PHYSICAL REVIEW VOLUME 156, NUMBER 5 25 APRIL 1967

Strong Initial- and Final-State Interactions in Nonmesonic Hypernuclear Decays and in Parity-Violating $N+N \to N+N$ Reactions*

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(Received 14 October 1966)

A distorted-wave Born calculation is used to show that the strong initial- and final-state interactions in the nonmesonic hypernuclear decays, $\Lambda+N \to N+N$, are of major importance, greatly suppressing certain transitions, and suggest the dominance of one-pion exchange in the explanation of hypernuclear decay rates. Weak exchange of heavier mesons is also suppressed by the strong interactions both in $\Lambda+N \to N+N$—badly breaking any $SU(3)$ symmetry of the reactions—and in the parity-violating $N+N \to N+N$. It is shown that the decay rate of heavy hypernuclei provides a measure of the Λ-N correlation function at short distances in nuclear matter and should depend significantly on the existence of a hard core in the Λ-N strong interaction.

I. INTRODUCTION

THE reaction $\Lambda+N \to N+N$ is an interesting example of one of the little understood nonleptonic weak interactions, It has been observed as the so-called nonmesonic decay mode of both light[1] and heavy[2] hypernuclei. On the basis of available data, Block and Dalitz[3] have made a preliminary empirical analysis of the structure of this weak interaction. In this paper we would like to extend the theory of the reaction by an attempt at an explicit calculation of the effects of strong interactions in these nonmesonic hypernuclear decays.

There have been several formulations of the theory of the structure of the $\Lambda+N \to N+N$ reaction. The idea that the reaction proceeds by the exchange of a pion between the baryon currents seems first to have been suggested by Cheston and Primakoff in a calculation of hypernuclear decay rates.[4] It was used by Ruderman and Karplus in a deduction of the Λ spin.[5] This work was extended by Cerulus to include effects of parity violation.[6] Ferrari and Fonda considered the effects of two-pion exchanges creating a virtual Σ-N state in the decay.[7] The reaction has been analyzed in the framework of $SU(3)$ symmetry. Exchanges of the entire pseudoscalar and vector-meson octet were calculated by Itoh[8] using $SU(6)$. Kohmura has used unitary symmetry to predict decay rates of double hypernuclei from the known decays of single hyperfragments.[9] A discussion of the reaction assuming octet baryon currents by Tamiya, Kawaguchi, and Sumi allowed for form factors at the meson-baryon vertices but was not able to account for observations.[10] Panchapakesan pointed out that the form proposed by Block and Dalitz requires a mixture of octet and 27 representation components in the current.[11]

In none of the above discussions was account taken of the strong interaction between the Λ and nucleon in the initial state or the two fast outgoing nucleons in the final state. It is our thesis here that these strong interactions are of major importance in the $\Lambda+N \to N+N$ reaction, that they select the one-pion-exchange mechanism as the dominant weak effect, badly breaking SU_3 symmetry, and that they allow for a crude explanation of experimental observations.[12]

In this paper the decay rate is computed for a Λ particle at rest in infinite nuclear matter. For such a Λ the normal decays $\Lambda \to N+\pi$, $\Lambda \to N+l+\nu$ are forbidden because the final nucleon's momentum ($\leqslant 176$ MeV) is less than a reasonable Fermi momentum (~ 250 MeV). Thus in the limit of large hypernuclei the decay rate should be purely nonmesonic.[13] Some comments about the applicability of these results to light hypernuclei are offered.

Calculations are carried out in an independent-pair approximation and are based on the assumption that

* Research sponsored by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Contract No. AF49(638)1389.

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[1] M. M. Block, R. Gessaroli, S. Ratti, L. Grimellini, T. Kikuchi, L. Lendinara, L. Monari, and E. Harth, Nuovo Cimento **28**, 299 (1963); M. M. Block, R. Gessaroli, J. Kopelman, S. Ratti, M. Schneeberger, L. Grimellini, T. Kikuchi, L. Lendinara, L. Monari, W. Becker, and E. Harth, in *Proceedings of the International Conference on Hyperfragments, St. Cerque, Switzerland, 1963* (CERN, Geneva, 1964).

[2] There have been a great number of studies. See, for example, D. A. Evans, D. T. Goodhead, A. Z. M. Ismail, and Y. Prakash, Nuovo Cimento **39**, 785 (1965); J. P. Lagnaux, J. Lemmonne, J. Sacton, E. R. Fletcher, D. O'Sullivan, T. P. Shah, A. Thompson, P. Allen, Sr., M. Heeran, A. Montwill, J. E. Allen, D. H. Davis, D. A. Garbutt, V. A. Bull, P. V. March, M. Yaseen, T. Pniewski, and T. Zakrzewski, Nucl. Phys. **60**, 97 (1964), and references contained therein.

[3] M. M. Block and R. H. Dalitz, Phys. Rev. Letters **11**, 96 (1963).

[4] W. Cheston and H. Primakoff, Phys. Rev. **92**, 1537 (1953).

[5] M. Ruderman and R. Karplus, Phys. Rev. **102**, 247 (1956).

[6] F. Cerulus, Nuovo Cimento **5**, 1685 (1957).

[7] F. Ferrari and L. Fonda, Nuovo Cimento **7**, 320 (1958).

[8] C. Itoh, Tokyo University of Education report (unpublished).

[9] T. Kohmura, Progr. Theoret. Phys. (Kyoto) **35**, 65 (1966).

[10] T. Tamiya, M. Kawaguchi, and Y. Sumi, Progr. Theoret. Phys. (Kyoto) **34**, 833 (1965).

[11] N. Panchapakesan, Phys. Letters **20**, 435 (1966).

[12] Preliminary reports of this work were given in J. B. Adams [Phys. Letters **22**, 463 (1966)]. This paper contains numerical errors corrected here; J. B. Adams and J. D. Walecka, in Proceedings of the International Conference on Nuclear Physics, 1966 (unpublished).

[13] H. Primakoff, Nuovo Cimento **3**, 1394 (1956).

effects of the strong interactions can be accounted for by interbaryon potentials. The weak one-meson exchange is then calculated in the distorted-wave Born approximation. The most important feature of the potentials assumed is a hard core which forces the S-wave radial wave functions to go to zero at the core radius rather than to a finite value at the origin. This effect and oscillations of the final-state radial wave function in the range of the weak interactions lead to major reductions in the radial integral in the matrix element. The spirit of the calculation will be to adopt the simplest forms of interaction and potential to show these effects.

In Sec. II we present the model weak interaction used and the resulting matrix elements. Section III is a discussion of the strong potentials and wave functions used. Numerical results and sensitivity to variations of some parameters are in Sec. IV. Results are compared to experiment in Sec. V. Section VI is a discussion of some of the shorter-range effects and of the applicability of this technique to parity-violating nucleon-nucleon reactions. Section VII is a critique of the work and conclusions.

II. FUNDAMENTAL INTERACTION AND RATES

The longest-range effect of the fundamental weak interactions in $\Lambda+N\rightarrow N+N$ should be one-pion exchange. Both dispersion relations and the field-theory point of view we adopt here include such an interaction because of the known weak π-Λ-N and strong π-N-N couplings. We assume effective Hamiltonians for these of the form

$$H_w=iG_w\bar{\psi}_N(1+\lambda\gamma_5)\boldsymbol{\tau}\psi_\Lambda\boldsymbol{\phi}_\pi\,,$$

$$H_s=iG_s\bar{\psi}_N\gamma_5\boldsymbol{\tau}\psi_N\boldsymbol{\phi}_\pi\,,$$

where ψ_Λ is the Λ spurion and boldface denotes vectors in isospace. This weak-interaction Hamiltonian is taken as a purely phenomenological model without prejudice as to what the underlying fundamental form may be. For first-order weak interactions between baryons on their mass shells, this form for H_w is equivalent to one in which a polar and an axial-vector current are coupled to the derivative of the pion field even if the pion is not on its mass shell. For baryons moving in a potential this is still true to the approximation that the potential can be neglected compared to $M_\Lambda+M_N$ and $M_\Lambda-M_N$. The form of the Hamiltonian adopted here gives the rate and angular correlations for free Λ decay[14] with the dimensionless coupling constants $G_w=9.0\times10^{-8}$, $\lambda=-6.7$. The pseudoscalar strong coupling constant is $G_s^2/(4\pi)=14$.

