# Model for the $\Delta I = \frac{1}{2}$ Rule*

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We show that a modification of the Cabibbo theory leads to identical results concerning the leptonic decays of hyperons but additionally contains the $\Delta I = \frac{1}{2}$ rule. The $\Delta I = \frac{1}{2}$ rule so obtained is weakly broken, but the deviations are not inconsistent with experiment. (The estimated rate for $K^+ \rightarrow \pi^+\pi^0$ is somewhat less than experimentally observed.) The model differs from Cabibbo's theory in that $G_V$ ($\beta$ decay)/$G_V$(muon decay) $=1$ rather than $\cos\theta = 0.966$. Branching ratios of the intermediate bosons are estimated.

## I. INTRODUCTION

A PERSISTENT problem in the theory of weak interactions has been to understand the existence of the $\Delta I = \frac{1}{2}$ rule[1] as a fundamental feature of the weak interactions. The two most popular proposals have been (1) to attribute the rule to "dynamical" corrections (effectively saying that the rule is a feature of the detailed kinematics, including also the strong interactions), or (2) to invoke the existence of a neutral intermediate boson which must then satisfy certain coupling rules if the $\Delta I = \frac{1}{2}$ rule is to result. We will show below that it is possible to develop a model for the weak interactions, following a modified version of approach (2), wherein the required coupling rules follow from a simple symmetry principle.

A second problem has been to understand the relationship between the $\Delta S = 0$ and $\Delta S = 1$ processes. A great advance has been made by Cabibbo[2] who showed that the $\Delta S = 1$ processes could be understood in terms of an octet-transforming baryon current which had suffered a unitary transformation that introduced the $\Delta S = 1$ amplitudes. The general idea being that violations of $SU_3$ are responsible for the $\Delta S = 1$ processes, rather than such processes being a fundamental feature of the weak interactions. The model we will discuss follows the spirit of the Cabibbo theory quite closely, but differs markedly in detail.

A third problem has been to justify the hypothesis of Feynman and Gell-Mann[3] that the leptonic decays of the hyperons obey the rule $\Delta S = \Delta Q = 1$. This rule is obtained in the Cabibbo theory by assuming octet transformation properties for the baryon currents. In our model the rule results for the same reason, but the transformation properties follow from a more fundamental postulate.

A few preparatory comments will be essential before going into the detailed model. As indicated above we intend to develop a neutral boson formulation of the $\Delta I = \frac{1}{2}$ rule, and such a model must therefore contain neutral currents. It is possible to get the impression that experiment has ruled out the possibility of neutral currents (in much the same way that it was once assumed that experiment confirmed the conservation of parity in these same interactions), while in fact many if not most of the theoretically interesting neutral currents are exceedingly difficult to observe.[4] Recall that the experimental evidence against a current transforming a nucleon into a lepton can be summarized by assigning these particles to different classes, each class separately obeying the conservation law: Particles minus antiparticles equals a constant. The experimental evidence similarly suggests that the lepton class is actually two classes: one containing the electron and the beta-decay neutrino, the other the muon and its associated neutrino. Thus the well-known absence of a neutral current coupling the muon to the electron (e.g., $\mu \rightarrow e + \gamma$, etc.) no more disproves the existence of neutral currents than does absence of the reaction $p \rightarrow \bar{e} + \gamma$, and certainly absence of $\nu_\mu + n \rightarrow p + e$ doesn't exclude all *charged* currents. Unfortunately, neutral currents clearly involving particles of the same class such as formed by transforming a particle into itself (e.g. $n + e \rightarrow e + n$) are difficult to detect[4] in general, mainly due to competition from other interactions that have similar properties. In the present theory only the hadron terms involve neutral currents, and the competition from the strong interactions makes direct detection of these currents exceedingly difficult. In view of the impressive list of theoretical precepts concerning weak interactions that have been shown to be incorrect (e.g., conservation of parity, equality of the $\Delta S = 0$ and $\Delta S = 1$ coupling strengths, and conservation of time-reversal invariance, to name a few), it is perhaps premature to consider the absence of neutral currents to be established.

A second problem area, insofar as commanding serious consideration to the model is concerned, involves the question of lepton symmetries. Our model assumes common symmetries of the hadrons and the weak intermediate bosons (little is experimentally known about the latter, not even whether or not they exist), but as a consequence it is difficult to satisfactorily treat the leptons. If one merely introduces *ad hoc* the

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[1] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. **7**, 407 (1957).

