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Quark Model for Photoproduction of Baryons

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A quark (Q) model of photoproduction of positive- and negative-parity baryons in association with a meson octet is proposed. According to this model, the basic mechanism for photoproduction is represented by a U-spin-invariant process $\gamma+Q\rightarrow Q+\Pi$ at each quark, Π being a meson octet. The matrix elements of this basic amplitude are taken between appropraite $3Q$ states representing the initial proton and a final baryon, according to the multiplet structure $\mathbf{56}^+$ of $SU(6)$ and $(\mathbf{70},3)^-$ of $SU(6)\otimes O(3)$ for the positive-parity and negative-parity baryonic states, respectively. The $SU(3)$ sum rules based on U-spin invariance of the amplitudes are verified. Several additional sum rules, valid separately for spin-flip and non-spin-flip amplitudes, are derived for the $\mathbf{56}^+$ states. For the negative-parity states, a number of nontrivial sum rules connecting different $SU(3)$ multiplets are obtained.

PHOTOPRODUCTION is one of the most important processes that can be studied in the quark model with a suitable assumption on the mode of introduction of the electromagnetic interaction. Becchi and Morpurgo[1,2] calculated the M_1 transition matrix element between a proton and the N_{33}^* resonance, each looked upon as a $3Q$ system belonging to the **56** representation of $SU(6)$. The electromagnetic operator in this modle is a sum of the quark magnetic moment operators, so that the calculation is essentially in the spirit of evaluation of a transition magnetic moment between certain states (N^{*+} and p) of a three-particle system, in analogy with an M_1 transition between two nuclear states. The basic amplitude in such a model is, therefore, something like a "form factor" rather than a conventional photoproduction amplitude, which is eventually obtained through the decay $N_{33}^*\rightarrow N+\pi$. The photoproduction process is thus assumed to proceed through an $N\pi$ resonant state, without, of course, suggesting any mechanism within the quark model for the pionic decay of N_{33}^*. We wish to propose here an alternative mechanism, which is more in the spirit of meson photoproduction without the latter proceeding through a resonant state. More specifically, we assume the photoproduction of the meson to occur at each individual quark, (through a suitable impulse-approximation scheme to be explained below), and add up to different amplitudes, taking account of the proper symmetries of the $3Q$ wave functions. In this model, the meson is essentially produced through the individual excitations of quarks, rather than through the decay of an excited $3Q$ system as a whole (like N^{*++}). This model, being a nonresonant mechanism for photoproduction, should in a sense, be regarded as complementary to the Morpurgo model[11,12] and should be more applicable to situations where the mass distribution of a final (two-particle) baryon-meson system does not show any significant peak. The model will also be shown to predict amplitudes for states like ($B_{10}^*M_8$) which subsequently decay to B_8+2M_8 systems. Thus it should be possible to apply the model to physical situations where the final three-particle B_8+2M_8 systems had originated through $B_{10}^*+M_8$ states, as judged by their mass-distribution plots.

A similar model for meson-baryon scattering and production (of baryon resonances plus mesons), through a sum over amplitudes of individual quark-meson scattering has been suggested recently[3] on the basis of a quark-model of baryons and their resonances, proposed by one of us.[4] Its predictions for elastic meson-baryon scattering agree with those of Lipkin and others,[5] while those for production processes are somewhat different, (though not comparatively disfavored by experiment). Since a short description of the $3Q$ wave functions in various states of interest (for both positive-parity baryons and negative-parity resonances) have been already given in JBM, the same will be used (without explanation of notation) in this work as well.

