Experimental Evaluation of Quark and Regge-Pole Models for High-Energy Scattering*

V. BARGER AND L. DURAND, III
Department of Physics, University of Wisconsin, Madison, Wisconsin
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Relations among high-energy cross sections derived from the simple quark model are found to be in systematic disagreement with existing experimental data. Regge-pole models with only $SU(3)$-symmetric vertices do not share these difficulties. The use of quark-model vertices with the Regge-pole model leads to the unsuccessful relations of the simple quark model.

I. INTRODUCTION

SIMPLE quark models for particle interactions have attracted enormous attention during the past year.[1–7] During this period, there has also been a striking renewal of interest in the Regge-pole description of high-energy scattering.[7–9] Both models have been successful to some degree, the quark model in predicting relations among experimental cross sections, the Regge-pole model in describing systematically the variation of those cross sections with energy and scattering angle. Some attempts have also been made to combine the two models.[10,11] The comparisons of the various theoretical predictions with experiment are unfortunately rather scattered in the literature, and it has been difficult to obtain a clear over-all impression of the successes and limitations of the models. In this article, we present a critical examination of these models on the basis of existing experimental data. Our conclusions are as follows:

(i) The relations among cross sections derived from the simple quark model are systematically in disagreement with experiment, in some cases by factors of 2 to 5. Some of these discrepancies have been noted previously; others are new. Taken as a whole, they cast serious doubt on the validity of the quark-model description of high-energy scattering.

(ii) The Regge-pole models which use $SU(3)$-symmetric vertices, but allow the trajectories within a multiplet to differ in accordance with the observed particle mass splittings, are generally successful in describing high-energy cross sections.[9] In particular, such models do not yield the unsuccessful relations of the quark model. However, possible consequences of Regge cuts and conspirator trajectories, and the implications of $SU(3)$ symmetry breaking for the Regge residues, have not been studied fully.

(iii) The use of quark-model vertices with the Regge-pole model[10,11] leads to the objectionable relations obtained with the simple quark model.

We will not attempt to give an exhaustive comparison of all quark-model relations with experiment, nor will we attempt to list the variety of assumptions used in different formulations of the quark model.[1–7] We will also confine our attention to forward scattering, although extensions of the quark model to nonforward processes have been suggested. Discrepancies similar to those to be discussed persist at nonforward angles.

II. QUARK MODEL

The simple quark model for high-energy hadron collisions assumes that the forward scattering amplitude (and, in some models, the nonforward amplitude) is given by the sum of the scattering amplitudes of the constituent quarks. $SU(3)$ symmetry is indirectly built into the model by use of a fundamental 3 representation for the quarks. In the direct channel, the $\bar{q}q$ scatterings involve only the 1 and 8 representations of $SU(3)$ and the qq scatterings involve only the $\bar{3}$ and 6 representations. [In the crossed (t) channel only 1 and 8 representations occur, with two possibilities for the charge-conjugation quantum number C.] Consequently, the 10 forward scattering amplitudes for πN, KN, $\bar{K}N$, NN, and $\bar{N}N$ scattering can be expressed in terms of four independent amplitudes, and six sum rules result for the total cross sections (optical theorem), cf. Table I: (ia), (iia), (iiia), any two of the Johnson-Treiman relations[12] (iv), and the symmetric sum rule (v). Since the quark model applies directly to amplitudes, relations (iib) and (iiib) between the forward differential cross sections are also obtained if the qq and $\bar{q}q$ amplitudes are spin-independent.[4] Although the quark model also

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[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz., Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).

[3] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).

[4] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966).

[5] J. J. J. Kokkedee, Phys. Letters **22**, 88 (1966).

[6] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters **17**, 412 (1966).

[7] See, for example, L. Van Hove CERN Report No. TH 714, 1966 (unpublished).

[8] R. J. N. Phillips and W. Rarita, Phys. Rev. **140**, B200 (1965).

[9] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

[10] N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters **22**, 336 (1966). Hereafter called CHN.

[11] N. Cabibbo, L. Horwitz, J. J. J. Kokkedee, and Y. Ne'eman, Nuovo Cimento **45A**, 275 (1966).

[12] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965); R. F. Sawyer, *ibid.* **14**, 471 (1965).

TABLE I. Experimental evaluation of quark- and Regge-pole-model predictions for cross sections. The differential cross section comparisons in the table are for forward scattering.