[2] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963); **12**, 62 (1964).

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[4] F. C. Michel, Phys. Rev. **138**, B408 (1965).

minimal lepton currents in order to satisfy the known experimental data, then the over-all theory including leptons lacks complete symmetry; on the other hand, if the symmetry is considered to be universal, then one is forced to seriously consider how the leptons might fit into that symmetry. Even if we were able to formulate the definitive theory of leptons, it would be pedagogically awkward to try to present such a theory in conjunction with a new theory of the weak interactions. Therefore our approach has been to present the *ad hoc* treatment of the leptons and defer consideration of possible lepton symmetries elsewhere. To the valid complaint that the *ad hoc* treatment is not completely symmetric, we can only point out that the conventional theory of weak interactions is not itself symmetric in that there are no lepton currents analogous to the $\Delta S=1$ baryon currents, and we have essentially perpetuated this same asymmetry.

## II. THE MODEL

It is simplest to discuss our model in comparison to the Cabibbo theory, since they are rather similar. In this way the reader can more easily compare the relative number and severity of the assumptions involved as well as the differences and similarities of the two theories.

Cabibbo employed three explicit assumptions as well as several implicit assumptions in framing his theory. We emphasize these assumptions to show that basically we only make slightly *different* assumptions, rather than additional assumptions. The Cabibbo assumptions are (1) $J_\alpha$ transforms according to the eightfold representation of $SU_3$, (2) the vector part of $J_\alpha$ is in the same octet as the electromagnetic current, and (3) $J_\alpha$ has "unit length," i.e., $a^2+b^2=1$. In the above notation $J_\alpha$ is the baryon current while $a$ and $b$ represent the $\Delta S=0$ and $\Delta S=1$ current strengths respectively. The major implicit assumptions are that the lepton current transforms as an $SU_3$ singlet (it seems easy to overlook the fact that excluding the leptons from transformation nevertheless constitutes a definite statement about their transformation properties), and that in the limit $\theta\to 0$ the basic weak baryon current is formed of the charged $\Delta S=0$ octet member.

We will now present the basic assumptions of our model.

(1) *The baryon currents are $SU_3$ symmetrically coupled to the intermediate bosons.* In other words, the weak interactions are unitary-spin-symmetric at least insofar as the hadronic currents are concerned. (Remember, we wish to here treat the leptons on an *ad hoc* basis.) This assumption establishes the symmetry principle on which the model is based, and corresponds most closely to Cabibbo's assumption of a $J_\alpha$ unitary octet behavior. Assumption (1) by itself could be satisfied by a current transforming as any one of the six possible baryon currents: $\bar{\mathbf{8}}\times\mathbf{8}=\mathbf{1}+\mathbf{8}+\mathbf{8}+\mathbf{10}+\overline{\mathbf{10}}+\mathbf{27}$, however only for an *octet* can the charged pion decay into an electron-neutrino pair (in the $SU_3$ limit) via an intermediate baryon loop. That is, the intermediate bosons ($W$) must belong to the same unitary multiplet as the charged pion. We cannot tell which octet, however, and must therefore consider both together. A direct consequence (unfortunately not of any present experimental significance) is that mesons in a different multiplet (e.g., the unitary singlet component[5] of the $\omega$ and $\varphi$) can not directly decay into leptons. Thus we arrive at Cabibbo's assumption of the baryon current transforming as an octet, however *all* components of the octet are present whereas Cabibbo invoked only those explicitly required by experiment. The "new" currents will be discussed, in due course, but first let us further develop the model according to assumption 1.

