Lowest ^1S and ^3P Scattering Resonances in the e-H System*

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A study of the position and width of the lowest ^1S and ^3P resonances in the e-H system was made by judiciously comparing the published data of McGowan, Clarke, and Curley with values of the cross section calculated for a variety of resonance positions E_0^+ and E_0^-, and widths Γ_0^+ and Γ_1^-. It is determined that the ^1S and ^3P resonance positions are $E_0^+=9.56\pm0.01$ eV and $E_1^-=9.71\pm0.03$ eV, while the respective widths are $\Gamma_0^+=0.043\pm0.006$ eV and $\Gamma_1^-\geq0.009$ eV. The resonance shapes are shown to depend critically upon both the angle of observation and the angular resolution.

I. INTRODUCTION

CONSIDERABLE theoretical effort has been spent on studies of three-body atomic systems (e-H) and (e-He$^+$), with a good measure of success in such areas as the prediction of elastic-scattering resonances,[1–15] the multiplicity of such resonances,[4,5,8,9,15] and the nonzero threshold value of the inelastic-scattering cross section.[16,17] The existence of the resonances[18–23] and the nonzero excitation threshold[24] have subsequently been verified experimentally. However, success in this area is far from universal, and often, as, for example, in the case of the total excitation cross section,[25,26] it has been demonstrated that the best theory[27] cannot yet reproduce reasonably substantiated experimental evidence.

Even in the case of (e-H) scattering resonances where various approximations appear to give good agreement as to resonance position, these same approximations lead to a variety of resonance widths. This point is discussed at length by Chen in the accompanying paper[15] for the singlet and triplet S-wave electron scattering resonances just below the $n=2$ level of atomic hydrogen. The purpose of the present paper is to give an experimental value for the position of the lowest ^1S resonance E_0^+ and its width Γ_0^+ and to give some experimental limits for similar quantities for the lowest ^3P resonance. With our present electron energy resolution, it is possible to investigate these two resonances. To study in detail the higher resonances is virtually impossible because the electron-energy resolution is not good enough to resolve other members of the ^1S and ^3P series in competition with a long series of resonances in the ^1P and ^3S systems, not to mention resonance structure[10,11] in the D partial wave which also lies in the small energy interval <0.2 eV below the $n=2$ thresholds. Structure associated with this large number of resonances has been identi-

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[5] M. Gailitis and R. Damburg, Proc. Phys. Soc. (London) **82**, 192 (1963); A. I. Baz, Zh. Eksperim. i Teor. Fiz. **36**, 1762 (1959) [English transl.: Soviet Phys.—JETP **9**, 1256 (1959)].

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[10] M. Gailitis, in *Proceedings of the Fourth International Conference on Physics of Electronic and Atomic Collisions, Quebec, 1965*, edited by L. Kerwin and W. Fite (Science Bookcrafters, Hastings-on-Hudson, New York, 1965), p. 10.

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[13] H. C. Taylor and J. K. Williams, J. Chem. Phys. **42**, 4063 (1965).

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[18] G. J. Schulz, Phys. Rev. Letters **13**, 583 (1964).

[19] H. Kleinpoppen and V. Raible, Phys. Letters **18**, 124 (1965).

[20] J. Wm. McGowan, E. M. Clarke, and E. K. Curley, Phys. Rev. Letters **15**, 917 (1965); **17**, 66 (1966).

[21] J. Wm. McGowan, Phys. Rev. Letters **17**, 1207 (1966).

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[26] H. Kleinpoppen, Z. Physik (to be published).

[27] M. J. Seaton, in *Atomic and Molecular Processes*, edited by D. R. Bates (Academic Press Inc., New York, 1962), Chap. 11 (references); also, M. R. H. Rudge, Proc. Phys. Soc. (London) **86**, 763 (1965).

fied,[20] but no resonances have yet been isolated experimentally in this energy interval.

For the study of resonance structure, Fano[28] has developed an analytic formalism for isolated Breit-Wigner[29] resonances superimposed on an unspecified nonresonant background. This has been applied several times with success[28–32] and is particularly useful where scattering phase shifts for the nonresonant portions of the scattering are not known. However, in the case of the (e-H) system below the $n=2$ level, the S, P, and D scattering phase shifts are thought by many to be well known.[33,34] In part, they are substantiated by total scattering experiments yielding cross sections which agree with theory to within the rather large uncertainty of the experiments.[35] Therefore, in the case of the ^1S and ^3P resonances in the (e-H) scattering system, it is desirable to compare the experiment with the more complete calculation involving the various partial waves in interference with each other and the truly isolated resonances. It is important to do this, since the magnitude and shape of the resonant differential cross sections will depend strikingly upon the interference. No such effect is expected for the total cross section.