Predictions for scattering cross sections $\Sigma_{AB}\equiv\sigma_t(\bar{A}B)+\sigma_t(AB)$ $\Delta_{AB}\equiv\sigma_t(\bar{A}B)-\sigma_t(AB)$	Quark model[a–f] Triplet quarks (q); additivity; $SU(3)$	CHN Regge model[g] (1+8) vector and tensor exchanges; quark model vertices	Regge-pole model[h] (1+8) vector and tensor exchanges; (1) Pomeranchuk; $SU(3)$ vertices	Comparison with experiment Typical data only; see references for more complete comparisons
(ia) $\Sigma_{pp}=\frac{1}{2}[3\Sigma_{\pi p}+\Sigma_{Kp}-\Sigma_{Kn}]$	Yes	Yes	No (no relation between meson and nucleon residues)	91 mb:76 mb:81 mb at 12 BeV/c[i] 88 mb:73 mb:80 mb at 16 BeV/c also cf. Fig. 1 of Ref. a
(ib) $=2\Sigma_{\pi p}-\frac{1}{2}\Sigma_{Kp}$	(ia)+(v)			
(iia) $\sigma_t(pp)-\sigma_t(pn)=\sigma_t(K^+p)-\sigma_t(K^+n)$	Yes (with spin-independent amplitudes[d])	Yes	Possible, but not necessary	$0\approx0$, data inconclusive[i]
(iib) $\frac{d\sigma}{d\Omega}(pn\to np)=\frac{d\sigma}{d\Omega}(K^+n\to K^0p)$	Yes	Yes (no, if conspirators are included)	No (conspirator trajectories[j])	No high-energy data on $K^+n\to K^0p$
(iiia) $\sigma_t(\bar{p}p)-\sigma_t(\bar{p}n)=\sigma_t(K^-p)-\sigma_t(K^-n)$	Yes (with spin-independent amplitudes[d])	Yes	Possible, but not necessary	Data inconclusive (large errors on neutron data)
(iiib) $\frac{d\sigma}{d\Omega}(\bar{p}p\to\bar{n}n)=\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)$	Yes	Yes (no, if conspirators are included)	No (conspirator trajectories)	Cf. Fig. 1 $\left[\frac{d\sigma}{d\Omega}(\bar{p}p\to\bar{n}n)\Big/\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\right]\simeq5$ to 10
(iv) $\frac{1}{2}\Delta_{Kp}=\Delta_{Kn}=\Delta_{\pi p}$ (Johnson-Treiman relations)[k]	Yes	Yes	No (F type $V\bar{B}B$ coupling not necessary)	$\Delta_{Kn}/\Delta_{\pi p}\simeq1.8$; $\Delta_{Kp}/2\Delta_{\pi p}\simeq1.4$; $\Delta_{Kp}/2\Delta_{Kn}\simeq0.8$[l] ratios are energy-dependent.
(v) $\Sigma_{Kp}=\frac{1}{2}[\Sigma_{\pi p}+\Sigma_{Kn}]$	Yes	Yes	No (F type $T\bar{B}B$ coupling not necessary)	39 mb:44 mb at 12 BeV/c[i] 38 mb:43 mb at 16 BeV/c also cf. Fig. 3 of Ref. a
(vi) $\Delta_{Kp}-\Delta_{Kn}=\Delta_{\pi p}$[m]	Yes, but (iv) is stronger	Yes, but (iv) is stronger	Yes (ρ exchange only)[m]	Agreement seems good[h] (see comment in text)
(viia) $\sigma_t(K^+p)=\sigma_t(K^+n)$ (viib) $\sigma_t(pp)=\sigma_t(pn)$	With asymptotic qq scattering	With exchange degeneracy[n]	Approximate relation due to small ρ, R residues[h]	Seems very good, but Glauber correction uncertain
(viiia) $\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)=2\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)$	With asymptotic qq scattering	No	No	Cf. Fig. 1, $\left[2\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)\Big/\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\right]\simeq2$

TABLE I (*continued*)

