W-Boson Contribution to the Anomalous Magnetic Moment of the Muon*

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The contribution of the intermediate vector boson to the muon anomalous moment is calculated in the ξ-limiting formalism. We find

$$\Delta\kappa_\mu=\Delta[(g_\mu-2)/2]=(g_W^2/8\pi^2)(M_\mu/M_W)^2[2(1-\kappa_W)\ln\xi+10/3],$$

where $\sqrt{2}(g_W/M_W)^2=G_F$ is the Fermi weak coupling constant and κ_W is the W anomalous moment. For $\kappa_W=0$, and ξ chosen according to the prescription of Lee, this contribution is numerically

$$\Delta\kappa_\mu\cong-1.0\times10^{-8}\cong-0.8(\alpha/\pi)^3.$$

I. INTRODUCTION

IN view of the increasing accuracy of current and projected measurements[1] of the muon's magnetic moment κ_μ, it is important to have reliable estimates of the weak-interaction contribution.[2] In fact, if the intermediate vector boson exists, one obtains a contribution to the anomalous magnetic moment that is first order in the Fermi weak coupling constant G_F and independent of the W mass. This contribution is of order $(G_FM_N^2)(M_\mu/M_N)^2$ and is nominally the same size as the strong interaction contributions via vacuum polarization to κ_μ, which are of order[3] $(\alpha^2/3\pi)(M_\mu/M_\rho)^2$, and the sixth-order electrodynamic corrections[4,5] of order $(\alpha/\pi)^3$. In contrast, the direct four-field Fermi interaction contributes negligibly to κ_μ in order G_F^2.

Actually, several theoretical calculations of the intermediate-vector-boson contribution to the muon's anomalous magnetic moment have already been given.[6] However, in view of their mutual disagreement (see Appendix A) we are presenting here yet another calculation of this quantity. The special features and checks included in our calculation, which we believe make it especially credible, are discussed below in Sec. III.

It should be remarked that the W contribution to κ_μ is model-dependent, since an arbitrary procedure must be used to cut off a logarithmically divergent integral. The coefficient of this term is, however, unambiguous. Our results agree with that of Shaffer[6] who calculated the case of zero anomalous moment for the W. Our choice for regularizing the divergence is the gauge-invariant ξ-limiting formalism of Lee and Yang,[7] which modifies both the propagators and the electromagnetic vertex of the W.

II. CALCULATION

The intermediate boson contribution to the muon's electromagnetic vertex is obtained from the Feynman diagram of Fig. 1. According to the rules listed in Appendix B, the matrix element is[8]

$$\mathfrak{M}_\mu=-\frac{ieg_W^2}{(2\pi)^4}\int\frac{d^4k}{i}\bar{u}(p+Q)A_\mu u(p-Q)\,,\quad(1)$$

where

$$A_\mu=\gamma_\beta(1+\gamma_5)(\not{p}-\not{k})^{-1}\gamma_\alpha(1+\gamma_5)\times P^{\beta\tau}(k+Q)V_{\mu\sigma\tau}P^{\sigma\alpha}(k-Q)\,,\quad(2)$$

$$V_{\mu\sigma\tau}=g_{\sigma\tau}2k_\mu-g_{\sigma\mu}(k-Q)_\tau-g_{\tau\mu}(k+Q)_\sigma+2\kappa_W(g_{\sigma\mu}Q_\tau-g_{\tau\mu}Q_\sigma)+\xi[g_{\sigma\mu}(k+Q)_\tau+g_{\tau\mu}(k-Q)_\sigma]\,,\quad(3)$$

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[1] F. J. M. Farley *et al.*, Report to the Thirteenth International Conference on High Energy Physics, Berkeley, 1966 (unpublished).

[2] S. D. Drell, Report to the Thirteenth International Conference on High Energy Physics, Berkeley, 1966, Stanford Linear Accelerator Center, Report No. SLAC-PUB-225 (unpublished).

[3] L. W. Durand III, Phys. Rev. **128**, 441 (1962). Durand's result, which assumes ρ and ω dominance in the strong interaction modifications of the photon propagator, is $\Delta\kappa_\mu=(5\pm1)\times10^{-8}$ if one uses the latest values for the mass and coupling constants of the vector mesons. See A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967). The 20% uncertainty here is an estimate of the uncertainty in the ρ-γ and ω-γ coupling constants and the non-resonant two-pion contributions. Before the weak interaction effects computed in this paper can be definitely established, it will be necessary to reduce this uncertainty as well as to know the sixth-order QED contributions.

