can be found to agree with experiment, it will give good support to the $SU(3)$ Regge-pole theory and the importance of the ρ and A_2 contributions to various charge-exchange scatterings.

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$SU(3)$ Sum Rules for Meson-Nucleon Charge-Exchange Reactions*

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Assuming that the dominant mechanism for high-energy meson-nucleon charge-exchange scattering is the exchange of $I=1$ members of $SU(3)$ octet meson states, the sum rule

$$\frac{d\sigma}{dt}(K^-p\to\bar{K}^0n)+\frac{d\sigma}{dt}(K^+n\to K^0p)=\frac{d\sigma}{dt}(\pi^-p\to\pi^0n)+3\frac{d\sigma}{dt}(\pi^-p\to\eta n)$$

is derived and discussed.

ACCUMULATING experimental evidence seems to support a dynamical picture of high-energy meson-baryon scattering at small momentum transfer mediated by exchanges of single-meson states in the crossed channel. Furthermore, the occurrence of only 1 and 8 representations of $SU(3)$ for the observed mesons suggests that exchanges of singlet and octet states should dominate the scattering amplitudes. It is unfortunate that most quantitative interpretations of the data depend rather sensitively on very specific assumptions concerning the nature of the meson-exchange mechanism (such as Regge-pole models with a limited number of trajectories). The purpose of this paper is to point out some sum rules between charge-exchange reactions that are independent of specific dynamical models. These sum rules are based on the assumptions of octet dominance in the crossed-meson channel with approximate $SU(3)$ invariance for the *individual* exchange amplitudes. From this set of rather weak assumptions some striking predictions are made for experimental quantities that should be measured in the near future.

Assuming that the (π,K,η) pseudoscalar mesons form an *octet* representation[1] of $SU(3)$, the meson-nucleon charge-exchange scattering amplitudes of definite helicity are expressible in terms of two amplitudes, $D(s,t)$ and $F(s,t)$, which represent the contributions from $(I=1,G=-)$ and $(I=1,G=+)$ meson exchanges, respectively. In terms of these two amplitudes, $SU(3)$ symmetry for charge-exchange scattering amplitudes gives

$$\begin{aligned}\langle\pi^-p|\pi^0n\rangle&=-\sqrt{2}F(s,t)\,,\\ \langle K^-p|\bar{K}^0n\rangle&=F(s,t)+D(s,t)\,,\\ \langle K^+n|K^0p\rangle&=-F(s,t)+D(s,t)\,,\\ \langle\pi^-p|\eta n\rangle&=(\sqrt{\tfrac{2}{3}})D(s,t)\,.\end{aligned}\tag{1}$$

Helicity indices are suppressed in Eq. (1) since corresponding relations hold for both helicity amplitudes. Two sum rules follow directly from Eq. (1):

$$\langle K^-p|\bar{K}^0n\rangle-\langle K^+n|K^0p\rangle=-\sqrt{2}\langle\pi^-p|\pi^0n\rangle\,,\tag{2}$$

$$|\langle K^-p|\bar{K}^0n\rangle|^2+|\langle K^+n|K^0p\rangle|^2=|\langle\pi^-p|\pi^0n\rangle|^2+3|\langle\pi^-p|\eta n\rangle|^2.\tag{3}$$

Note also that Eqs. (2) and (3) hold separately for the real and imaginary components of the amplitudes. We make all experimental comparisons of Eqs. (2) and (3) at equal values of the Mandelstam variables (s,t). Nevertheless, the primary conclusions which will be drawn are qualitatively independent of this prescription.

Applying the optical theorem to Eq. (2) yields the well-known $SU(3)$ sum rule for total-cross-section differences,[2]

$$[\sigma_t(K^-p)-\sigma_t(K^-n)]-[\sigma_t(K^+p)-\sigma_t(K^+n)]=[\sigma_t(\pi^-p)-\sigma_t(\pi^+p)]\,,\tag{4}$$

which has previously been shown to be in excellent agreement ($\sim10\%$) with the experimental data above $P_{\rm lab}\sim5$ BeV/c.[3] [Note that factors of c.m. momentum from the optical theorem are essentially equal above $P_{\rm lab}\sim3$ BeV/c and therefore have not been retained in

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[1] This is commensurate with the commonly held notion that (η,η') mixing effects are negligible.

[2] V. Barger and M. Rubin, Phys. Rev. **140**, B1365 (1965).

[3] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965); Phys. Rev. **146**, 1080 (1966).