An expression for the nonmesonic decay rate of a Λ particle in nuclear matter by the one-pion-exchange mechanism can now be given, leaving open for the moment the choice of strong interbaryon potentials and corresponding wave functions. Because the correlations are most easily discussed in terms of the various partial waves, the complete set of final states summed over will be spherical waves. Correspondingly the initial Λ-N state is analyzed into partial waves. In our nuclear matter calculation we neglect all but the S waves for the initial Λ-N state. Below we give a numerical check which suggests that this is indeed a good approximation. (In the light hypernuclei with $A<5$ the ground state is mainly S wave.) The initial Λ-N S wave function can have spins arranged either in a singlet state which can make transitions to the final nucleon states ${}^1S_0$, ${}^3P_0$ or in a triplet state which can make transitions to ${}^3P_1$, ${}^1P_1$, and states ${}^3S_1+{}^3D_1$ which may be mixed by a tensor force. It is the total transition rate which is experimentally measurable of course, but for the purposes of displaying the effects of correlations we shall also compute quantities $R(\alpha\rightarrow\beta)$ defined in Appendix A which are proportional to the square of the transition matrix element for state α to state β for some relative Λ-N momentum q. Computation of these is straightforward. The results are conveniently expressed in terms of certain derivative operators which can be defined to be

$$D_1=\frac{d}{dr},\quad D_2=\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr},\quad D_3=r\frac{d}{dr}\frac{1}{r}\frac{d}{dr},\quad D_4=\frac{1}{r^2}\frac{d}{dr}r^2,$$

$$D_5=\frac{d}{dr}\frac{1}{r^2}\frac{d}{dr}r^2,\quad D_6=r\frac{d}{dr}\frac{1}{r},\quad D_7=\frac{1}{r^3}\frac{d}{dr}r^3,\quad D_8=\frac{1}{r^2}\frac{d}{dr}\frac{1}{r}\frac{d}{dr}r^3.$$

These act on the functions $f_{1S}(t_0,r)$, $f_{3S}(t_0,r)$, $f_{1P}(t_0,r)$, $f_{3P}(t_0,r)$, and $f_{3D}(t_0,r)$ which are the final-state ${}^1S_0$, ${}^3S_1$, ${}^1P_1$, ${}^3P_1$ ${}^3D_1$ radial wave functions which in the absence of internucleon potential would be the spherical Bessel functions $j_0(t_0r)$, $j_0(t_0r)$, $j_1(t_0r)$, $j_1(t_0r)$, and $j_2(t_0r)$, respectively. Here t_0 is the magnitude of the relative momentum of the final nucleons for a given q. The same radial function is used for the ${}^3P_0$ and ${}^3P_1$ states. For the initial Λ-N state we use only one radial function $f_i(q,r)$ for both triplet and singlet cases for reasons discussed below.

[14] J. W. Cronin and O. E. Overseth, Phys. Rev. 129, 1795 (1963).

The rates for $\Lambda+p\to n+p$ are then

$$R({}^1S_0\to{}^1S_0)=\tfrac{1}{4}h\left\{\int_0^\infty dr\, re^{-mr}[b(f_i)(D_2f_{1S})+(b+c)(D_1f_i)(D_1f_{1S})+c(D_2f_i)(f_{1S})]\right\}^2,$$

$$R({}^1S_0\to{}^3P_0)=\tfrac{1}{4}h\left\{\int_0^\infty dr\, re^{-mr}[-(D_1f_i)(-aD_5f_{3P}+f_{3P})+(D_4f_{3P})(-aD_2f_i+f_i)]\right\}^2,$$

$$R({}^3S_1\to{}^3P_1)=\tfrac{1}{6}h\left\{\int_0^\infty dr\, re^{-mr}[-a(D_1f_i)(D_5f_{3P})-\tfrac{1}{3}a(D_2f_i)(D_4f_{3P}) -\tfrac{4}{3}a(D_3f_i)(D_6f_{3P})+(f_i)(D_4f_{3P})+(D_1f_i)(f_{3P})]\right\}^2,$$

$$R({}^3S_1\to{}^1P_1)=\tfrac{3}{4}h\left\{\int_0^\infty dr\, re^{-mr}[-a(D_1f_i)(D_5f_{1P})+\tfrac{1}{3}a(D_2f_i)(D_4f_{1P}) -\tfrac{4}{3}a(D_3f_i)(D_6f_{1P})+(f_i)(D_4f_{1P})+(D_1f_i)(f_{1P})]\right\}^2,$$

$$R({}^3S_1\to{}^3S_1+{}^3D_1)=h\left\{\int_0^\infty dr\, re^{-mr}[\tfrac{1}{2}[b(f_i)(D_2f_{3S})+(b+c)(D_1f_i)(D_1f_{3S})+c(D_2f_i)(f_{3S})] +\sqrt{2}[-b(f_i)(D_8f_{3D})-(b+c)(D_1f_i)(D_7f_{3D})-c(D_3f_i)(f_{3D})]]\right\}^2.$$

Here we have used

$$a\equiv 1/(4M_NM_\Lambda),\quad b\equiv\lambda/(2M_N),\quad c\equiv\lambda/(2M_\Lambda),$$
$$h\equiv 2(t_0/M_N)G_w^2(G_s^2/4\pi),$$
$$m\equiv\{M_\pi^2-\tfrac{1}{4}(M_\Lambda-M_N)^2[1+q^2/(2M_\Lambda M_N)]^2\}^{1/2}\cong[M_\pi^2-\tfrac{1}{4}(M_\Lambda-M_N)^2]^{1/2}.$$

For $\Lambda+n\to n+n$, transitions are not possible to the states ${}^3S_1$, ${}^3D_1$, ${}^1P_1$ which have isotopic spin zero, but transition rates to the isospin 1 states ${}^1S_0$, ${}^3P_0$, ${}^3P_1$ are just twice the corresponding $\Lambda+p\to n+p$ rates because the Hamiltonian in our model obeys the $\Delta I=\frac{1}{2}$ rule. In deriving these expressions the kinetic energy of the particles has been neglected compared to their rest mass.

In the expressions for the R's one can recognize the radial integral of the Born matrix element. The integrand consists of the radial weighting function r^2 times the Yukawa-like "weak potential" e^{-mr}/r times a product of the initial and final wave functions operated on by various derivatives. These derivative couplings come from the $\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ terms inherent in pseudoscalar interaction which has been assumed. The total decay rate for a Λ in nuclear matter is

$$\tau^{-1}=3(2\pi)^{-2}[(M_\Lambda+M_N)/M_\Lambda]^3\times\int_0^{q_M}dq\, q^2[R_{p0}+R_{p1}+R_{n1}],$$

where $q_M=k_FM_\Lambda/(M_\Lambda+M_N)$ and R_{p0} is the sum of the $R(\alpha\to\beta)$ for a p-Λ spin-zero state, R_{p1} is the sum of the $R(\alpha\to\beta)$ for a p-Λ spin-1 state, etc. This expression is derived in Appendix A.

III. INTERBARYON POTENTIALS AND WAVE FUNCTIONS

To proceed with the calculation one must choose potentials and derive the corresponding wave functions for the initial and final states. The potentials chosen are the simplest analytical forms which we hope will describe the important physics of the situation.

The lambda-nucleon potential seems to consist of a short-range $[<1/(2M_\pi)]$ attractive part and a strong, repulsive core region. "Empirical" evidence for a repulsive core comes from various studies of light hypernuclei.[15] Indeed on general theoretical grounds one would expect the exchange of a heavy vector meson such as the ω to give rise to a short-range repulsive potential, and detailed calculation shows this to be so.[16] The rates shall be computed for a Λ particle at rest in nuclear matter. In such a situation the Pauli principle prevents scattering of nucleons in the Fermi sea from the Λ-particle, and for a given Λ-N potential the wave function is described not by Schrödinger's equation but rather by the Bethe-Goldstone equation. The solution to the Bethe-Goldstone equation with a hard core has been given by Gomes.[17] Gomes and Walecka have shown that addition

[15] J. J. DeSwart and C. K. Iddings, Phys. Rev. **128**, 2810 (1962); J. H. Hetherington and L. H. Schick, *ibid.* **139**, B1164 (1965); R. C. Herndon, Y. C. Tang, E. W. Schmid, *ibid.* **137**, B294 (1964); A. Deloff and J. Wizecionko, Nuovo Cimento **34**, 1195 (1965); A. R. Bodmer, Phys. Rev. **141**, 1387 (1966); R. C. Herndon and Y. C. Tang, *ibid.* **149**, 735 (1966).

[16] B. W. Downs and R. J. N. Phillips, Nuovo Cimento **36**, 120 (1965).