The most important property of the electromagnetic interaction is its U-spin invariance, which can be incorporated in the λ-matrix formalism[6] through the special combination

$$\lambda_3+\tfrac{1}{3}\sqrt{3}\lambda_8\equiv Z\,, \tag{1}$$

which behaves like a scalar under U-spin transformations. The charge-hypercharge structure of a photoproduction matrix element for the process

$$\gamma+Q\rightarrow\Pi+Q\,, \tag{2}$$

where Π is a member of the pseudoscalar octet and Q is a quark-triplet, is expressible in terms of two basic combinations

$$[Z,\lambda_\alpha]_\pm \tag{3}$$

where the respective operators Z and λ_α arise out of the vertices for the electromagnetic (U-spin-invariant)

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[1] C. Becchi and G. Morpurbo, Phys. Letters **17**, 3 (1965); **17**, 352 (1965).
[2] C. Becchi and G. Morpurgo, Phys. Rev. **140**, B687 (1965); W. Thirring, Phys. Letters **16**, 335 (1965).
[3] G. Joshi, V. S. Bhasin, and A. N. Mitra, preceding paper, hereafter referred to as JBM.
[4] A. N. Mitra, Phys. Rev. **151**, 1168 (1966).
[5] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).
[6] M. Gell-Mann, in *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964).

and mesonic [$SU(3)$-spin invariant] interactions with quarks. The general structure for the amplitude (2) is then of the form

$$A^{(+)}[Z,\lambda_\alpha]_+ + A^{(-)}[Z,\lambda_\alpha]_-, \tag{4}$$

where $A^{(\pm)}$ are the operators independent of the λ matrices, but dependent on the spin, polarization, momentum, etc., of the particles involved. In terms of the f and d matrices of Ref. 6, we have

$$[Z,\lambda_\alpha]_+ = \tfrac{4}{3}[\delta_{3\alpha}+\tfrac{1}{3}\sqrt{3}\delta_{8\alpha}]+2[d_{3\alpha\beta}\lambda_\beta+\tfrac{1}{3}\sqrt{3}d_{8\alpha\beta}\lambda_\beta], \tag{5}$$

$$[Z,\lambda_\alpha]_- = 2i[f_{3\alpha\beta}\lambda_\beta+\tfrac{1}{3}\sqrt{3}f_{8\alpha\beta}\lambda_\beta]. \tag{6}$$

For the spin and spatial structure of the amplitudes $A^{(+)}$ and $A^{(-)}$, one may proceed as in the case of ordinary photoproduction[7,8] processes for pseudoscalar meson production, and express them in terms of the following independent combinations (in the nonrelativistic limit for quarks):

$$(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}),\ \boldsymbol{\sigma}\cdot[\mathbf{q}\times(\mathbf{k}\times\boldsymbol{\varepsilon})],\ (\mathbf{q}\cdot\boldsymbol{\varepsilon})(\boldsymbol{\sigma}\cdot\mathbf{k}),\ (\mathbf{q}\cdot\boldsymbol{\varepsilon})(\boldsymbol{\sigma}\cdot\mathbf{q}), \tag{7}$$

where $\boldsymbol{\sigma}$ is the quark-spin operator, $\mathbf{q}$ and $\mathbf{k}$ are the momentum of the meson and photon, respectively, and $\boldsymbol{\varepsilon}$ is the polarization vector for the photon, so that $\mathbf{k}\cdot\boldsymbol{\varepsilon}=0$.

The same formalism can be adapted in principle for vector meson production as well, except for a longer list of invariants than (7), because of the appearance of an extra vector representing the state of the meson polarization. It is more convenient, however, to represent the amplitudes $A^{(+)}$ and $A^{(-)}$ in (4), each in terms of the 2×2 spin matrices $\boldsymbol{\sigma}$. Thus we write

$$A^{(\pm)} = If^{(\pm)}+\boldsymbol{\sigma}\cdot\mathbf{A}^{(\pm)}, \tag{8}$$

defining two scalars $f^{(\pm)}$ and two vectors $\mathbf{A}^{(\pm)}$ in terms of the set (7). This does not involve sacrifice of any information, since both the representations (7) and (8) involve four parameters each. However, (8) has the advantage that an identical formalism would suffice for the treatment of the photoproduction of vector mesons as well, though the latter involves more degrees of freedom and hence a larger number of scalar functions of the available vectors. Moreover, the form (8) does not require any specific commitment on the relativistic structure of the mesons, but merely one on the nonrelativistic structure of the quarks.

