where the "other terms" arise from the last term in (17) and include any electromagnetic corrections which may arise in $[A^i(0),\dot{A}^j(0)]=(1/\sqrt{3})\delta_{ij}(\sqrt{2}u_0+u_8)$. However, since, for the s-wave part,

$$A_{s\text{ wave}}(\eta^0\to\pi^+\pi^-\pi^0)=\tfrac{1}{3}A(\eta\to 3\pi^0)\,,$$

we can conclude that "other terms" in (18) must cancel each other. Thus the two definitions of PCAC, namely (1) and (17), lead to identical results for the s wave, at least for the Gell-Mann model.[2] For the p-wave contribution to $\eta\to\pi^+\pi^-\pi^0$, this cancellation is not obvious.

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Quark Model for Meson-Baryon Processes

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An $SU(3)$-invariant model of the elastic quark (Q) meason (pseudoscalar) scattering amplitude is proposed in the spirit of an impulse approximation, for the description of meson-baryon scattering and production processes involving both the positive- and negative-parity baryons looked upon as $3Q$ composites. The mesons are looked upon as "elementary" particles, rather than $Q\bar{Q}$ composites. The positive-parity baryons are assumed to belong to the **56** representation of $SU(6)$, and the negative parity ones to the (**70**,3) representation of $SU(6)\otimes O(3)$. The results for the scattering and production amplitudes are expressed in the form of sum rules connecting physically interesting processes. We obtain several sum rules for the elastic scattering amplitudes which agree with the Johnson-Treiman and Lipkin relations. For production processes within the **56** of baryons, some of our results, which do not agree with $SU(6)$ or $SU(6)_W$, are at least not disfavored by experiment compared with the latter. A number of relations among amplitudes connecting production of negative-parity baryonic states are also obtained, but these cannot be tested experimentally at this stage. A slight variant of the model, in which the initial meson is replaced by a spurion octet, reproduces in a natural way all the known sum rules for nonleptonic decays of Σ, Λ, and Ξ. For the Ω^- decay to $\Xi\pi$ and $\Xi^*\pi$ systems, the predictions of the model are, however, different from those of $SU(6)$ or of partially conserved axial-vector current with equal-time commutators.

I. INTRODUCTION

IN spite of the continued failure so far to detect quarks experimentally,[1] the quark model has had a surprising number of successes which apart from the familiar ones involving the static properties of baryons,[2–4] now include more sophisticated phenomena like electromagnetic decays of vector mesons,[5] photoproduction of the baryon decuplet,[6] nuclear beta-decay[3] and mass splittings within and among $SU(3)$ multiplets.[7] These results suggest that there is perhaps more to the model than is conceptually understood at this stage, and it should therefore encourage investigations of a wider class of phenomena, using suitable variants of the model. Thus Lipkin and his collaborators,[8,9] making essentially a combinatorial analysis within the quark model (which merely takes account of the possible QQ and $Q\bar{Q}$ pairs that can scatter, without any references even to the symmetries of the $3Q$ and $Q\bar{Q}$ systems that make up the baryons and mesons), have found several interesting sum rules involving high-energy elastic scattering processes, some of which agree very well with experiment. These sum rules go much beyond some of the $SU(6)$ results like the Johnson-Treiman[10] and Barger-Rubin[11] relations for elastic scattering amplitudes. Such a model is however not adequate for describing production processes, even

[1] See, e.g., (i) W. A. Chupka, J. P. Schiffer, and C. M. Sterms, Phys. Rev. Letters **17**, 60 (1966); (ii) R. Adair, in *Proceedings of the 1964 Coral Gables Conference on Symmetry Principles at High Energy*, edited by B. Kursunoğlu and A. Perlmutter (W. H. Freeman and Company, San Francisco, 1965).

[2] G. Morpurgo, Physics **2**, 95 (1965); A. N. Mitra, Phys. Rev. **142**, 1119 (1966).

[3] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[4] A. N. Tavkhelidze, in *Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

[5] C. Becchii and G. Morpurgo, Phys. Rev. **140**, B687 (1965); see also W. Thirring, Phys. Letters **16**, 335 (1965).

[6] C. Becchi and G. Morpurgo, Phys. Letters **17**, 352 (1965).

[7] G. Zweig, in *Symmetries in Elementary Particle Physics*, edited by A. Ziehichi (Academic Press Inc., New York, 1964); A. P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Letters **22**, 208 (1966); H. R. Rubinstein, *ibid.* **22**, 210 (1966).

[8] H. J. Lipkin and F. Scheck, Phys. Letters **16**, 73 (1966).

[9] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).

[10] K. Johnson and S. B. Trieman, Phys. Rev. Letters **14**, 189 (1965).

[11] V. Barger and M. H. Rubin, Phys. Rev. Letters **14**, 713 (1965).

among the familiar **56** of baryons, for which one needs a more detailed (relativistic) mechanism, e.g. $SU(6)_W$.[12,13]

To extend the quark model to these last-named processes, it is necessary to make more specific assumptions involving the detailed spin, $SU(3)$ and spatial structures of the baryons and mesons as $3Q$ and $Q\bar{Q}$ composites. The purpose of this paper is to propose a model in which the baryons are regarded as $3Q$ states with appropriate quantum numbers and symmetries in all the available degrees of freedom, but no attempt is made to relate the mesons to their $Q\bar{Q}$ structures. Rather, the mesons are regarded as "elementary" particles which interact with the baryons through the individual quarks, *one at a time*, in the spirit of an impulse approximation.[14] The basic quantity is thus a phenomenological quark-meson scattering amplitude in an $SU(3)$-invariant form, and expressed as an operator in spin and $SU(3)$ variables, whose matrix elements must be evaluated between the desired $3Q$ states representing the initial and final baryons. It is clear that such quark-meson scattering amplitudes can be generated only through the exchange of mesons and/or quarks but *not baryons*.[15] We should therefore expect amplitudes for processes involving *more than one unit* of charge, isospin or strangeness transfer between the initial and final baryons, to vanish identically.[16] Such a mechanism which is characteristic of rather high-energy phenomena could, however, fall short of observations for moderate energies, and a comparison with experimental data could perhaps give an indication of the relative importance of baryon exchange forces in the actual situations.