The interaction can be written

$$L_{\text{int}}=(4\pi)^{1/2}fJ_{\alpha\lambda}W_{\alpha\lambda}; \tag{1}$$

in words, a vector ($\alpha$) and unitary-octet ($\lambda$) baryon current ($J$) couples with an octet of vector bosons ($W$) to form a relativistically invariant unitary-spin scalar. The pion octet would couple as

$$L_{\text{effective}}=g_{\Phi W}(\partial_\alpha\Phi_\lambda)W_{\alpha\lambda}. \tag{2}$$

For the leptons, we know that we need a current of the form $J_\alpha=(4\pi)^{1/2}f(\bar{e}\gamma_\alpha a\nu_e)$ that couples to the charged triplet member of the $W$ octet. Clearly we can construct other lepton currents [e.g., $(\bar{\nu}\gamma_\alpha a\nu)$], but we must at least have the coupling

$$L_{\text{lepton}}=(4\pi)^{1/2}f(\bar{e}\gamma_\alpha a\nu_e)W_{3\alpha}^{+}, \tag{3}$$

where the muon coupling is given by the substitution $e\to\mu$ in Eq. (3), and the $W$ octet has been labeled according to the notation $W_1$ for the isosinglet, $W_2$ and $\bar{W}_2$ for the isodoublets, and $W_3$ for the isotriplets. So long as $M_W^2\gg k^2$ where $k_\alpha$ is the 4-momentum transfer, Eqs. (1) and (3) reduce to the current-current formulation,[3]

$$L_{\text{int}}=8^{1/2}G(J_\alpha+j_\alpha)(J_\alpha+j_\alpha)^\dagger, \tag{4}$$

with $8^{1/2}\,G\equiv 4\pi f^2/M_W^2$ and $J_\alpha$ is the charged $\Delta S=0$ current. The normalization of $J_\alpha$ is taken so that the $(\bar{n}p)W_3^-$ coupling coefficient is unity.

What cannot be accomplished without further assumptions is the construction of the other lepton couplings: In general a neutral current such as $(\bar{\nu}\nu)$ would couple to both $W_3^0$ and $W_1$ and the relative coupling strengths are theory-dependent. In either our model or Cabibbo's the $\Delta S=\Delta Q=1$ rule obtains because the $\Delta S=1$ leptonic decays can only occur via an *effective* coupling $(\bar{e}\nu)W_2^+$; we will presently show how such an effective coupling is obtained.

Having seen that the usual current-current formulation is contained within Eq. (1) after Eq. (3), let us consider the additional terms in Eq. (1) [and (2)]. Unfortunately it would be difficult to experimentally

[5] J. J. Sakurai, Phys. Rev. Letters 9, 472 (1962).

verify the existence of any of these $\Delta S=0$ hadronic couplings because the same interaction can also be obtained via strong-interaction corrections to the $J_\alpha J_\alpha^\dagger$ part of Eq. (4). Furthermore, the strong interactions compete with the weak $\Delta S=0$ hadronic couplings and thereby mask the existence of the latter: The only practical way to detect, for example, the weak interaction $J_\alpha J_\alpha^\dagger$ is to look for the tiny parity admixtures[6] in nuclear states. The analogous predictions of Eqs. (1) and (2) are parity admixtures in the states of hypernuclei.

(2) *The Conserved-Vector-Current theory holds.* This is Cabibbo's second assumption, which we also retain.

(3) *The W-octet mass degeneracy is removed by a mass splitting that transforms as $F_6$.* This assumption does not look very much like Cabibbo's assumption of "unit length," but we will see that it leads to a rather similar statement. Nondegenerate multiplet masses of the hadrons are an experimental fact and are theoretically well reproduced[7] by assuming a mass splitting that transforms as $F_8$. By introducing a similar mass splitting (but along a different "axis," $F_6$ instead of $F_8$), we follow quite closely Cabibbo's conjecture that $SU_3$ enters into both the strong and weak interactions, but enters in a different aspect. In both theories the distinction between the strong and weak interactions is not only the relative strengths but also the "axis" along which the symmetry is broken. Only a perturbation transforming as $F_6$ or $F_7$ will lead to violation of $S$, and, since the two give essentially the same results, the strangeness violation is uniquely specified in this theory. (It is also uniquely specified in Cabibbo's theory. However, we cannot use his trick— a unitary transformation—because a unitary transformation will leave a symmetric interaction symmetric: hence no $\Delta S=1$ amplitudes would be introduced with our assumption 1.) We have no reason not to break the symmetry along some more complex axis composed, say, of both $F_6$ and $F_8$, except that this necessarily introduces additional arbitrary parameters and we would like to present only the simplest and most economical model (even at the risk of introducing simplifications that are not respected by nature).

