letters **17**, 730 (1966). The authors discussed $\Delta I > 1$ for the C-violating part of $\eta \rightarrow 3\pi$. In connection with the C-conserving part of $\eta \rightarrow 3\pi$, $\Delta I > 1$ for the electromagnetic interaction has also been discussed by M. Veltman and J. Yellin, [Phys. Rev. **154**, 1469 (1967)], who term the possibility "ugly."

extensively by Brown and Singer.[10] The lowest value of R achieved in their σ model is of the order 1.3.[11] Without committing ourselves to a specific final-state interaction first, we will concentrate here on the more intrinsic kind of energy dependence.

Although the distinction between the "intrinsic" energy dependence and the energy dependence arising from final-state interactions is often somewhat vague, we will follow the usual prescription of assigning the "intrinsic structure" to the subtraction terms in, say, the Khuri-Treiman (KT) equations.[12] Centrifugal-barrier considerations would indicate that because of the limited phase space available, the presence of an intrinsic p-wave part in the 2π system is of more consequence than higher partial waves. Although the same consideration would also indicate the dominance of the s wave over the p wave, the three considerations listed at the beginning offer arguments why the s-wave part may be suppressed in magnitude for this particular case. On the other hand, a large p wave is in the right direction to ameliorate all three difficulties: (1) The p wave contributes only to the charged mode, and hence is in the right direction to reduce R. (2) Sutherland shows from current commutation relations that the $\eta\rightarrow 3\pi$ amplitude has the isotopic-spin structure $\epsilon_{0\alpha\mu}\epsilon_{\mu\beta\gamma}$ (where α, β, γ refer to the isotopic-spin indices of the 3 pions), at the unphysical points $E_{\pi^+\pi^-}=m_\pi$ and m_η. This is different from the totally symmetric isotopic-spin wave function. Extrapolating by linear matrix elements, he then argues that $\eta\rightarrow 3\pi$ is forbidden. However, a p-wave part would have just this isotopic-spin structure, and need not vanish at these points. Even when the p wave is only comparable to the s wave in the physical region, it would become dominant at these unphysical points because the p-wave contribution grows with the distance away from the center of the Dalitz plot. With the usual uncertainties in the current commutation relation considerations, we believe that the dominance of these "asymmetric" contributions at the unphysical points is what one should deduce from Sutherland's result, and not the strict absence of a symmetric part. (3) Whereas the pseudoscalar pole contribution to $\eta\rightarrow 3\pi$ vanishes in the $SU(3)$ limit for constant vertices, the vector-meson pole contributions do not vanish; this has been discussed by Oneda, Kim, and Kaplan.[13] Thus all three considerations are favorable to the presence of a large asymmetric part in the amplitude. What we need to do now is to estimate the lowest values of R obtainable in such a theory.

We use the usual notation,[12] and denote

$$M(\eta\rightarrow 3\pi)=A(s_1,s_2,s_3)\delta_{0\alpha}\delta_{\beta\gamma}+B(s_1,s_2,s_3)\delta_{0\beta}\delta_{\gamma\alpha}+C(s_1,s_2,s_3)\delta_{0\gamma}\delta_{\alpha\beta}\,,$$

where (1)

$$A(s_1,s_2,s_3)=B(s_2,s_1,s_3)=C(s_3,s_2,s_1)\,,$$
$$s_i=(P-p_i)^2\,,\quad P=\textstyle\sum_i p_i\,.$$

α, β, γ is the isotopic-spin index of π_1, π_2, and π_3, respectively. We will also use the variables

$$y=(s_0-s_1)(\tfrac{2}{3}m_\eta^2-2m_\pi m_\eta)^{-1}\,,$$
$$x=(s_2-s_3)(\tfrac{2}{3}\sqrt{3}m_\eta^2-2\sqrt{3}m_\pi m_\eta)^{-1}\,,$$

where

$$s_0=\tfrac{1}{3}m_\eta^2+m_\pi^2\,.$$

From what was said above, we will use the twice-subtracted Khuri-Treiman equations. As to the π-π interactions, we note that the p-wave π-π scattering phase is quite small at these energies if one extrapolates from the ρ-meson resonance formula. The $I=2$ s-wave phase shift is usually considered to be smaller than the $I=0$ s-wave phase shift, and of the opposite sign.[14] This is deduced from the forward-backward asymmetry in pion-production experiments. However, a recent analysis by Jacobs and Selove[15] pointed out that the previous data had some bias, and that when the bias is removed a still smaller $I=2$ phase shift is deduced for the low energies. Thus both the p-wave phase shift and the $I=2$ s-wave phase shift seem small.[16] (In any case, if either of these is large at these energies, the $\eta\rightarrow\pi^+\pi^-\pi^0$ Dalitz plot would have more x^2 dependence; whereas a large $I=0$ s-wave phase shift alone does not rise to x^2 dependences. The data so far do not show any significant x^2 dependence.)