Perhaps a word of explanation is in order regarding the assumption that the mesons are "elementary," but the baryons composite in this model. Such an assumption would, of course, be immediately acceptable if the mesons as $Q\bar{Q}$ states were much lighter than the baryons as $3Q$ states, indicating tighter structures of the former. Unfortunately, this does not seem to be the physical picture, since the average mass of a baryon is hardly about twice that of a pseudoscalar meson. While this no doubt indicates more binding energy *per particle* in a meson than in a baryon, this fact by itself may well be too restrictive a condition on the qualitative validity of the model. Indeed, if direct three-body forces among quarks are ruled out in favor of two-body Q-Q or Q-$\bar{Q}$ interactions to generate mesons and baryons, in analogy with usual ideas on forces between more familiar particles, the physical plausibility of the above model may be translated to the requirement that the Q-Q force be appreciably weaker than the Q-$\bar{Q}$ force. This last requirement is, of course, well satisfied for the B-B and B-$\bar{B}$ systems as manifest, e.g., from the low mass of the pion as a ${}^1S_0$ $N\bar{N}$ state compared to the loosely bound deuteron as a ${}^3S_1$ NN state. This probably is an extreme picture, but the parallel requirement might at least be approximately satisfied in the quark case. Indeed, it has been shown how with the assumption of "long-range" Q-Q forces (compared to M_Q^{-1}) which produce bound $2Q$ states as massive as quarks ($\sim$5–10 BeV), it is possible to generate $3Q$ states at the level of baryon masses ($\sim$1 BeV) without necessarily violating the saturation requirement at the $3Q$ level. Such Q-Q forces are clearly much weaker than the Q-$\bar{Q}$ forces needed to produce the mesons ($\sim\frac{1}{2}$ BeV). This is a much weaker assumption than one which unconditionally requires the mesons to be much tighter structures than the baryons. However, it seems to us that *under the assumption of two-body forces*, the more relevant objects for comparison for the present purposes are the QQ and $Q\bar{Q}$ systems rather than $Q\bar{Q}$ and QQQ. This distinction provides the main *raison d'etre* for our model which regards the parametrization in terms of quarks and mesons as more efficient (and perhaps not more violent) than one in terms of quarks and antiquarks. It is recognized, of course, that the model, like other contemporary ones, has its speculative content (which is perhaps unavoidable for such mysterious objects as quarks). We merely take it as a working hypothesis whose success or failure must ultimately be judged by confronting its predictions with experiment.

The processes which can in principle be described by the model involve both scattering and production, not only among the **56** of baryons, but of higher baryon resonances as well. Thus it will be shown that the scattering relations of Lipkin[8,9] follow in a natural manner in this model. Production processes like $\pi+p\rightarrow N^*+\pi$, Y^*+K, etc., in which the initial and final baryons are within the **56** representation, will be shown to obey certain interesting sum rules which can be confronted with experiment, though in a limited manner at the present stage. Similar sum rules will be derived for production processes involving a large variety of negative parity baryonic states, although the latter are not yet at a stage for comparison with experiment.

A further interesting possibility which can be explored by a slight variant of the model is the correlation of several results on nonleptonic decays of hyperons. For example, the famous Lee–Rosen–Gell-Mann–Sugawara[17] (LRGS) sum rule, which had previously been obtained under varying degrees of physical assumptions, follows in a simple way in this model, provided

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[12] J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler, and H. J. Lipkin, Phys. Rev. Letters **15**, 373 (1965).

[13] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[14] See, e.g., L. Van Hove, CERN report (unpublished).

[15] Since quark exchange would result in an extremely short-range force compared to meson exchange, its contribution is expected to be unimportant compared to the latter. However, these respective contributions cannot be identified in a phenomenological Q-meson scattering amplitude, without further dynamical assumptions.

[16] We are neglecting double scattering of mesons within the $3Q$ systems representing baryons.

[17] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); S. P. Rosen, *ibid.* **12**, 408 (1964); M. Gell-Mann, *ibid.* **12**, 155 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964), referred to as LRGS in the text.

that the initial meson octet is replaced by a spurion octet which formally restores the $SU(3)$ invariance of the decay process via the $\Delta I=\frac{1}{2}$, $\Delta U=\frac{1}{2}$ rule. Clearly, this assumption is consistent with the main features of the model which do not allow the transfer of more than one unit of charge, isospin, or strangeness between the baryons.

The calculation involves the use of initial and final $3Q$ states of correct symmetry, taking all the degrees of freedom into account. The method of construction of these functions is described elsewhere[18] in the context of a dynamical model of baryons as $3Q$ states generated by Q-Q forces. This model, which predicts the usual octet and decuplet of baryons to belong to the representation **56** of $SU(6)$, requires, however, the use of parastatistics which allows spatially symmetric functions to go with **56**, since it is the S-type state of $L=0$ that can be shown to have a strongly attractive kernel. An antisymmetric (A) function of $L=0$, on the other hand, is found to have a repulsive kernel, and therefore disfavors Fermi statistics dynamically.[19] For our present purposes, however, in which we are mainly interested in finding sum rules for different processes rather than their absolute rates, the distinction between Fermi and parastatistics is academic, since as long as the radial integrals are kept as phenomenological parameters, both types of statistics give identical results.[20] The model also provides a set of negative-parity $L=1$ baryonic states belonging to the (70,3) representation of $SU(6)O(3)$,[21] whose structure in terms of spin, $SU(3)$ spin, and angular functions is described in Ref. 18. Again, the distinction between Fermi and parastatistics for these states is unimportant for the present investigation.[20]

In Sec. II, we describe the necessary steps in the formulation of the model, including its distinction from $SU(6)$ theory and summarize, for convenience, the kinematical structures of the various $3Q$ wave functions.[18] In Sec. III, we obtain results for scattering and production processes with the **56** of baryons in the form of sum rules for physically realizable processes. These results are compared with experiment, with the corresponding results obtained by other authors. Section IV deals with the calculation of amplitudes for production processes involving negative parity baryonic states. Finally, Sec. V gives the results for nonleptonic hyperon decays in the form of several interesting sum rules which compare favorably with experiment.

II. NECESSARY FORMALISM

The basic scattering amplitude

$$\Pi+Q\rightarrow\Pi+Q\,,\qquad(2.1)$$

where Π represents a pseudoscalar octet of mesons, and Q is a quark triplet, can be expressed as[22]

$$M=f(\theta)+i\boldsymbol{\sigma}(\mathbf{k}\times\mathbf{k}')g(\theta)\,,\qquad(2.2)$$

where $f(\theta)$ and $g(\theta)$ are, respectively, the non-spin-flip and spin-flip parts of the amplitude, $\mathbf{k}$ and $\mathbf{k}'$ are the initial and final momenta in the c.m. system, and $\boldsymbol{\sigma}$ is the spin of the quark (assumed nonrelativistic). The $SU(3)$ structures of $f(\theta)$ and $g(\theta)$ may be recognized through the decomposition

$$\mathbf{3}\otimes\mathbf{8}=\mathbf{15}\oplus\mathbf{6}^*\oplus\mathbf{3}\,.\qquad(2.3)$$