Eq. (4)]. Analogously, the nonlinear sum rule of Eq. (3) gives the following relation for charge-exchange cross sections[4]:

$$\frac{d\sigma}{dt}(K^-p\rightarrow \bar{K}^0n)+\frac{d\sigma}{dt}(K^+n\rightarrow K^0p)$$

$$=\frac{d\sigma}{dt}(\pi^-p\rightarrow \pi^0n)+3\frac{d\sigma}{dt}(\pi^-p\rightarrow \eta n). \quad (5)$$

Equation (5) should hold at either fixed s as a function of t, or fixed t as a function of s. Furthermore, a corresponding relation should be valid for total charge-exchange cross sections in the forward hemisphere.

Experimental data now exist at high energy on three of the four reactions in Eq. (5).[5] Consequently, some general qualitative predictions can be made for the fourth process, $K^+n\rightarrow K^0p$. In Fig. 1, the predicted value of $d\sigma(K^+n\rightarrow K^0p)/dt$ at $t=0$ is plotted as a function of laboratory momentum. For comparison, the upper limit of the contribution of the imaginary amplitude to the forward differential cross section has been computed from the optical theorem[6] as shown in Fig. 1. We wish to draw attention to the qualitative fact that the sum rule in Fig. 1 predicts that the $K^+n\rightarrow K^0p$ amplitude at $t=0$ *must be* predominantly real. This conclusion is independent of the number of octet meson exchanges in the crossed channel, or of the dynamical details. Furthermore, the same conclusion is obtained if the reactions were compared at equal Q value instead of equal s.

In Fig. 2, the predicted total charge-exchange cross section for $K^+n\rightarrow K^0p$ is shown as a function of labora-

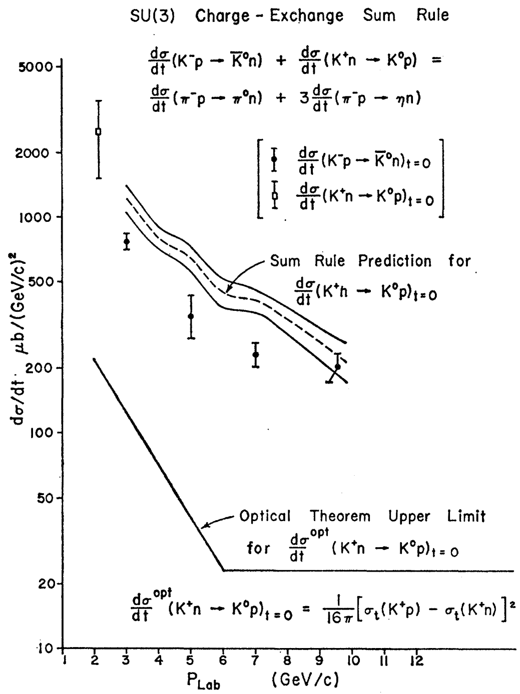

FIG. 1. The dashed curve represents the prediction for the $K^+n\rightarrow K^0p$ near forward differential cross section obtained from the $SU(3)$ charge-exchange sum rule using the experimental data of Ref. 5. The solid curves represent a crude estimate for the error corridor of this prediction. Also shown by the straight solid line is the optical theorem upper limit for the forward differential cross section. For comparison, the available data points from Ref. 5 for the $K^-p\rightarrow \bar{K}^0n$ and $K^+n\rightarrow K^0p$ near forward differential cross sections are plotted.

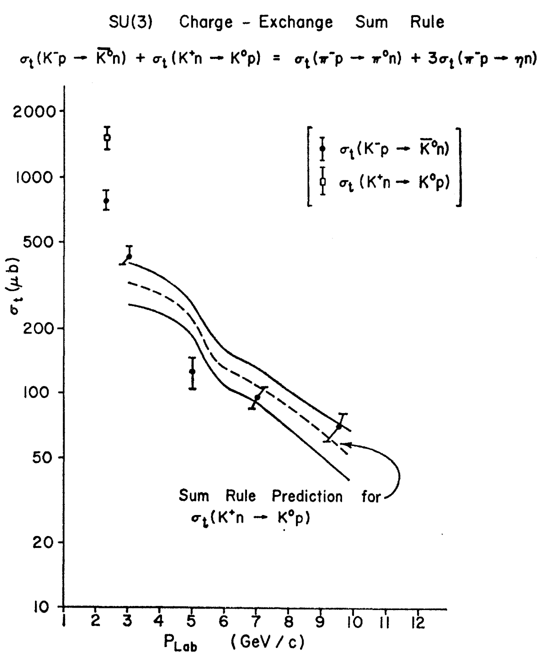

FIG. 2. Prediction for the $K^+n\rightarrow K^0p$ total cross section in the forward hemisphere from the $SU(3)$ charge-exchange sum rule using the data of Ref. 5.