II. ANALYTICAL APPROACH

In Table I we list calculated and measured values for resonance width Γ_l^ϵ and position E_l^ϵ for the lowest (e-H) resonances in S and P channels. Here ϵ refers to the singlet (+) or triplet (−) spin state and l refers to the orbital angular momentum 0, 1, or 2. Only the lowest ^1S and ^3P resonances lend themselves readily to electron scattering studies, because they are well isolated from other resonances and do not interfere with one another, and are therefore most likely to be resolved with presently obtainable electron-energy resolution.

In the following study, particular attention is focused upon the lowest ^1S resonance, but some attention must be given to the ^3P resonance which is nearby. If a measurement could have been made at $\theta=90$ deg with a detector which subtended an infinitesimally small solid angle, there would have been no contribution in the data from P-wave scattering. However, in the work of McGowan, Clarke, and Curley,[20] the angular width of the detector $\Delta\theta$ was such that some contribution from the P wave was detected, as well as from S and D partial waves.

The cross section which we calculate below is to be compared with experiment and must therefore reflect only that portion of the total cross section which is sampled in the experimental solid angle. Once comparison with the experiment has permitted us to estimate a value for Γ_l^ϵ and E_l^ϵ, the total cross section through the resonance region can be obtained. For comparison with experiment, integration is carried out over the experimental solid angle $\Delta\Omega(\Delta\theta,\Delta\varphi)$. The calculated scattering cross section[36] is

$$\sigma=\tfrac{1}{4}(\sigma^+ + 3\sigma^-)\,, \tag{1}$$

where the contribution for either singlet or triplet scattering is written

$$\sigma^\epsilon=\int_{\exp}\int |f^\epsilon(\theta)|^2 \sin\theta d\theta d\varphi$$
$$=\int_{\varphi+(1/2)\Delta\varphi}^{\varphi-(1/2)\Delta\varphi}\int_{\theta-(1/2)\Delta\theta}^{\theta+(1/2)\Delta\theta}|f^\epsilon(\theta)|^2 d(\cos\theta)d\varphi\,. \tag{2}$$

$f(\theta)$ is the scattering amplitude given by

$$f^\epsilon(\theta)=\frac{1}{2ik}\sum_l (2l+1)\{\exp(2i\eta_l^\epsilon)-1\}P_l(\cos\theta)\,,$$
$$=\frac{1}{k}\sum_l (2l+1)\sin\eta_l^\epsilon \exp(-i\eta_l^\epsilon)P_l(\cos\theta)\,, \tag{3}$$

where the η_l^ϵ are the total scattering phase shifts for each partial wave and spin state, and k^2 (or E_e) is the energy of the incident electrons.

The lowest ^1S and ^3P resonances in our analysis, as in Fano's, are considered as isolated Breit-Wigner resonances with phase shifts ρ_l^ϵ which are functions of Γ_l^ϵ and E_l^ϵ. These are added to a known contribution to the total phase shift from nonresonance scattering δ_l^ϵ such that the total phase shifts η_l^ϵ are

$$\eta_l^\epsilon=\delta_l^\epsilon+\rho_l^\epsilon\,, \tag{4}$$

$$\eta_l^\epsilon=\delta_l^\epsilon-\tan^{-1}\left\{\frac{\Gamma_l^\epsilon}{2(k^2-E_l^\epsilon)}\right\}\,. \tag{5}$$

The values for $\delta_0^\pm$ used are the variational values derived by Schwartz,[34] while those for $\delta_1^\pm$ and $\delta_2^\pm$ are close-coupling values[2,7] which have been linearly extrapolated through the resonance energy interval. It follows from Eqs. (2) and (3), above, that the differential cross section

$$d\sigma/d\Omega=|f^\epsilon(\theta)|^2 \tag{6}$$

for each symmetry ϵ is

$$|f^\epsilon(\theta)|^2=\frac{1}{k^2}\sum_l\sum_{l'}(2l+1)(2l'+1)\sin\eta_l^\epsilon \sin\eta_{l'}^\epsilon$$
$$\times\cos(\eta_l^\epsilon-\eta_{l'}^\epsilon)P_l(\cos\theta)P_{l'}(\cos\theta)\,. \tag{7}$$

[28] U. Fano, Phys. Rev. **124**, 1866 (1961).
[29] Breit-Wigner theory reviewed, for example, in the book by J. M. Blatt and V. F. Weisskipf, *Theoretical Nuclear Physics* (John Wiley & Sons Inc., New York, 1952).
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[32] U. Fano and J. W. Cooper, Phys. Rev. **137**, A1364 (1965).
[33] S. Geltman, Astrophys. J. **141**, 376 (1965).
[34] C. Schwartz, Phys. Rev. **124**, 1468 (1961).
[35] R. T. Brackmann, W. L. Fite, and R. H. Neynaber, Phys. Rev. **112**, 1157 (1958).
[36] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, England, 1965), Chap. 16.

TABLE I. Theoretical and experimental results.