[4] Actually, the sixth-order $(\alpha/\pi)^3$ corrections can be enhanced by factors of $[\ln(M_\mu/M_e)]^2$ and $\ln(M_\mu/M_e)$. These have been studied by S. D. Drell and J. S. Trefil (unpublished), A. Peterman (to be published), and T. Kinoshita (to be published). Kinoshita's result for the terms of these orders is $\Delta\kappa_\mu=1.721(\alpha/\pi)^3=(2.16)\times10^{-8}$.

[5] The recent measurement of the ac Josephson effect by W. H. Parker, B. N. Taylor, and D. N. Langenberg [Phys. Rev. Letters **18**, 787 (1967)] gives a new value for the fine-structure constant $\alpha^{-1}=137.0359\pm0.0004$ and introduces an uncertainty $\delta\kappa_\mu=\delta(\alpha/2\pi)=\pm0.3(\alpha/\pi)^3$ in the muon magnetic moment.

[6] N. Byers and F. Zachariasen, Nuovo Cimento **18**, 1289 (1960); R. D. Amado and L. Holloway, *ibid.* **30**, 1083 (1963); **30**, 1572 (1963); G. Segrè, Phys. Letters **7**, 357 (1963); Ph. Meyer and D. Schiff, *ibid.* **8**, 217 (1964); H. Pietschmann, Z. Physik **170**, 409 (1964); R. A. Shaffer, Phys. Rev. **135**, B187 (1964).

[7] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

[8] The notation is that of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1964). It should be noted that the one-photon vertex of the vector boson is given with the wrong sign in this reference. Our normalization for $\mathfrak{M}_\mu$ is such that the muon vertex in lowest order is $\mathfrak{M}_\mu=-ie\bar{u}(p+Q)\gamma_\mu u(p-Q)$.

$$P^{\beta\tau}(l)=\frac{1}{l^2-M_W{}^2}\left[g^{\beta\tau}-l^\beta l^\tau\frac{1-\xi}{M_W{}^2-\xi l^2}\right], \tag{4}$$

$$Q^\mu=\tfrac{1}{2}q^\mu,$$

and

$$\sqrt{2}g_W{}^2=G_F M_W{}^2\cong 10^{-5}(M_W/M_N)^2.$$

After the d^4k integration, $\mathfrak{M}_\mu$ must take the form[9] ($M\equiv M_\mu$):

$$\begin{aligned}\mathfrak{M}_\mu=&-ie\bar{u}(p+Q)[F_1(q^2)\gamma_\mu+F_2(q^2)(i/2M)\sigma_{\mu\nu}q^\nu\\&+F_3(q^2)\gamma_5(\gamma_\mu q^2-\not{q}q_\mu)(1/4M_W{}^2)]u(p-Q)\\ \equiv&-ie\bar{u}(p+Q)J_\mu u(p-Q),\end{aligned} \tag{5}$$

because of current conservation and CP invariance. $F_1(0)$ is part of the charge renormalization and $F_2(0)$ is the contribution to the anomalous moment that we seek:

$$\Delta\kappa_\mu=F_2(0)=\Delta(g_\mu-2)/2. \tag{6}$$

The form factors F_1 and F_2 can be obtained by the identities

$$\mathrm{Tr}\{\Lambda_j{}^\mu(\mathbf{p}+Q+M)J_\mu(\mathbf{p}-Q+M)\}=F_j(q^2),\ j=1,2 \tag{7}$$

where

$$\begin{aligned}\Lambda_1{}^\mu=&[1/16(M^2-Q^2)^2][-(M^2-Q^2)\gamma^\mu+3Mp^\mu],\\ \Lambda_2{}^\mu=&[1/16Q^2(M^2-Q^2)^2][M^2(M^2-Q^2)\gamma^\mu\\&-M(M^2+2Q^2)p^\mu].\end{aligned} \tag{8}$$

Thus

$$\begin{aligned}F_j(q^2)=&\frac{g_W{}^2}{(2\pi)^4}\int\frac{d^4k}{i}\\&\times\mathrm{Tr}\{\Lambda_j{}^\mu(\not{p}+Q+M)A_\mu(\not{p}-Q+M)\}.\end{aligned} \tag{9}$$

We compute all traces by computer.[10]

Upon integration, we obtain

$$\Delta\kappa_\mu=\frac{g_W{}^2}{8\pi^2}\left(\frac{M_\mu}{M_W}\right)^2\left\{2(1-\kappa_W)\ln\xi+\frac{10}{3}\right\}. \tag{10}$$

In writing Eq. (10), terms of higher order in $M_\mu{}^2/M_W{}^2$ and ξ have been neglected. The fact that $\Delta\kappa_\mu$ remains logarithmically divergent when $\kappa_W\neq 0$, instead of the expected quadratic divergence, is due to an accidental cancellation.