Thus we will neglect the $I=1$ p-wave and $I=2$ s-wave π-π scattering in the integrals of the dispersion equations, although their effect may be partially included in the subtraction constants. (Also for this reason we do not require the subtraction constants to be real even in the absence of any $I=0$ s-wave scattering.) With only the $I=0$ s-wave π-π interaction explicitly taken into account in the dispersion integrals, $A(s_1,s_2,s_3)$

[10] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962); Phys. Rev. **133**, B812 (1964); L. M. Brown and H. Faier, in *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energy*, edited by B. Kurşunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman and Company, Inc., San Francisco, 1965).

[11] Brown and Faier (Ref. 10) found that by using a modified propagator, the value of R is reduced from their unmodified propagator value to $R=1.19$ for $M_\sigma=400$ MeV, $\Gamma_\sigma=100$ MeV. However Barrett *et al.* [Phys. Rev. **141**, 1342 (1966)] found that, also using the modified propagator, with $M_\sigma=420$ MeV and $\Gamma_\sigma=100$ MeV, the value $R=1.3$. Chiu *et al.* (Ref. 5), moving M_σ further down to 350 MeV, obtained $R\approx1.26$.

[12] N. N. Khuri and S. B. Treiman, Phys. Rev. **119**, 1115 (1960).

[13] S. Oneda, Y. S. Kim, and L. M. Kaplan, Nuovo Cimento **34**, 655 (1964). An estimate of the vector-meson contribution much smaller than obtained by these authors has been published [P. Mobius and H. Pietschmann, Phys. Letters **22**, 684 (1966)]. However, the latter result is strongly dependent on a special dynamical model (the static quark model).

[14] See, for instance, Ref. 19.

[15] L. D. Jacobs and W. Selove, Phys. Rev. Letters **16**, 669 (1966).

[16] See also M. G. Olsson, University of Wisconsin report, 1966 (unpublished).

depends only on y, and one has[12]

$$A(y)=a+by+\frac{y^2}{\pi}\int_{y_0}^{-\infty}\frac{[A+\frac{1}{3}(\tilde{B}+\tilde{C})]f_0^*}{(y')^2(y'-y)}dy', \quad (2)$$

where

$$f_0=\sin\delta_0\, e^{i\delta_0}, \quad y_0=(s_0-4m_\pi^2)(\tfrac{2}{3}m_\eta^2-2m_\pi m_\eta)^{-1}.$$

Or, if one defines

$$F(y)\equiv A(y)+\tfrac{1}{3}[\tilde{B}(y)+\tilde{C}(y)],$$

$$F(y)=a+by+\tfrac{1}{3}[\tilde{B}(y)+\tilde{C}(y)] \times\frac{y^2}{\pi}\int_{y_0}^{-\infty}\frac{F(y')f_0(y')}{(y')^2(y'-y)}dy'. \quad (3)$$

Here

$$\tilde{B}(y)=\frac{1}{4\pi}\int d\Omega\, B(s_1,s_2,s_3),$$
$$\tilde{C}(y)=\frac{1}{4\pi}\int d\Omega\, C(s_1,s_2,s_3). \quad (4)$$

The integrations are over the angles between the momentum of π_1 and that of π_2 in the rest system of π_2 and π_3. It has been pointed out by Bronzan and Kacser[17] that in certain ranges of integration in Eq. (3), $\tilde{B}$ and $\tilde{C}$ should be obtained by an analytic continuation which results in contributions to $\tilde{B}$ and $\tilde{C}$ additional to that from averaging over the physical angles. In view of the somewhat crude approximations used below in solving the equation, we will neglect this additional contribution here.

For a simple p-wave amplitude $A=a+by$, $(\tilde{B}+\tilde{C})$ is of the form $c+dy$ from Eq. (4). For a crude approximation, we will use the ansatz $(\tilde{B}+\tilde{C})=c+dy$ for the inhomogeneous term on the right-hand side of Eq. (3). This is essentially the same approximation as that used by Barrett and Truong[18] in their study of $K\to 3\pi$. We will relate the parameters c and d to a and b by self-consistency requirements.