Predictions for scattering cross sections $\Sigma_{AB}\equiv\sigma_t(\bar{A}B)+\sigma_t(AB)$ $\Delta_{AB}\equiv\sigma_t(\bar{A}B)-\sigma_t(AB)$	Quark model[a–f] Triplet quarks (q); additivity; $SU(3)$	CHN Regge model[g] (1+8) vector and tensor exchanges; quark model vertices	Regge-pole model[h] (1+8) vector and tensor exchanges; (1) Pomeranchuk; $SU(3)$ vertices	Comparison with experiment Typical data only; see references for more complete comparisons
(viiib) $\frac{d\sigma}{d\Omega}(K^+n\to K^0p)=0$	With asymptotic qq scattering	No	No (predicts sizeable cross section)	Not true at 2.3 BeV/c[o], where $[d\sigma/d\Omega]_{0°}=3.2$ mb/sr no high-energy data
(ix) $\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)=3\frac{d\sigma}{d\Omega}(\pi^-p\to\eta n)$	With asymptotic qq scattering	With exchange degeneracy	No	Experimental ratio $\left[\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)\Big/\frac{d\sigma}{d\Omega}(\pi^-p\to\eta n)\right]$ is energy-dependent, i.e., $\alpha_\rho\simeq0.56$, $\alpha_R\simeq0.35$[p]
(xa) $\mathrm{Im}\langle K^-p\|\bar{K}^0n\rangle=\mathrm{Re}\langle K^+n\|K^0p\rangle$	...	With exchange degeneracy	Predicted to hold approximately (from viiia)	Predicts $\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)=\frac{d\sigma}{d\Omega}(K^+n\to K^0p)$[n]
(xb) $\mathrm{Re}\langle K^-p\|\bar{K}^0n\rangle=\mathrm{Im}\langle K^+n\|K^0p\rangle=0$	...	With exchange degeneracy	Predicted to hold approximately (from viiia)	Not well tested, but seems to disagree with prediction from (xv)
(xi) $\Delta_{pp}=5\Delta_{\pi p}=(5/4)\Delta_{pn}$ (Freund relations)[q]	With isosinglet $\bar{q}q$ annihilation dominant[e]	Yes	No (Cf. i and iv)	Δ_{pp} systematically ~40% larger than $5\Delta_{\pi p}$; errors large on neutron cross sections. Also cf. Table I of Ref. q.
(xii) $\Sigma_{pp}+\Sigma_{pn}=3\Sigma_{\pi p}$	...	Yes	No (Cf. i)	185 mb:150 mb at 12 BeV/c[i] 181 mb:146 mb at 16 BeV/c
(xiii) $\sigma_t(\pi p)=\sigma_t(Kp)=\frac{2}{3}\sigma_t(pp)$	With Pomeranchuk limit for qq and $\bar{q}q$ scattering	In Pomeranchuk limit	No (Cf. i, broken symmetry for Pomeranchuk coupling)	Pomeranchuk limit from Regge-pole analysis of Ref. h: $\sigma_t(\pi p)=20.9$ mb, $\sigma_t(Kp)=17.6$ mb, $\frac{2}{3}\sigma_t(pp)=25.2$ mb.
(xiva) $\sigma_t(\pi^-p)=\sigma_t(K^-p)$	...	With trajectory degeneracy	No	25 mb:21 mb at 16 BeV/c[i]
(xivb) $\sigma_t(\pi^+p)=\sigma_t(K^-n)$	...	With trajectory degeneracy	No	23 mb:20 mb at 16 BeV/c
(xv) $\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)+\frac{d\sigma}{d\Omega}(K^+n\to K^0p)=\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)+3\frac{d\sigma}{d\Omega}(\pi^-p\to\eta n)$	(viii) and (ix) are stronger	(ix) is stronger	Yes[r] (with η a pure octet member and conspirators unimportant)	Predicts sizeable cross section for $K^+n\to K^0p$[r]

[a] Reference 1. [b] Reference 2. [c] Reference 3. [d] Reference 4. [e] Reference 5. [f] Reference 6. [g] Reference 10. [h] Reference 9. [i] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965). [j] Reference 20. [k] Reference 12. [l] Reference 16. [m] Reference 18. [n] Reference 30. [o] I. Butterworth *et al.*, Phys. Rev. Letters **15**, 734 (1965). [p] Reference 31. [q] Reference 13. [r] V. Barger and D. Cline, this issue, Phys. Rev. **156**, 1522 (1967).

gives the $SU(3)$ sum rule (vi), the Johnson-Treiman relations (iv) constitute a more stringent prediction, inasmuch as these relations follow directly from the F-type coupling of the t-channel octet to the $\bar{N}N$ system that is built into the model. Relations (ia) and (ii) depend on the additive quark scattering assumption and isospin conservation, but are otherwise independent of $SU(3)$ symmetry.

If the quark-quark scattering is asymptotic in the sense that qq charge-exchange scattering is negligible, then the additional relations (vii), (viii), (ix), and (xv) of Table I are obtained.[3] The further assumption that the $q\bar{q}$ annihilation channel is dominantly isosinglet[5] leads to the Freund relations,[13] (xi). Finally in the Pomeranchuk limit for both qq and $q\bar{q}$ scattering, equal πN and KN total cross sections are predicted along with the celebrated $\frac{2}{3}$ ratio of meson-nucleon to nucleon-nucleon total cross sections,[1] (xiii).