The representations on the right-hand side are most conveniently expressed in terms of the λ matrices of Gell-Mann[23] by noting that **15** and **6*** are, respectively, associated with the usual D and F combinations of the λ matrices, viz., $d_{\alpha\beta\gamma}\lambda_\gamma$ and $f_{\alpha\beta\gamma}\lambda_\gamma$, where each index (α,β,γ) runs over the values 1–8. In the same notation, the representative of **3** in (2.3) is $\delta_{\alpha\beta}$. The most general $SU(3)$ structure of the quark-meson scattering amplitude (2.2) is thus an 8×8 matrix in the indices α, β $(=1$–$8)$ operating on the meson states each element in turn being a 3×3 λ matrix operating on the quark states. Thus we write

$$f_{\alpha\beta}(\theta)=\sum_{i=1}^{3}\left[A_1\delta_{\alpha\beta}\mathbf{1}+iB_1f_{\alpha\beta\gamma}\lambda_\gamma^{(i)}+C_1d_{\alpha\beta\gamma}\lambda_\gamma^{(i)}\right],\qquad(2.4)$$

$$g_{\alpha\beta}(\theta)=\sum_{i=1}^{3}\left[A_2\delta_{\alpha\beta}\mathbf{1}+iB_2f_{\alpha\beta\gamma}\lambda_\gamma^{(i)}+C_2d_{\alpha\beta\gamma}\lambda_\gamma^{(i)}\right],\qquad(2.5)$$

where $\mathbf{1}$ represents a 3×3 unit matrix, A, B, C are scalar functions of the momenta $\mathbf{k}$ and $\mathbf{k}'$, and the superscript (i) on the λ matrices indicates which of the 3 quarks within the baryon scatters the meson. We may thus regard the total $Q\Pi$ amplitude M, given by (2.3)–(2.5), as a sort of "scattering operator" whose matrix elements should be evaluated between appropriate $3Q$ states representing the initial and final baryons, in close analogy, e.g., to the problem of evaluation of the magnetic moment of a nucleus by taking the expectation value of the sum of individual magnetic moment operators for the various nucleons constituting the nuclear state.

Insofar as the baryon states, as $3Q$ composites of proper symmetry, do not involve any new parameters

[18] A. N. Mitra, Ann. Phys. (N. Y.) (to be published).

[19] A second reason for the choice of parastatistics concerns the universal shape (Sachs) of the baryon form factor which is predicted to have nodal structure with A-type functions, in discord with experiment, but smooth monotonic behavior with S-type functions, in agreement with experiment. See A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).

[20] Indeed, the results of Fermi statistics can be derived from those of parastatistics with the following replacement of the spatial functions: $S\rightarrow A$, $M''\rightarrow M'$, $M'\rightarrow -M''$, where M' and M'' are the two independent functions of mixed symmetry.

[21] K. T. Mahanthappa and E. G. C. Sudarshan, Phys. Rev. Letters **14**, 163 (1965).

[22] J. Hamilton, *The Theory of Elementary Particles* (Clarendon Press, Oxford, England, 1959), p. 381.

[23] M. Gell-Mann and Y. Neeman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964).

over and above the six quantities $A_1B_1C_1A_2B_2C_2$, our model is characterized by six independent amplitudes. This may be contrasted with the situation in a mere $SU(3)$ invariant theory in which the decomposition

$$8\otimes 8=27\oplus 10\oplus 10^*\oplus 8^s\oplus 8^a\oplus 1 \qquad (2.6)$$

accounts for a total of *twelve* amplitudes, since each term on the right-hand side of (2.6) is a sum of two parts (spin-flip and non-spin-flip). An $SU(6)$ theory, on the other hand, gives only *four* amplitudes[24] according to the decomposition

$$56\otimes 35=56\oplus 700\oplus 1134\oplus 70. \qquad (2.7)$$

Our model is thus intermediate between $SU(3)$ and $SU(6)$, and as such is expected to yield somewhat weaker results than $SU(6)$. Yet it will be shown to produce a large number of correlations, and, in particular, to predict most of the (experimentally verifiable) $SU(6)$ results for elastic scattering. On the other hand, for production processes, some of our results will be found to disagree with the corresponding predictions of $SU(6)$ or its limited relativistic version $SU(6)_W$, while being within the general $SU(3)$ framework. In view of the remarks made above about the difference between $SU(6)$ and our model, this need not be surprising.

As has been pointed out already in the Introduction, for the calculation of the matrix elements of the operator M defined by (2.2)–(2.5) between $3Q$ states, we do not need a detailed dynamical knowledge of the radial structures of the $3Q$ wave functions, but merely their spin, angular, and $SU(3)$ spin dependence. We shall formally list these structures under parastatistics, though *formally identical* results are obtainable under Fermi statistics, by making the replacement noted in Ref. 20.

Under parastatistics, the **56** of baryons have a common S function ψ^S. In the notation of Verde,[25] let χ and ζ represent, respectively, the spin and unitary spin functions of the $3Q$ states. χ^S and (χ',χ'') are, respectively, the symmetric (quartet) and mixed (doublet) spin functions. Similarly, ζ^S, (ζ', ζ''), and ζ^a represent, respectively, the **10** (symmetric), **8** (mixed), and **1** (antisymmetric) unitary-spin functions. Then[18] for the stretched states,

$$\chi_{m=3/2}{}^S=\alpha_1\alpha_2\alpha_3, \qquad (2.8)$$

$$\zeta^S(N^{*++})=u_1u_2u_3, \quad \zeta^S(\Omega^-)=w_1w_2w_3, \qquad (2.9)$$

where (α,β) are the two usual spin states of a quark and (u,v,w) its three charge-hypercharge states. Similarly,

$$\chi'_{m=1/2}=(\sqrt{\tfrac{2}{3}})T'(\alpha_2\alpha_3\beta_1),$$
$$\chi''_{m=1/2}=(\sqrt{\tfrac{2}{3}})T''(\alpha_2\alpha_3\beta_1), \qquad (2.10)$$

and for a "proton" state,

$$\zeta'=(\sqrt{\tfrac{2}{3}})T'(v_1u_2u_3), \quad \zeta''=(\sqrt{\tfrac{2}{3}})T''(v_1u_2u_3), \qquad (2.11)$$

where T' and T'' are the usual combinations of permutation operators.[25] Other ζ states are constructed in a similar way.[26] For example,

$$\zeta^a=(1/\sqrt{6})\det(u_iv_jw_k). \qquad (2.12)$$

The appropriate normalized combinations of the spin-cum-unitary spin functions are then

$$\chi^S\zeta^S \quad\text{and}\quad \tfrac{1}{2}\sqrt{2}(\chi'\zeta'+\chi''\zeta'') \qquad (2.13)$$

for **10** and **8** states, respectively.