Within this approximation, Eq. (3) has the solution

$$F(y)=\left\{(a+\tfrac{1}{3}c)+(b+\tfrac{1}{3}d)y-(a+\tfrac{1}{3}c)y \times\left[\frac{\partial D^{-1}(y)}{\partial y}\right]_{y=0}\right\}D^{-1}(y), \quad (5a)$$

where

$$D(y)=\exp\left\{-\frac{y}{\pi}\int\frac{\delta_0(y')}{(y'-y)y'}dy'\right\}.$$

If one tries to represent Eq. (5a) approximately by a linear function of y for small y, one simply expands $D^{-1}(y)$ around $y=0$ and obtains

$$F(y)\simeq(a+\tfrac{1}{3}c)+(b+\tfrac{1}{3}d)y. \quad (5b)$$

On the other hand, if δ_0 vanishes at infinity, $D^{-1}(y)\to 1$ for large negative y. In this case for large $-y$ a linear approximation to Eq. (5a) would be

$$F(y)\simeq(a+\tfrac{1}{3}c)+(b+\tfrac{1}{3}d)y-(a+\tfrac{1}{3}c)\left[\frac{\partial D^{-1}(y)}{\partial y}\right]_{y=0}y. \quad (5c)$$

For intermediate values of y, Eq. (5c) is still a somewhat better approximation than Eq. (5b) for most phase shifts, simply because $D(y)\simeq 1$ is a better approximation than $D(y)\simeq 1+y[\partial D^{-1}(y)/\partial y]_{y=0}$. Although for the decay process only small values of y are explicitly involved, in principle the values of $(\tilde{B}+\tilde{C})$ for larger $-y$ are also involved through the integrands in Eq. (2) or Eq. (3). So we will use Eq. (5c) as the linear approximation in which the input ansatz $(\tilde{B}+\tilde{C})=c+dy$ is to be recovered. The hope is that Eq. (5c) will provide a rough approximation to Eq. (5a) up to intermediate values of y, even though some accuracy is lost near $y=0$. Using Eq. (4), one has in the linear approximation

$$(\tilde{B}+\tilde{C})=2A(0)-y\left[\frac{\partial A(y)}{\partial y}\right]_{y=0}=c+dy.$$

Comparison with Eq. (5c) gives

$$a+\tfrac{1}{3}c=(5/3)A(0);\quad b+\tfrac{1}{3}d-(a+\tfrac{1}{3}c) \times\left[\frac{\partial D^{-1}(y)}{\partial y}\right]_{y=0}=\frac{2}{3}\left[\frac{\partial A(y)}{\partial y}\right]_{y=0}.$$

Combining the three equations above, one finds

$$c=2a,\quad d=-b+(5/3)a\left[\frac{\partial D^{-1}(y)}{\partial y}\right]_{y=0}.$$

If we had used the approximation (5b), a slightly different relation would have been obtained. This really indicates the weakness of the "Omnes" approximation to the KT equation. However, the parameter d will be varied in the application below; and we checked that the final conclusion about the branching ratio R does not change if Eq. (5b) had been adopted to recover c and d.

Eliminating c and b, we have

$$F(y)=[(5/3)a-\tfrac{2}{3}dy]D^{-1}(y),$$

and the approximate $A(y)$ is

$$A(y)=[(5/3)a-\tfrac{2}{3}dy]D^{-1}(y)-\tfrac{2}{3}a-\tfrac{1}{3}dy.$$

Since neither the absolute magnitude nor the over-all phase of $A(y)$ will concern us here, we can choose $a=1$. Thus

$$A(y)=[5/3-\tfrac{2}{3}dy]D^{-1}(y)-\tfrac{2}{3}-\tfrac{1}{3}dy. \quad (6)$$

[17] J. B. Bronzan and C. Kacser, Phys. Rev. **132**, 2703 (1963).
[18] B. Barrett and T. N. Truong, Phys. Rev. Letters **17**, 880 (1966).

