High-Energy np Charge-Exchange Scattering and One-Pion Exchange

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It is shown that the "droplet" model with long-range π exchange included can account for the observed np charge-exchange differential cross sections in the GeV region for $0<-t<0.5$ (GeV/c)^2. The very steep rise in $d\sigma/dt$ for $-t<0.02$ (GeV/c)^2 is due to long-range π exchange and is similar to the steep rise near $t=0$ in elastic scattering of charged particles due to the Coulomb interaction. From the droplet-model viewpoint, π exchange can be identified only in large-impact-parameter collisions. For smaller-impact-parameter collisions, the model differs from the absorptive OPE model; however, the resulting amplitudes also have the one-pion pole at $t=\mu^2$. The reaction $np\rightarrow pn$ in the GeV region is remarkable in that it may be the only reaction in which the effect of long-range π exchange can be easily seen. In elastic scattering, long-range π^0 exchange scattering is small compared to diffraction scattering, and is masked by Coulomb effects. It might be observed in high-precision measurements of np elastic scattering. In $\bar{p}p\rightarrow\bar{n}n$, the effect of long-range π^- exchange is likely to be more accessible to experimental observation. The anomaly near $t=0$ due to π exchange may tend to vanish at high energies, due to the rapid decrease of the π-exchange amplitude with increasing energy. Fits to the data in the region $0.1<-t<0.5$ (GeV/c)^2 indicate strong spin dependence in np and $\bar{p}p$ charge exchange.

EXPERIMENTS in the GeV region[1] show that the neutron-proton charge-exchange differential cross section rises very steeply for momentum transfers $-t<0.02$ (GeV/c).[2] Below 600 MeV, strong peaking for np charge exchange (elastic scattering near 180°) is also observed.[2] Many years ago, Chew[3] suggested that the nucleon-nucleon interaction at large distances is dominated by single-pion exchange. This is supported by the phase-shift analyses of the lower-energy data which include one-pion exchange (OPE) in the higher angular-momentum states.[4] In this article, we analyze the high-energy data from an impact parameter, or eikonal, point of view[5] and report a calculation which indicates that the data near $-t=0$ may be a direct observation of long-range one-pion exchange.

High-energy elastic-scattering diffraction peaks[6] indicate that strong interaction between hadrons occurs in collisions with impact parameter[5,7] $b<5$ (GeV/c)$^{-1}$; for example, the diffraction peaks are fitted (see below, and Refs. 5 and 8) by partial-wave amplitudes $\alpha(b)$ of the form (const)$\times e^{-b^2/a^2}$, where $a\approx4.5$. Since the range of one-pion exchange is about 7 in these units, it is a relatively long-range interaction. The Born approximation for π^- exchange in np scattering [see Eqs. (2) and (4)] gives, for $b>\mu^{-1}$,

$$\alpha_{\text{OPE}}\sim i\frac{g^2}{4\pi}\frac{\mu^2}{k\sqrt{s}}\left(\frac{\pi}{2}\right)^{1/2}\frac{e^{-\mu b}}{(\mu b)^{1/2}},\qquad(1)$$

when $k\gg\mu$; $\mu=$pion mass, $k=$c.m. momentum, M $=$nucleon mass, $s=4(k^2+M^2)$, and $g^2/4\pi\cong14$.

We shall assume that the Born approximation for

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[1] See, e.g., J. L. Friedes, H. Palevsky, R. L. Stearns, and R. J. Sutter, Phys. Rev. Letters **15**, 38 (1965); see also G. Manning, A. G. Parham, J. D. Jafar, H. B. van der Raay, D. H. Reading, D. G. Rejan, B. D. Jones, J. Malos, and N. H. Lipman, Nuovo Cimento **49**, 189 (1966). The paper of Manning *et al.*, in addition to reporting the results of an experiment at 8 GeV/c, summarizes the results of previous experiments in the GeV region and gives references to earlier theoretical discussions of these data.

[2] See, e.g., R. R. Wilson, *The Nucleon-Nucleon Interaction* (John Wiley & Sons, Inc., New York, 1963), Fig. 6.3.

[3] G. F. Chew, Phys. Rev. **112**, 1380 (1958).