[17] L. C. Gomes, thesis, Massachusetts Institute of Technology, 1958 (unpublished). See, however, the criticisms of this form in B. W. Downs and W. E. Ware, Phys. Rev. **133**, B132 (1964); H. S. Köhler, *ibid.* **137**, B1145 (1965); B. W. Downs and M. E. Grypeos, Nuovo Cimento **44B**, 306 (1966).

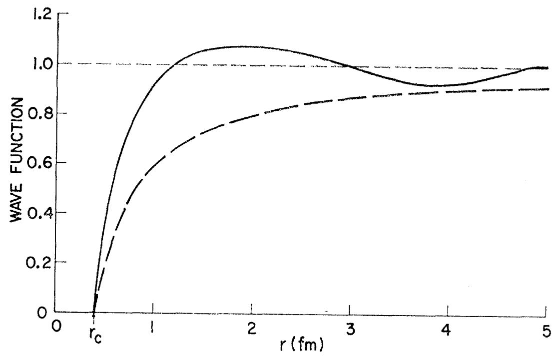

FIG. 1. Initial wave functions. The solid line is $f_i(q,r)$. The long-dashed line is the solution of Schrödinger's equation with a hard core. The short-dashed line is the solution of Schrödinger's equation with no correlations. All are for $q=0.1$ MeV.

of a weak attractive potential to the hard core changes the wave function but little.[18] For the Λ-N potential then we adopt simply a hard core, making no distinction between singlet and triplet states, and use an approximation to the exact solution for the wave function given by Gomes[17]

$$f_i(q,r)=\left[j_0(qr)-\frac{\sin(qr_c)\,\mathrm{si}(\beta r)}{qr\,\mathrm{si}(\beta r_c)}\right]\theta(r-r_c),$$

where $r_c=0.4$ F is the core radius, θ is the Heaviside step function, and

$$\mathrm{si}(x)\equiv-\int_x^\infty\frac{\sin t}{t}dt.$$

The parameter β is related to the healing distance r_H by $\beta r_H=1.9265$, where r_H is defined as the radius of the first zero of $f_i(q,r)-j_0(qr)$ and measures the distance in which the hard-core wave function differs significantly from the free wave function. The healing length given by Gomes is $r_H=1.18$ F. The function is sketched in Fig. 1.

For the nucleon-nucleon potential we shall not use the full complexity of the phenomenological potentials now available[19] but rather confine ourselves to rather simple central and tensor potentials. One can in principle choose four different central potentials, one for each pair of the two isospin states and the two spin states. We shall use only two, both with a hard-core r_c with the same radius used for the Λ-N potential and both with a constant potential extending beyond the core to a radius r_w. In the one of these potentials used for the $^3S_1$, $^3D_1$, $^1S_0$ states, the potential V_S is taken to be attractive giving a square well, while in the other potential used for the $^1P_1$, $^3P_0$, $^3P_1$ states, the potential is taken as repulsive giving a shoulder. These are sketched in Fig. 2. The depth of the S, D well $V_S=-28.2$ and the radius $r_w=2.3$ F are chosen to give the $^1S_0$ N-N wave function an almost bound state at zero energy and an effective range of 2.7 F. The repulsive shoulder for the P-wave potentials V_P is a crude attempt to imitate the phenomenological potentials indicated at these energies by the $^1P_1$ phase shifts.[20] The tensor potential will be of the same shape as the central potential for the S, D states but the ratio of their strength $\gamma\equiv$ (tensor well depth)/(central well depth) will be taken as 2. As justification for a stronger tensor than central potential can be cited the results of the calculation one-pion-exchange potentials. These potentials are somewhat arbitrary, but will be varied below to show that our principal results do not depend on the exact choice of parameters.

Because of the rather large energy release in the reaction $\Lambda+N\to N+N$, the outgoing nucleons are going so fast that the presence of the spectator nucleons in the Fermi sea can be ignored, so Schrödinger's equation can be used for their relative wave function. Analytic solutions are perfectly straightforward for all the states except the $^3S_1$ and $^3D_1$ which are coupled by the tensor force. For these states, we generate numerically two orthonormal solutions $(^3S_1+{}^3D_1)_1$ and $(^3S_1+{}^3D_1)_2$ which in the absence of the tensor force are pure $^3S_1$ and $^3D_1$, respectively. The procedure used is as follows. All the wave functions are set equal to zero at r_c. The solution $(^3S_1+{}^3D_1)_1$ is generated by setting $df_{3S}/dr=1$ and $df_{3D}/dr=0$ at r_c and integrating the equations numerically to r_w. The normalization of the solution is then adjusted to match a normalized pair of phase-shifted exterior S- and D-wave solutions at r_w.

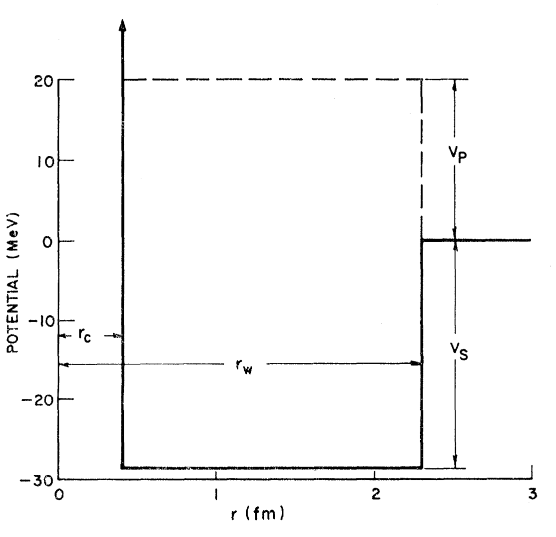

FIG. 2. Form of nucleon-nucleon potential used. Broken line is for P-wave states and solid line is central potential used in S- and D-wave states.

[18] L. C. Gomes and J. D. Walecka (unpublished).

[19] See, for example, the review monograph: M. J. Moravcsik, *The Two-Nucleon Interaction* (Clarendon Press, Oxford, England, 1963).

[20] G. Breit, M. M. Hull, Jr., K. E. Lassila, and K. D. Pyatt, Jr., Phys. Rev. **120**, 2227 (1960); M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, *ibid.* **122**, 1606 (1960); R. A. Arndt and M. H. MacGregor, *ibid.* **141**, 873 (1966).

An independent solution is generated similarly starting with $df_{3S}/dr=0$ and $df_{3D}/dr=1$ at r_c. From this the solution $({}^3S_1+{}^3D_1)_2$ is created by using the Schmidt procedure in the exterior region.

IV. NUMERICAL RESULTS

The actual rates predicted by our model are derived by carrying out the necessary integrals numerically. The partial rates are given in τ_Λ^{-1} F^3 where for the free-Λ decay rate we have used $\tau_\Lambda^{-1}=3.83\times10^9$ sec^{-1}.[21] In Table I in the row (i) labeled "noncorrelated" are the transition rates computed for $q=0.1$ MeV with no interbaryon potentials in the initial or final states, using free-particle wave functions. In the row labeled (ii) are the rates computed with the potentials given in Sec. III. Comparison of these two rows shows the dramatic effect of the introduction of the interactions in the initial and final states.

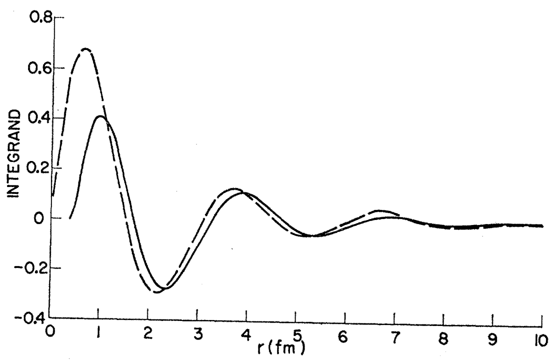

FIG. 3. Radial integrand for ${}^1S_0\to{}^3P_0$ at $q=0.1$ MeV. The solid line is with the correlations of set (ii) and the broken line is with no correlations, set (i).

The reasons for these large changes are found in investigating the form of the radial integrand in the expressions for the R's. The potentials change the values and the derivatives of the S wave functions near the origin. As an example the effect of the core on the S wave radial function is shown in Fig. 1. The weighting function $r^2(e^{-mr}/r)$ has quite a long range. It peaks at 1.8 F and falls to one quarter of its peak value only at 6.7 F. In this range N-N radial wave function and its derivatives oscillate several times while the Λ-N radial wave function does not change sign. This means that the integral is the sum of several cancelling contributions and so is very sensitive to relatively modest changes in the form of the integrand. This is illustrated for the ${}^1S_0\to{}^3P_0$ transition in Fig. 3.