Applications of the quark model to isobar production,[1,6] photoproduction of mesons,[14] and nucleon-antinucleon annihilation into mesons[15] have been considered, but will not be discussed here. Marked discrepancies between the predictions of the model and experiment are known to exist in these cases.[14,15]

The relations predicted by the simple quark model are compared with the available experimental data in Table I. It is important in these comparisons to make use of the *systematic trends* of the data over the energy range available. A case in point is provided by the Johnson-Treiman relations. Because of the uncertainties in the cross-section differences,[16] these relations might be considered as marginally consistent with experiment if compared point by point. However, a smooth parametrization of the data leads to the large systematic discrepancies noted in Table I (the uncertainties are less than $\sim 5\%$[16]). That is, the *complete set* of cross-section differences from 6 to 18 BeV/c would have to be changed systematically by the ratios indicated to obtain satisfactory results.

It should be noted also that the comparisons of the quark-model predictions with experiment are given in Table I *in the form which tests the relevant features of the quark (or Regge-pole) model*. For example, the Johnson-Treiman relations test the notations of $C=-1$ octet exchange in the t channel and F-type coupling to the $N\bar{N}$ system.[12] Hence the comparison of these relations with experiment should be made using cross-section differences as in (iv). Rearrangement of these equations so that only sums of cross sections are involved[3] introduces large $C=+1$ singlet exchange contributions to both sides of the equation. For example, the second relation in (iv),

$$\sigma_t(K^-n)-\sigma_t(K^+n)=\sigma_t(\pi^-p)-\sigma_t(\pi^+p)\,, \qquad (1)$$

may be rearranged as

$$\sigma_t(K^-n)+\sigma_t(\pi^+p)=\sigma_t(\pi^-p)+\sigma_t(K^+n)\,. \qquad (2)$$

The left-hand side of Eq. (1) is systematically larger than the right-hand side by at least 70%. On the other hand if Eq. (2) is used, this discrepancy appears only as a $<5\%$ deviation from equality. Thus, the rearrangement of the equality of Eq. (1) tends to mask the discrepancy.

The discrepancies between the predictions of the simple quark model and experiment evident in Table I cast serious doubt on the validity of the model. The $SU(3)$-independent equality (ia) fails systematically by ~ 15 mb in the present energy range; the uncertainties in the cross-section sums which enter this relation are ≤ 1 mb. It consequently seems difficult to justify the additivity assumptions basic to the model.[17] The comparisons for relations (ii) and (iiia) are inconclusive because of inadequate data. The failure of (iiib) is spectactular: *The forward $\bar{p}p\to\bar{n}n$ and $K^-p\to\bar{K}^0n$ differential charge exchange cross sections shown in Fig. 1 differ by factors of 5–10 for momenta of 3 to 9 BeV/c.* This difference in magnitude persists at nonforward angles; the shapes of the angular distributions are also quite different. The Johnson-Treiman relations (iv) are seriously in error for momenta of 6 to 18 BeV/c; furthermore, the ratios in (iv) are energy-dependent. The symmetric sum rule (v) fails systematically by ~ 5 mb for momenta of 6 to 18 BeV/c. The experimental uncertainties in the sums of cross sections are ~ 1 mb. This sum rule follows from the assumptions of 1+8, $C=+1$ exchanges in the t channel with F-type coupling of the octet to baryons.

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[13] P. G. O. Freund, Nuovo Cimento **43A** 1171 (1966); Phys. Rev. Letters **16**, 291 (1966).

[14] Photoproduction: J. Kupsch, Phys. Letters **22**, 690 (1966). The quark-model predictions in the forward direction are

$$27\frac{d\sigma}{d\Omega}(\gamma p\to K^+\Sigma^0)=\frac{d\sigma}{d\Omega}(\gamma p\to K^+\Lambda),$$

$$\frac{d\sigma}{d\Omega}(\gamma p\to \pi^+n)=50\frac{d\sigma}{d\Omega}(\gamma p\to K^+\Sigma^0),$$

whereas the experimental cross sections for these processes seem to be about the same size: V. B. Elings *et al.* Phys. Rev. Letters **16**, 474 (1966).