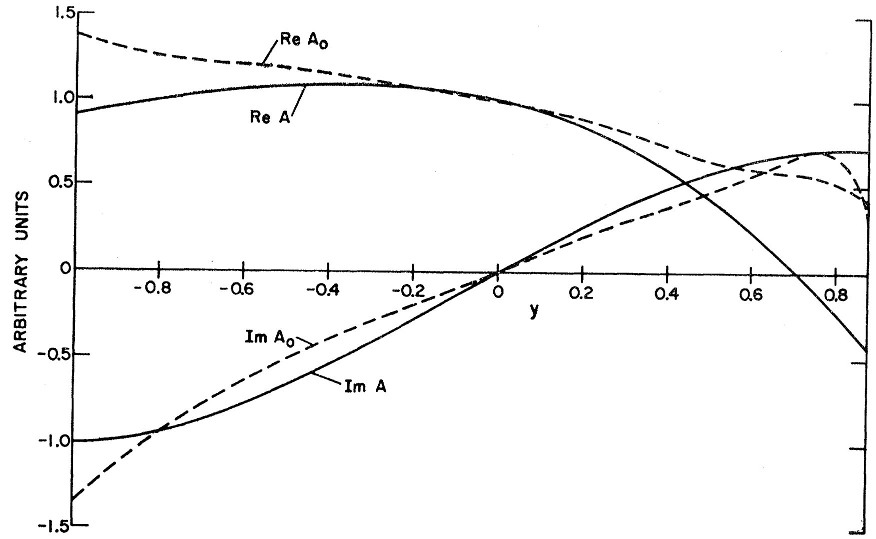

FIG. 1. The real and imaginary parts of the amplitude as a function of y. The dashed lines A_0 correspond to the solution with Wolf's phase shifts. The solid lines represent the phenomenological solution.

Hence, given the π-π $I=0$, s-wave phase shift, one can vary the one (complex) parameter d to minimize the branching ratio R, with of course the constraint that there should be reasonable agreements of the spectrum with the experiments.

If we use the s-wave phase shift deduced by Wolf[19] from analyzing pion-production experiments, we find the best solution to correspond to $d=1.23-i1.08$. The magnitude of d is roughly a measure of the asymmetric amplitude relative to the symmetric amplitude. The resulting amplitude $A(y)$, which alone contributes to the $\eta\rightarrow\pi^+\pi^-\pi^0$ mode, is shown as dashed lines in Fig. 1, and the spectrum is shown in Fig. 2. The branching ratio R for this parameter is 1.28.

We realize that the low-energy s-wave phase shift is in fact not well known. There are suggestions ranging from fairly large positive scattering lengths (which is incorporated in Wolf's phase shifts), to very small scattering lengths,[20] to very rapidly decreasing phase shifts.[21] All are claimed to be compatible with the present experimental data (such as the K_{e4} spectrum). We found that between $a_0=-0.6m_\pi^{-1}$ and $a_0=2m_\pi^{-1}$, none of the scattering lengths give rise to a $D(y)$ which reduces R below $R\approx 1.25$. Even for very rapidly varying phase shifts, such as suggested in Ref. 21, we have not found it possible[22] to reduce R below $R\approx 1.1$. The amplitudes corresponding to the best solution for each scattering length have essentially the same features as the dotted lines in Fig. 1, and will not be reproduced. We only note that in all these cases, ReA and ImA each has at most one node along the y axis of the Dalitz plot in the physical region. In view of the crude nature of our solution to the dispersion equation, the fact that a particular set of phase gives rise to a slightly lower R than another set is probably not significant. For the same reason, it may not seem convincing that $R\approx 1$ really is the lower limit obtainable in theories with large asymmetric amplitudes. We have therefore also made the following purely phenomenological check. We represent $A(y)$ by

$$A(y)=1+g_1y+g_2y^2+g_3y^3\,,$$

and vary the 3 complex parameters g_1, g_2, and g_3 with the constraint that ReA and ImA each has at most one node in the physical region. Our best solution[23] (in the sense of small R) compatible with the spectrum is

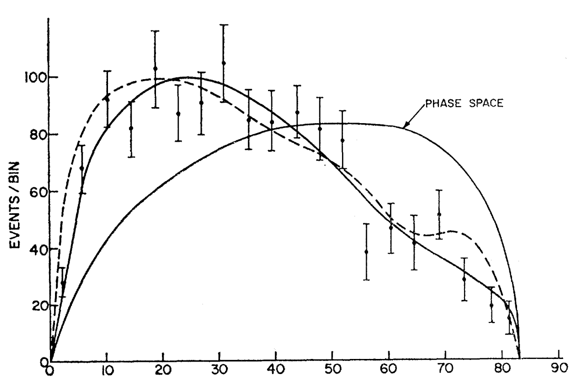

FIG. 2. The spectrum as a function of the π^0 kinetic energy T_0. The dashed line and the solid line correspond to the same solutions as explained in the caption to Fig. 1. The data is taken from Ref. 13, and the energy range is divided into twenty equal bins.

[19] G. Wolf, Phys. Letters **19**, 328 (1965).

[20] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); N. N. Khuri, Phys. Rev. **153**, 1477 (1967).

[21] L. F. Cook, Phys. Rev. Letters **17**, 212 (1966).