[4] Inclusion of OPE in the higher angular-momentum states is now standard procedure. It was introduced by M. J. Moravcsik, University of California Radiation Laboratory Report No. UCRL-5886-T, 1958 (unpublished); it was also introduced by A. F. Grashin, Zh. Eksperim. i Teor. Fiz. **36**, 1717 (1959) [English transl.: Soviet Phys.—JETP **9**, 1223 (1959)]. For more recent discussions, see, e.g., R. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966); see also G. Breit, in Proceedings of the Williamsburg Conference on Nucleon-Nucleon Scattering, 1966 (unpublished).

[5] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966); hereafter referred to as I.

[6] See, e.g., K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **11**, 425 (1963).

[7] Throughout this paper we take $b=(J+\frac{1}{2})/k$, momenta in GeV/c, and $\hbar=c=1$.

[8] M. Perl and M. C. Corey [Phys. Rev. **136**, B787 (1964)] show how this behavior develops in the 1–3-GeV/c region for πp scattering.

[9] Note that partial-wave amplitudes which are smooth functions of b and *asymptotically* go to (1) at large b yield scattering amplitudes $A(t)$ whose dependence on t for fixed s has the pion pole at $t=\mu^2$. To see this, consider the partial-wave expansion

$$A(t)=\left(\frac{i}{2k}\right)\sum_{J=0}^{\infty}(2J+1)\alpha_J P_J(\cos\theta),$$

with $t=-2k^2(1-\cos\theta)$. This sum converges in a neighborhood of $t=0$ [H. Lehmann, Nuovo Cimento **10**, 579 (1958)]. For large J and small $t>0$,

$$P_J(\cos\theta)\approx J_0(b\sqrt{(-t)})\rightarrow[(2/\pi)b\sqrt{t}]^{1/2}\cosh(b\sqrt{t}),\quad\text{as}\quad b\rightarrow\infty.$$

If α_J has the asymptotic form (1) at large J, the sum diverges as $\sqrt{t}\rightarrow\pm\mu$. To see that this corresponds to a simple pole, consider the approximation to the sum given by

$$A(t)\cong ik\int_0^\infty bdb\,\alpha(b)J_0(b\sqrt{(-t)}).$$

Replacing $\alpha(b)$ by (1) and $J_0(b\sqrt{(-t)})$ by its asymptotic form above, one obtains, for the two cases $\sqrt{t}\rightarrow\pm\mu$, the two terms

$$-\frac{g^2}{8\pi\sqrt{s}}\left(\frac{\mu}{\mu-\sqrt{t}}+\frac{\mu}{\mu+\sqrt{t}}\right)=-\frac{g^2}{4\pi\sqrt{s}}\frac{\mu^2}{\mu^2-t},$$

which give precisely the π^- pole term for np scattering [see Eq. (4)].

[10] If one assumes that the Born approximation for OPE accounts for the charge exchange for all b, even with absorption included, one obtains amplitudes for $b\lesssim5$ which are too large to account for the data; see G. A. Ringland and R. J. N. Phillips, Phys. Letters **12**, 62 (1964).

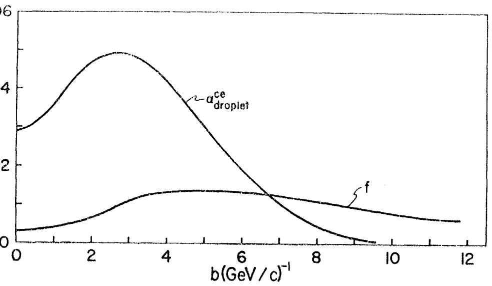

FIG. 1. Magnitudes of contributions to the nonflip partial-wave amplitude α^{ce} at 8 GeV/c [see Eq. (5)]; $b=(J+\frac{1}{2})/k$; f is Eq. (13) with $g^2/4\pi=14$, and agrees with (1) for $b>7$; $\alpha_{\text{droplet}}{}^{ce}$ is Eq. (9) with $K=0.133$ corresponding to the fit to the data in Fig. 2. Strong absorption in the central region [the factor $(1-\alpha)$ in Eqs. (9) and (13)] gives the dip near $b=0$.