[15] $\bar{p}p$ annihilation: H. R. Rubinstein and H. Stern, Phys. Letters **21**, 447 (1966); J. Kirz, *ibid.* **22**, 524 (1966); J. Harte, R. H. Socolow, and J. Vandermuelen, CERN Report No. TH. 697, 1966 (unpublished). The last authors show that the quark rearrangement model for nucleon-antinucleon annihilation is in serious disagreement with experiment.

[16] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965), especially Figs. 2 and 3.

[17] That the average meson-nucleon and nucleon-nucleon cross sections should be of similar magnitudes is hardly surprising. The hadron couplings are sufficiently strong that many intermediate states are excited in any high-energy collision, leading to a partial loss of identity of the incoming particles. Thus, one may expect the cross sections to be determined by essentially geometrical considerations, with similar "sizes" for the interaction regions irrespective of the incident particles. The Pomeranchuk theorem, $\sigma_t(\bar{p}p)=\sigma_t(pp)$ at infinite energy, may be regarded as an example of this phenomenon. Thus ratios of total cross sections on the order of unity are to be expected intuitively. It is conceivable, in fact, that *all* total cross sections approach a common limit at infinite energy, a possibility which is apparently not precluded by present analyses, especially if the experimental cross sections continue to decrease.

The relation is trivially true for the singlet exchanges with $SU(3)$-symmetric couplings. Because singlet exchange gives the dominant contribution to the cross sections, this relation does not provide a sensitive test of the F-type coupling for the octet. Of the testable relations, only the antisymmetric sum rule (vi), derivable assuming only $SU(3)$ symmetry and octet dominance in the t channel,[18] is in agreement with experiment. The accuracy of this relation is difficult to assess because of the rather large fractional errors in the cross-section differences (Ref. 9, Fig. 4). However, a systematic fit to the cross section from 6 to 16 BeV/c gives $[\Delta_{Kp}-\Delta_{Kn}]/\Delta_{\pi p}=1.14\pm0.12$.[16]

The addition of the asymptotic assumptions necessary to derive the remaining quark-model relations weakens the conclusions which can be drawn from any discrepancies. Nevertheless, the apparent equality of the K^+p and K^+n total cross sections predicted by relation (viia) has been cited as a striking success of the model. Similar assumptions lead to the unsuccessful relations (viii) and (ix). The charge-exchange equality (viiia) is compared with the available data in Fig. 1, and shows a systematic discrepancy of a factor of 2 for the forward differential cross sections. The Freund relations[13] (xi), systematically in error by $\sim$40%, again require a new assumption. Finally, the comparison with experiment of the asymptotic predictions (xiii) for the ratios of meson-nucleon and nucleon-nucleon total cross sections is model-dependent. If the cross sections are asymptotically constant, present analyses[9] suggest a failure of the $\frac{2}{3}$ ratio of meson to nucleon cross sections[17] (Table I), and the presence of $\sim$20% $SU(3)$ violation in pion-nucleon and kaon-nucleon scattering. If the cross sections continue to decrease as the energy becomes infinite, as suggested by Cabibbo *et al.*,[11] no test is presently possible.

The discrepancies noted between the predictions of the quark model and present experimental results constitute rather strong evidence against the validity of that model in its simple form. A number of attempts have been made to relax the assumptions of the model to avoid the most striking difficulties either by introducing $SU(3)$ symmetry breaking or relaxing the additivity assumption. (See, for example, Refs. 3 and 6.) Unfortunately, the model has lost in the process much of its intuitive appeal and its predictive power. (It is difficult, for example, to relate the requisite symmetry breaking to that known for the particle mass spectra, a relation which is at least partially understood in the Regge-pole model.) In this sense, recent work represents what is perhaps more properly regarded as an exploration of the quark model than a derivation of relations among physical quantities.[3,6] On a more fundamental level, it seems difficult to give any convincing theoretical justification for the additivity assumption, especially for nonforward scattering. Finally, spin-