Resonance	$E_l^{\pm}$ (eV) Theory	Expt.	$\Gamma_l^{\pm}$ (eV) Theory	Expt.	Reference
^1S	9.61		0.109		Burke and Schey[a]; Smith *et al.*[b]
	9.61		...		Herzenberg *et al.*[c]
	9.559		...		O'Malley and Geltman[d]
	9.557		...		Bhatia *et al.*[e]
	~9.56[f]		...		Gailitis[g]
	9.55		...		Holøien and Midtdal[h]
	9.560		0.048		Burke and Taylor[i]
			0.051		Chen[j] (sine function) (static exchange)
			0.041		
		9.56±0.01		0.043±0.006	This report
^3P	9.78		0.009		Burke and Schey[a]
	9.727		...		O'Malley and Geltman[d]
	~9.72[f]				Gailitis[g]
		9.71±0.03		>0.009	This report[k]
		9.73±0.12		>0.009[k]	Kleinpoppen and Raible[l]
		9.7 ±0.15			Schulz[m]

[a] Reference 2.
[b] Reference 1.
[c] Reference 6.
[d] Reference 8.
[e] Reference 12.
[f] Only a rough estimate of the resonance position can be gotten from the report of Gailitis, since only a few calculated points define it.
[g] Reference 10.
[h] Reference 11.
[i] Reference 14.
[j] Reference 15.
[k] From estimates made in the report and in J. Wm. McGowan, Phys. Rev. Letters **17**, 1207 (1966).
[l] Reference 19.
[m] Reference 18.

Unfortunately, the energy width of the electrons at half-maximum Δ_s from the electron scattering spectrometer is greater than the resonance widths Γ_l^{ϵ} of interest in this report. Therefore, in order to make comparison between theory and experiment, one must either unfold the energy distribution of the electrons from the data or fold the energy distribution into the calculated cross sections. Even when the noise in data is minimal, the former procedure is difficult. For the results analyzed here, the unfolding procedure was impossible because of the relatively large statistical errors in the data. On the other hand, the folding in of a Gaussian distribution (Δ_s eV wide at half-maximum)

$$\sigma_{\exp}(k^2)=(\pi\Delta_s^2)^{-1/2}\int\sigma(E')\exp\left[\frac{-(k^2-E')^2}{\Delta_s^2}\right]dE' \quad (8)$$

is readily accomplished with a fast computer.

The primary object of this study is to obtain E_0^+ and Γ_0^+ and some idea of E_1^- and Γ_1^-.

III. DESCRIPTION OF EXPERIMENT

The experimental results of McGowan, Clark, and Curley[20] (hereafter referred to as MCC) pertinent to this discussion and the scattering spectrometer[37] have been partially described previously. The geometry of the scattering pertinent to this discussion is given in Fig. 1. Scattered electrons come from the region where the atomic hydrogen and electron beams intersect. In the final stage of the electron detection, electrons pass through a nearly square aperture. Although the solid angle subtended by the detector does not vary much from point to point in the scattering volume, the angle θ which defines the axis of the scattering pyramid varies slightly from the center to the top or bottom of the volume. As a result, the effective angular openings $\Delta\varphi$ around $\varphi=0$ deg and $\Delta\theta=90$ deg are slightly larger than those measured from the center of the scattering volume. From the geometric center, the face of the counter was such that $\varphi=\varphi\pm\frac{1}{2}\Delta\varphi=0\pm7.5$ deg, and $\theta=\theta\pm\frac{1}{2}\Delta\theta=90\pm7.5$ deg.

Rather than integrate over the experimental volume, we have chosen for this analysis to fix the scattering center and vary the opening $\Delta\theta$ in order to study the change produced in the effective contribution of the P-wave scattering resonance to the structure near 9.56 eV. This simply changes the calculated resonance

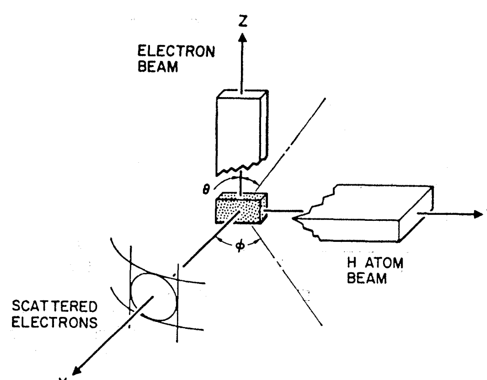

FIG. 1. A sketch showing the geometrical arrangement used in the experiment of McGowan, Clarke, and Curley (Ref. 20). The electrons scattered out of the shaded scattering volume defined by the intersection of the electron and hydrogen atom beams finally pass through a rectangular surface in the lower left-hand corner of the figure. The distorted rectangle shown is a good approximation to the true surface.

[37] J. Wm. McGowan, M. A. Fineman, E. M. Clarke, and H. P. Hanson, General Atomic Report No. GA-7387 (unpublished).

structure in the vicinity of 9.7 eV (Fig. 2) and does not affect our analysis of the ^1S resonance. Using this model, a reasonably good fit in the region of 9.7 eV can be obtained when $\Delta\theta$ is set at 25 deg while $\Delta\varphi$ is kept fixed at 15 deg.[38] However, $\Delta\theta=25$ deg is unreasonably large, so we must find another parameter to explain the results of MCC. The observable contribution of the ^3P resonance can also be increased in our calculation by increasing the width Γ_1^-. The implications of such a change are discussed in Sec. IV.