As for the negative-parity states, it would be reasonable to assume, according to the Dalitz classification,[3] that they all belong to $L=1$, with mixed (M) functions. As has been pointed out already in Sec. I, this is what the model of Ref. 18 also suggests. For present purposes, however, we shall require only the kinematical structures of these wave functions. In the notation

$$(SU_3 \text{ multiplet};\ {}^{2S+1}P_J)$$

for the various $SU(3)$ multiplets of $L=1$ and negative parity, the following members make up the $(\mathbf{70},3)$ representation of $SU(6)\otimes O(3)$:

$$(1,{}^2P_{1/2}),\quad (1,{}^2P_{3/2}),\quad (8,{}^2P_{1/2}),\quad (8,{}^3P_{1/2}),$$
$$(8,{}^2P_{3/2}),\quad (8,{}^4P_{3/2}),\quad (8,{}^4P_{5/2}),$$
$$(10,{}^2P_{1/2}),\quad (10,{}^2P_{3/2}). \qquad (2.14)$$

The spin-cum-angular structures of these wave functions that are associated with the appropriate $SU(3)$ representations under parastatistics, are shown in Table I. Here ψ_μ, ψ_{1m} are, respectively, the Cartesian and spherical forms of a vector function (of type M). The doublet and quartet spin functions (χ_μ',χ_μ'') and $\chi_\mu{}^S$ are the ones appropriate for P states with $J=\frac{1}{2}$.[27] Similarly, the doublet and quartet spin functions $(\sigma_\mu',\sigma_\mu'')\chi_{3/2}{}^S$ and $\sigma_\mu{}^S\chi_{3/2}{}^S$ are suitable for $J=\frac{3}{2}$ states.[18,28] The octet states for both $J=\frac{1}{2}$, $\frac{3}{2}$ of course get mixed up under $SU(3)$-invariant spin-orbit forces, and these mixing parameters, which can be determined dynamically, turn out to be simple (geometrical-looking) numbers.[18] However, since our understanding of these negative parity resonances still leaves a lot to be desired at this stage, we shall, in the spirit of our earlier remarks, avoid a dynamical approach to the problem of negative-parity baryon production, and content ourselves with calculations on the basis of the doublet and

[24] A. Pais, Rev. Mod. Phys. **38**, 215 (1966).

[25] M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 170.

[26] These definitions already agree with Zweig's [CERN Report, 1964 (unpublished)] list of the baryons for the χ' states, so that the correctness of our definition for the (conjugate) χ'' states is established *a fortiori*.

[27] See, e.g., R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953).

[28] $\sigma_\mu'=\frac{1}{2}\sqrt{3}(\sigma_{3\mu}-\sigma_{2\mu})$,
$\sigma_\mu''=-\sigma_{1\mu}+\frac{1}{2}(\sigma_{2\mu}+\sigma_{3\mu})$,
$\sigma_\mu{}^S=\sigma_{1\mu}+\sigma_{2\mu}+\sigma_{3\mu}$; $\chi_{3/2}{}^S=\alpha_1\alpha_2\alpha_3$.

TABLE I. Negative-parity wave function under parastatistics. For simplicity only the 8′ states are listed.

L	S	J	[10]	[8]	[1]
1	$\frac{1}{2}$	$\frac{1}{2}$	$\psi_\mu'\chi_\mu'+\psi_\mu''\chi_\mu''$	$\psi_\mu'\chi_\mu''+\psi_\mu''\chi_\mu'$	$\psi_\mu'\chi_\mu''-\psi_\mu''\chi_\mu'$
1	$\frac{3}{2}$	$\frac{1}{2}$	0	$\psi_\mu'\chi_\mu^S$	0
1	$\frac{1}{2}$	$\frac{3}{2}$	$(\psi_\mu'\sigma_\mu'+\psi_\mu''\sigma_\mu'')\chi_{3/2}^S$	$(\psi_\mu'\sigma_\mu''+\psi_\mu''\sigma_\mu')\chi_{3/2}^S$	$(\psi_\mu'\sigma_\mu''-\psi_\mu''\sigma_\mu')\chi_{3/2}^S$
1	$\frac{3}{2}$	$\frac{3}{2}$	0	$\psi_\mu'\sigma_\mu^S\chi_{3/2}^S$	0
1	$\frac{3}{2}$	$\frac{5}{2}$	0	$\psi_{1,1}'\chi_{3/2}^S$	0

quartet spin states for the octets as they appear in Table I.

III. SCATTERING AND PRODUCTION PROCESSES WITHIN THE 56 OF BARYONS

We shall evaluate here the amplitudes for (1) elastic scattering of mesons by members of the baryon octet, and (2) production processes involving octet and decuplet members in the final state. The calculation of such processes requires mainly the evaluation of the spin-cum-unitary spin matrix elements of the "scattering operator" M, (2.3)–(2.5), among the states defined by (2.13). Since these states are totally symmetric, it is sufficient to consider only the first term $M^{(1)}$ of the full operator M, corresponding to "quark number 1." This index is particularly convenient for calculations since the functions of mixed symmetry, viz., (χ',χ'') and (ζ',ζ'') have been constructed on this basis. We indicate the steps with the example of the process $\pi^+p\to\Sigma^+K^+$ which is represented by the following matrix element[23] of $M^{(1)}$:

$$\langle\Sigma^+(2)^{-1/2}(\Pi_4-i\Pi_5)|M^{(1)}|p(2)^{-1/2}(\Pi_1-i\Pi_2)\rangle, \quad (3.1)$$

which is clearly expressible in the alternative form

$$\tfrac{1}{2}\langle\Sigma^+|M_{41}^{(1)}+M_{52}^{(1)}-iM_{51}^{(1)}+iM_{42}^{(1)}|p\rangle \quad (3.2)$$

in terms of the elements $M_{\alpha\beta}^{(1)}$ which still are matrices in the unitary spin indices of "quark number 1." Now using the tables of the $d_{\alpha\beta\gamma}$ and $f_{\alpha\beta\gamma}$ elements,[23] it is easily seen from (2.4) and (2.5) that each of the combinations

$$M_{41}^{(1)}-iM_{51}^{(1)} \quad\text{and}\quad M_{52}^{(1)}+iM_{42}^{(1)} \quad (3.3)$$

is proportional to the λ matrix

$$\tfrac{1}{2}(\lambda_6^{(1)}-i\lambda_7^{(1)}) \quad (3.4)$$

which takes the "proton-type state" of quark number 1 to its "lambda-type state." According to our definitions (2.11) for the unitary spin structures of various baryonic states, it is found that

$$\langle\Sigma^{+\prime}|\tfrac{1}{2}(\lambda_6^{(1)}-i\lambda_7^{(1)})|p'\rangle=0, \quad (3.5)$$

$$\langle\Sigma^{+\prime\prime}|\tfrac{1}{2}(\lambda_6^{(1)}-i\lambda_7^{(1)})|p''\rangle=\tfrac{2}{3}. \quad (3.6)$$