One can now calculate the photoproduction matrix elements for various meson-baryon combinations, using a proton target. The calculation is on identical lines to JBM, where the amplitude (4)–(8) is regarded as an operator whose matrix elements must be evaluated in appropriate $3Q$ states representing the cases of interest. In all cases, there will be an accompanying meson, arising out of the basic process (2). Thus we shall have final states like $B_8+\Pi$ and $B_{10}^*+\Pi$, where the former does not arise as a result of $B_{10}^*\to B_8+\Pi$. Similar amplitudes can be written down for negative-parity baryonic states as well.

With the help of the wave functions for the $\mathbf{56}^+$ and $(\mathbf{70},3)^-$ baryons which are given in JBM, one obtains the following matrix elements after contraction of the $SU(3)$ variables alone:

Photoproduction of the 8^+ States

$$A(\pi^+n)=\sqrt{2}\langle\chi'|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi'\rangle - \tfrac{1}{3}\sqrt{2}\langle\chi''|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi''\rangle, \tag{9}$$

$$A(\Sigma^0K^+)=\tfrac{2}{3}\langle\chi''|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi''\rangle, \tag{10}$$

$$A(\Lambda^0K^+)=\tfrac{2}{3}\sqrt{3}\langle\chi'|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi'\rangle, \tag{11}$$

$$A(\pi^0p)=6\langle\chi'|A^{(+)}|\chi'\rangle+(50/9)\langle\chi''|A^{(+)}|\chi''\rangle, \tag{12}$$

$$A(\eta p)=2\sqrt{3}\langle\chi'|A^{(+)}|\chi'\rangle + (14/3)2\sqrt{3}\langle\chi''|A^{(+)}|\chi''\rangle, \tag{13}$$

$$A(\Sigma^+K^0)=-(4/9)\sqrt{2}\langle\chi''|A^{(+)}|\chi''\rangle. \tag{14}$$

As long as we do not evaluate the spin matrix elements explicitly, these are essentially $SU(3)$ results where the above amplitudes are given in terms of the four-parameters

$$\langle\chi'|A^{(\pm)}|\chi'\rangle,\quad \langle\chi''|A^{(\pm)}|\chi''\rangle, \tag{15}$$

leading to the connections

$$\sqrt{2}A(\pi^+n)+A(\Sigma^0K^+)=\sqrt{3}A(\Lambda^0K^+) \tag{16}$$

$$\sqrt{2}A(\Sigma^+K^0)+A(\pi^0p)=\sqrt{3}A(\eta p) \tag{17}$$

Equations (16) and (17) are just the $SU(3)$ results of Likpin *et al.*,[9] and of Fujii,[10] obtained on the basis of U-spin invariance. Similar results are also obtained within an $SU(6)$ formalism.[11]

Photoproduction of 10^+ States

$$A(\Delta^0\pi^+)=\tfrac{2}{3}\langle\chi^S|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi''\rangle, \tag{18}$$

$$A(\Delta^+\pi^0)=(2/9)\sqrt{2}\langle\chi^S|A^{(+)}|\chi''\rangle, \tag{19}$$

$$A(\eta\Delta^+)=(\tfrac{2}{3})^{3/2}\langle\chi^S|A^{(+)}|\chi''\rangle, \tag{20}$$

$$A(Y_1^{*0}K^+)=\tfrac{1}{3}\sqrt{2}\langle\chi^S|\tfrac{1}{3}A^{(+)}-A^{(-)}|\chi''\rangle, \tag{21}$$

$$A(Y_1^{*+}K^0)=(4/9)\langle\chi^S|A^{(+)}|\chi''\rangle. \tag{22}$$

This lead to the relations

$$A(\Delta^0\pi^+)=\sqrt{2}A(Y_1^{*0}K^+) \tag{23}$$

$$\sqrt{3}A(\Delta^+\pi^0)=A(\Delta^+\eta)=\sqrt{\tfrac{3}{2}}A(Y_1^{*+}K^0). \tag{24}$$

Equation (23), but not Eq. (24), was given in Ref. 9. This may not be surprising, since our (quark-model) assumption for the baryon states is somewhat stronger than the mere U-spin-invariance assumption of Ref.