An attractive choice for the value of the cutoff ξ has been given by Lee.[11] He proposes that when all electrodynamic radiative corrections to Fig. 1 are included, $\Delta\kappa_\mu$ will be finite for $\xi\to 0$. With this assumption one obtains for the case $\kappa_W=0$,

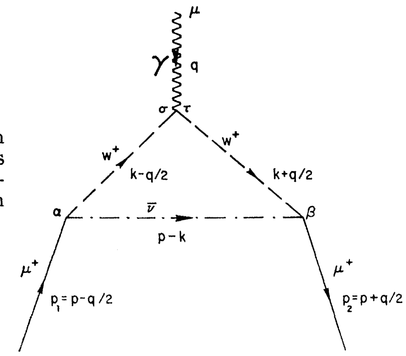

FIG. 1. The Feynman diagram which gives the W-boson contribution to the muon anomalous moment.

$$\Delta\kappa_\mu=(g_W{}^2/8\pi^2)(M_\mu/M_W)^2[2\ln\alpha+10/3+G(0)] \tag{11}$$

$$\cong -1.0\times 10^{-8}\cong -0.8(\alpha/\pi)^3, \tag{12}$$

where $G(0)$ is an unknown constant which can be obtained only by summing the leading divergence of all higher-order electrodynamic graphs. Presumably, $G(0)$ is of order 1, as has been assumed in writing Eq. (12).

Alternatively, one might choose a value for ξ based on intuitive physical arguments. One identifies $\xi^{-1}=(\Lambda/M_W)^2$, where Λ is the regulator mass in the usual cutoff approach. Choosing $\Lambda\cong 300$ BeV, which is the energy at which the four-field interaction violates unitarity, and is therefore presumably an upper limit for the cutoff, and taking $M_W\cong 2$ BeV, the present lower limit to the W mass,[12] one finds a result that is a factor of 2 larger than Eq. (12).

III. CONCLUSION

As stated in the Introduction, our result disagrees with all previous calculations except that in the case $\kappa_W=0$ our answer confirms the result of Shaffer.[6] Appendix A lists these previous results.

As a check of our expression, Eq. (2), and our use of the computer trace program, we have set $M_\mu=0$ and projected $F_1(q^2)$ from Eq. (9) and find the leading $\ln\xi$ term in the neutrino's electrodynamic form factor in agreement with the results of Bernstein and Lee,[13] and of Meyer and Schiff.[14] In addition, we note that for the case $\kappa_W=1$, the logarithmically divergent term cancels from Eq. (10). This same behavior was obtained[15] in the $\mu^\pm\to e^\pm+\gamma$ calculations, which were carried out in the (now disproven) one-neutrino theory. Finally, the trace routine[10] employed by us has been extensively checked in many other calculations.[16]

[9] See, for example, H. Pietschmann, Ref. 6. The two other diagrams in which the photon couples to the muon line make a contribution only to $F_1(0)$ and may be ignored here.

[10] Stanley M. Swanson, Stanford University Report No. ITP-120 (unpublished).

[11] T. D. Lee, Phys. Rev. **128**, 899 (1962).

[12] M. M. Block *et al.*, Phys. Letters **12**, 281 (1964); G. Bernardini *et al.*, *ibid.* **13**, 86 (1964).

[13] J. Bernstein and T. D. Lee, Phys. Rev. Letters **11**, 512 (1963).

[14] Ph. Meyer and D. Schiff, Phys. Letters **8**, 217 (1964).

[15] M. E. Ebel and F. J. Ernst, Nuovo Cimento **15**, 173 (1960).

[16] R. G. Parsons (private communication). We have also checked the required trace using the algebraic computation program of A. C. Hearn, Stanford University Report No. ITP-247 (unpublished).