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[4] An analogous sum rule can be derived for isobar production cross sections. The relation is

$$3d\sigma(K^-p\rightarrow \bar{K}^0N^{*0})+d\sigma(K^+p\rightarrow K^0N^{*++})$$
$$=d\sigma(\pi^+p\rightarrow \pi^0N^{*++})+3d\sigma(\pi^+p\rightarrow \eta N^{*++}).$$

At present, there do not exist enough data for a significant comparison of this sum rule with experiment. Supplementary information can be obtained from the equalities

$$d\sigma(\pi^+p\rightarrow \pi^0N^{*++})=3d\sigma(\pi^-p\rightarrow \pi^0N^{*0}),$$
$$d\sigma(K^-p\rightarrow \bar{K}^0N^{*0})=d\sigma(K^-p\rightarrow K^-N^{*+}),$$

which also follow from this meson-exchange model.

[5] *$\pi^-p\rightarrow \pi^0n$ data.* A. Stirling *et al.*, Phys. Rev. Letters **14**, 763 (1965); I. Mannelli *et al.*, *ibid.* **14**, 408 (1965); P. Sonderegger *et al.*, Phys. Letters **20**, 75 (1966).

$\pi^-p\rightarrow \eta n$ data. O. Guison *et al.*, Phys. Letters **18**, 200 (1965).

$K^-p\rightarrow \bar{K}^0n$ data. P. Astbury *et al.*, Phys. Letters **16**, 328 (1965); in Proceedings of the Thirteenth International Conference on High Energy Physics at Berkeley, 1966 (unpublished); J. Badier *et al.*, Saclay Report, 1966 (unpublished).

$K^+n\rightarrow K^0p$ data. I. Butterworth *et al.*, Phys. Rev. Letters **15**, 734 (1965).

A number of data points exist for the first three reactions. However, in some cases the data are not for the same values of laboratory momenta. In these instances reasonable interpolations of the cross sections were used. For this reason, a strict interpretation should not be made for the precise numerical predictions of the $K^+n\rightarrow K^0p$ cross sections.

[6] W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965); R. L. Cool *et al.*, Phys. Rev. Letters **17**, 102 (1966); W. F. Baker *et al.*, Phys. Rev. **129**, 2285 (1963).

tory momentum. It is well known that $\sigma_t(K^+p)$ and $\sigma_t(K^+n)$ are equal within experimental errors and are constant over the laboratory momentum range of 4–18 GeV/c.[6] A diffraction scattering picture for this empirical fact would imply pure imaginary and equal amplitudes for K^+p and K^+n elastic scattering and thus a zero $K^+n \rightarrow K^0p$ cross section. In contrast to this picture, the sum rule of Eq. (5) predicts a sizeable charge-exchange cross section as shown in Figs. 1 and 2. We wish to emphasize the need for an early experimental measurement of the cross section at high energy for the reaction $K^+n \rightarrow K^0p$.

Of course, more restrictive assumptions can be made to obtain additional relations among the charge-exchange cross sections. For example, the dynamics of the charge-exchange reactions can be formulated in terms of Regge poles. The usual models assume the existence of a single ρ trajectory (even G parity) and a single R trajectory (odd G parity). In this case, the t-channel amplitudes of definite helicity in Eq. (1) can be written as

$$D(s,t)=\{-\cot[\tfrac{1}{2}\pi\alpha_R(t)]+i\}\alpha_R(t)d(s,t)\,, \qquad (6)$$
$$F(s,t)=\{\tan[\tfrac{1}{2}\pi\alpha_\rho(t)]+i\}f(s,t)\,,$$

where $d(s,t)$ and $f(s,t)$ are real amplitudes.[7] Here $\alpha(t)$ denotes the appropriate Regge trajectory. If the assumption of "weak" exchange degeneracy is made, namely, $\alpha_\rho(t)=\alpha_R(t)$, then $D^*F+F^*D=0$ and we obtain from Eqs. (1) and (6) the relation

$$\frac{d\sigma}{dt}(K^-p\rightarrow \bar{K}^0n)=\frac{d\sigma}{dt}(K^+n\rightarrow K^0p)\,. \qquad (7)$$