OPE is valid *only at large b*.[9,10] For $b\lesssim 5$, we assume that many processes conspire to give the scattering and that the droplet model[5] describes their effect. This model gives partial-wave amplitudes which decrease at large b like the elastic-scattering amplitudes and comparison with experiment (see below) shows that they are small compared to (1) for $b>7$. In Fig. 1, a function f which agrees with (1) for $b\geq 7$ is compared with the corresponding droplet-model amplitude. Since these curves can be smoothly joined, we assume that the droplet model gives the scattering for $b\lesssim 5$ and that the amplitudes have the form (1) at large b.[11]

In addition to accounting for the np charge-exchange peak (see Fig. 2), this model predicts the π^0 contribution to elastic np and pp scattering. Calculations (see below) give, at 3 GeV/c, a spike of about 2 mb (GeV/c)$^{-2}$ rising above about a 100-mb (GeV/c)$^{-1}$ background when $-t<0.02$, if the elastic amplitudes, aside from OPE, are purely imaginary. (If constructive interference occurs, the effect may be larger.) The OPE contribution (1) to differential cross sections $d\sigma/dt$ decreases with increasing energy like $(k^2s)^{-1}$. It is masked by Coulomb effects in pp elastic scattering. Precise measurement of np elastic scattering for $-t\lesssim 0.02$ may reveal it.

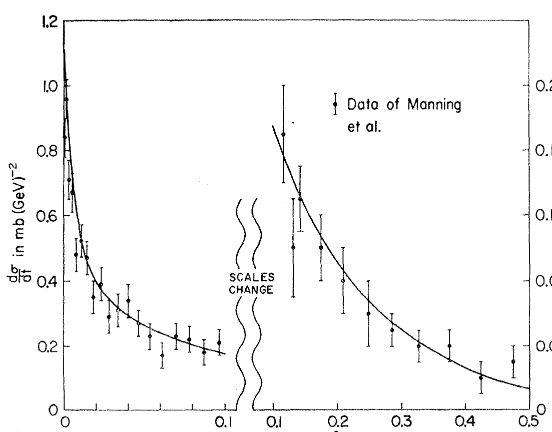

FIG. 2. $d\sigma/dt$ versus $-t$ for np charge-exchange scattering at 8 GeV/c. The data are those of Manning *et al.* (Ref. 1). The curve is Eq. (21) with $C=-0.079$, $|K|=0.133$, and $|K_{df}|=0.0050$ (GeV/c)^2 (see text). At $t=0$, it gives $d\sigma/dt=1.11$ mb (GeV/c)$^{-2}$.

To see whether droplet amplitudes with the asymptotic form (1) could account for the np charge-exchange data, we made the following analysis. In nucleon-nucleon scattering, there are five independent amplitudes[12] for each isospin state. Three are non-spin-flip amplitudes, and two describe one and two units of spin flip, respectively. As discussed in I, the spin flip and non-spin-flip charge-exchange amplitudes may be comparable. To take spin effects into account, we use the helicity amplitudes which are discussed in detail in Ref. 12. They have the expansions

$$\phi_1=\langle+\tfrac{1}{2}+\tfrac{1}{2}|\phi_1|+\tfrac{1}{2}+\tfrac{1}{2}\rangle=(i/2k)\sum_{J=0}^{\infty}(2J+1)\alpha_{J(1)}d_{00}{}^J(\theta)\,,$$

$$\phi_2=\langle+\tfrac{1}{2}+\tfrac{1}{2}|\phi_2|-\tfrac{1}{2}-\tfrac{1}{2}\rangle=(i/2k)\sum_{J=0}^{\infty}(2J+1)\alpha_{J(2)}d_{00}{}^J(\theta)\,,$$

$$\phi_3=\langle+\tfrac{1}{2}-\tfrac{1}{2}|\phi_3|+\tfrac{1}{2}-\tfrac{1}{2}\rangle=(i/2k)\sum_{J=1}^{\infty}(2J+1)\alpha_{J(3)}d_{11}{}^J(\theta)\,,$$

$$\phi_4=\langle+\tfrac{1}{2}-\tfrac{1}{2}|\phi_4|-\tfrac{1}{2}+\tfrac{1}{2}\rangle=(i/2k)\sum_{J=1}^{\infty}(2J+1)\beta_J d_{-11}{}^J(\theta)\,,$$

$$\phi_5=\langle+\tfrac{1}{2}+\tfrac{1}{2}|\phi_5|+\tfrac{1}{2}-\tfrac{1}{2}\rangle=(i/2k)\sum_{J=1}^{\infty}(2J+1)\gamma_J d_{10}{}^J(\theta)\,. \tag{2}$$