There are theoretical reasons to believe that the anomalous moment κ_W of the W must in fact be zero. The assumption of minimal electrodynamic coupling and the Lagrangian for $\kappa_W=0$ given in Ref. 7 leads to the most convergent set of Feynman rules (listed in Appendix B for reference). If we accept this version of the minimal interaction principle for the W, then any anomalous moment would arise only from dynamics. The dynamics would at the same time generate form factors for the W's anomalous moment interactions. A further argument that $\kappa_W=0$ is given by Bernstein and Lee.[13] They show that the assumption of $\kappa_W\neq 0$, together with the assumption that the sum of all radiative corrections is finite for $\xi\to 0$, would imply a neutrino charge radius that is independent of α. The inclusion of a possible quadrupole moment for the W, which we have neglected everywhere, leads to an even more unacceptable behavior. However, we understand so little about the W boson (the origin of its large mass, for example), that we cannot afford to be too prejudiced about its behavior.

Finally, we remark that even if nature has chosen some way other than the W boson model to give structure to the four field Fermi interaction, one should expect weak interaction corrections to κ_μ that are of first order in G_F and are quite possibly of the same order of magnitude as the result in Eq. (12).

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APPENDIX A

We list below previous calculations[6] of $\Delta\kappa_\mu$ expressed in the notation of this paper. Here $\Delta\kappa_\mu$ is the part of the anomalous magnetic moment of the muon which arises from its first-order weak interactions with the W boson. The units of $\Delta\kappa_\mu$ are muon magnetons, $e\hbar/2M_\mu c$. For convenience we have defined

$$A=\left(\frac{g_W^2}{8\pi^2}\right)\left(\frac{M_\mu}{M_W}\right)^2=\frac{1}{8\pi^2}\left(\frac{G_F M_N^2}{\sqrt{2}}\right)\left(\frac{M_\mu}{M_N}\right)^2\cong 1.0\times 10^{-9}.$$

For zero anomalous moment of the W, $\kappa_W=0$:

Byers and Zachariasen[17]: $\Delta\kappa_\mu=-A\{\frac{1}{8}\ln\xi\}$,
Amado and Holloway[17]: $\Delta\kappa_\mu=-A\{\frac{1}{2}\ln\xi\}$,
Segrè: $\Delta\kappa_\mu=-A\{3\ln\xi+1\}$,
Meyer and Schiff: $\Delta\kappa_\mu=A\{\ln\xi+1\}$,
Pietschman: $\Delta\kappa_\mu=A\{4\ln\xi+10/3\}$,
Schaffer: $\Delta\kappa_\mu=A\{2\ln\xi+10/3\}$,
Brodsky and Sullivan (this paper); also Burnett and Levine[18]: $\Delta\kappa_\mu=A\{2\ln\xi+10/3\}$.

For $\kappa_W\neq 0$:

Segrè: $\Delta\kappa_\mu=-A\{(3+2\kappa_W)\ln\xi+1+4\kappa_W\}$,
Brodsky and Sullivan (this paper); also Burnett and Levine[18]: $\Delta\kappa_\mu=A\{2(1-\kappa_W)\ln\xi+10/3\}$.

APPENDIX B

For reference, we give the Feynman rules for the vector boson, using the ξ-limiting regularization, in the metric

$$g^{\mu\nu}=(1,-1,-1,-1).$$

The rules for the propagators and vertices illustrated in Fig. 2 are:

(1) vector boson propagator:

$$-i\{g^{\mu\nu}-m^{-2}k^\mu k^\nu)(k^2-m^2+i\epsilon)^{-1}+(m^{-2}k^\mu k^\nu)(k^2-\xi^{-1}m^2+i\epsilon)^{-1}\};$$

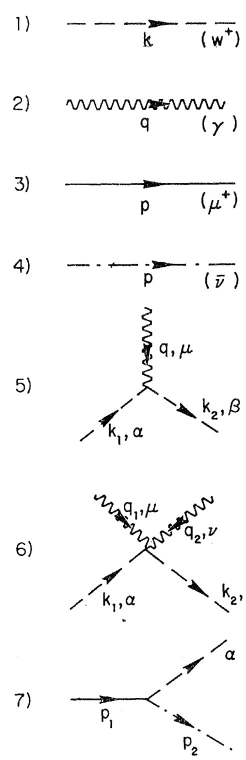

FIG. 2. Propagators and vertices for the W boson, the photon, the muon, and the antineutrino.