The spin dependence of the matrix element (3.1) is similarly evaluated by using the spin functions of Eq. (2.10). The final result for the non-spin-flip part of the amplitude $\pi^+p\to\Sigma^+K^+$ is

$$\langle K^+\Sigma^+|\pi^+p\rangle=\tfrac{1}{3}[B_1+C_1-\tfrac{1}{3}iq_z(B_2+C_2)], \quad (3.7)$$

where q_z is the z component of the vector $\mathbf{q}=\mathbf{k}\times\mathbf{k}'$. In this manner it is possible to derive expressions for the other amplitudes in terms of the six parameters A_i, B_i, C_i $(i=1,2,)$ which in turn provide a number of interesting sum rules. Some of the more important ones for elastic scattering are

$$\tfrac{1}{2}[(K^+p)-(K^-p)]=(\pi^+p)-(\pi^-p)=(K^0p)-(\bar{K}^0p), \quad (3.8)$$

$$2[(K^-p)+(K^+p)]=(K^-n)+(K^+n)+(\pi^+p)+(\pi^-p), \quad (3.9)$$

$$(\pi^+p)+(K^-p)+(K^0p)=(\pi^-p)+(K^+p)+(\bar{K}^0p), \quad (3.10)$$

$$(K^-p)+2(K^+n)=(K^+p)+2(K^-n), \quad (3.11)$$

$$(\pi^+p)+(K^-n)=(\pi^-p)+(K^+n)=(K^-p)+(K^+p). \quad (3.12)$$

Equation (3.8) is the Johnson-Treiman[24] relation, valid for the forward direction, whose experimental success has already been noted. Equations (3.9) and (3.10) represent, respectively, the "symmetric" and "antisymmetric" sum rules obtained by Lipkin and Scheck[8] (who also discussed their experimental status). Equation (3.11) agrees with a more recent one given by Lipkin,[9] on the basis of his combinatorial model. Equation (3.12) is not an independent relation, but follows from (3.9) and (3.11). The experimental status of the last two forms (3.11) and (3.12) has also been discussed by Lipkin.[9] Our results of elastic scattering, being in conformity with the $SU(6)$ and Lipkin models, share their experimental successes and failures.

Several sum rules involving production amplitudes can also be derived on this model. Among the nontrivial ones, there is a sum rule connecting members of the baryon octet in the final state

$$\langle\Sigma^+K^+|\pi^+p\rangle=-\sqrt{2}\langle\Sigma^0K^0|\pi^-p\rangle, \quad (3.13)$$

which is valid for any direction, and includes *both* spin-flip and non-spin-flip amplitudes. Unfortunately, this relation does not check well with the experimental figure of (0.105±0.01) mb given by Yamamoto *et al.*[29]

[29] S. S. Yamamoto, L. Bertanza, G. C. Moneti, D. C. Rahm, and I. O. Skillicorn, Phys. Rev. 134, B383 (1964).

for the total cross section of $\pi^+p\to\Sigma^+K^+$ at 2.77 BeV, and the corresponding figure of (0.086±0.025) mb given by Wangler *et al.*[30] for $\pi^-p\to\Sigma^0K^0$ at 3 BeV. The corresponding $SU(6)$ prediction[11] for the cross sections is

$$\sigma(\pi^-p\to\Sigma^0K^0)+\tfrac{1}{3}\sigma(\pi^-p\to\Lambda K^0) =\sigma(\pi^-p\to\Sigma^-K^+)+\sigma(\pi^+p\to\Sigma^+K^+)\,, \quad (3.14)$$

which again does not agree well with experiment.[31] Our model of course yields a zero value for $\langle\Sigma^-K^+|\pi^-p\rangle$, which does not compare too unfavorably with a ratio $\approx 1:6$ for its experimental cross section[30] with respect to $\langle\Sigma^0K^0|\pi^-p\rangle$ at 3 BeV. We obtain an additional relation among two of these processes in the forward direction, viz.,

$$\sqrt{3}\langle\Sigma^0K^0|\pi^-p\rangle=\langle\Lambda K^0|\pi^-p\rangle \quad (3.15)$$

by neglecting the spin-flip terms (B_2,C_2) in the quark meson scattering amplitude. Strictly speaking, this cannot be tested experimentally in the absence of data on the differential cross sections in the forward direction. It is, however, curious to note its numerical consistency with the experimental total cross sections for $\pi^-p\to\Sigma^0K^0$ and $\pi^-p\to\Lambda K^0$ which are 86 μb and 31 μb, respectively,[30] and this could imply the smallness of the spin-flip terms (B_2,C_2) compared with the non-spin-flip ones (B_1,C_1), at the energy considered ($\sim$3 BeV).

This model gives *zero* values for the amplitudes

$$\langle K^+\Xi^-|K^-p\rangle\,,\quad \langle K^0\Xi^0|K^-p\rangle\,,\quad \langle K^0\Xi^-|K^0n\rangle \quad (3.16)$$

involving as they do, two units of strangeness transfer to the baryon. We note in this connection that the $SU(6)$ predictions[11] on their ratios do not agree at all with experiment.[31]

For decuplet production, we obtain the sum rule

$$M_a=\sqrt{3}M_c+M_b\,, \quad (3.17)$$

where[32]

$$M_a=\langle K^0N^{*++}|K^+p\rangle\,,\quad M_b=\langle N^{*++}\pi^0|\pi^+p\rangle\,,$$
$$M_c=\langle N^{*++}\eta|\pi^+p\rangle\,,\quad M_d=\langle Y_1^{*+}K^+|\pi^+p\rangle\,. \quad (3.18)$$

The $SU(3)$ prediction for these amplitudes[32] is

$$|M_a|^2=|M_b|^2+3|M_c|^2-3|M_d|^2\,, \quad (3.18')$$

with which (3.17) is consistent in the limit $M_d\approx 0$. However, we get an additional relation

$$M_a=\sqrt{3}M_d \quad (3.19)$$

which makes M_a vanish with M_d. The experimental curves in Ref. 31 for the cross sections of these processes shows that $|M_a|^2$ exceeds $|M_d|^2$ much more than merely 3 times implied by (3.19). Therefore, our detailed prediction does not conform to experiment, but it at least maintains the relative order of magnitudes between $|M_a|^2$ and $|M_d|^2$. On the other hand, the corresponding $SU(6)_W$ results are[12]

$$M_a=0\,,\quad \sqrt{2}M_b=\sqrt{3}M_d=(\sqrt{6})M_c\,, \quad (3.20)$$

which even reverses the experimental order of magnitudes of M_a and M_d. Physically a difference between the $SU(6)_W$ predictions and ours could arise from the fact that the former depends strongly on the baryon exchange force,[33] a mechanism which is totally absent in our model. The discrepancy of $SU(6)_W$ predictions from experiment[32] seems therefore to suggest that the role of baryon exchange in these processes should at least not be dominant.