A crude evaluation of this relation is shown in Fig. 1, where data points for $K^-p \rightarrow \bar{K}^0n$ at $t=0$ are compared with the predictions for $K^+n \rightarrow K^0p$ obtained from the sum rule of Eq. (5). Although the experimental errors are large, the trend of the data seems to disagree with the prediction of Eq. (7). The assumption of "strong" exchange degeneracy as formulated by some authors[8] uses $\alpha_\rho(t)=\alpha_R(t)$ and requires $|\alpha_R(t)d(s,t)|=|f(s,t)|$ for both t-channel helicity amplitudes.[9] This relation implies an absolute *zero* in $d\sigma(\pi^-p\rightarrow\pi^0n)/dt$ at the t value for which $\alpha_\rho(t)=0$. There is no experimental indication that such a zero is present in the differential cross section.[5] Therefore, it appears that the useful results of exchange degeneracy may simply be the realization of the more general sum rule of Eq. (5).[10]

In conclusion, the sum rules derived here present an excellent test of the validity of $SU(3)$ symmetry for the meson exchange amplitude as a function of momentum transfer. Although it is usually said that $SU(3)$ is expected to be broken for scattering processes, there is very little concrete evidence on this question for meson octet exchange models of high-energy scattering. In fact, the excellent agreement with experiment of the sum rule of Eq. (4) would seem to indicate that symmetry breaking effects for individual meson exchanges are not very important. Equation (5) is expected to be equally valid. It is an independent relation and will serve as another test of the octet dominance hypothesis for high-energy scattering reactions.

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Note added in manuscript. After submitting this paper for publication, we received an unpublished report by S. Okubo in which the charge-exchange sum rule was also derived. In addition, F. S. Chen-Cheung has informed us that she has obtained this sum rule.

[7] In terms of the t-channel helicity amplitudes, the conventional α parametrizations of the amplitudes are

$$f_{++}\sim 1,\qquad f_{+-}\sim\alpha_\rho(t),$$
$$d_{++}\sim\alpha_R(t),\quad d_{+-}\sim\alpha_R(t)\ \text{ or }\ [\alpha_R(t)]^2.$$

One factor of $\alpha(t)$ is kinematically required for the flip $(+-)$ amplitudes. This factor gives rise to a nonzero minimum in $d\sigma(\pi^-p\rightarrow\pi^0n)/dt$ at the t value for which $\alpha_\rho(t)=0$. The factor of $\alpha_R(t)$ in d_{++} is required to avoid a ghost state at $\alpha_R(t)=0$.

[8] A. Ahmadzadeh and C. H. Chan, Phys. Letters **22**, 692 (1966); R. C. Arnold, Phys. Rev. **153**, 1506 (1967).

[9] Recent Regge-pole analyses of total-cross-section data have demonstrated that the approximate experimental equality $\sigma_t(K^+p)\simeq\sigma_t(K^+n)$ does not require exchange degeneracy in either "weak" or "strong" form. Instead, these analyses indicate that the ρ and R Regge-pole amplitudes make anomalously small contributions (which are of opposite sign) to the KN total cross sections when compared with the contributions of the $I=0$ Regge exchanges. See, for example, V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966). Model-independent determinations of the ρ trajectory from the experimental energy dependence of $d\sigma(\pi^-p\rightarrow\pi^0n)/dt$ and the R trajectory from the experimental energy dependence of $d\sigma(\pi^-p\rightarrow\eta n)/dt$ indicate that the ρ and R trajectories differ for $t\leq 0$. See, for example, R. Phillips and W. Rarita, Phys. Letters **19**, 598 (1965). P. K. Mitter and R. F. Sawyer [Phys. Rev. **154**, 1382 (1967)] have discussed the arbitrariness of the exchange degeneracy assumption for Regge residues.

[10] Similar sum rules are obtainable from the quark model. See, for example, G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters **17**, 412 (1966). For the scattering of quarks on nucleons, the t channel can contain only the $SU(3)$ representations given by $\bar{3}\times 3=8\oplus 1$; thus octet dominance in the t channel follows for charge exchange. However, in the quark model it must also be assumed that the scattering amplitude for a meson-nucleon reaction is given by the sum of the constituent quark-nucleon and antiquark-nucleon scattering amplitudes. No such assumption is required in the derivation of the sum rules presented here which makes this derivation somewhat more aesthetically pleasing. The quark model formulated by H. J. Lipkin [Phys. Rev. Letters **16**, 1015 (1966)] predicts $d\sigma(K^+n\rightarrow K^0p)/d\Omega=0$, in contrast to the prediction of the $SU(3)$ charge-exchange sum rule.