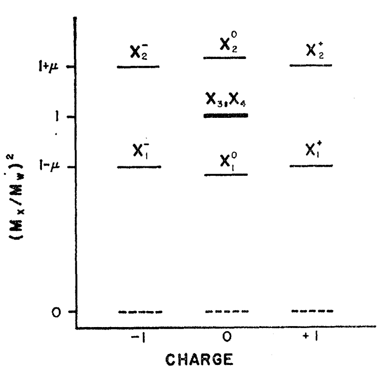


Fig. 1. Mass (squared) spectrum of the $X$ bosons for $\mu\approx\frac{1}{4}$.

Table I. Eigenstates of the $X$, as defined in Fig. 1, are given in terms of $S$, $I$ eigenstates of the $W$. An entry such as $-\frac{1}{2}$ is to be read $-\sqrt{\frac{1}{2}}$, etc. The mass correction $\delta M^2$ is in units of $\mu(M_W{}^2)$.

| State | $W_2{}^\pm$ | $W_3{}^\pm$ | $W_1$ | $W_2{}^0$ | $\bar{W}_2{}^0$ | $W_3{}^0$ | $\delta M^2$ |
|---|---|---|---|---|---|---|---|
| $X_1{}^\pm$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | | | | | $-1$ |
| $X_2{}^\pm$ | $\frac{1}{2}$ | $\frac{1}{2}$ | | | | | $+1$ |
| $X_1{}^0$ | | | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $-2/\sqrt{3}$ |
| $X_2{}^0$ | | | $\frac{1}{8}$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | $\frac{3}{8}$ | $+2/\sqrt{3}$ |
| $X_3$ | | | 0 | $\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 |
| $X_4$ | | | $-\frac{3}{4}$ | 0 | 0 | $\frac{1}{4}$ | 0 |

The relation to Cabibbo's theory is that his postulate of "unit length" for $J_\alpha$ (a weak version of universality in his own estimation) enabled him to introduce in a unique way the $\Delta S=1$ interactions via the unitary transform $U=\exp(2iF_7)$. In our model universality is represented by a universal value for $f$ (just as for the coupling constants of the electromagnetic, gravitational, and presumably the strong interactions), which seems to be a stronger version of universality and more consistent with universality as evidenced in the other interactions. Having specified the mass perturbation to transform as $F_6$, we therefore can obtain the mass spectrum by diagonalizing the Hamiltonian

$$H^2=M_W{}^2(F_0+\mu F_6)\,, \tag{5}$$

where $F_0$ is the unit matrix, and we have employed the empiricism that boson mass corrections are quadratic, hence a quadratic Hamiltonian, although for our purposes the results are essentially the same if a linear Hamiltonian is used. The new eigenstates are designated $X$, and Fig. 1 shows the mass spectrum while Table I lists the $X$ states in terms of the $W$. Note that Eqs. (1) to (3) *remain unchanged*, the only new consideration is that the $W$ are no longer eigenstates and the $X$ are not degenerate.

## III. RESULTS

### A. Cabibbo Equivalent

For low-momentum-transfer interactions involving only virtual intermediate bosons, the appropriate coupling constant is no longer $G$ as defined for Eq. (4) but rather $G'=CG$, where $C$ introduces the net correction due to mass nondegeneracy and intermixing of a given $W$ state into two or more $X$ states.

For charged $\Delta S=0$ interactions (e.g., beta decay) we have the factor

$$\begin{aligned} C_0&=\sum_X (W_3{}^\pm|X)(X|W_3{}^\pm)M_W{}^2M_X{}^{-2}\\ &=(1-\mu^2)^{-1}\,, \end{aligned} \tag{6}$$

where (see Table I) only $X_1{}^\pm$ and $X_2{}^\pm$ contribute to the sum over $X$ states in Eq. (6). The sum represents those

[6] Compare F. C. Michel [Phys. Rev. **133**, B329 (1964)], which gives references to earlier work.
[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1961).