An interesting relation connecting octet and decuplet production in the *backward direction*, which follows from our model, is

$$(\sqrt{6})\langle\Sigma^+\pi^-|K^-p\rangle=\langle Y_1^{*+}\pi^-|K^-p\rangle\,. \quad (3.21)$$

An examination of the histogram data of Ref. 33 for these processes in the backward direction shows a qualitative agreement with this prediction.

IV. PRODUCTION OF ODD-PARITY BARYONIC RESONANCES

Our model is capable of yielding a large number of sum rules for the production of negative-parity resonances, on the lines of Sec. III. However, the experimental data on these resonances at this stage are essentially confined to the knowledge of their existence, and are still a long way from the possibility of any meaningful comparison with theory. Dalitz[3] has made a classification of the experimentally observed states, and their general pattern seems to follow a (70,3) representation of $SU(6)\otimes O(3)$. The actual identifications of these states are, however, still very incomplete. Thus for the two $J^P=\frac{1}{2}^-$ octets, only the $I=\frac{1}{2}$, $Y=\frac{1}{2}$ components are so far known [$N^*(1510)$, $N^*(1700)$]. One $J^P=\frac{3}{2}^-$ octet(?) seems to have been established, with $N^*(1518)$, $Y_1^*(1660?)$, and $\Xi^*(1816)$ known. For the $\frac{1}{2}^-$ decuplet, the $N_{3/2}^*$ component is known. There is a $\frac{5}{2}^-$ octet with known $N_{1/2}^*$ and $Y_1^*(1765)$ components. Finally, the two singlets $Y_0^*(1405)$ and $Y_0^*(1520)$ are the ones appropriate for the $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ singlets, respectively, in the series (2.14). This still leaves a second $J^P=\frac{3}{2}^-$ octet and a $J^P=\frac{3}{2}^-$ decuplet completely unidentified, apart from many important members in the otherwise identified multiplets. Moreover, because of the close promixity in masses among many of these resonances, there is the possibility of many $SU(3)$-breaking effects, in addition to mixing effects which conserve $SU(3)$. In the absence of any proper physical criteria at the present stage, to simulate these mixing effects we shall be content merely with a description of the amplitudes obtained without taking account of any possible ad-

[30] T. P. Wangler, A. R. Erwin, and W. D. Walker, Phys. Rev. **137**, B414 (1965).

[31] T. Binford, D. Clive, and M. Olsson, Phys. Rev. Letters **14**, 715 (1965).

[32] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

[33] L. Lyons, Nuovo Cimento **43**, 904 (1966).

mixtures [$SU(3)$ conserving and violating] and just keep to the (theoretical) classification given in Table I.

The calculation of the $SU(3)$ part of the amplitudes is identical to that in Sec. III. As for the spin-cum-spatial structure, we note that the part $f(\theta)$ cannot contribute to these processes. As for the part $g(\theta)$, it is clear that the term A_2 will appear only in processes involving identical mesons and identical $SU(3)$ structures for the final baryons, e.g., $\pi^+p\to\pi^+N_{1/2}^*$, where $N_{1/2}^*$ is the $Y=\frac{1}{2}$, $I_3=\frac{1}{2}$ member of the (negative parity) baryon octet. The spin matrix elements can be calculated in a straightforward manner, and the final results for the production amplitudes are all expressible in terms of six parameters ($\bar{R}_S,R_S',R_S''$) and ($\bar{R}_D,R_D',R_D''$) which are certain radial integrals connecting the spatial parts of the initial and final wave functions.

Thus

$$\bar{R}_S=\int\psi^S A_2\mathbf{q}\cdot\psi'', \tag{4.1}$$

$$\bar{R}_D=\int\psi^S A_2(q_z\psi_z''-\tfrac{1}{3}\mathbf{q}\cdot\psi''), \tag{4.2}$$

$$\mathbf{q}=\mathbf{k}\times\mathbf{k}'. \tag{4.3}$$

The subscripts S and D bringing out the respective S- and D-wave structures of the above integrands corresponding to the s- and d-wave mesons which are the only possibilities available for the production of $L^P=1^-$ baryons. Similarly, the quantities (R_S',R_D') and (R_S'',R_D'') are the corresponding integrals with A_2 in (4.1) and (4.2) replaced by B_2 and C_2, respectively.[34]

We can calculate the matrix elements for the elastic-scattering-type processes (only from the point of charge-hypercharge classification), which yield sum rules analogous to Eqs. (3.8)–(3.12). For example, one finds the sum rule, cf. Eq. (3.8),

$$\tfrac{1}{2}[\langle K^0p_{1/2^-}|K^0p\rangle-\langle\bar{K}^0p_{1/2^-}|\bar{K}^0p\rangle] =\langle K^+p_{1/2^-}|K^+p\rangle-\langle K^-p_{1/2^-}|K^-p\rangle +\langle\pi^-p_{1/2^-}|\pi^-p\rangle-\langle\pi^+p_{1/2^-}|\pi^+p\rangle, \tag{4.4}$$

where for the octet states the same notation is used as for the corresponding members of the ordinary octet, and the subscripts stand for J^P values. Relations like (4.4) which are valid for any direction can be written down for $J^P=\frac{3}{2}^-$ and $\frac{5}{2}^-$ states as well. Similarly, one may obtain results which are counterparts of the symmetrical sum rule (3.9) for the different J^P octets.

As for the other production processes, instead of writing all the possible amplitudes, we choose only those which involve certain specified members of an $SU(3)$ multiplet, since the results for other members belonging to the same multiplet are a matter of mere $SU(3)$ Clebsch-Gordan coefficients. Thus we always take the target as proton (no choice) and the meson beams as K^- or π^- only. The final-state mesons are taken as π^0, η, and K^0 to go with the appropriate baryon states. Table II is a catalog of these matrix elements in which the two octet states each of $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$ are written as arbitrary mixtures of the doublet and quartet functions of Table I. A few typical sum rules are

$$2A_2+(\sqrt{15})A_3=0, \tag{4.5}$$

$$2A_2+(3\sqrt{15})A_6=0, \quad (a=0) \tag{4.6}$$

$$4A_1+\sqrt{2}A_2-18\sqrt{3}A_6=0, \quad (b=0) \tag{4.7}$$

$$2\sqrt{2}A_1+3\sqrt{3}A_9=0, \quad (a'=0) \tag{4.8}$$

$$2\sqrt{2}A_2-A_1-(9\sqrt{6})A_9=0, \quad (b'=0) \tag{4.9}$$

$$4A_{12}+2\sqrt{2}A_{13}+A_{15}+A_{16}-(3\sqrt{5})\ A_5=0. \tag{4.10}$$

At the moment, all these relations do not have more than academic interest. We hope, however, that it may not be too long before at least some of these come within the realm of experimental possibility.