In terms of these, the differential cross section is

$$d\sigma/dt=\tfrac{1}{2}\pi\lambda\!\!\!^{-2}[|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+4|\phi_5|^2]. \tag{3}$$

Consideration of the $d_{\mu\nu}{}^J$ functions shows that ϕ_1, ϕ_2, and ϕ_3 may be nonvanishing at $\theta=0$, whereas $\phi_4\sim\theta^2$ and $\phi_5\sim\theta$ as $\theta\to 0$. For small θ, ϕ_5 describes one unit of spin flip and ϕ_4 two units of spin flip.

The Born approximation for π^- exchange contributes only to ϕ_2 and ϕ_4, and gives[12]

$$(\phi_2)_{\text{OPE}}=(\phi_4)_{\text{OPE}}=-\left(\frac{g^2}{4\pi\sqrt{s}}\right)t/\mu^2-t\,. \tag{4}$$

By inverting the relations (2), one may evaluate the corresponding $\alpha_{J(2)}$ and β_J. For large J and $k\gg\mu$, they have the form (1).

As in I, we estimate droplet contributions to the charge-exchange amplitudes from the elastic-scattering data. Since, for small θ, $d_{11}{}^J$ behaves like $d_{00}{}^J=P_J$, we represent contributions from the amplitudes ϕ_1, ϕ_2, and ϕ_3 by one function A having the expansion

$$A=(i/2k)\sum_{J=0}^{\infty}(2J+1)\alpha_J d_{00}{}^J(\theta)\,. \tag{5}$$

[11] Differential cross sections for $-t\lesssim 0.5$ are insensitive to the detailed behavior with b of partial-wave amplitudes for $b\lesssim 7$. Consequently, any *smooth* continuation of (1) to small b which gives α^{ce} the form $\alpha_{\text{droplet}}{}^{ce}$ for $b\lesssim 5$ will yield the same results.

[12] M. C. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960).

The pp elastic-scattering data[6] give, for $0\leq -t\leq 1$,

$$(d\sigma/dt)_{\text{elastic}}\approx 106e^{10t}\text{ mb }(\text{GeV}/c)^{-2}. \tag{6}$$

Neglecting spin flip, we assume

$$(d\sigma/dt)_{\text{elastic}}=\pi\lambda\!\!\!^{-2}|A|^2. \tag{7}$$

If one neglects the real part of A, one finds from (7)

$$\alpha_J\approx 0.91e^{-b^2/20}\quad\text{with}\quad b=(J+\tfrac{1}{2})/k. \tag{8}$$

Using these elastic amplitudes $\alpha_J\equiv\alpha$, we also represent the charge-exchange non-spin-flip amplitudes ϕ_1^{ce}, ϕ_2^{ce}, and ϕ_3^{ce} by an A^{ce} of the form (5) with

$$\alpha^{\text{ce}}{}_{\text{droplet}}=(1-\alpha)K\ln(1-\alpha). \tag{9}$$

For ϕ_4^{ce} and ϕ_5^{ce}, we take (2) with[13]

$$\beta_{\text{droplet}}{}^{\text{ce}}=(1-\alpha)K_{df}b^2\ln(1-\alpha), \tag{10}$$

$$\gamma_{\text{droplet}}{}^{\text{ce}}=(1-\alpha)K_f b\ln(1-\alpha). \tag{11}$$

Here, K, K_{df}, and K_f are (complex) energy-dependent parameters to be evaluated by comparison with data.