[7] G. Chew, M. Goldberger, F. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).
[8] K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1963).
[9] C. A. Levinson, H. J. Lipkin, and S. Meshkov, Phys. Letters **7**, 81, (1963).
[10] L. Holloway and A. Fujii, Nuovo Cimento, **28**, 1514 (1963).
[11] K. C. Tripathy, Phys. Rev. **141**, 1350 (1966).

9. Equations (16) and (23) have been verified experimentally.[12]

The more specific $SU(6)$-type results connecting the amplitudes for the octet and decuplet production can be obtained only after taking explicit account of the spin matrix elements of the operators (8), in the initial and final wave functions. It is convenient to analyze the elements for octet production in terms of spin flip and non-spin-flip amplitudes, which are respectively calculated by taking $J_z=\pm\frac{1}{2}$ for the final baryon states and $J_z=+\frac{1}{2}$ for the initial state. For decuplet production, we shall consider merely the production amplitudes for the $J_z=\frac{1}{2}$ state, as for the initial proton.

The non-spin-flip amplitudes for octet production give merely the $SU(3)$ results (16) and (17). The following sum rules are also obtained for the connection between the non-spin-flip amplitudes for octet production and those for decuplet production:

$$\sqrt{3}[A(\pi^+n)-A(\pi^+\Delta^0)]=\sqrt{2}A(\Lambda^0K^+), \tag{25}$$

$$A(\pi^+n)-\sqrt{2}A(\Sigma^0K^+)=3\sqrt{2}A(Y_1{}^{*0}K^+)=3A(\Delta^0\pi^+), \tag{26}$$

$$A(\eta_1p)+\sqrt{3}A(\Sigma^+K^0)=2\sqrt{3}A(Y_1{}^{*+}K^0)=6\sqrt{\tfrac{2}{3}}A(\Delta^+\pi^0). \tag{27}$$

Further, the spin-flip amplitudes for octet production give the additional relations

$$A(\pi^+\eta)=(5/3)\sqrt{\tfrac{2}{3}}A(\Lambda^0K^+)=-\tfrac{5}{2}\sqrt{2}A(\Sigma^0K^+) \tag{28}$$

With spin-flip amplitudes, we also obtain fairly simple results for the mixing effects of the η_8, as a member of the $SU(3)$ octet, with the "heavy eta" (η_1) as an $SU(3)$ singlet. The production of this extra particle can be incorporated in our general formalism simply by extending the index α in (4) to run over the values 0, 1, 2, $\cdots$, 8 instead of merely $(1,2,\cdots,8)$. The inclusion of this effect gives for the spin-flip nonet amplitudes the relation

$$A(\pi^0p)=(14/5\sqrt{3})A(\eta_8p)=(14/3)\sqrt{\tfrac{2}{3}}A(\eta_1p). \tag{29}$$

The $SU(3)$-breaking mixing effect between η_1 and η_8 can be taken into account through the combinations

$$\eta_8\to\eta=\eta_8\cos O_p-\eta_1\sin O_p;$$
$$X_0=\eta_8\sin O_p+\eta_1\cos O_p, \tag{30}$$

where O_p has been estimated to be 10° on the linear mass formula and 24° on the quadratic mass formula.[13] From (29) and (30), we obtain a sum rule for the spin-flip amplitude for physical η production in the form

$$\sqrt{3}[5\cos O_p-\tfrac{3}{2}\sqrt{2}\sin O_p]A(\pi^0p)=14A(\eta p). \tag{31}$$

The experimental ratio[14] of the cross sections for π^0 to η production at $O_{\text{c.m.}}=105°$ and at lab energy of 978 ± 22 MeV for the photon, is 8.0, to be compared with a value of 4.8 obtained with $O_p=+24°$ and 3.2 with $O_p=10°$, thus indicating a preference for the estimate from the quadratic mass formula.

The same formula (31) could in principle be used to calculate the ratio of ρ^0 and ω photoproduction, for which the corresponding mixing angle is $O_V\approx39°$.[13] Thus from the known data on ρ^0 photoproduction,[15] one could estimate a cross section for the photoproduction of ω which from (31) works out as 10% of ρ^0 production. It would be interesting to check this estimate against experimental data for ω production when available.