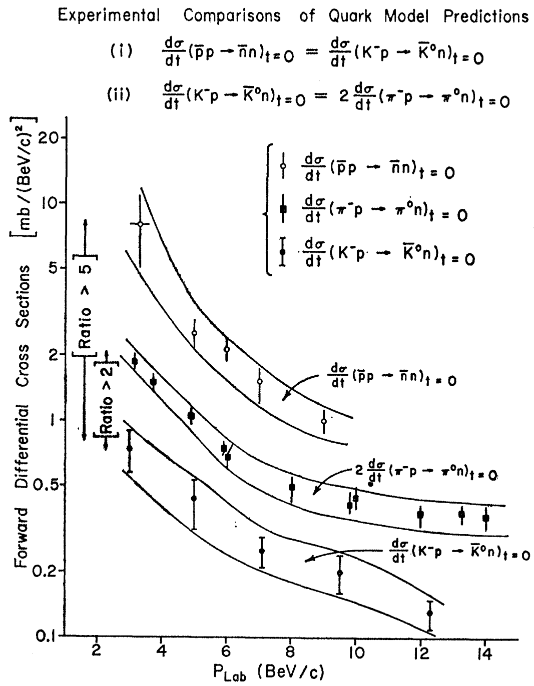

FIG. 1. Comparison of quark-model predictions for charge-exchange cross sections with experiment. $\bar{p}p\to\bar{n}n$ *data*: O. Czyzewski *et al.*, Phys. Letters **20**, 554 (1966); P. Astbury *et al.*, *ibid.* **22**, 537 (1966); P. Astbury *et al.*, in Proceedings of the Thirteenth International Conference on High Energy Physics at Berkeley, 1966 (unpublished). $\pi^-p\to\pi^0n$ *data:* A. Stirling *et al.*, Phys. Rev. Letters **14**, 763 (1965); I. Mannelli *et al.*, *ibid.* **14**, 408 (1965); P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966). $K^-p\to\bar{K}^0n$ *data:* P. Astbury *et al.*, Phys. Letters **16**, 328 (1965); P. Astbury *et al.*, *ibid.* **23**, 396 (1966); J. Badier *et al.*, Saclay Report, 1966 (unpublished).

dependent phenomena have not yet been explored in the quark model. This may be of particular interest in connection with the equalities (iib) and (iiib) of Table I.

III. REGGE-POLE MODEL

The Regge-pole model regards hadron scattering amplitudes as sums of amplitudes associated with specific crossed (t) channel exchanges. Considerable, but by no means complete, theoretical justification can be given for such a model. Since only singlet and octet meson states are presently known experimentally, it is customary in discussing forward scattering to confine the model to 1+8 exchanges associated with the observed 1^- and 2^+ mesons. (Successful models for backward scattering have also been obtained by including Reggeized baryon exchanges.)[19] In addition, the existence of a unitary singlet Pomeranchuk trajectory with $\alpha(0)=1$ is generally assumed. Contributions

[18] V. Barger and M. H. Rubin, Phys. Rev. **140**, B1365 (1965).

[19] See, for example, C. B. Chiu and J. Stack, Phys. Rev. **153**, 1575 (1967); V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966). The second paper provides evidence that the numerous πN resonances lie on the Regge trajectories which are exchanged. To date, the quark model has not provided a framework suitable for the description of backward or large-angle scattering.

from the known 0^- and possible 1^+ octets do not contribute to total cross sections. However, it has recently been shown that such trajectories, with their associated secondary trajectories (conspirators) may give important contributions to differential cross sections even at $t=0$.[20] Possible contributions from moving cuts in the angular-momentum plane[21] have generally been ignored.

It is usually assumed that the factored Regge-pole residues satisfy $SU(3)$ symmetry. However, the symmetry breaking implied by the different masses of the observed particles in the 1^- and 2^+ nonets is normally taken into account by allowing nondegenerate trajectories for those particles. No restrictions are imposed with respect to the D/F ratios for the couplings of the Regge poles to the $\bar{N}N$ system,[22] and (ω,ϕ) and (f,f') mixing may also be permitted. Since at least some symmetry breaking ($\sim$10 to 15%) in the residues is anticipated, and since some contributions to the cross sections are of much different size (for example, at 12 BeV/c the contribution of the Pomeranchuk trajectory to total cross sections is larger by factors of 5–50 than the contributions of the lower-lying trajectories[9]), considerable care must be taken in making symmetry tests. An example is provided by the $SU(3)$-symmetry relation among the amplitudes

$$A(K^-p\to K^-p)-A(\pi^-p\to\pi^-p)=A(K^-p\to\pi^-\Sigma^+)\,, \quad (3)$$

which leads to a set of triangle inequalities for the differential cross sections. The right-hand side of this relation involves only nonet exchange. On the other hand, the separate amplitudes on the left-hand side involve both nonet exchange and singlet Pomeranchuk exchange. The indicated equality requires $SU(3)$ symmetry for the Regge residues and degeneracy of the octet trajectories. However, if a realistic deviation from exact symmetry is permitted in the Pomeranchuk couplings to $\pi\pi$ and $\bar{K}K$,[9] a significant Pomeranchuk contribution remains, and the relation need not hold even approximately. Similar restrictions apply to other symmetry tests in the Regge-pole model.