To get some idea of the sensitivity of the results to our choice of potentials the rates can be evaluated with variant values of the parameters specifying the potentials. Sets of these parameters together with M_π, M_Λ, and q are defined in Table II and the corresponding decay rates

[21] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 37, 633 (1965).

TABLE I. Partial-transition rates for various correlations in τ_Λ^{-1} F^3.

	Case	$R({}^1S_0\to{}^1S_0)$	$R({}^1S_0\to{}^3P_0)$	$R({}^3S_1\to({}^3S_1+{}^3D_1)_1)$	$R({}^3S_1\to({}^3S_1+{}^3D_1)_2)$	$R({}^3S_1\to{}^1P_1)$	$R({}^3S_1\to{}^3P_1)$	R_{n1}	R_{p1}	R_{p0}
i	Noncorrelated	0.81	0.38	0.81	6.5	1.2	0.26	0.51	8.7	1.2
ii	Standard correlation	$\sim10^{-4}$	0.023	$\sim10^{-5}$	0.23	0.49	0.16	0.32	0.88	0.024
iii	r_c	$\sim10^{-5}$	$\sim10^{-3}$	0.016	0.22	0.41	0.14	0.27	0.78	$\sim10^{-3}$
iv	r_H	$\sim10^{-4}$	$\sim10^{-3}$	$\sim10^{-4}$	0.20	0.35	0.12	0.24	0.67	$\sim10^{-3}$
v	V_S, V_P variants	$\sim10^{-4}$	0.047	0.069	0.13	0.43	0.19	0.38	0.82	0.048
vi	r_w	$\sim10^{-5}$	0.019	0.23	0.096	0.56	0.14	0.28	1.0	0.019
vii	γ	$\sim10^{-4}$	0.023	0.15	0.074	0.49	0.16	0.32	0.88	0.24
viii	M_π	$\sim10^{-4}$	0.028	$\sim10^{-4}$	0.19	0.42	0.13	0.26	0.73	0.028
ix	M_Λ	$\sim10^{-6}$	0.049	$\sim10^{-4}$	0.21	0.53	0.17	0.35	0.91	0.049
x	No tensor	$\sim10^{-4}$	0.023	$\sim10^{-5}$	4.1	0.49	0.16	0.32	4.8	2.4
xi	Core only final	$\sim10^{-6}$	$\sim10^{-3}$	$\sim10^{-6}$	4.3	0.60	0.13	0.26	5.0	$\sim10^{-3}$
xii	Schrödinger solution initial	$\sim10^{-4}$	$\sim10^{-3}$	$\sim10^{-4}$	0.12	0.20	0.067	0.13	0.38	$\sim10^{-3}$
xiii	Noncorrelated initial	0.61	0.26	0.98	0.19	0.56	0.18	0.35	1.9	0.88
xiv	Attractive ${}^1P$	$\sim10^{-4}$	0.023	$\sim10^{-5}$	0.23	0.80	0.16	0.32	1.2	0.024
xv	Schrödinger solution initial, core final	$\sim10^{-6}$	$\sim10^{-6}$	$\sim10^{-6}$	1.9	0.24	0.053	0.11	2.2	$\sim10^{-5}$
xvi	Large q	$\sim10^{-4}$	0.031	$\sim10^{-3}$	0.17	0.42	0.13	0.26	0.73	0.031
xvii	M_K noncorrelated	0.16	0.074	0.16	1.2	2.2	0.049	0.098	1.7	0.23
xviii	M_K correlated	0.020	0.0039	0.0027	0.012	0.055	0.013	0.025	0.082	0.024

TABLE II. Definitions of sets of parameters used in computing rates.

		r_c (F)	V_S (MeV)	V_P (MeV)	r_w (F)	γ	r_H (F)	M_π (MeV)	M_Λ (MeV)	q (MeV)
i	Noncorrelated	...[a]	...[a]	...[a]	...[a]	...[a]	...[a]	140	1115	0.1
ii	Standard correlation	0.4	−28.2	20	2.3	2	1.18	140	1115	0.1
iii	r_c variants	0.5	−28.2	20	2.3	2	1.18	140	1115	0.1
iv	r_H variants	0.4	−28.2	20	2.3	2	2.0	140	1115	0.1
v	V_S, V_P variants	0.4	−40	40	2.3	2	1.18	140	1115	0.1
vi	r_w variants	0.4	−28.2	20	3.0	2	1.18	140	1115	0.1
vii	γ variants	0.4	−28.2	20	2.3	4	1.18	140	1115	0.1
viii	M_π variants	0.4	−28.2	20	2.3	2	1.18	180	1115	0.1
ix	M_Λ variants	0.4	−28.2	20	2.3	2	1.18	140	1090	0.1
x	No tensor	0.4	−28.2	20	2.3	0	1.18	140	1115	0.1
xi	Core only final	0.4	0	0	...[a]	0	1.18	140	1115	0.1
xii	Schrödinger solution initial	0.4	−28.2	20	2.3	2	...[a]	140	1115	0.1
xiii	Noncorrelated initial	0.4	−28.2	20	2.3	2	...[a]	140	1115	0.1
xiv	Attractive ^1P	0.4	−28.2	−28.2	2.3	2	1.18	140	1115	0.1
xv	Schrödinger initial core final	0.4	0	0	...[a]	0	...[a]	140	1115	0.1
xvi	Large q	0.4	−28.2	20	2.3	2	1.18	140	1115	125
xvii	M_K noncorrelated	...[a]	...[a]	...[a]	...[a]	...[a]	...[a]	498	1115	0.1
xviii	M_K correlated	0.4	−28.2	20	2.3	2	1.18	498	1115	0.1

[a] See text for explanation.

given in Table I. Set (ii) is the standard set given above to which the others should be compared. In sets (iii), (iv), (v), (vi), and (vii) the core size, healing length, potential strength, radius of the potential well, and ratio of the tensor to central strength are varied. In set (viii) we vary the pion mass imagining that the pion propagator in nuclear matter might be modified. In set (ix) an approximate binding energy for a Λ particle in nuclear matter has been subtracted from the Λ mass. As one might expect, there are considerable fluctuations in the rates as the parameters are varied, but the gross effects seen by comparing cases (ii)–(ix) to the noncorrelated case (i) remain. Although the effects are the results of large cancellations, they do not seem to be a quirk of our particular choice of parameters. These gross features are a large reduction in the transition rates from angular-momentum-zero states and a suppression of transitions to the isospin-zero states as compared to the isospin-one states.

Other features of the calculations can be illustrated by different sets of parameters. The importance of the tensor force is illustrated by setting $\gamma=0$ in set (x) which makes $({}^3S_1+{}^3D_1)_2$ pure D wave and shows how little this is suppressed without the tensor force. Since the radial integrands have terms proportional to the second derivatives of radial wave functions they have discontinuities at the edge of the square well. Because of the size of the tensor coupling and the value of the S-wave radial wave function in the coupled differential equations this discontinuity is substantial for $({}^3S_1+{}^3D_1)_2$ at r_w for the standard set (ii). This is perhaps not physical but more sophisticated potentials might be expected to show similar cancellations even without a discontinuity. Apart from suppression of this D wave most of the gross features are due to the hard-core assumption alone. This can be seen in set (xi) where the core is the only potential used for the N-N state. The set of rates (xii) is calculated using for the Λ-N wave function a solution of the Schrödinger equation with the hard-core potential rather than the Bethe-Goldstone equation. This gives a further idea of the insensitivity of the gross form of results to the detailed form of the Λ-N wave function as long as it goes to zero at the core radius (see Fig. 1) so the main features of the results should remain even if the approximation to the solution of the Bethe-Goldstone equation is not a good one. That this boundary condition is important is illustrated by comparing sets (ii) and (xii) to set (xiii) which is calculated with the noncorrelated initial wave function (Λ-N potential equal zero) and the standard final-state wave functions. In set (xiv) we try using the same attractive central potential for the P wave as for the S- and D-wave N-N wave functions. Set (xv) is perhaps the simplest set of correlations with solutions to Schrödinger's equation with a hard core used for both the Λ-N and N-N wave functions. The rates are not very sensitive to the relative momentum of the Λ-N pair. As illustration of this in set (xv) we have used the standard set of potentials but changed q from 0.1 to 125 MeV. Finally for the purposes of discussion below, in sets (xvii) and (xviii) we compare the noncorrelated and correlated rates as in sets (i) and (ii) but with the pion mass replaced by the K-meson mass in both.

The rates are of course all proportional to G_w^2 and G_s^2. The rates of the parity-conserving transitions (S wave to S or D wave) are proportional to λ^2 whereas rates of parity the parity-violating transitions (S wave to P wave) are not functions of λ.