The model also enables us to calculate the amplitudes for the photoproduction of negative-parity baryons in association with mesons. For this purpose, we can use the wave functions for negative-parity states, as listed in Table I of JBM. The calculational procedure is identical to JBM, according to which the amplitudes are obtained in terms of certain radial integrals corresponding to S- and D-wave mesons. Thus the spin-cum-spatial matrix elements for the coefficients $A^{(\pm)}$ in (4) are expressible in terms of three types of integrals

$$R_S{}^{(\pm)}=\int\boldsymbol{\psi}''\cdot\mathbf{A}^{(\pm)}\psi^s,$$

$$R_D{}^{(\pm)}=\int(\psi_z''A_z{}^{(\pm)}-\tfrac{1}{3}\boldsymbol{\psi}''\cdot\mathbf{A}^{(\pm)})\psi^S, \tag{32}$$

$$R_0{}^{(\pm)}=\int\psi_z''f^{(\pm)}\psi^S.$$

The terms $R_s{}^{(\pm)}$ and $R_D{}^{(\pm)}$, which correspond respectively to S and D wave mesons, arise from the vector coefficients $\mathbf{A}^{(\pm)}$ in (8) and the integrals $R_0{}^{(\pm)}$ relate to the scalar coefficients $f^{(\pm)}$. Further evaluation of these integrals would require the detailed momentum dependence of the functions $f^{(\pm)}$ and $\mathbf{A}^{(\pm)}$ and would, in general, depend on recoil effects.[16] However, their explicit evaluations, which depend on the dynamical details, are not of so much interest as the algebraic forms of the matrix elements in terms of these radial integrals which we take as free parameters for purposes of this paper.

For transitions within a given $SU(3)$ multiplet, we of course obtain merely the $SU(3)$ relations (rather, U-spin invariant relations) (16) and (17) for **8**, and (23) and (24) for the **10**. However, the inclusion of the spin-cum-spatial structures clearly gives several additional relations, even without going into the detailed dynamics. Thus for the transition to the $J^P=\frac{3}{2}^-$ states

[12] V. S. Elings *et al.*, Phys. Rev. Letters **16**, 474 (1966).

[13] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[14] C. Bacchi *et al.*, Phys. Rev. Letters **11**, 37, (1963).

[15] L. J. Larzerotti *et al.*, Phys. Rev. Letters **15**, 210 (1965).

[16] These effects can be taken into account by invoking the Galilean invariance of the basic $\gamma+Q\to Q+\Pi$ amplitude, which requires the explicit dependence of the quantities $f^{(\pm)}$ and $\mathbf{A}^{(\pm)}$ on the actual momenta of the various particles involved.

in any $SU(3)$ multiplet, the matrix elements are all expressible only in terms of four-parameters $R_D^{(\pm)}$ and $R_0^{(\pm)}$. Similarly, the transition to the $J^P=\frac{5}{2}^-$ octet is expressible only in terms of the two parameters $R_D^{(\pm)}$. For $J^P=\frac{1}{2}^-$ states of any $SU(3)$ multiplet, the parameters needed are merely $R_S^{(\pm)}$ and $R_0^{(\pm)}$. These features lead to nontrivial sum rules [going beyond $SU(3)$] like the following[17]:

$$2a'\sqrt{2}A(Y_0^*(\tfrac{1}{2}^-)K^+)=-(6a'^2+\tfrac{3}{2}b'^2)^{1/2}\times A(\Lambda^0(\tfrac{1}{2}^-)K^+), \quad (33)$$

$$2a\sqrt{2}A(Y_0^*(\tfrac{3}{2}^-)K^+)=-(6a^2+15b^2)^{1/2}\times A(\Lambda^0(\tfrac{3}{2}^-)K^+), \quad (34)$$

$$5^{3/2}A(\Sigma^0(\tfrac{5}{2}^-)K^+)-4\sqrt{2}A(Y_0^*(\tfrac{3}{2}^-)K^+)=12A(Y_1^{*0}(\tfrac{3}{2}^-)K^+)=6\sqrt{2}A(\Delta^0(\tfrac{3}{2}^-)\pi^+), \quad (35)$$