To include the exponential tail due to OPE, we take

$$\alpha^{\text{ce}}=\alpha_{\text{droplet}}{}^{\text{ce}}+if, \tag{12}$$

where, to smoothly join (1) to α^{ce} droplet,[11]

$$f=(1-\alpha)\left(\frac{g^2}{4\pi}\right)\left(\frac{\mu^2}{k\sqrt{s}}\right)\left(\frac{\pi}{2}\right)^{1/2}\times[\mu(b^2+20)^{1/2}]^{-1/2}\exp[-\mu(b^2+20)^{1/2}], \tag{13}$$

and $g^2/4\pi=14$. Aside from the absorption factor $(1-\alpha)$, f is roughly constant in the region where $\alpha_{\text{droplet}}{}^{\text{ce}}$ is appreciable (see Fig. 1), and in this "droplet region," α^{ce} and $\alpha_{\text{droplet}}{}^{\text{ce}}$ have the same form. Similarly, to give β^{ce} an exponential tail, we take

$$\beta^{\text{ce}}=\beta^{\text{ce}}{}_{\text{droplet}}+if_4, \tag{14}$$

with

$$f_4=b^2(b^2+\mu^{-2})^{-1}f. \tag{15}$$

To compare with the data, we computed A^{ce}, ϕ_4^{ce}, and ϕ_5^{ce} using (8), (11), (12), and (14). To discuss our results, we write in correspondence with (12) and (14),

$$A^{\text{ce}}=iK\mathcal{A}+F, \tag{16}$$

$$\phi_4^{\text{ce}}=iK_{df}\mathcal{G}_4+F_4, \tag{17}$$

$$\phi_5^{\text{ce}}=iK_f\mathcal{G}_5. \tag{18}$$

Our model does not distinguish OPE for $b<7$; consequently, F and F_4 are significant only in that they contain the OPE tail (1). The t dependence of F, $\mathcal{A}$, F_4, and $\mathcal{G}_4$ is shown in Fig. 3; $\mathcal{G}_5{}^2$ is very similar to the product $F_4\mathcal{G}_4$ in the region $0.02<-t<0.2$, where these functions are appreciable. Estimations of the functions F, $\mathcal{A}$, F_4, $\mathcal{G}_4$, and $\mathcal{G}_5$ for $-t\ll 4k^2$ and $p_{\text{lab}}\gtrsim 3$ GeV/c may be obtained by using the approximation $d_{\mu\nu}{}^J(\theta)\approx J_{|\mu-\nu|}(b\sqrt{(-t)})$ and replacing the sums (2) by integrals over b. In this approximation, they are functions only of t aside from normalization. Neglecting the factor $(1-\alpha)$ in (13), one finds

$$F=-\frac{g^2}{4\pi\sqrt{s}}\frac{\mu^2}{\mu^2-t}e^{-\mu a(1-t/2\mu^2)}h(a,t), \tag{19}$$

where $a=4.5$ is the "droplet radius" and $h(a,t)$ has the limiting values

$$\begin{aligned} h &\underset{a\to 0}{\longrightarrow} (\pi/2\sqrt{2})\,{}_2F_1(+\tfrac{1}{4},-\tfrac{1}{4};1;t/\mu^2)\\ &\underset{-t\to 0+}{\longrightarrow} (\pi/2)^{1/2}\{\Gamma(\tfrac{3}{2})-\gamma(\tfrac{3}{2},\mu a)\}\\ &\underset{-t,a\to\infty}{\longrightarrow} (\mu a\pi/2)^{1/2}(1-t/\mu^2). \end{aligned} \tag{20}$$

Here γ and Γ are the incomplete and complete gamma functions, respectively; ${}_2F_1(a,b;c;z)$ is the hypergeometric function. The most complete data available to us were those of Manning *et al.*[1] at 8 GeV/c. A fit to those data is shown in Fig. 2. It is given by

$$d\sigma/dt=\tfrac{1}{2}\pi\lambda\!\!\!^{-2}[F^2+CF\mathcal{A}+|K|^2\mathcal{A}^2+F_4{}^2+|K_{df}|^2\mathcal{G}_4{}^2], \tag{21}$$

with $C=-0.079$, $|K|=0.133$, and $|K_{df}|=0.0050$. [From (10), one sees that K_{df} has the dimension of inverse length squared; in units of the "droplet radius," $a=4.5$ and $|K_{df}|=0.11$.] The contributions from the various terms in (21) are shown in Fig. 3. The curve in Fig. 2 is a visual fit; the value of $|K_{df}|$ was determined from the data near $-t\approx 0.3$ and the value of $|K|$ from the data near $-t\approx 0.08$. The data for $-t\lesssim 0.05$ require destructive interference between F and $\mathcal{A}$ $(C<0)$. The data do not seem to require single-flip scattering (ϕ_5);