Table III presents the results of integrating the partial rates to get τ^{-1}, the decay rate in nuclear matter. The cases referred to are those defined in Table II except of course that the integration is carried out over all q appropriate to the Fermi sea. The Fermi momentum used is 250 MeV. The lifetimes are then 0.51 τ_Λ^{-1} for noncorrelated wave functions and 0.056 τ_Λ^{-1} for correlated wave functions. It is interesting to note that the combination $R_{p0}+R_{p1}+R_{n1}$ is only a weak function of the relative Λ-N momentum. Over the range of q for the Λ at rest in the Fermi sea ($0\leqslant q\leqslant 114$ MeV) the sum $R_{p0}+R_{p1}+R_{n1}$ changes by less than 20%. This means that the rate τ^{-1} is roughly proportional to the cube of the Fermi momentum. Comparing cases (i) and (xiii) to cases (ii)-(ix) in Table III one sees that τ^{-1} is a measure of the short range Λ-N correlation in nuclear matter. The ratio of $\Lambda+p\rightarrow n+p$ decays to $\Lambda+n\rightarrow n+n$ decays in our model with correlations set (ii) is about 2.6 compared to 7.0 without correlations. The approximation of nuclear matter one expects to be better and better in the limit of large hypernuclei. The large hypernuclei should decay predominantly by the nonmesonic $\Lambda+N\rightarrow N+N$ mode. Any experimental possibility of measuring large hypernuclear lifetimes would be of great interest as providing direct information on the Λ-N nuclear correlations.

In order to check the effect of omission of higher partial waves in the Λ-N state and to check relativistic corrections, the nonmesonic decay rate was computed using plane-wave initial and final states with a full relativistic formalism. The expression for this is given in Appendix B. The result obtained is 0.45 τ_Λ^{-1} which is actually smaller than the value of 0.51 τ_Λ^{-1} we obtained neglecting all higher partial waves. This discrepancy can be traced to the 10% approximation $\omega_t\approx M_N$ made in the computation of rates with partial waves. This indicates that effects of higher partial waves are small.

V. SHORTER-RANGE EFFECTS AND PARITY-VIOLATING NUCLEON-NUCLEON INTERACTIONS

The distorted-wave Born approximation can also be applied to the exchanges of heavier mesons in the reaction $\Lambda+N\rightarrow N+N$. For pseudoscalar mesons the analytic forms for the rates will be like those given above for pion exchange. However the shape of the radial integrand will be substantially different. For heavier mesons the "weak potential" e^{-mr}/r becomes negligible for r less than the first zero of the final-state wave function, so there is essentially no cancellation in the radial integral due to the oscillations of the final-state wave function. The effects of the introduction of interbaryon potentials then are due only to the changes they introduce in the innermost Fermi or two of the radial wave functions. One can expect the hard core to induce a significant reduction in rates.

TABLE III. Total nonmesonic decay rate for a Λ in nuclear matter for various choices of initial and final wave function.

	Case	$\tau^{-1}/\tau_\Lambda^{-1}$
i	Noncorrelated	0.51
ii	Standard	0.056
iii	Core variant	0.047
iv	Healing length variant	0.040
v	Potential strength variant	0.057
vi	r_w variant	0.060
vii	γ variant	0.055
viii	M_π variant	0.046
ix	M_Λ variant	0.060
x	No tensor	0.23
xiii	Noncorrelated initial	0.15
xiv	Attractive ^1P	0.059
xvii	M_K noncorrelated	0.11
xviii	M_K correlated	6.0×10^{-3}

As an example consider K exchange. The strong K-Λ-N interaction predicted by $SU(3)$ with $F/D=\frac{2}{3}$ has a strength $(6/25)^{1/2}$ times the strong π-N-N interaction. Any model of a weak K-N-N interaction is rather speculative, so from among the variety of K-N-N couplings possible in $SU(3)$ let us choose as a model a set of couplings which gives the weak K-N-N just the same form and strength as the weak π-Λ-N, motivated by the anticipation of utilizing the same forms already computed for pion exchange. One can reasonably hope that such a choice will give rough and physically reasonable results. Rates calculated for K exchange are shown as cases (xvii) and (xviii) in Tables I and III. The rates shown in the tables are computed merely by substituting the K mass for the pion mass in the expressions given above for the rates. They should be further reduced by a factor of 6/25 if one wishes to allow for the reduced K-Λ-N strong coupling strength. In set (xvii) noncorrelated wave functions (potentials all equal zero) were used for both initial and final states. Comparing this set to set (i) we see that even without correlations or allowing for different coupling strengths the effect of the π, K mass difference is substantial. This difference is even further increased if we compare the K exchange computed with the standard set of interbaryon potentials, set (xviii), to the corresponding pion set (ii) still not allowing for the reduced coupling strength. These results indicate that the K exchange contribution to nonmesonic Λ decay is <10% of the pion exchange.

One can also attempt to estimate the effect of initial- and final-state interactions on a vector meson exchange in $\Lambda+N\rightarrow N+N$. For a model we suppose that a vector meson with quantum numbers of the ρ is coupled to the nucleon current with an effective Hamiltonian

$$H_s'=G_s'\bar{\psi}_N\gamma_\mu\boldsymbol{\tau}\psi_N\phi^\mu.$$

For G_s^1 we shall use the value 0.95 given by Kantor.[22] A

[22] P. B. Kantor, Phys. Rev. Letters 12, 52 (1964).

model for ρ-Λ-N weak interactions is more speculative even than the K-N-N, for there is no known example of a weak vector-meson–baryon current interaction. The existence of such an interaction is conjectured in analogy to a presumed vector meson-lepton current interaction which can account for the baryon weak form factors. For the sake of estimating an order of magnitude let us suppose then that the weak coupling is

$$H_w' = G_w \bar{\psi}_N \gamma_\mu (1+\lambda\gamma_5) \boldsymbol{\tau} \psi_\Lambda \boldsymbol{\phi}^\mu,$$

where G_w and λ have the same values used for the π-Λ-N vertex. In order to economize in algebra and computer time, rates were calculated only for the parity-violating transitions in the expectation that these should suffice to estimate the magnitude of the rates and to indicate the effects of initial and final state interactions. The partial rates defined as above for $\Lambda+p \to n+p$ for vector-meson exchange are

$$R(\alpha\to\beta) = (2/\pi) G_w^2 G_s'^2 \lambda^2 (t_0/M_N) |I(\alpha,\beta)|^2,$$

$$I(^1S_0, {}^3P_0) = \int_0^\infty dr\, re^{-mr} [(f_i + \tfrac{1}{3}\xi^2 D_2 f_i)(D_4 f_{3P})],$$

$$I(^3S_1, {}^3P_1) = (\tfrac{1}{6})^{1/2} \int_0^\infty dr\, re^{-mr} [(f_i - \tfrac{1}{3}\xi^2 D_2 f_i)(D_4 f_{3P}) - (D_1 f_i)(f_{3P} - \xi^2 D_5 f_{3P})],$$

$$I(^3S_1, {}^1P_1) = \sqrt{3} \int_0^\infty dr\, re^{-mr} [(D_1 f_i)(f_{1P} - \xi^2 D_5 f_{1P})],$$

where all symbols are as defined above except that now $m = [M_v^2 - (M_\Lambda - M_B)^2/4]^{1/2}$ and $\xi = 1/(2M_N)$. In these expressions the difference between $1/2M_\Lambda$ and $1/2M_N$ has been neglected for the coefficients of terms in square brackets. Table IV compares the rates R_v computed with correlated and noncorrelated wave functions for the values of q and M_v indicated. The first six rows are computed with noncorrelated wave functions and the last six rows are computed with the correlated wave functions corresponding to the standard set of potentials (ii) used above.

The large difference between $I(^3S_1, {}^1P_1)$ for the correlated and noncorrelated cases at low q is due to the fact that the integrand is proportional to the first derivative of the initial radial wave function. In the noncorrelated case this derivative is $-q^2 r/3$ which goes to zero for $q \to 0$ whereas for the correlated case the derivative is finite near r_c even for $q=0$ (see Fig. 1). We see that for the coupling constants conjectured in our model that these correlated vector meson-exchange rates are much smaller than the important pion-exchange rates. Independent of the choice of coupling constants one can see that the effect of correlations is to suppress the important vector-meson exchange rates by a factor of 10 or more depending on M_v. Unlike the pion-exchange rates, the R_v do not depend on large cancellations in the radial integral and so the R_v should not be as sensitive to the choice of potentials outside the core as are the pion-exchange rates.