[22] A large negative scattering length may not be compatible with general principles, as pointed out by C. Goebel [in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, California, 1966* (University of California Press, Berkeley, California, 1967)]. I wish to thank Professor T. N. Truong for this information.

[23] The solution is obtained by first analytically locating small ranges of the parameters where the best solution is expected to lie, and then numerically checking a number of possible solutions in these ranges. It does not represent the result of a computer search.

given by: $g_1=-0.56+i1.37$, $g_2=-0.96-i0.16$, $g_3=-0.33-i0.53$.

The ReA and ImA corresponding to these parameters are shown as solid curves in Fig. 1, and the corresponding spectrum[24] is shown as a solid curve in Fig. 2. The value of R in this case is 1.05. Since the 4 additional parameters in the phenomenological fit represent a rather liberal allowance as to what the final-state interaction can do in producing desirable energy variations in the amplitude $A(y)$, we believe that $R=1$ indeed represents the lower limit of what one can reasonably achieve in theories with a rapidly energy-dependent decay amplitude.

We conclude therefore that it is of great interest to have the experimental differences on the branching ratio R resolved, because the two present extremes are in the right range to allow some definite deductions to be made.

(i) If $R\approx 0.5$, our study indicates strongly that this cannot be made consistent with the usual theory even when large energy dependences in the decay amplitudes are taken into account. The conclusion is then almost unavoidable that a part with $\Delta I>2$ is present in $\eta\to 3\pi$.[25] We have considered other obvious, though equally radical, departures from the usual theory, including modifications of the quantum number of η (including the 2^{-+} assignment), or the presence of a large C-violating amplitude. None of these alternatives are satisfactory in explaining both the Dalitz plot and the branching ratio. So we believe that the case for $\Delta I>2$, electromagnetic (second order) or otherwise, is very strong if $R\approx 0.5$.

(ii) If $R\gtrsim 1.1$, our study shows that it is not yet imperative to give up $\Delta I\leq 2$ in $\eta\to 3\pi$. The three difficulties mentioned at the beginning are no longer very serious when a large asymmetric part is included in the amplitude. In this case the decay amplitude will necessarily have a large variation in phase along the y axis of the Dalitz plot, and this must be taken into account in such questions as deducing the isospin character of any C-violating amplitude by the charge asymmetry variation from sextant to sextant,[26] if such asymmetry should exist. A rapid energy dependence in the decay amplitude may still be due to either a σ resonance as considered by Brown and Singer, or due to mainly an intrinsic structure as discussed here. The two possibilities will lead to different consequences in other processes such as K_{e4} and $K\to 3\pi$. In the σ model, the resonant final-state interaction effect should be felt elsewhere. The peak in the 2π spectrum in Ke_4 will be moved towards higher energies (as compared to phase space), and the $\tau\to 3\pi$ amplitude should have a structure very similar to that of $\eta\to 3\pi$. On the other hand, if the energy dependence in $\eta\to 3\pi$ is due essentially to the intrinsic structure as studied here, there is no reason at all for the $\tau\to 3\pi$ amplitude to have the same structure as the $\eta\to 3\pi$ amplitude. The similarity in their spectrum is then accidental, and the marked difference in their respective R values seems to bear this out. As far as the intrinsic energy dependence is concerned, from either current-commutation-relation or vector-pole-model considerations,[27] the two processes are of a very different nature. Thus future experiments such as further improvements in the K_{e4} spectrum should be able to determine which alternative is to be preferred.

I am grateful to Professor S. Oneda and Professor C. Kacser for numerous discussions and suggestions. I also wish to thank Dr. D. Sutherland for an enlightening correspondence. Financial support from the General Research Board of the University of Maryland is gratefully acknowledged.

---

[24] The experimental points are taken from Columbia-Berkeley-Purdue-Wisconsin-Yale collaboration, Phys. Rev. **149**, 1044 (1966). See also M. Forster *et al.*, *ibid.* **138**, B652 (1964).

[25] Yellin and Veltman (Ref. 8) state that a deviation of 10–15% from $R=1.7$ would imply a $|\Delta I|>2$ contribution. This we do not agree with. We would draw the line at $R\approx 1.1$.

[26] See, for instance, Barrett *et al.* (Ref. 11); T. D. Lee, Phys. Rev. **140**, B957 (1965).

[27] The vector-pole contributions to $\tau\to 3\pi$ also vanishes in the $SU(3)$ limit, unlike the $\eta\to 3\pi$ case. See Oneda, Kim, and Kaplan, Ref. 13.