V. NONLEPTONIC HYPERON DECAYS

As already remarked in Sec. I, a simple variant of the "scattering model" is capable of describing weak (nonleptonic) decays of hyperons as well. In the formalism of Sec. II for the scattering model, we must now replace the vector $\mathbf{k}\times\mathbf{k}'$ by the relative momentum $\mathbf{q}$ of the decay products (in the rest frame of the hyperon). Secondly, the two scattering amplitudes $f(\theta)$ and $g(\theta)$ in (2.2), which will now be independent of any angle, must be interpreted as the amplitudes for s-wave and p-wave decays, respectively. With this modification, the representations (2.4) and (2.5) for these amplitudes are valid, noting that the replacement of the initial meson by a spurion formally restores the $SU(3)$ invariance of the process.

The calculational techniques for the matrix elements in spin and $SU(3)$ space are formally identical to the scattering case (Sec. III). The matrix elements for the chief modes of decay within the octet family are all expressible in terms of two parameters

$$f=B_1+C_1,\quad g=iq_z(B_2+C_2), \tag{5.1}$$

where q_z is the component of $\mathbf{q}$ in the direction of quantization. The results are, in the usual notation,[35]

$$\Sigma_+{}^+=0, \tag{5.2}$$

$$3\sqrt{2}\Sigma_0{}^+=f-\tfrac{1}{3}g, \tag{5.3}$$

$$\sqrt{6}\Xi_-{}^-=f-\tfrac{2}{3}g, \tag{5.4}$$

$$\sqrt{6}\Lambda_-{}^0=-f+g, \tag{5.5}$$

[34] It is important to note that in all the integrals the dominant term is due to the effect of quark recoil in the final $3Q$ state because of its odd-parity structure. The recoil term is taken into account by considerations of Galilean invariance according to which the final meson momentum $\mathbf{K}'$ must be replaced by $\mathbf{K}'-(w_\pi/M_Q)\mathbf{P}$, where w_π is the energy of the meson and M_2, $\mathbf{P}$ represent, respectively, the mass and momentum of the quark. It is this second term that contributes to the integrals (4.1) and (4.2). No such correction is necessary for the initial momentum $\bar{K}$.

[35] See, e.g., K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1963).

TABLE II. Matrix elements for odd-parity baryon states.[a]

$A_1 \equiv \langle Y_{01/2}^{-*}\pi^0 \vert K^- p\rangle$	$\frac{1}{6}(R_S''-R_S')$
$A_2 \equiv \langle Y_{03/2}^{-*}-\pi^0 \vert K^- p\rangle$	$\frac{1}{4}\sqrt{2}(R_D''-R_D')$
$A_3 \equiv \langle \Sigma_{5/2}^{-0}\pi^0 \vert K^- p\rangle$	$-(1/\sqrt{30})(R_D''-R_D')$
$A_4 \equiv \langle \Sigma_{5/2}^{-0}K^0 \vert \pi^- p\rangle$	$-(1/\sqrt{60})(R_D'+R_D'')$
$A_5 \equiv \langle n_{5/2}^{-}\eta \vert \vert \pi^- p\rangle$	$(1/3\sqrt{5})R_D''$
$A_6 \equiv \langle \Sigma_{3/2}^{-0}\pi^0 \vert K^- p\rangle$	$-\frac{1}{6(6a^2+15b^2)^{1/2}}\left[-\frac{a}{3\sqrt{2}}\{(R_D''+\frac{4}{3}R_S'')-(R_D'+\frac{4}{3}R_S')\}+b\sqrt{2}(R_D''-R_D')\right]$
$A_7 \equiv \langle \Sigma_{3/2}^{-0}K^0 \vert \pi^- p\rangle$	$-\frac{1}{6\sqrt{2}(6a^2+15b^2)^{1/2}}\left[-\frac{a}{3\sqrt{2}}\{(R_D''+\frac{4}{3}R_S'')+(R_D'+\frac{4}{3}R_S')\}+b\sqrt{2}(R_D''+R_D')\right]$
$A_8 = \langle n_{3/2}^{-}\eta \vert \pi^- p\rangle$	$\frac{1}{3\sqrt{6}(6a^2+15b^2)^{1/2}}\left[-\frac{a}{3\sqrt{2}}(R_D''+\frac{4}{3}R_S'')+b\sqrt{2}R_D''\right]$
$A_9 = \langle \Sigma_{1/2}^{-0}\pi^0 \vert K^- p\rangle$	$-\frac{1}{6(6a'^2+\frac{3}{2}b'^2)^{1/2}}\left[-\frac{a'}{3}\{(2R_D''-\frac{1}{3}R_S'')-(2R_D'-\frac{1}{3}R_S')\}+\frac{2b'}{3}(R_S''-R_S')\right]$
$A_{10} \equiv \langle \Sigma_{1/2}^{-0}K^0 \vert \pi^- p\rangle$	$-\frac{1}{6\sqrt{2}(6a'^2+\frac{3}{2}b'^2)^{1/2}}[-\frac{1}{3}a'\{(2R_D''-\frac{1}{3}R_S'')+(2R_D'-\frac{1}{3}R_S')\}+\frac{2}{3}b'(R_S''+R_S')]$
$A_{11} \equiv \langle n_{1/2}^{-}\eta \vert \pi^- p\rangle$	$\frac{1}{3\sqrt{6}(6a'^2+\frac{3}{2}b'^2)^{1/2}}[-\frac{1}{3}a'(2R_D''-\frac{1}{3}R_S'')+\frac{2}{3}b'R_S'']$
$A_{12} \equiv \langle \Sigma_{1/2}^{-0*}\pi^0 \vert K^- p\rangle$	$(1/18)[2R_D''-\frac{1}{3}R_S''-2R_D'+\frac{1}{3}R_S']$
$A_{13} \equiv \langle \Sigma_{1/2}^{-0*}K^0 \vert \pi^- p\rangle$	$(1/9\sqrt{2})[2(R_D''+R_D')-\frac{1}{3}(R_S''+R_S')]$
$A_{14} \equiv \langle N_{1/2}^{-*0}\eta \vert \pi^- p\rangle$	$(\frac{1}{9}\sqrt{\frac{2}{3}})[2R_D''-\frac{1}{3}R_S'']$
$A_{15} \equiv \langle \Sigma_{3/2}^{-0*}\pi^0 \vert K^- p\rangle$	$(1/18\sqrt{2})[(R_D''-R_D')+\frac{4}{3}(R_S''-R_S')]$
$A_{16} \equiv \langle \Sigma_{3/2}^{-0*}K^0 \vert \pi^- p\rangle$	$(1/18)[(R_D''+R_D')+\frac{4}{3}(R_S''+R_S')]$
$A_{17} \equiv \langle N_{3/2}^{-0*}\eta \vert \pi^- p\rangle$	$(1/9\sqrt{3})(R_D''+\frac{4}{3}R_S'')$