[17] These authors computed only the leading logarithmic divergence.

[18] *Note added in proof.* The result Eq. (10) has been independently obtained by T. Burnett and M. J. Levine (to be published). The authors wish to thank them for pointing out an error in a preliminary version of this paper which gave the constant term as 7/3 rather than the correct 10/3 value.

(2) photon propagator:

$$-ig^{\mu\nu}(q^2+i\epsilon)^{-1};$$

(3) fermion propagator (the positive charged fermion is taken to be the particle):

$$i(\not{p}+m)(p^2-m^2+i\epsilon)^{-1};$$

(4) antineutrino propagator:

$$i\not{p}(p^2+i\epsilon)^{-1};$$

(5) 1-photon W^+ vertex:

$$ie\{g^{\alpha\beta}(k_1+k_2)^\mu-g^{\alpha\mu}(k_1+\kappa_W k_1-\kappa_W k_2-\xi k_2)^\beta - g^{\beta\mu}(k_2+\kappa_W k_2-\kappa_W k_1-\xi k_1)^\alpha\};$$

(6) 2-photon W^+ vertex:

$$-ie^2\{2g^{\mu\nu}g^{\alpha\beta}-(1-\xi)g^{\alpha\mu}g^{\beta\nu}-(1-\xi)g^{\alpha\nu}g^{\beta\mu}\};$$

(7) muon$^+\rightarrow W^+$+antineutrino vertex:

$$-ig_W\gamma^\mu(1+\gamma_5).$$

The Lagrangian and derivations are given in Ref. 7.

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K^--d Breakup Scattering*†

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Cross sections for the reactions $K^-+d\rightarrow K^-+n+p$ and $K^-+d\rightarrow \bar{K}^0+n+n$ are calculated by means of the Faddeev equations and separable potentials. Kaon-nucleon interactions were based on either the Kim or the Sakitt scattering lengths. Derivations of the equations for these processes and of the technique for obtaining these cross sections after solving the integral equation for complex momenta are presented. The cross section for $K^-+d\rightarrow \bar{K}^0+n+n$ so calculated is somewhat larger than that obtained in recent experiments.

I. INTRODUCTION

THE K^--d elastic-scattering cross section which we previously calculated[1,2] could not easily be compared with experiment. We had obtained only the elastic scattering cross section, while experimental cross sections[3,4] included a "pseudoelastic" contribution consisting of $K^-+d\rightarrow K^-+p+n$ scattering. Recently, experimental results[5] have been obtained for the charge-exchange reaction $K^-+d\rightarrow \bar{K}^0+n+n$. We have extended our previous work on K^--d interaction to include calculations of the cross section into these two breakup channels. As before, we have used the Faddeev formalism and solved the three-body problem exactly after parametrizing the two-body interactions with s-wave nonlocal-separable potentials. Absorption into hyperon channels has been accounted for by allowing complex values for the strength of the K^--N potentials. Coulomb effects have been neglected. We have not included mass splitting between the $K^--\bar{K}^0$ as we did in HS4. Although this could be an important correction, we have delayed inclusion of this effect because it entails substantial revision of our procedure.

Section II contains a derivation of the two breakup cross sections integrated over all final-state configurations. These cross sections are expressed in terms of integrations over the imaginary part of modified Green's functions. It is also shown how this result can be fit into the matrix formulation which we have used in HS1.

Our procedure for solving the integral equation numerically provides solutions for certain complex values of the momentum,[1] while the breakup cross sections as derived in Sec. II require knowledge of the solutions for real values of momentum. The technique needed for returning to the real axis is given in Sec. III. The notations in Secs. II and III, unless otherwise specified, are that of HS1.

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† A preliminary report of this work was given in Bull. Am. Phys. Soc. 11, 381 (1966).

[1] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965), hereafter referred to as HS1.

[2] J. H. Hetherington and L. H. Schick, Phys. Rev. **141**, 1314 (1966), hereafter referred to as HS4.

[3] L. Alvarez, in *Proceedings of the Ninth Annual Conference on High-Energy Physics, Kiev, 1959* (Academy of Sciences, USSR, 1960), p. 471.

[4] An experiment in progress at the University of Illinois will also include this pseudoelastic contribution [R. D. Hill (private communication)].

[5] Robert M. Lansford, Robert B. Muir, Alexander Shapiro, and R. D. Hill, Bull. Am. Phys. Soc. **11**, 37 (1966).