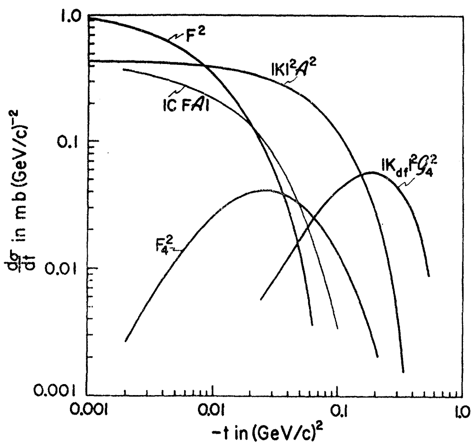

FIG. 3. Log-log plots of the contributions to $d\sigma/dt$ versus $-t$ (see Eq. 21) corresponding to the fit to the data shown in Fig. 2. The interference term $CF\mathcal{A}$ is negative.

[13] The relation of the helicity partial-wave amplitudes to the usual singlet and triplet partial-wave amplitudes is given in Ref. 12. Considerations similar to those given in I[4] for the spin-flip amplitude in πp scattering (proportional to $\boldsymbol{\sigma}\cdot\mathbf{L}$) lead us to assume that if matrix elements have no central singularities, $\beta\sim b^2$ and $\gamma\sim b$ as $b\to 0$.

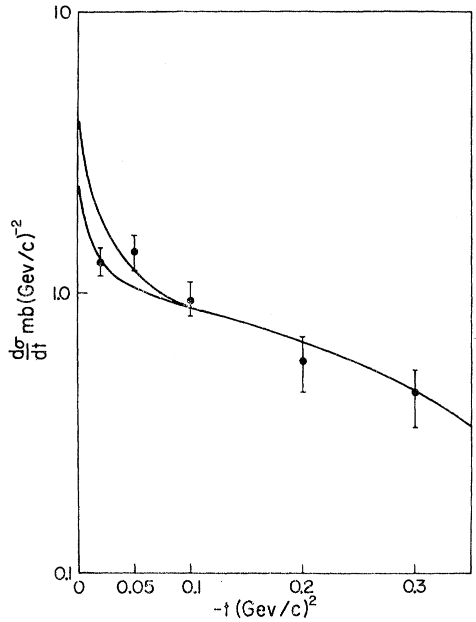

FIG. 4. $d\sigma/dt$ versus $-t$ for the charge-exchange reaction $\bar{p}p \rightarrow \bar{n}n$ at 7 GeV/c. The data are representative points taken from the graph in Ref. 17. The curves are calculated using Eq. (21), assuming $|C/K|$ has the same value as in the fit to the np data (Fig. 2); $|K|=0.26$ and $|K_{df}|=0.016$. The upper and lower curves correspond, respectively, to $C<0$ and $C>0$. For $\bar{p}p \rightarrow \bar{n}n$, the OPE amplitudes are positive near $t=0$.

note also that an $F_4\mathcal{G}_4$ interference term is missing in (21). Since $\mathcal{G}_5{}^2$ and $F_4\mathcal{G}_4$ are similar functions of t, they may both be appreciable and cancel.[14]

We use the parameter C rather than the phase of K because [see Eq. (3)] $|K|^2\mathcal{A}^2$ may represent contributions from $|\phi_1|^2$ and $|\phi_3|^2$ as well as the droplet contribution to $|\phi_2|^2$. Unitarity and isospin invariance require

$$\mathrm{Im}[\phi_1{}^{\mathrm{ce}}(0)+\phi_3{}^{\mathrm{ce}}(0)]=(k/2\pi)(\sigma_{pp}-\sigma_{pn}), \qquad (22)$$

where σ_{pp} and σ_{pn} are proton-proton and proton-neutron total cross sections, respectively. Data[15] give $\sigma_{pp}-\sigma_{pn}=-1.8\pm1.8$ mb at 8 GeV/c. Our values for $|K|$ and C are consistent with these data. To measure the magnitudes and phases of ϕ_1 and ϕ_3 relative to ϕ_2 at small t, spin correlations must be studied (for example, charge exchange of polarized neutrons with polarized proton target).