It has been possible with Regge-pole models to achieve very accurate fits to the high-energy data on all NN, $\bar{N}N$, πN, KN, and $\bar{K}N$ total cross sections.[8,9] It has also been possible to fit both the energy and momentum transfer dependence of the elastic and charge-exchange differential cross sections. Fairly direct evidence for Regge behavior is in fact provided by the observed correlations of minima in the differential cross sections with their energy dependence [i.e., with the trajectories $\alpha(t)$].[8,23] The predictions of the model for the real parts of the forward pp, pn, and πp scattering amplitudes are in reasonable accord with the experimental results.[24] The predictions of the Regge-pole model for the polarization in $\pi^\pm p$ elastic scattering are remarkably successful.[25] The difficulties in the model with the np and $\bar{p}p$ charge-exchange cross sections noted in the past have apparently been removed with the discovery of contributions to the forward differential cross sections associated with secondary Regge trajectories with singular residues (conspirators).[20] Finally, it has been suggested that the outstanding problem with the model, the existence of polarization in the charge-exchange reaction $\pi^-p\to\pi^0n$,[26] zero for single ρ exchange, may arise from the interference of the dominant ρ contributions with small contributions from the tails of low-energy πN resonances,[27] lower-lying 1^- trajectories,[28] or small Regge-cut terms.[29] It is not clear which, if any, of these explanations for the measured polarization[26] is correct. Resonance or secondary trajectory contributions are unimportant for the high-energy total cross sections of primary interest in the present paper. However, if cuts exist, it may well be necessary to consider their contributions to total cross sections. This problem remains open. Although there are still some problems outstanding, the Regge model seems in general to provide a satisfactory framework for a detailed description of high-energy phenomena.

Of the relations in Table I, only the $SU(3)$ sum rule (vi) is obtained unambiguously in the general Regge-pole model; as noted previously, this is successful. The approximate experimental equalities in (viia) and (viib) arise in the model from the smallness of the ρ and A_2 residues compared to the $I=0$ residues, and the tendency of even these small contributions to cancel. [If the ρ and A_2 couplings are universal, then relation (iia) is also obtained. This relation cannot be tested with present data.] The same parametrization[8,9] leads to the *prediction* that, at present energies, the amplitude for $K^-p\to\bar{K}^0n$ is predominantly imaginary, the amplitude for $K^+n\to K^0p$ is predominantly real, and that (xa) and (xb) should hold approximately.

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[20] Loyal Durand, III, Phys. Rev. Letters **18**, 58 (1967).

[21] S. Mandelstam, Nuovo Cimento **30**, 1127 (1963); **30**, 1148 (1963).

[22] It has been shown, for example, by Y. T. Chiu and L. Durand [University of Wisconsin Report, 1965, (unpublished)] that the forward πN scattering amplitude and, hence, the total πN scattering cross sections, depend on both the vector and tensor types of ρNN coupling for $\alpha(0)\neq 1$. The vector coupling may be pure F type, as would be expected in a theory with a conserved current. However, the tensor coupling is unrestricted, and is known to contain a large D-type admixture at the ρ pole. A nonzero D/F ratio for the resultant forward scattering amplitude may therefore be expected in a Regge-type theory, even in the limit of exact $SU(3)$ symmetry.

[23] F. Arbab and C. B. Chiu, Phys. Rev. **147**, 1045 (1966); S. C. Frautschi, Phys. Rev. Letters **17**, 722 (1966); C. B. Chiu and J. Stack, Phys. Rev. **153**, 1575 (1967); L. L. Wang, Phys. Rev. Letters **16**, 756 (1966).

[24] V. Barger and M. Olsson, Phys. Rev. Letters **16**, 545 (1966); S. J. Lindenbaum, in Proceedings of the 1967 Coral Gables Conference on Symmetry Principles at High Energies, BNL Report No. 11175 (unpublished).

[25] C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. **153**, 1485 (1967).

[26] P. Bonamy *et al.*, Phys. Letters **23**, 499 (1966).

[27] R. J. N. Phillips, Nuovo Cimento **45A**, 245 (1966); R. K. Logan and L. Sertorio, Phys. Rev. Letters **17**, 834 (1966).

[28] H. Högaasen and W. Fischer, Phys. Letters **22**, 516 (1966).

[29] V. M. deLany, D. J. Gross, I. J. Muzinich, and V. L. Teplitz, Phys. Rev. Letters **18**, 148 (1967).