Included in the total measured scattering signal is a contribution due to scattering from H_2 which remained in our beam. Because of the presence of H_2 in the beam, the magnitude of the resonance structure relative to the total scattering background is slightly distorted. Crude mass spectrometric analysis places the H_2 content in the H beam at approximately 15%. By using the differential scattering data of Ramsauer and Kollath[39] for H_2, an approximate correction has been made. As a result the magnitude of resonance structure relative to background has been changed very slightly from the measured results in the data to be used here.

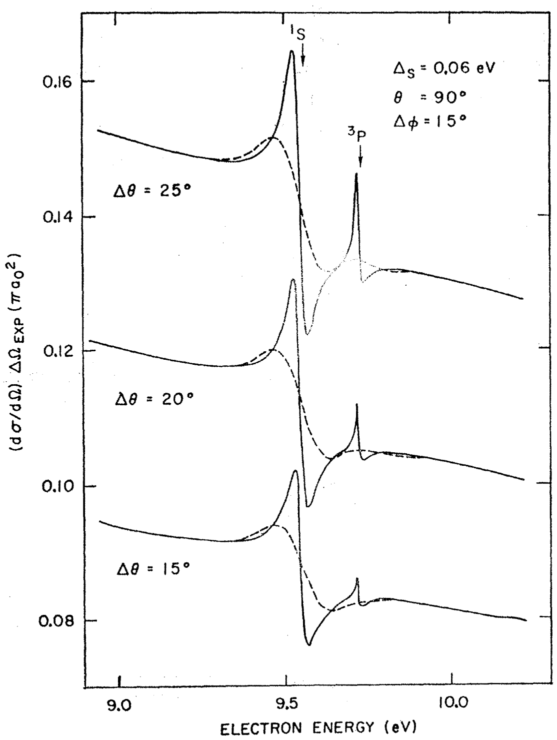

FIG. 2. The calculated scattering cross section in the vicinity of the lowest ^1S and ^3P resonances as a function of the acceptance angle $\Delta\theta$. In the figure, the solid curves relate to the calculated cross section $|d/d\Omega|^2\Delta\Omega_{exp}$, while the broken curves have the electron energy distribution folded into them. As $\Delta\theta$ is increased, the constructive interference portion of the ^3P resonance increases. For this calculation, $\Gamma_0^+=0.040$ eV and $\Gamma_1^-=0.010$ eV.

[38] In our geometry, $\Delta\phi$ is but a scaling parameter and, since we normalize our results, it is rather unimportant.

[39] C. Ramsauer and R. Kollath, Ann. Physik **12**, 529 (1932).

In their publication, MCC displayed the average of the data from a number of experiments. In Fig. 3, however, we show instead the individual data points from which the average was derived. There are a number of reasons for doing this, but perhaps the most salient is the fact that certain data points can be readily identified as unreal when they define structure which cannot exist within the electron energy resolution. Clear examples of this are seen at 9.63 and 9.86 eV. In particular, the inclusion of the former points in the averaging with others tends to make the destructive interference portion of the ^1S resonance structure sharper than is experimentally possible. Accordingly, these points are given a lower weight in the analysis.

The two critical parameters in the resonance experiment, as in all high-resolution electron-scattering studies, are the calibration of the energy scale and the measurement of the electron energy resolution Δ_s. The electron energy scale has been calibrated with reference to the ionization potential of H ($1s$).[37] In the publication of MCC,[20] this scale was based upon a linear extrapolation of the linear portion of the ionization efficiency curve to intersect the energy axis at a point which was taken to be 13.60 eV. This was never quite satisfactory, since the tailing in the ionization efficiency curve was always in excess of that expected for the measured energy resolution of the source monochromator. More recent studies[37] have shown that the original method used for calibration is most likely in error, since the ionization threshold is apparently nonlinear and follows surprisingly well an $(E_0\text{-IP})^{1.127}$ power law for the first $\sim$0.3 eV above the threshold. Accordingly, the energy scale has been shifted downward by 0.03 eV. This correction has not only brought the measured ^1S resonance into register with the calculated position (refer to Table I), but it has led to a more satisfactory fit of the H_2^+ auto-ionization data[37] with spectroscopic values.

In most experiments, we measure our electron energy distribution with a second electrostatic analyzer. However, in these experiments, an upper limit to the breadth was obtained by measuring the width of the lowest helium resonance at $\theta=90$ deg and by using the estimates of the resonance width published by Simpson and Fano[40] as the real width, and then extracting from the data an estimate of Δ_s. In this way, we conservatively estimated the resolution to be $\Delta_s\approx0.08$ eV. Subsequent and more complete analysis of the He data suggests that the resolution approaches $\Delta_s=0.06$ eV in most experiments performed.