The 3-GeV/c data of Friedes *et al.*[1] can also be fit with (21). Corresponding parameter values are $|K|\approx0.51$, $C\approx-0.18$, and $|K_{df}|\approx0.026$. These values reflect the fact that the differential cross sections for $-t>0.05$, in the range of incident momenta 1.26 to 8 GeV/c, decrease with increasing energy more rapidly than the extreme forward peak (see, e.g., Fig. 9 in Manning *et al.*, Ref. 1).

Owing to the behavior with t of the functions displayed in Fig. 3, our model yields the following interpretation of the np charge-exchange differential cross sections for $-t\lesssim0.5$. This forward-peak region may be divided into three subregions: For $0.2<-t<0.5$, the double-flip amplitude $\mathcal{G}_4$ dominates; for $0.02<-t<0.2$, the zero-flip amplitude $\mathcal{A}$ dominates (single-flip scattering may also occur in this region; if so, its contribution to $d\sigma/dt$ is cancelled by $F_4\mathcal{G}_4$ interference); for $0<-t<0.02$, the rapid variation of $d\sigma/dt$ is due to long-range one-pion exchange. There are then two distinct effects which may be verified experimentally: important contributions from OPE for $-t<0.02$ and important spin dependence in the scattering for $-t\gtrsim0.1$. The spin dependence can be checked by spin-correlation measurements.[16]

If OPE contributes appreciably to np charge exchange, it should also be observable in the charge-exchange reaction $\bar{p}p \rightarrow \bar{n}n$. The differential cross section for this reaction is of order 1 mb (GeV/c)$^{-1}$ at 7 GeV/c. Presently available data[17] do not extend into the region $-t<0.02$. The OPE effect here may be less striking than in the np case because the droplet contributions appear to be relatively larger (see Fig. 4). For $\bar{p}p$ charge exchange, one might expect the interference between droplet and OPE amplitudes to be constructive because the OPE amplitudes change sign for particle $\rightarrow$ antiparticle. However, the droplet amplitude may also have a different phase than in the np case. (If the effective droplet amplitude also changes sign and the interference is destructive and large, there might be a pronounced *dip* near $t=0$.) To exhibit the region of $-t$ where the OPE tail might be observed, we assumed that $|C|$ and $|K|$ are in the same ratio as in the np case and, allowing both signs for C, computed $d\sigma/dt$ using (21) with $|K|$ and $|K_{df}|$ evaluated by

[14] If single-flip scattering is appreciable, it would give observable effects in polarization measurements; e.g., if K_f is real, the right-left asymmetry for charge exchange from a target polarized perpendicular to the incident-beam direction might be as large as $0.5P$ (P=target polarization) for $-t\approx0.15$. In terms of the amplitudes ϕ_i, this asymmetry is given by

$$\frac{N_R-N_L}{N_R+N_L}=\frac{4}{\pi}P\frac{\mathrm{Im}[(\phi_1+\phi_2+\phi_3-\phi_4)\phi_5{}^*]}{[|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+4|\phi_5|^2]}.$$

[15] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, and A. L. Read, Phys. Rev. **139**, B913 (1965).

[16] For example, our fit predicts that for $-t\simeq0.3$ the outgoing nucleons produced by charge exchange with a proton target polarized along the beam direction will be longitudinally polarized with polarization nearly equal to the target polarization. In terms of helicity amplitudes, the longitudinal polarization of the final particles is

$$P=\pm P_{\mathrm{target}}\frac{[|\phi_4|^2\mp|\phi_2|^2\pm|\phi_1|^2-|\phi_3|^2]}{[|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2+4|\phi_5|^2]},$$

where the upper (lower) sign corresponds to proton (neutron). From Fig. 3, one sees that for $-t\simeq0.3$, $|\phi_4|^2$ dominates, and one has $P\simeq P_{\mathrm{target}}$. A more complete discussion of spin correlation effects is given by G. Thomas (to be published).