$X$ states that contain an admixture of $W_3^{\pm}$ [i.e., that couple to $(\bar{n}p)$ or $(\bar{e}\nu)$], while the factor $M_X^{-2}$ gives the effect of the mass splitting on the boson propagator in the low-momentum-transfer approximation. The factor $C_0$ is equivalent to Cabibbo's $\cos\theta$ except that $C_0$ applies to *all* $\Delta S=0$ processes involving leptons [i.e., to the coefficient $G$ of Eq. (4)], whereas $\cos\theta$ applied only to $J_\alpha$ in Eq. (4). Therefore, $C_0$ will not account for a deviation of the $O^{14}$ decay rate from the CVC (conserved-vector-current) theory prediction.[3,8]

The coupling factor appropriate to the observed charged $\Delta S=1$ processes (e.g., leptonic $K$ decay) is

$$C_1=\sum(W_3^{\pm}|X)(X|W_2^{\pm})M_W^2M_X^{-2} \qquad (7)$$
$$=-\mu(1-\mu^2)^{-1},$$

which vanishes as expected for $\mu\rightarrow 0$. In Cabibbo's notation $C_1^2/C_0^2=\tan^2\theta$; thus $\mu\approx\theta$ and from the data[9] on $K^+\rightarrow\bar{\mu}\nu$ versus $\pi^+\rightarrow\bar{\mu}\nu$,

$$\mu=0.273\pm0.005.$$

The observation[2] that the data can be understood in terms of a single value of $\theta$ (or $\mu$) is a necessary consequence of either theory, and the interrelation among the leptonic hyperon decays can be obtained in exactly the same way as illustrated by Cabibbo: $J_\alpha$ is composed of an axial and polar vector part, each of which can further be decomposed into a symmetric and antisymmetric octet contribution, and the CVC theory (assumption 2) fixes the amplitude of the two polar vector amplitudes at $\frac{1}{2}$ and 0, respectively. (With our definition of $G$, the interactions are $\frac{1}{2}V-\frac{1}{2}A$.) The axial-vector contributions can be determined experimentally to interrelate the various leptonic decays. It has been proposed that for the axial vector current $(F/D)_{\text{weak}}=(F/D)_{\text{strong}}$ which would further reduce the number of arbitrary parameters in either theory.[10]

### B. This Model

The neutral currents contained in Eq. (1) now become readily observable insofar as they violate strangeness, the coupling factor for neutral $\Delta S=1$ interactions being

$$C_1^0=\sum(W_3^0|X)(X|W_2^0)M_W^2M_X^{-2}$$
$$=-C_1(1-\mu^2)(1-4\mu^2/3)^{-1}2^{-1/2}, \qquad (8)$$

(only $X_1^0$ and $X_2^0$ contribute to the sum) and in the limit $\mu\rightarrow 0$, *the exact* $\Delta I=\frac{1}{2}$ *rule is obtained*. In other words $C_1=-\sqrt{2}C_1^0$, which is equivalent[11] to the weak couplings $K^0\pi^0=-K^+\pi^-/\sqrt{2}$ or $(\bar{\Lambda}n)(\bar{n}n)=(\bar{\Lambda}p)(\bar{p}n)$. We also see from Eq (8) that the $\Delta I=\frac{1}{2}$ rule is violated owing to the finite value of $\mu(\approx 0.273)$; however, the factor $(1-\mu^2)(1-\frac{4}{3}\mu^2)^{-1}\approx 1.028$ and therefore the $\Delta I=\frac{1}{2}$ rule holds to a quite good approximation. It is not too disappointing that we do not get an exact $\Delta I=\frac{1}{2}$ rule, since there are indications (e.g., the decay rate for $K^+\rightarrow\pi^0\pi^+$) that the experimental deviations are larger than can readily be explained exclusively on the basis of electromagnetic violations of $I$. We can make a rough estimate of the $K^+\rightarrow\pi^0\pi^+$ rate in the following way: Since all particles obey $SU_3$ in the appropriate limit we can (in that limit) directly compare amplitudes that lead to $K_1^0\rightarrow\pi^+\pi^-$, with the analogous amplitudes that lead to $K^+\rightarrow\pi^0\pi^+$, and in this way we have

$$A^+/A_1^0=(\sqrt{2}C_1^0+C_1)/2C_1$$
$$=\mu^2/6(1-\tfrac{4}{3}\mu^2)\approx 1.4\times 10^{-2}. \qquad (9)$$