IV. RESULTS

Our procedure is then to fit experimental results with Eq. (8) and to obtain from this fit, values of the position and width of the ^1S and ^3P lowest resonances. In our analysis, our resonance parameters are considered E_0^+

[40] A. J. Simpson and U. Fano, Phys. Rev. Letters **11**, 158 (1963).

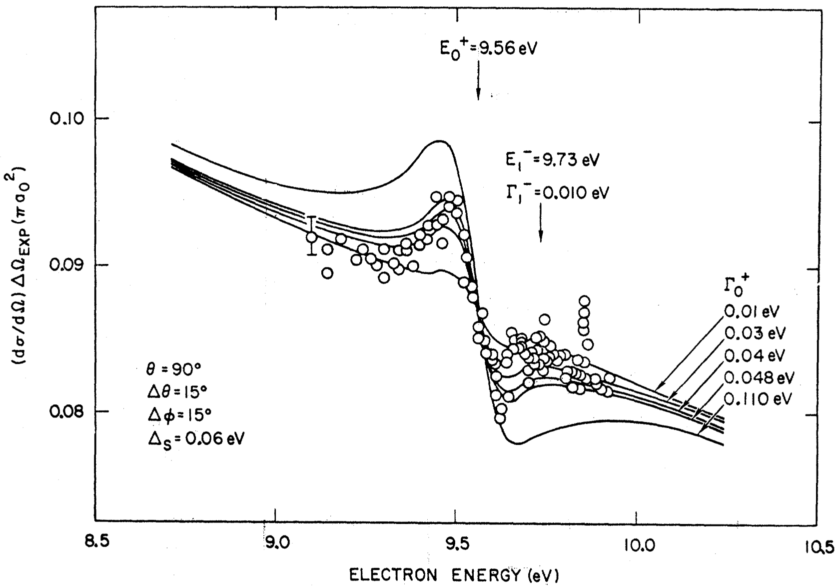

FIG. 3. A comparison between experimental and calculated cross sections, where Γ_0^+ is chosen as a variable parameter while the other parameters E_0^+, E_1^-, Γ_1^-, $\Delta\theta$ are kept constant.

Γ_0^+, E_1^-, and Γ_1^-, while the effects of the two experimental parameters, $\Delta\theta$ and Δ_s are tested.

Lowest ^1S Resonance

Through our calculation we find immediately that only one of the values, $E_0^+\cong 9.56$ eV, given in Table I fits extremely well with the data (see Fig. 3). As a result, the only parameter remaining to be varied for the ^1S resonance study was Γ_0^+. Values ranging from 0.010 to 0.110 eV were introduced into the calculation, and the results were compared with the data. One set of such results for different values of Γ_0^+ and fixed values of $\Delta_s=0.06$ eV, $\Delta_\theta=15$ deg, $E_1^-=9.73$ eV, and $\Gamma_1^-=0.01$ eV is shown in Fig. 3. From this comparison and many others, it is possible to estimate the value of Γ_0^+ giving best fit. We find it to be $\Gamma_0^+=0.043\pm 0.006$ eV.

Keeping Γ_0^+ constant, it is then possible to study the effect of varying Δ_s, and this is shown in Fig. 4, where it can be seen that a variation of Δ_s between 0.06 to 0.08 eV does not change the cross section enough to affect our estimate of Γ_0^+, which remains well within the errors quoted, i.e., ± 0.006 eV.

The value of Γ_0^+ thus obtained may now be compared with the various theoretical estimates (see Table I). It is immediately clear that the value of 0.109 eV, obtained under a $1s2s2p$ close-coupling approximation to the wave function,[1,2] is too high. The value of 0.048 eV recently obtained by Burke and Taylor[14] when a number of electron-electron correlation terms were added to the approximation gives a marked improvement. However, it is not clear that the inclusion of correlation gives the complete answer. Even though this value is included within the uncertainty of our estimate of Γ_0^+, it is definitely on the high side of this estimate. The sensitivity of Γ_0^+ to various trial wave functions is discussed at length in the accompanying paper by Chen.[15] His sine trial wave function, surprisingly enough, gives a value 0.051 eV, which approaches the width for the lowest ^1S resonance calculated by Burke and Taylor.[14] Chen's value 0.041 eV, based upon a static exchange approximation, is in good agreement with the magnitude of our measured value.

Lowest ^3P Resonance

At $\theta=90$ deg and $\Delta\theta=0$, there should be no contribution to the scattering signal from the ^3P resonance. For nonzero values of $\Delta\theta$, some contribution not only from interference, but from the resonance itself, appears in the data. The apparent contribution will depend both upon the angular resolution of the apparatus $\Delta\theta$

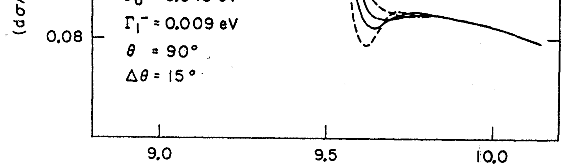

FIG. 4. The dependence of the calculated cross section upon the energy distribution of the bombarding electrons. The width of the bombarding electrons at half-maximum lies between 0.06 and 0.08 eV, giving values of the cross section which must lie within the solid curves above. The broken curves define limits outside the scope of this paper.