It should perhaps be emphasized that the results noted for differential cross sections and polarizations constitute a much more stringent test of the Regge-pole model than is yet available for the quark model; the experimental results for total cross sections are reproduced without difficulty by current Regge-pole models.

IV. CABIBBO-HORWITZ-NE'EMAN MODEL

Special assumptions can be made which reduce the number of parameters in the Regge-pole model. One such model has been proposed by Cabibbo-Horwitz-Ne'eman (CHN).[10] This model allows only (1+8) tensor and (1+8) vector exchanges (no additional singlet Pomeranchauk exchange) and invokes quark-model results for the residue factors. Relations (i) through (vi) of Table I follow immediately from these assumptions [relations (iib) and (iiib) could be eliminated by the addition of conspirator trajectories to the model]. Relations (xi) and (xii) of Table I also follow without further assumptions. Both of these relations are in substantial disagreement with experiment. In addition to the foregoing equalities, CHN quote a number of inequalities among total cross sections which are consistent with experiment. However, it is readily shown that these inequalities follow from their *assumption* that the Regge residues have the same signs at $t=0$ as at the physical particle poles, where the signs are known. Although commonly made, and apparently true empirically, this assumption is nontrivial as may be seen from the fact that one of the ρ residues changes sign rather close to $t=0$ [at $t\sim-0.1$ to -0.3 $(\mathrm{BeV}/c)^2$]; in fact this nearby zero accounts for the anomalously small value of the ρ residue at $t=0$ mentioned previously.

It has also been proposed by Arnold and Ahmazadeh[30] that certain even- and odd-signature Regge poles might have degenerate trajectories and equal residues (exchange degeneracy). With the assumption that the ρ and A_2 (sometimes called R) are exchange degenerate, CHN obtained predictions (vii), (ix), and (x) of Table I. As noted before, (vii) follows approximately from the smallness of the ρ and A_2 residues without the assumption of exact exchange degeneracy. The ratio of the cross sections in (ix) is energy-dependent[31]; detailed fits give $\alpha_\rho(0)\simeq0.56$, $\alpha_R(0)\simeq0.35$. These ratios indicate that exchange degeneracy is at best an approximate relation, albeit one which may be useful for approximate predictions of cross sections. If the assumption of (ρ,A_2) exchange degeneracy is supplemented by (f_8,ω_8) exchange degeneracy, the cross sections in (xiv) are predicted to be equal at finite momenta[10]; this approximation is clearly rather crude.

[30] R. C. Arnold, Phys. Rev. Letters **14**, 657 (1965); A. Ahmadzadeh, *ibid.* **16**, 952 (1966); Phys. Letters **22**, 669 (1966); A. Ahmadzadeh and C. H. Chan, *ibid.* **22**, 692 (1966); R. C. Arnold, Phys. Rev. **153**, 1506 (1967).

[31] See, for example, the detailed analysis of V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967), especially Fig. 1.

V. FUTURE POSSIBILITIES

The unrestricted Regge-pole model permits very accurate parametrizations of cross-section data, and appears not to lead to undesirable predictions. However, the possible consequences of Regge cuts, and the effects of conspirator trajectories on differential cross sections, have yet to be explored in detail. In addition, the potential provided by this model for the study of $SU(3)$ symmetry-breaking and particle-mixing phenomena has not been exploited fully. The following points may be of particular interest: (1) Trajectory (or particle) mixing occurs at equal values of the complex angular momentum $\alpha(t)$. Because, for example, the (ω,ϕ) trajectories are not degenerate, the effective mixing angles derived from cross sections at equal values of the momentum transfer (e.g., at $t=0$) will differ in general from the actual mixing angles, and modification of previous analyses involving $I=0$ trajectories may be necessary. (2) The Regge residues are functions of both t and $\alpha(t)$. Potential scattering suggests that the major dependence is on $\alpha(t)$. Consequently, $SU(3)$ symmetry should perhaps be required for the residues at equal $\alpha(t)$ rather than at equal t. (3) Regge-pole models may apply directly to πd and Kd scattering. If so, direct use of the deuteron data would permit a more accurate analysis for the $I=0$ trajectories than can be performed using the neutron data, the accuracy of the latter being limited by the uncertainties in the Glauber screening corrections. In particular, a more stringent test of the suggestion of Cabibbo-Horwitz-Kokkedee-Ne'eman (CHKN),[11] that the highest-lying $I=0$ trajectory may have $\alpha(0)<1$ [$\alpha(0)=0.925$] may be possible. Higher-energy deuteron data would help to clarify this point.

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