The technique of distorted-wave Born approximation can also be applied to the parity-violating $N+N \to N+N$ weak interaction in order to estimate the importance of initial- and final-state interactions. If one assumes a current-current model of the weak interaction then one-pion exchange will not contribute to parity-violating effects.[23] Again models of the weak vector meson interaction are highly speculative but let us assume the interaction forms H_w' and H_s'. Although one is interested ultimately in parity nonconservation in nuclear states, the one-meson-exchange scattering matrix acts only between two pairs of nucleons. If we imagine expanding the relative wave functions of each of these pairs into partial waves and keeping only the lowest partial waves which contribute to parity violation, one sees that matrix elements between S and P states should measure the effective parity violating

TABLE IV. Partial nonmesonic decay rates computed with vector-meson exchange.

Parameters			Rates in τ_Λ^{-1} F^3		
q (MeV)	M_v (MeV)	r_c (F)	$R(^1S_0 \to {}^3P_0)$	$R(^3S_1 \to {}^1P_1)$	$R(^3S_1 \to {}^3P_1)$
0.1	550	Nonc.[a]	4.7×10^{-2}	3.0×10^{-16}	7.0×10^{-2}
30	550	Nonc.	4.7×10^{-2}	2.4×10^{-6}	7.1×10^{-2}
125	550	Nonc.	4.8×10^{-2}	6.8×10^{-4}	7.6×10^{-2}
0.1	780	Nonc.	1.7×10^{-2}	3.8×10^{-17}	2.5×10^{-2}
30	780	Nonc.	1.7×10^{-2}	3.4×10^{-7}	2.5×10^{-2}
125	780	Nonc.	1.8×10^{-2}	9.3×10^{-5}	2.7×10^{-2}
0.1	550	0.4	4.9×10^{-3}	1.8×10^{-2}	3.0×10^{-3}
30	550	0.4	4.8×10^{-3}	1.8×10^{-2}	3.0×10^{-3}
125	550	0.4	4.4×10^{-3}	1.4×10^{-2}	2.9×10^{-3}
0.1	780	0.4	7.9×10^{-4}	3.4×10^{-3}	4.3×10^{-4}
30	780	0.4	7.8×10^{-4}	3.4×10^{-3}	4.3×10^{-4}
125	780	0.4	7.3×10^{-4}	3.0×10^{-3}	4.3×10^{-4}

[a] Noncorrelated.

TABLE V. Results of radial integration in matrix element for parity-violating $N+N \to N+N$.

Parameters			Integrals in F^2		
q (MeV)	M_v (MeV)	r_c (F)	$I(^1S_0, {}^3P_0)$	$I(^3S_1, {}^1P_1)$	$I(^3S_1, {}^3P_1)$
0.1	550	Nonc.[a]	2.2×10^{-5}	-8.0×10^{-13}	2.7×10^{-5}
30	550	Nonc.	6.5×10^{-3}	-2.1×10^{-5}	7.9×10^{-3}
125	550	Nonc.	2.4×10^{-2}	-1.3×10^{-3}	3.1×10^{-2}
0.1	780	Nonc.	1.1×10^{-5}	-2.0×10^{-13}	1.3×10^{-5}
30	780	Nonc.	3.2×10^{-3}	-5.5×10^{-6}	4.0×10^{-3}
125	780	Nonc.	1.3×10^{-2}	-3.6×10^{-4}	1.6×10^{-2}
0.1	550	0.4	1.5×10^{-5}	6.2×10^{-6}	1.4×10^{-5}
30	550	0.4	4.4×10^{-3}	1.8×10^{-3}	4.2×10^{-3}
125	550	0.4	8.6×10^{-3}	2.7×10^{-3}	8.8×10^{-3}
0.1	780	0.4	5.1×10^{-6}	2.7×10^{-6}	4.3×10^{-6}
30	780	0.4	1.5×10^{-3}	7.7×10^{-4}	1.3×10^{-3}
125	780	0.4	2.9×10^{-3}	1.3×10^{-3}	2.6×10^{-3}
30	780	0.5	1.1×10^{-3}	6.5×10^{-4}	9.0×10^{-4}

[a] Noncorrelated.

[23] F. C. Michel, Phys. Rev. 133, B329 (1964).

coupling strength. These matrix elements will be proportional to the three integrals I given above with M_Λ replaced by M_N and the final relative momentum set equal to the initial. In Table V the first four rows show the integrals for no final or initial state interaction. The last six rows show the integrals with the correlations induced by the standard set of potentials used above (ii) except for the parameters q, M_v, r_c indicated.

One can see that this distorted-wave Born calculation indicates that the effective weak parity-violating nucleon-nucleon interaction is considerably reduced by initial- and final-state interactions. For much of the range of q the reduction is by more than a factor of 2. (The reduction due to correlations is smaller in parity violating $N+N\rightarrow N+N$ weak effects which are proportional to the weak transition matrix element than in $\Lambda+N\rightarrow N+N$ rates which are proportional to the square of the matrix element. The energy of the final nucleon pair is also different in the two cases.) This is somewhat larger than the reduction factor 1.25 estimated by Michel.[23] One might note in passing that our calculation suggests that it may not be a good approximation to neglect the finite range of the weak interaction even for the exchange of a meson with $M_v=780$ MeV in computing parity violation in nuclei.

These calculations applying the distorted-wave Born approximation to the short-range weak interactions are perhaps of only dubious value. While they may in principle correctly take into account strong interactions in the initial and final states, they do not include the sort of process indicated by the Feynman graph of Fig. 4. Such processes may be significant at the short distances ($\leqslant 0.5$ F) which are important in the weak exchange of heavy particles. Nevertheless one can take the calculations of short-range effects in this section as an indication that correlations will substantially reduce the effective weak interbaryon interactions due to exchange of heavy mesons.

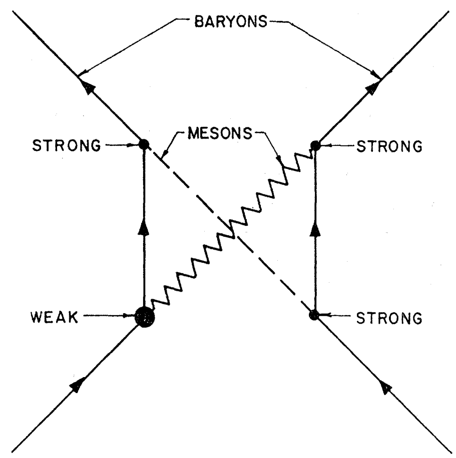

FIG. 4. Feynman graph of one type of process which is not accounted for in a distorted-wave Born approximation.

VI. COMPARISON WITH EXPERIMENT

The little available knowledge of the structure of the reaction $\Lambda+N\rightarrow N+N$ rests upon the analysis of Block and Dalitz of the light hypernuclear decays.[3] Assuming theoretical values for mesonic decay rates they deduce

$$\tau^{-1}{}_{\text{nonmesonic}}({}_\Lambda\text{He}^4)=(0.14\pm0.03)\tau_\Lambda^{-1},$$
$$\tau^{-1}{}_{\text{nonmesonic}}({}_\Lambda\text{H}^4)=(0.29\pm0.14)\tau_\Lambda^{-1}.$$

It is stretching a point to compare these rates to our nuclear matter calculation; however, it should be noted that even though the Λ is only loosely bound to the core nucleus it is nonetheless usually within the range of the pion-exchange weak "potential," so it is encouraging that our nuclear matter decay rate ($0.056\ \tau_\Lambda^{-1}$) is of the same order of magnitude as the light hypernuclear decay rates. If one assumes that in the hypernuclei ${}_\Lambda\text{H}^4$ and ${}_\Lambda\text{He}^4$ the relative wave functions of the Λ and the protons are the same as the relative wave functions of the Λ and the neutrons, then one can conclude with Block and Dalitz that the transition rates from the spin-0 Λ-p states are small compared to transition rates from the spin-1 Λ-p states and that transition rates from the spin-1 Λ-n states are about twice as large as for spin-1 Λ-p states. Because the shape of our radial wave function $f_i(q,r)$ is different from the shape of the relative Λ-N wave function in a light hypernucleus, comparison of these empirical ratios with the ratios of the R_{NJ} we have computed for nuclear matter is a poor procedure; however, one notes that the effect of correlations is greatly to suppress R_{p1} and R_{p0} compared to R_{n1} bringing their ratios closer to those of Block and Dalitz. These tests of the theory are unfortunately qualitative at best. To put it in a somewhat more quantitative way, Block and Dalitz find $R_{p1}:R_{n1}:R_{p0}=1:(2\pm1):(0.4\pm0.2)$ whereas the noncorrelated one-pion exchanges gives $R_{p1}:R_{n1}:R_{p0}=1:0.06:0.14$ whereas the correlated one-pion exchange gives $R_{p1}:R_{n1}:R_{p0}=1:0.4:0.03$. Block and Dalitz's derivation of absolute rather than relative "empirical" values for the R_{NJ} hinges on the neglect of the range of the weak interaction compared to the range in which the Λ-N wave function changes significantly as explained in Appendix A. Because this is not a reasonable assumption for one-pion exchange with hard-core baryon-baryon potentials this step in the analysis of Block and Dalitz would seem to be invalid. At present it does not seem possible to exclude the one-pion-exchange mechanism as the cause of the nonmesonic light hypernuclear decays as did Block and Dalitz, and indeed one can hope that it will provide a complete explanation. A decisive confrontation of theory with experiment must await a detailed calculation using good correlated hypernuclear wave functions and realistic nucleon-nucleon potentials.