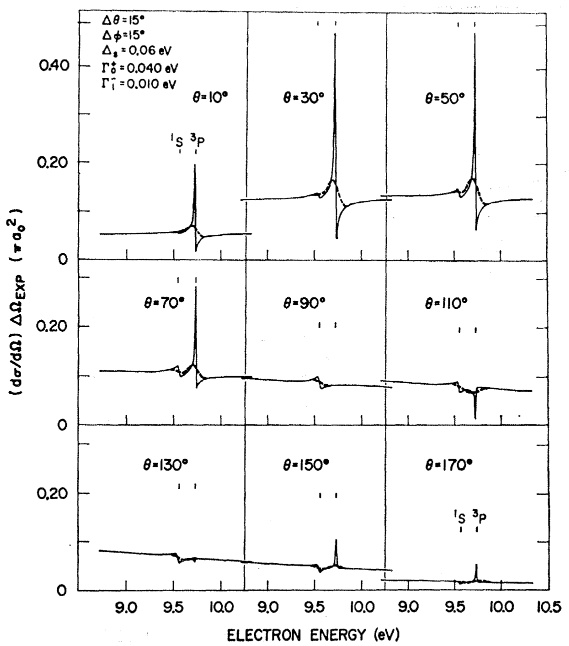

FIG. 5. The solid curves are calculated $(d\sigma/d\Omega)\Delta\Omega_{\mathrm{exp}}$ for a sampling of scattering angles. The broken curves are the same cross sections with an electron energy distribution $\Delta_s=0.06$ eV folded in.

and upon the resonance parameters themselves, E_1^- and Γ_1^-. It is clear in the data (Fig. 3) that to the right of the main ^1S resonance structure and near an energy of 9.7 eV, there is some further structure due to the lowest ^3P resonance.

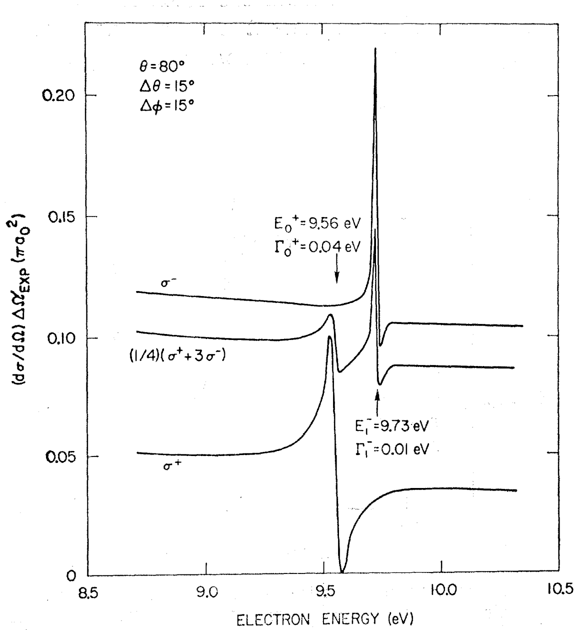

FIG. 6. The total calculated cross section $\frac{1}{4}(\sigma^+ + 3\sigma^-)$ for the experimental solid angle $\Delta\Omega_{\mathrm{exp}}$, but for $\theta=80°$. Shown also are the cross sections associated with each symmetry.

In our calculations shown in Figs. 2 to 6, we have taken for E_1^-, 9.73 eV, which is the value calculated by O'Malley and Geltman[8] and approximately that calculated by Gailitis.[10] These values agree very well with what we would estimate the position to be from our data, i.e., $E_1^-=9.71\pm0.03$ eV. The close-coupling calculation[2] giving 9.78 eV for E_1^- is clearly too high.[41] Corroborating experimental values for E_1^-, are obtained from an analysis of the work of Schulz[18] and Kleinpoppen and Raible[19] which will be discussed below.

As we mentioned in the previous section, the ^3P contribution to the resonance structure can be changed in the calculations in two main ways: by changing $\Delta\theta$, or by varying the resonance width Γ_1^-. In Fig. 2, it was shown that in order to increase the ^3P contribution sufficiently to agree roughly with the data, $\Delta\theta$ had to be at least 25 deg. However, this $\Delta\theta$ is unquestionably too large to correspond with the real experimental situation. Consequently, to fit the data, a value of Γ_1^- greater than the theoretically estimated value 0.009 eV appears to be necessary. This conclusion can also be drawn from the work in Professor Kleinpoppen's laboratory.[19,42]

Differential Scattering

Several extensive experimental studies of differential electron scattering from targets other than atomic H have recently been reported.[43,44] To analyze these systems in detail, however, is impossible. Only the (e-H) and (e-He$^+$) systems lend themselves to both experimental and theoretical treatment. Recently McGowan,[21] using the analysis reported here, has discussed the results of the three experiments[18–20] thus far performed with atomic hydrogen. He compared the various results with the differential cross section $d\sigma/d\Omega=|f(\theta)|^2$, associated with a finite experimental solid angle $\Delta\Omega_{\mathrm{exp}}$ [Eq. (2)] and with the experimental energy distribution $\Delta_s=0.06$ eV folded into the calculated curve [Eq. (8)].