[17] G. Finocchiaro, A. Michelini, W. Beusch, W. E. Fischer, B. Gobbi, and E. Polgar, in Proceedings of the Stony Brook Conference on High-Energy Two-Body Reactions, 1966 (to be published).

fitting the data at larger $-t$. The results[18] are shown in Fig. 4 along with some representative data points.

Observation of rapid t variation in the differential cross section for $\bar{p}p$ charge exchange with $-t<0.02$ and/or verification of the energy dependence of the rapidly varying contribution for $-t<0.02$ in np charge exchange would be important confirmation of the presence of long-range OPE in nucleon-nucleon collisions. Note that we have assumed in the foregoing that the pion is "elementary"; i.e., the amplitudes f and f_4 decrease with increasing energy like (1). If the pion is, on the other hand, "Reggeized,"[19] these amplitudes would have a slightly different energy dependence. This effect is expected to be small.[19]

[18] For more accurate estimations, the $\bar{p}p$ elastic scattering data should be used.

[19] G. F. Chew and S. C. Frautschi, Phys. Rev. Letters **8**, 41 (1962).

It will be interesting to learn more about the energy dependence of the structure of these charge-exchange peaks. Assuming our model to be correct, we expect the magnitudes of the parameters K, K_f, and K_{df} to decrease at high energies like, e.g., some power of energy.[20] We do not know whether they obey the same or different power laws.

We would like to take this opportunity to thank C. N. Yang and R. P. Feynman for stimulating and informative discussions, and M. E. Parkinson for help with computations.

[20] The phases of these parameters may also be measured (see, e.g., Refs. 14 and 16). A. A. Lozunov, N. van Hieu, and I. T. Todorov [Ann. Phys. (N.Y.) **31**, 203 (1965)] showed that in the asymptotic region ($s\rightarrow\infty$ for fixed t), analyticity and crossing symmetry of scattering amplitudes relate their phases to their energy dependence. It would be interesting to compare such relations with experimental data. M. LeBellac [Nuovo Cimento, **42**, 443 (1966); CERN report (unpublished)] discusses such a comparison for πp scattering.

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Pion-Pion Scattering in a K-Matrix Model Incorporating Crossing Symmetry*

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A crossing-symmetric generalization of the K-matrix formalism is developed and used to construct a model for π-π scattering. The full amplitude satisfies elastic unitarity in the s channel and has the correct singularity structure at the elastic thresholds in the t and u channels. The two parameters present in the model are determined by requiring approximate satisfaction of the crossing relations in the neighborhood of the symmetry point. The resulting p-wave phase shift exhibits a resonance with mass about 800 MeV and width about 250 MeV. The f^0 is not reproduced. The s-wave scattering lengths in the $I=0$ and $I=2$ channels are $(a_0,a_2)=(-0.67,-0.30)$ and the effective Chew-Mandelstam coupling constant is found to be 0.18.

1. INTRODUCTION

IT has been known for many years that in potential theory the product $q\cot\delta$, considered as a function of energy, is regular at threshold and that consequently a power-series expansion in q^2 is valid there. This fact has frequently enabled useful parametrizations of scattering data in the form of the scattering-length and effective-range approximations.[1]

It is also well known that this result may be thought of from a somewhat different point of view, in which use is made of the K-matrix formalism. The characteristic feature of the K matrix, defined by

$$S=\frac{1+\frac{1}{2}iK}{1-\frac{1}{2}iK}, \tag{1.1}$$

lies in the fact that its Hermiticity is a necessary and sufficient condition for the S matrix to be unitary. To obtain further properties of K the scattering process under consideration should be specified in some detail. Thus we consider the elastic scattering of two identical particles of unit mass, with charge and spin both zero; the usual Mandelstam variables are defined as shown in Fig. 1. Taking two-particle matrix elements of K in the

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[1] R. H. Dalitz, *Strange Particles and Strong Interactions* (Oxford University Press, New York, 1962).

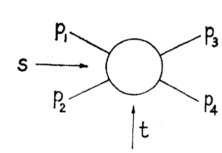

FIG. 1. Elastic pion-pion scattering.