$$A(\Lambda^0(\tfrac{3}{2}^-)K^+)+\sqrt{3}A(\Sigma^0(\tfrac{3}{2}^-)K^+)=-(2a-b)(6a^2+15b^2)^{-1/2}\sqrt{5}\times A(\Sigma^0(\tfrac{5}{2}^-)K^+), \quad (36)$$

$$[A(\Lambda^0(\tfrac{1}{2}^-)K^+)+\sqrt{3}A(\Sigma^0(\tfrac{1}{2}^-)K^+)][6a'^2+\tfrac{3}{2}b'^2]^{1/2}=-(2a'-b')[\sqrt{2}A(Y_0^*(\tfrac{1}{2}^-)K^+)+3A(Y_1^{*0}(\tfrac{1}{2}^-)K^+)]. \quad (37)$$

Here, (a,b) and (a',b') represent the mixing parameters for doublet and quartet spin states of the $J^P=\frac{3}{2}^-$ and $\frac{1}{2}^-$ octets, respectively.[18]

At the present stage, the absence of any experimental results on the photoproduction amplitude for negative-parity states prevents even a crude comparison with observations. However, it is perhaps time to put these results on record, in the hope that photoproduction data for such states would be available in the not-too-distant future. What characterizes these sum rules is their independence from any detailed dynamics except an (LSJ) classification of spatial functions of mixed symmetry in association with the multiplet structure of $SU(3)$ so as to give a total of 70×3 states, which according to Dalitz analysis[13] seems to be in qualitative accord with observation.

Moreover, though these results have been formally obtained under parastatistics, identical relations follow from Fermi statistics as well.[19] A comparison of these relations with experiment would therefore be a good test of the classification scheme within U-spin invariance. Now, the effect of $SU(3)$ violation within the wave functions which has not been considered here could well be important enough to distort the above predictions of pure U-spin invariance. However, such questions will remain speculative till enough data on the cross-sections are available.

After this work was completed we were informed of a similar work by Baldin[20] on the absolute cross sections for the photoproduction of the 56^+ baryons at threshold energies, using a field-theoretical model of gauge-invariant electromagnetic interaction. In a sense, Baldin's model is more in the spirit of the "nonresonant" mechanism proposed in this paper, rather than the "form-factor" model of Morpurgo[1] which gives essentially $SU(6)$ results. Baldin finds that the absolute magnitudes of his cross sections for several processes like $A(\Sigma^0K^+)$, $A(\Lambda^0K^+)$, $A(\Delta^{++}\pi^-)$, show sharp differences, a feature he claims is in agreement with experiment but not with the predictions of $SU(6)$. On the other hand, the present model, based as it is on much weaker assumptions than Baldin's cannot predict the absolute cross sections but only certain $SU(3)$ relations among the amplitudes, whose experimental status has already been noted.[12]

The more detailed results like (25)–(28) which are predicted in this model for the spin-flip and non-spin-flip amplitudes are unfortunately not amenable to comparison with the published experimental data which are as yet available only in a very limited form. Baldin's analysis[20] does not extend to photoproduction of negative-parity baryons, which has been considered by Moorhouse[21] in a resonant model,[1] to which the results of the present nonresonant model may be regarded as complementary.

We would like to thank Professor R. C. Majumdar for his interest in this work.

[17] In $SU(3)$ structures, we use exactly the same symbols as for the **8** and **10** baryons, and the letter Y_0^* for the $SU(3)$ singlets. The J^P values are indicated in brackets following the $SU(3)$ symbols.

[18] A determination of their ratio is a dynamical problem [A. N. Mitra, Ann. Phys. (N. Y.) (to be published)] but this detail is unnecessary in the spirit of this paper.

[19] This change is merely equivalent to $S\rightarrow A$, $A\rightarrow S$, $M'\rightarrow M''$, $M''\rightarrow -M'$, and as long as the radial integrals are left as free parameters, the formal expressions for the matrix elements for the various transitions do not change.

[20] A. M. Baldin, Joint Institute for Nuclear Research Report No. P-2556, Dubna, 1965 (unpublished).

[21] R. G. Moorhouse, Phys. Rev. Letters **16**, 771 (1966).