In Fig. 5, we show 18 such calculated curves for values of the scattering angle 10 through 170 deg. The nine solid curves are the cross sections for the specified angle, while the nine broken curves which are superimposed on the solid curves are the cross section as it

[41] Although theoretical distinction can be made between the exact position of a resonance $E_l^{\pm}$ and a prediction of the eigenvalue of the compound state $\mathcal{E}_l^{\pm}$, because of the remaining uncertainties in our energy resolution and small uncertainty in the energy calibration which remain, it is impossible for us to determine the difference between them. Our uncertainty though small, embraces both values. The calculated difference appears to be no greater than a few millielectron volts (Refs. 9, 12, 28).

[42] Our proposition that Γ_1^- may be greater than 0.009 eV is further supported in Ref. 21 by the magnitude of the structure measured at 94° by Kleinpoppen and Raible (Ref. 19). It is difficult to imagine that this structure should be so pronounced for an energy resolution of 0.1 eV unless the measured resonance is considerably broader than 0.009 eV.

[43] H. Ehrhardt and G. Meister, Phys. Letters 14, 200 (1965).

[44] D. Andrick and H. Ehrhardt, Z. Physik 192, 99 (1966).

would be measured with an electron energy resolution of $\Delta_s=0.06$ eV.

The work of Schulz[18] deals with the total transmission of electrons and shows the constructive interference portion of the ^3P resonance (Fig. 5). In his data, this appears as a broad peak centered at 9.7 ± 0.15 eV. On the other hand, the differential scattering results of Kleinpoppen and Raible[19] for $\theta=94\pm2$ deg show a broad depression in the scattered signal, the minimum of which is located at 9.73 ± 0.12 eV. It is now clear that this depression is also associated with the ^3P resonance, but in this case, the destructive-interference portion of the resonance (see Fig. 5, $\theta=110$ deg). The maximum in the Schulz data and the minimum in the Kleinpoppen-Raible data is recorded in the table under E_1^- and support the value given in this report and the calculated resonance positions of O'Malley and Geltman[8] and Gailitis.[10]

The most striking feature in Fig. 5 is the rapid change in the magnitude and the shapes of the resonances with θ. For example, the change in the ^3P resonance as θ goes from just below $\theta=90$ deg to just above could not have been forseen. At $\theta=90$ deg, there is no contribution of the P-wave or ^3P resonance to $|f(\theta)|^2$, but the shape of the ^1S resonance, though no doubt primarily due to interference between the ^1S resonance phase shift and the S-wave phase shift, must reflect some interference with the D-partial wave. The contribution from the D-partial wave disappears when $P_2(\cos\theta)=0$, i.e., near $\theta\cong54.5$ and 125.5 deg. The latter angle approximately corresponds to the angle at which the ^3P resonance changes from one dominated by destructive interference to one again dominated by constructive interference. This might imply that the incoming P partial wave couples strongly with an outgoing D wave.

One surprising feature of the ^1S resonance is the dependence of its shape upon angle. The constructive interference portion of the resonance which dominates below $\theta=90$ deg virtually disappears for θ between 90 and 180 deg.

In this report we have been interested in the lowest ^1S and ^3P resonances which are completely separated both by symmetry and because they appear at different electron energies. Even in this case, we have found a very strong dependence of the differential cross section upon interference between the nonresonance background δ_l' and the resonance itself ρ_l. At slightly higher electron energies, just below the $n=2$ threshold, there are not only singlet and triplet S and P resonances, but singlet and triplet D resonances as well.[10,16] Experimentally it is difficult and perhaps impossible to separate these resonances because of our finite energy resolution and because at any scattering angle and energy where, in a given symmetry more than one resonance exists, there is not only interference between the various background partial waves, and the background partial waves and the resonances, but also interference between the resonances themselves. As a result one would expect to find even with infinite energy resolution, a very complicated spectrum. Such structure smoothed by the finite energy resolution has already been identified by MCC[20] just below and reaching above the $n=2$ level of atomic hydrogen. However, Chen[15] has pointed out that when the resonance is very narrow compared to the electron energy distribution its contribution directly or through interference to the measured cross section is negligible. In the experiments of MCC,[20] the scattering angle is 90 deg so that the P wave contribution is small. From the resonance widths quoted in Chen's paper, it would seem then that the structure observed may be due simply to the second ^1S resonance and the first few ^1D resonances.[10]

Furthermore, for the same reasons as above, it is impossible for us to determine experimentally the number of resonances below the $n=2$ onset. Gailitis and Damburg[16,17] have intimated that there are an infinite number of resonances, while Temkin and Walker[9] have shown, that if the $2s$ and $2p$ states are degenerate, the number of singlet and triplet S eigenvalues is infinite. Also, Chen[15] has demonstrated that these resonances are truly isolated under the assumption of degeneracy. But it is not yet known how many resonances there are or whether or not they will overlap when the finite separation between the $2s$ and $2p$ levels is considered. It is, however, clear that this will not be an experimentally measured quantity.