[a] Here (a,b) are the mixing coefficients for the doublet and quartet states of the $J=\frac{3}{2}^-$ octets, in the same normalization as listed in Table I. Similarly, (a',b') are the corresponding coefficients for the two $J=\frac{1}{2}^-$ octets.

the last three immediately yielding the LRGS[17] sum rule

$$2\Xi_-^- + \Lambda_-^0 = \sqrt{3}\Sigma_0^+, \tag{5.6}$$

which is in agreement with experiment, with the following phases of the different amplitudes[36]

$$\begin{aligned} \Lambda_-^0 &= -(0.31\pm0.01), \quad \Sigma_0^+ = (0.36\pm0.04), \\ \Xi_-^- &= (0.41\pm0.02), \quad \Sigma_+^+ = (0.01\pm0.03); \\ &\qquad \Sigma_-^- = (0.39\pm0.02). \end{aligned} \tag{5.7}$$

The (weaker) isospin relations under the $\Delta I=\frac{1}{2}$ rule, viz.,

$$\begin{aligned} \sqrt{2}\Lambda_0^0 + \Lambda_-^0 &= 0, \\ \sqrt{2}\Sigma_0^+ &= \Sigma_-^-, \\ \sqrt{2}\Xi_0^0 + \Xi_-^- &= 0, \end{aligned} \tag{5.8}$$

are of course satisfied.

The result (5.2), which is a strict consequence of the model, is in good agreement with experiment[36] as well as with the conclusions of Sugawara and Suzuki,[37] who point out that a nonzero value for this amplitude would require a 27-plet contribution. Since, on the other hand, a three-quark model cannot give a **27**, the null result (5.2) is unavoidable.

As for the decuplet decays, the only one of physical interest is Ω^-, for which we obtain the following amplitudes:

$$A(\Omega^- \to \Xi^-\pi^0) = -\frac{1}{3}(\tfrac{2}{3})^{1/2}g, \tag{5.9}$$

$$\sqrt{6}A(\Omega^- \to \Xi^{-*}\pi^0) = -f - \tfrac{1}{6}g, \tag{5.10}$$

$$A(\Omega^- \to \Lambda^0 + K^-) = 0, \tag{5.11}$$

and the isotopic sum rules

$$\begin{aligned} A(\Omega^- \to \Xi^0\pi^-) + \sqrt{2}A(\Omega^- \to \Xi^-\pi^0) &= 0, \\ A(\Omega^- \to \Xi^{*0}\pi^-) + \sqrt{2}A(\Omega^- \to \Xi^{*-}\pi^0) &= 0. \end{aligned} \tag{5.12}$$

[36] These magnitudes are taken from M. A. Gaillard *et al.*, Phys. Letters **20**, 533 (1966).

[37] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); M. Suzuki, *ibid.* **15**, 986 (1965).

The result (5.11), which is a consequence of two units of strangeness transfer to the baryon, compares favorably with that of Hara,[38] who obtains $\sim 1/30$ for its branching ratio with respect to $\langle\Omega^-|\Xi^0\pi^-\rangle$ using the hypothesis of partially conserved axial-vector current (PCAC) and equal-time commutation relations.

For the cascade decays of Ω^-, the further assumption $|f|\ll|g|$, leads to the geometrical relation

$$\langle\Omega^-|\Xi^{*-}\pi^0\rangle|\langle\Omega^-|\Xi^-\pi^0\rangle=\tfrac{1}{4}, \tag{5.13}$$

yielding a ratio of 1:16 for the decay rates, compared with Hara's[38] of 1:3 and the $SU(6)$ prediction[39] of 1:10 for this mode. However, the condition $|f|\ll|g|$ is not compatible with the decay results for other hyperons, since, to satisfy these figures with (5.3–(5.5) one requires $(5/3)g\lesssim f\lesssim 2g$. Of course, such a magnitude of f would almost reverse the ratio (5.13), making the rate for $\Omega^-\to\Xi^{*-}\pi^0$ much faster than $\Omega^-\to\Xi^-\pi^0$, in total disagreement with both PCAC and $SU(6)$. Experiment alone can set these speculations at rest.

VI. CONCLUSIONS

The biggest limitation of the model is in its neglect of baryon exchange forces, which kills the amplitudes with 2 units of charge or strangeness transfer. This is not bad for elastic scattering at high energies, but clearly inadequate for many production processes (if not all). For example, processes involving Ξ production, though strongly inhibited in the model, are experimentally observable. To make the amplitudes for such phenomena nonzero within the model, one requires at least a *double scattering* of the meson by two different quarks in the baryon state which would be in the nature of a correction to the main (single-scattering) amplitude. Yet it is remarkable that for several production processes the predictions of such a simple model are better than those of $SU(6)$ or $SU(6)_W$, in relation to experiment.

The production of negative-parity baryons is strongly suppressed in comparison with positive-parity ones because of recoil effects which make the former proportional to the inverse mass of the quarks. Unfortunately, these predictions do not have any experimental data to be compared with at this stage. However, a similar model of an $SU(3)$-invariant $(\bar{Q}Q\Pi)$ vertex has recently been shown to predict a large number of ***strong*** decay widths of negative parity resonances, which are in rather good agreement with experiment.[40]

The variant of the model used for the understanding of weak decays is similar in spirit to the quark model for β decay, in which the individual quarks decay *one at a time*.[41] This is the usual type of assumption made for the calculation of decay amplitudes of many composite systems, and corresponds to a sort of "sudden approximation" for the decay of the basic entities (quarks). The introduction of the spurion octet is formally equivalent to the assumption of octet dominance, so that the identity of most of the decay results with the familiar ones is expected. The prediction for Ω^- is, however, different and therefore of experimental interest.

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[38] O. Hara, ICTP Report (Trieste), 1966 (unpublished).
[39] S. Pakwasa and S. P. Rosen, Phys. Rev. **147**, 1166 (1966).
[40] A. N. Mitra and M. H. Ross, Phys. Rev. (to be published).
[41] M. A. B. Bég and A. Pais, Phys. Rev. Letters **14**, 51 (1965).