Experimentally these amplitudes are in the ratio $4.7\times 10^{-2}$, so in a sense our $\Delta I=\frac{1}{2}$ rule may be too good. To calculate Eq. (9), we considered only the process

$$K\overset{s}{\rightarrow}K\pi;\quad K\rightarrow W\rightarrow\pi$$

(see Fig. 2) and ignored the breaking of $SU_3$ in the strong interactions. [Just correcting for the meson mass differences where they enter into the propagator gives a factor $(M_K^2-M_\pi^2)_{Q=1}/(M_K^2-M_\pi^2)_{Q=0}$ multiplying $C_1^0$ that alone doubles the numerical result of Eq. (9).] We present the above calculation of the $K^+\rightarrow\pi^0\pi^+$ decay amplitude mainly as a service to the reader who might understandably wonder how serious the deviations form our $\Delta I=\frac{1}{2}$ rule might be. Lacking more accurate comparison of these decay amplitudes, we can only say that the question of $\Delta I=\frac{1}{2}$ rule *violation* remains open.

To summarize at this point, we have already obtained all the confirmed predictions of the Cabibbo theory plus additionally the $\Delta I=\frac{1}{2}$ rule.

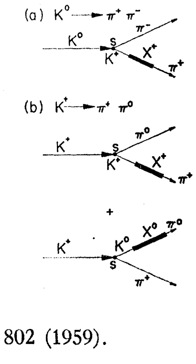


FIG. 2. Feynman diagrams for the decays (a) $K_1^0\rightarrow\pi^+\pi^-$ and (b) $K^+\rightarrow\pi^+\pi^0$. Since strong-interaction corrections will not affect the *relative* amplitudes (in the $SU_3$ limit), these amplitudes are used to estimate the relative decay amplitudes [Eq. (9)].

[8] Cabibbo's theory (Ref. 1) gives the correction $G_V^\beta=G_V^\mu\cos\theta$, which is sometimes cited as support for that theory because the correction has the correct sign to explain the $O^{14}$ decay rate. However, even if the $O^{14}$ matrix elements have perfect overlap, the predicted rate is then *less* than observed, a discrepancy much more difficult to resolve than too high a predicted rate, since imperfect overlap of nuclear matrix elements can reduce but cannot increase the decay rate. Furthermore, the absolute discrepancy remains about the same even after applying this correction.

[9] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).

[10] Such a prediction can be made on the basis of a generalized Goldberger-Trieiman relationship. See R. P. Feynman, in *Symmetries in Elementary Particle Physics*, edited by A. Zichichi (Academic Press Inc., New York, 1965), p. 159.

[11] Cf. R. H. Dalitz, Rev. Mod. Phys. **31**, 802 (1959).

TABLE II. Comparison of this model and the Cabibbo theory.

| Theory | Cabibbo's theory | This model |
|---|---|---|
| Assumptions (applications) | (1) $J_\alpha$ is octet (only one term enters) | (1) Complete unitary symmetry ($J_\alpha$ is octet, all terms enter) |
| | (2) CVC theory | (2) CVC theory |
| | (3) $J_\alpha$ has unit length ($\Delta S=1$ via gauge transformation) (mixing parameter $\theta$) | (3) Mass splitting ($f$ is universal) (mixing parameter $\mu$) |
| Consequences | (1) Interrelation of leptonic hyperon decays | (1) Same |
| | (2) $G_V^\beta/G_V^\mu=0.96$ | (2) $G_V^\beta/G_V^\mu=1$ |
| | | (3) $\Delta I=\frac{1}{2}$ rule |
| | | (4) Boson mass spectrum and branching ratios |

Table II summarizes the comparison between this model and the Cabibbo Theory.

The remaining currents that result do not seem to lead to any immediate experimental consequences. There is an additional neutral $\Delta S=1$ coupling factor

$$C_1^{00}=\sum(W_1|X)(X|W_2^0)M_W^2M_X^{-2}=3^{-1/2}C_1^0,\quad(10)$$

and a neutral $\Delta S=0$ coupling factor

$$C_0^{0'}=\sum(W_1|X)(X|W_3^0)M_W^2M_X^{-2}=(\tfrac{2}{3})^{1/2}\mu C_1^0.\quad(11)$$

The most direct application of the coupling factors 10 and 11 would be to weak decays involving the $\eta$, but that particle already decays rapidly via the electromagnetic interactions.