It is worthy of note in Fig. 5 that with the present electron-energy resolution ($\Delta_s=0.06$ eV), a complete study of the differential scattering cross section would be fruitless, since most of the important details would be blurred. However, it is desirable to measure the differential cross section at a few angles, perhaps $\theta\cong50$, 110, and 150 deg, where even crude information can give further support to theoretical values of E_1^- and Γ_1^-, and the calculated values of the scattering phase shifts. The qualitative agreement found here and in Ref. 21 between the experimentally measured cross sections and calculated cross sections based upon published phase shifts[2,7,33,34] gives partial support to the calculations. A better test will no doubt follow when more accurate measurements are available for the relative differential cross section and the total scattering cross section over a large energy interval. It should be recognized that the scattering phase shifts used here for P- and D-wave contributions are close-coupling values and that higher values calculated by the method of polarized orbitals[45] and variational methods[10,46,47] are

[45] A. Temkin and J. C. Lamkin, Phys. Rev. **121**, 788, 1961. The polarized orbital P-wave calculation has been amended, I. H. Sloan, Proc. Roy. Soc. (London) **A281**, 151 (1964).

[46] A. J. Taylor, in *Proceedings of the Fourth International Conference on Physics of Electronic and Atomic Collisions, Quebec, 1965*, edited by L. Kerwin and W. Fite (Science Bookcrafters, Hastings-on-Hudson, New York, 1965), p. 27.

[47] R. L. Armstead, University of California Radiation Laboratory Report No. UCRL-11628, 1964 (unpublished).

more accurate. However, it is expected that the results obtained here are qualitatively, and at worst semiquantitatively, correct.

V. SUMMARY

By judiciously comparing the results of theory and experiment, we have been able to extract information from experiment as to position and width for the lowest ^1S and ^3P resonances, even though the widths of those resonances are of the same magnitude as the energy resolution of the electrons used in the experiment. The very strong dependence of resonance structure in the (e-H) system upon θ and $\Delta\theta$ is found when S-, P-, and D-wave scattering is considered.

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Radiation from Nonlinearly Excited Plasmas

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Longitudinal plasma oscillations can be excited nonlinearly by two strong incident electromagnetic waves. The reradiation from such nonlinearly excited plasma is calculated for the cases of a thin plasma film and a small plasma column. Finite-temperature effects are neglected, but the dominant mechanism of coupling between longitudinal and transverse waves by boundaries is properly taken into account. Resonances in such reradiation from both the thin film and the small column are found at frequencies ω_p and $\omega_p/\sqrt{2}$, respectively, where ω_p is the plasma frequency. The intensity of such reradiation is compared with the scattered intensity of another electromagnetic wave from the same nonlinearly excited plasma. It is found quite generally that the reradiation dominates the scattering, thereby explaining the result of a recent experiment.

I. INTRODUCTION

THROUGH nonlinear interaction, two strong electromagnetic waves may induce longitudinal current or density fluctuations in matter. Considerable attention has been focused on the incoherent scattering of an electromagnetic wave by such nonlinearly induced density fluctuations in a plasma.[1] (From now on, we shall refer to such incoherent scattering as the *optical-mixing effect*). In this paper, we shall study instead the reradiation from such nonlinearly excited plasmas. We find that the reradiation from a nonlinearly excited thin film exhibits resonant behavior when the sum or the difference of the frequencies of the two electromagnetic waves is equal to the plasma frequency, although no such resonant behavior exists for a semi-infinite medium. The ratio of the intensity of the reradiation to that of one of the incoming waves is in fact found to be of the same order as the ratio of *optical mixing* to the reradiation. A similar conclusion is obtained for a small plasma column, with the resonant frequency at $\omega_p/\sqrt{2}$, where ω_p is the plasma frequency. As an application of our findings we see why, in a recent experiment,[2] reradiation from a nonlinearly excited plasma column was detected while the *optical-mixing effect* eluded observation.

II. THIN PLASMA FILM

Consider two electromagnetic waves with electric vectors, $\mathbf{E}_1 \exp(-i\omega_1 t+i\mathbf{k}_1\cdot\mathbf{x})$ and $\mathbf{E}_2 \exp(-i\omega_2 t+i\mathbf{k}_2\cdot\mathbf{x})$ respectively, entering into a bounded electron gas. For simplicity we adopt the collisionless-cold-plasma model, thereby neglecting the fine structure due to temperature. However, the dominant mechanism of coupling

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[2] R. A. Stern and N. Tzoar, Phys. Rev. Letters **16**, 785 (1966).