VII. CRITIQUE AND CONCLUSIONS

The calculations in this paper are subject to a good many legitimate criticisms. Most obvious improvement would come with the use of more realistic interbaryon potentials. In addition the work involves a good many implicit assumptions which may be subject to criticism. Among them are the following: Schrödinger's equation and nonrelativistic quantization of the spin has been applied to both the initial and final states; but although kinetic energy of the two outgoing nucleons is only 10% of their rest mass, they are moving with half the velocity of light. The notion of a potential may not be applicable down to the half a Fermi distance to which we use it. No allowance has been made for possible form factors at the π-Λ-N vertex which might change the effective coupling as the pion goes off its mass shell. Effects of a Σ component in the initial wave function are ignored. Possible modifications of the pion propagator due to presence of other nucleons were not considered. The independent pair approximation is certainly not good out to the distances of $\sim$6 F to which we use it; however, most important cancellation effects come from the first few Fermis of the radial wave function. The relative importance of higher partial waves should be more carefully studied. In the spirit of this as a crude first calculation all these approximations are taken to be fair.

One can certainly make a more sophisticated distorted-wave Born calculation using better initial- and final-state wave functions and a more complicated form for the weak interaction. However, the technique seems to face certain limitations. Because details of the results are sensitive to the potentials used and since experiment leaves some uncertainty about the potentials particularly at short distances, it would seem difficult to make convincing predictions with an accuracy better than 10–20%, and the Born approximation itself may break down at short distances as indicated at the end of Sec. VI.

The calculations of this paper can be used to draw the following conclusions. The strong final- and initial-state interactions are of great importance in the reaction $\Lambda+N\rightarrow N+N$. Certainly no calculation of hypernuclear decays can reasonably neglect them. They can enormously suppress transitions between certain states and should make it possible to explain the nonmesonic decays of light hypernuclei in terms of one-pion exchange. The effective weak interaction due to exchange of K and other heavy mesons is suppressed, and any underlying $SU(3)$ symmetry of the weak interaction should be badly obscured by the baryon correlations. Parity-violating $N+N\rightarrow N+N$ interactions due to exchange of heavy mesons are probably substantially reduced by the effects of strong initial- and final-state interactions. Experimental knowledge of heavy hypernuclear lifetimes would provide in principle a measure of the Λ-N correlation function at short distances in nuclear matter.

ACKNOWLEDGMENT

It is a pleasure to thank J. Dirk Walecka for the motivating suggestion that hypernuclear decay rates should depend on the Λ-N correlation and for valuable criticism.

APPENDIX A

In the rest frame of the Λ particle in nuclear matter the rate for the reaction $\Lambda+p\rightarrow n+p$ is given by

$$\tau_p^{-1}=(2\pi)^{-1}\int_0^{k_F}d^3\mathrm{k}\int_{(\infty)}d^3\mathrm{u}'\times\int_{(\infty)}d^3\mathrm{t}'\delta^4(P_f-P_i)\tfrac{1}{2}\sum_{\sigma_i,\sigma_f}|\langle f|M|i\rangle|^2,$$

where P_f, P_i are the final and initial total four-momenta of the systems; σ_i, σ_f are initial and final spins,

$$\delta^4(P_f-P_i)\langle f|M|i\rangle=\langle f|S|i\rangle.$$

$\mathbf{k}$, $\mathbf{u}'$, $\mathbf{t}'$ are the three-momenta of the initial and final protons and the neutron, respectively; and k_F is the Fermi momentum of the nuclear matter which we assume to be the same for protons and neutrons. The wave functions of the states $|i\rangle$ and $|f\rangle$ are asymptotically plane waves, but at short distances may exhibit the effects of correlations. A transformation to relative and total three momenta for both the initial and final pairs of particles and integrations of the final total momentum casts the expression for the rate into the form

$$\tau_p^{-1}=(4\pi)^{-1}(M_+/M_\Lambda)^3\int_0^{q_M}d^3q\times\int_{(\infty)}d^3t\delta(E_f-E_i)\sum_{\sigma_i,\sigma_f}|\langle f|M|i\rangle|^2,$$

where $M_+\equiv M_\Lambda+M_N$, $q_M\equiv k_FM_\Lambda/M_+$, $\mathbf{q}$ is the relative Λ-p momentum, and $\mathbf{t}$ is the relative n-p momentum. For the purposes of considering the effects of correlations it is useful to analyze the initial and final relative wave functions in states which are asymptotically spherical rather than plane waves. The discrete quantum numbers for the total spin, orbital angular momentum, total angular momentum, and z component of total angular momentum can be denoted collectively by α for the initial state and by β for the final state (so in spectroscopic notation α might stand for ${}^3P_0$, for example). The rate can then be written

$$\tau_p^{-1}=(4\pi)^{-1}(M_+/M_\Lambda)^3\int_0^{q_M}dq\,q^2\times\int_0^\infty dt\,t^2\delta(E_f-E_i)\sum_\alpha\sum_\beta|\langle\beta,t|M|\alpha,q\rangle|^2.$$

One can now define partial rates

$$R(\alpha\to\beta)\equiv\pi\int_0^\infty dt\, t^2\delta(E_f-E_i)|\langle\beta,t|M|\alpha,q\rangle|^2$$
$$=\tfrac{1}{2}\pi t_0 M_N|\langle\beta,t_0|M|\alpha,q\rangle|^2,$$

which are functions of q. In the expression for τ_p^{-1} we make the approximation of retaining in the sum on α only those sets α' in which the relative orbital angular momentum is zero, neglecting all higher partial waves for the initial state. One can then write

$$\tau_p^{-1}=(2\pi)^{-2}(M_+/M_\Lambda)^3\int_0^{q_M} dq\, q^2(R_{p0}+3R_{p1}),$$

where

$$R_{pJ}\equiv\sum_\beta R(\alpha'\to\beta)$$

and J is the total angular momentum of the set α'. Similar expressions give the rate $\Lambda+n\to n+n$:

$$\tau_n^{-1}=(2\pi)^{-2}(M_+/M_\Lambda)^3\int_0^{q_M} dq\, q^2(R_{n0}+3R_{n1}).$$

The total rate is simply the sum of the neutron and proton rates. The $\Delta I=\frac{1}{2}$ rule, which we assume, implies that $R_{n0}=2R_{p0}$, so

$$\tau^{-1}=\tau_p^{-1}+\tau_n^{-1}=3(2\pi)^{-2}(M_+/M_\Lambda)^3 \times\int_0^{q_M} dq\, q^2(R_{p0}+R_{p1}+R_{n1}).$$

In order to give an interpretation to the partial rates defined above we consider the nonmesonic decay of a light hypernucleus in the impulse approximation. The decay rate by the emission of a neutron and one of the nucleons N is

$$\tau_N^{-1}=(2\pi)^{-1}\int_0^\infty dt\, t^2\delta(E_f-E_i)\sum_\beta|\langle\beta,t|M|i\rangle|^2.$$

A complete set of noncorrelated spherical states can be inserted in the matrix element

$$\langle\beta,t|M|i\rangle=\sum_\alpha\int_0^\infty dq\, q^2\langle\beta,t|M|q,\alpha\rangle\langle q,\alpha|i\rangle.$$