## IV. EXPERIMENTS

There are several new experimental consequences that may be more accessible than the difficult-to-detect $\Delta S=0$ neutral currents (in our view, of course, the neutral currents are in fact evidenced in the $\Delta I=\frac{1}{2}$ rule decays). Figure 1 has already shown the expected mass spectrum for the $X$ bosons, and we can compute several relative partial decay widths for the charged $X$ bosons from unitary symmetry ($\mu$ does not enter and we neglect the small phase-space corrections) which gives

$$\Gamma(X_{1\text{ or }2}^+\to K^+\pi^0)=\tfrac{1}{3}\Gamma(K^+\eta)=\tfrac{1}{2}\Gamma(K^0\pi^+)$$
$$=\tfrac{1}{2}\Gamma(K^+\bar{K}^0)=\tfrac{1}{4}\Gamma(\pi^+\pi^0),\quad(12)$$

and the lepton coupling [Eq. (3)] gives $\Gamma(\bar{e}\nu_e)=\Gamma(\bar{\mu}\nu_\mu)$. The CVC theory (assumption 2) enables us to interrelate the leptonic and nonleptonic decay modes since $\Gamma(\pi^+\pi^0)$ would equal $\Gamma(\bar{e}\nu_e)$ if the form factors were independent of energy. Actually the leptonic widths are increased by a factor

$$E=F_e^2F_\pi^{-2}\approx F_{1V}^{-2}(M_X^2),\quad(13)$$

where $F_e$ and $F_\pi$ are the electron and pion charge form factors, respectively, (normalized to unity at zero momentum transfer) and as a reasonable approximation $F_e\approx1$, $F_\pi\approx F_{1V}$, the isovector nucleon-charge form factor.[12]

The neutral bosons $X_1^0$, $X_3$, and $X_4$ will result ($X_2^0$ is too massive) even if they are not directly produced, via

$$X_2^\pm\to K^\pm X^0\to\pi^\pm X^0,\quad(14)$$

such couplings being weaker perhaps (roughly a factor of $f^2M_X^2/M_\pi M_K$, which need not be less than unity) than the above. The partial decay widths for the neutral bosons are

$$\Gamma(X_3\to K^0\pi^0)=\Gamma(\bar{K}^0\pi^0)=\tfrac{1}{2}\Gamma(K^+\pi^-)=\tfrac{1}{2}\Gamma(K^-\pi^+)$$
$$=\tfrac{1}{3}\Gamma(K^0\eta)=\tfrac{1}{3}\Gamma(\bar{K}^0\eta),\quad(15)$$
$$\Gamma(X_4\to\pi^+\pi^-)=\Gamma(K^-K^+)=\tfrac{1}{4}\Gamma(K^0\bar{K}^0),$$

and

$$\Gamma(X^0_{1\text{ or }2}\to K^-K^+)=\Gamma(\pi^+\pi^-).$$

These relations follow directly from the $SU_3$ coupling rules and the CVC theory. Note the distinctive decay modes of the otherwise degenerate $X_3$ and $X_4$: It seems quite probable that they would have different lifetimes.

## V. CONCLUSIONS

We do not pretend that all possible experimental tests have been thought of, but every test the author has thought of has either been (1) given essentially correctly by the present model, or (2) not a clear test of either. The clear distinction (if one chooses to discount the $\Delta I=\frac{1}{2}$ rule) between the Cabibbo theory and our model is the value of $G_V$ ($\beta$ decay)/$G_V$(muon decay)$=1$ (here) or $\approx0.966$ (Cabibbo), neither of which is unambiguously supported at present by experiment.

It is possible that one might regard the $\Delta I=\frac{1}{2}$ rule as being an implicit assumption (certainly it follows from our assumptions and is therefore contained within the theory); however it seems clear that the development is direct given the symmetry hypothesis (assumption 1). What we feel has been accomplished here is the formulation of a model of the weak interactions on the basis of a very general principle (unitary symmetry) which yields as a consequence the rather parochial and empirical $\Delta I=\frac{1}{2}$ rule. Assumption 3 (the $F_6$ mass perturbation) is simply the explicit way of introducing the $\Delta S=1$ processes.

In our opinion it would have been quite interesting simply to show that it is *possible* to construct a satisfactory model of weak interactions on the basis of $SU_3$: The fact that it is not only possible but also that the (weakly broken) $\Delta I=\frac{1}{2}$ rule is furthermore obtained seems particularly suggestive.

[12] Cf. R. Carhart and J. Dooher, Phys. Rev. **142**, 1214 (1966), for calculations using the Cabbibo theory and references to earlier work.