If we now assume that $\langle\beta,t|M|q,\alpha\rangle$ is only a weak function of q for q in the range where $\langle q,\alpha|i\rangle$ is significantly different from zero then

$$\langle\beta,t|M|i\rangle=\sum_\alpha\langle\beta,t|M|q=0,\alpha\rangle\int_0^\infty dq\, q^2\langle q,\alpha|i\rangle$$
$$=\sum_\alpha\langle\beta,t|M|q=0,\alpha\rangle\langle\alpha|\sigma_i\rangle\psi(0)\pi\sqrt{2},$$

where $\psi(\mathbf{x})$ is the relative Λ-N wave function and $\langle\alpha|\sigma_i\rangle$ denotes the inner product between the Λ-N spin configuration in the hypernucleus and the configuration in α. The assumption made in this step amounts to neglecting the range of the weak interaction compared to distances in which the Λ-N relative wave function changes appreciably. This is probably not a good approximation for light hypernuclei. However, if one proceeds and assumes further that all the particles in the ground state of a light hypernucleus are in relative S states (which is probably true), then one gets

$$\tau_N^{-1}=(2\pi)^{-1}\int_0^\infty dt\, t^2\delta(E_f-E_i)\sum_{\alpha'}\sum_\beta|\langle\beta,t|M|q=0,\alpha'\rangle|^2 \times|\langle\alpha'|\sigma_i\rangle|^2 2\pi^2|\psi(0)|^2$$
$$=\sum_{\alpha'} R_{NJ}|\psi(0)|^2|\langle\alpha'|\sigma_i\rangle|^2,$$

where R_{NJ} are just the partial rates defined above with $q=0$ and noncorrelated wave functions. From this last expression we see that, under the assumptions made, the R_{NJ} can be interpreted as the transition probability for an S-wave Λ-N system with total spin J per unit nucleon density at the position of the Λ particle. We repeat, however, that the approximations involved in arriving at this sort of interpretation of the light hypernuclear decay rates are probably not good because the one-pion exchange has a long range and the Λ-N potential probably has a strong repulsive core making $\psi(0)=0$.

Finally, in this Appendix we write down the final proton-neutron states needed in the computation. In second quantization these are

$$|{}^1S_0,t_0\rangle=\int_{(\infty)} d^3t\, g_{1S}(t,t_0)(\tfrac{1}{2})^{1/2} \times Y_{00}(a_+{}^pa_-{}^n-a_-{}^pa_+{}^n)|\text{vac}\rangle,$$

$$|{}^3S_1,t_0\rangle=\int_{(\infty)} d^3t\, g_{3S}(t,t_0)Y_{00}a_+{}^pa_+{}^n|\text{vac}\rangle,$$

$$|{}^1P_1,t_0\rangle=\int_{(\infty)} d^3t\, g_{1P}(t,t_0)(\tfrac{1}{2})^{1/2} \times Y_{11}(a_+{}^pa_-{}^n-a_-{}^pa_+{}^n)|\text{vac}\rangle,$$

$$|{}^3P_0,t_0\rangle=\int_{(\infty)} d^3t\, g_{3P}(t,t_0)(\tfrac{1}{3})^{1/2} \times[Y_{11}a_-{}^pa_-{}^n-(\tfrac{1}{2})^{1/2}Y_{10}(a_+{}^pa_-{}^n+a_-{}^pa_+{}^n) +Y_{1-1}a_+{}^pa_+{}^n]|\text{vac}\rangle,$$

$$|{}^3P_1,t_0\rangle=\int_{(\infty)} d^3t\, g_{3P}(t,t_0)(\tfrac{1}{2})^{1/2} \times[(\tfrac{1}{2})^{1/2}Y_{11}(a_+{}^pa_-{}^n+a_-{}^pa_+{}^n) -Y_{10}a_+{}^pa_+{}^n]|\text{vac}\rangle,$$

$$|{}^3D_1,t_0\rangle=\int_{(\infty)} d^3t\, g_{3D}(t,t_0)[(\tfrac{3}{5})^{1/2}Y_{22}a_-{}^pa_-{}^n -(3/20)^{1/2}Y_{21}(a_+{}^pa_-{}^n+a_-{}^pa_+{}^n) +(\tfrac{1}{10})^{1/2}Y_{20}a_+{}^pa_-{}^n]|\text{vac}\rangle,$$

where the spherical harmonics Y_{lm} are functions of the direction of $\mathbf{t}$ and $a_{\pm}{}^{p}$ is the creation operator for a proton with z component of spin $=\pm\frac{1}{2}$ and momentum $\frac{1}{2}\mathbf{P}_f+\mathbf{t}$ and $a_{\pm}{}^{n}$ is the creation operator for a neutron with z component of spin $=\pm\frac{1}{2}$ and momentum $=\frac{1}{2}\mathbf{P}_f-\mathbf{t}$. Here $\mathbf{P}_f$ is the total momentum of the n-p pair in the final state. The g's are defined as the Fourier-Bessel transforms of the radial wave functions

$$g_{1S}(t,t_0)=(2/\pi)\int_0^\infty dr\, r^2 j_0(r,t) f_{1S}(r,t_0)\,,$$

$$g_{3S}(t,t_0)=(2/\pi)\int_0^\infty dr\, r^2 j_0(rt) f_{3S}(r,t_0)\,,$$

$$g_{1P}(t,t_0)=(2/\pi)\int_0^\infty dr\, r^2 j_1(rt) f_{1P}(r,t_0)\,,$$

$$g_{3P}(t,t_0)=(2/\pi)\int_0^\infty dr\, r^2 j_1(rt) f_{3P}(r,t_0)\,,$$

$$g_{3D}(t,t_0)=(2/\pi)\int_0^\infty dr\, r^2 j_2(rt) f_{3D}(r,t_0)\,.$$

APPENDIX B

We record here the expression for the decay rate of a Λ particle at rest in nuclear matter via the one-pion-exchange mechanism using noncorrelated plane waves for the initial and final states and a relativistic formalism. Here we call $\mathbf{r}$, $\mathbf{t}$, $\mathbf{u}$ the momenta of the incident nucleon and the two final nucleons respectively and ω_r, ω_t, ω_u the corresponding energies. We define

$$\begin{aligned}
B_1=&-(P_u^2-M_\pi^2)^{-2}[(1+\lambda^2)+(1-\lambda^2)M_N/\omega_u]\\
&\times[-1+(\mathbf{t}\cdot\mathbf{r}+M_N^2)/(\omega_t\omega_r)]\,,\\
B_2=&-(P_t^2-M_\pi^2)^{-2}[(1+\lambda^2)+(1-\lambda^2)M_N/\omega_t]\\
&\times[-1+(\mathbf{u}\cdot\mathbf{r}+M_N^2)/(\omega_t\omega_r)]\,,\\
B_3=&(P_u^2-M_\pi^2)^{-1}(P_t^2-M_\pi^2)^{-1}\\
&\times\{(1+\lambda^2)[-1+(\mathbf{r}\cdot\mathbf{u}+M_N^2)/(\omega_r\omega_s)\\
&+(\mathbf{r}\cdot\mathbf{t}+M_N^2)/(\omega_r\omega_t)-(\mathbf{t}\cdot\mathbf{u}+M_N^2)/(\omega_t\omega_u)]\\
&+(1-\lambda^2)[(1-\mathbf{t}\cdot\mathbf{u}/(\omega_u\omega_t))M_N/\omega_r\\
&-(1-\mathbf{r}\cdot\mathbf{u}/(\omega_r\omega_u))M_N/\omega_t-(1-\mathbf{r}\cdot\mathbf{t}/(\omega_r\omega_t))M_N/\omega_u\\
&+M_N^3/(\omega_r\omega_s\omega_t)]\}\,,
\end{aligned}$$

where P_u, P_t are the four-vectors

$$P_u\equiv(\omega_u-M_\Lambda,\mathbf{u})\,,\quad P_t\equiv(\omega_t-M_\Lambda,\mathbf{t})\,.$$

The decay rates in the Λ rest frame are then

$$\begin{aligned}
\tau_n^{-1}=&\tfrac{1}{4}(2\pi)^{-5}G_w^2G_s^2\int_0^{k_F}d^3r\int_{(\infty)}d^3t\int_{(\infty)}d^3u\,\delta^3(\mathbf{r}-\mathbf{t}-\mathbf{u})\\
&\times\delta(M_\Lambda+\omega_r-\omega_u-\omega_t)[B_1+B_2+\tfrac{1}{2}B_3]\,,
\end{aligned}$$

$$\begin{aligned}
\tau_p^{-1}=&\tfrac{1}{2}(2\pi)^{-5}G_w^2G_s^2\int_0^{k_F}d^3r\int_{(\infty)}d^3t\int_{(\infty)}d^3u\,\delta^3(\mathbf{r}-\mathbf{t}-\mathbf{u})\\
&\times\delta(M_\Lambda+\omega_r-\omega_u-\omega_t)[B_1+B_2-4B_3]\,.
\end{aligned}$$