Vector-Meson Dominance and Current Algebra in the Parity-Violating Nonleptonic Decays of K Mesons and Hyperons

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As far as the parity-violating nonleptonic decays of K mesons and hyperons are concerned, many of the results obtained from the algebra of currents can be understood in a completely trivial manner using vector-meson dominance without recourse to the current commutation relations. In particular, we show that the current-algebraic approach applied to an octet Hamiltonian supplemented by the hypothesis of the universal spurion coupling leads to exactly the same physical consequences as the K^*-dominance model of Lee and Swift. We also indicate how a vector-meson-dominance model of nonleptonic decays may emerge out of a current-current interaction involving neutral as well as charged currents.

RECENTLY, several authors have pointed out striking similarities between the predictions based on Gell-Mann's current algebra and those based on vector meson dominance—specifically in the radiative decays of heavy mesons,[1] pion scattering,[2] and K_{e4} decay.[3] In this paper we wish to compare the current-algebraic results with the consequences of the vector-meson-dominance approach in $K_{2\pi}$ decay and s-wave (parity-violating) hyperon decays.

Using the current commutation relations,[4] partially conserved axial-vector current (PCAC)[5], and the $|\Delta\mathbf{T}|=\frac{1}{2}$ rule, Hara and Nambu[6] have derived the following relations for $K_{2\pi}$ decay[7]:

$$\begin{aligned} A(K^+\to\pi^++\pi^0;\,q(\pi^0)=0) &= -A(K^+\to\pi^++\pi^0;\,q(\pi^+)=0)\\ &= -iA(K_1{}^0\to\pi^++\pi^-;\,q(\pi^+)=0)\\ &= -iA(K_1{}^0\to\pi^++\pi^-;\,q(\pi^-)=0) \end{aligned} \tag{1a}$$

$$A(K^+\to\pi^++\pi^0;\,q(K^+)=0)=0\,. \tag{1b}$$

At first glance these relations look rather puzzling and even paradoxical; it is therefore natural to ask if there is any dynamical mechanism that may account for the peculiar behavior of the off-mass-shell $K_{\pi 2}$ amplitudes. Let us consider a model of $K_{\pi 2}$ decay based on Fig. 1. At the ρ-K and K^*-π junctions we insert matrix elements that follow from the effective Lagrangian density

$$2\lambda d_{6\alpha\beta}\partial_\mu P^\alpha V_\mu{}^\beta\,, \tag{2}$$

where P^α and V^α are, respectively, the αth component of the pseudoscalar and the vector meson octets in Gell-Mann's notation.[8] We are then led to

$$\begin{aligned} A(K^+\to\pi^++\pi^0) &= -(i\lambda f_V/2m_V{}^2)[q(\pi^+)^2-q(\pi^0)^2]\,,\\ A(K_1{}^0\to\pi^++\pi^-) &\\ &= (\lambda f_V/2m_V{}^2)[2q(K)^2-q(\pi^+)^2-q(\pi^-)^2]\,, \end{aligned} \tag{3}$$

where A on the mass shell is related to the $K_{2\pi}$ decay rate via

$$\Gamma(K\to\pi+\pi)=(|\mathbf{q}_{\text{c.m.}}|/8\pi m_K{}^2)|A(K\to\pi+\pi)|^2\,, \tag{4}$$

and f_V stands for the vector-meson coupling constant[9] $(f_V{}^2/4\pi\approx 2.5)$. We see that all the current-algebraic requirements (1a) and (1b) are indeed satisfied in our simple model.[10,11] Note also that when all three particles are on the mass shell, the two-pion decay mode of K_1 is forbidden in the $SU(3)$ limit $m_K{}^2=m_\pi{}^2$, in agreement with more general considerations based on CP invariance together with the assumption of the weak Lagrangian transforming like the sixth component of an octet.[12]

Having reproduced the current-algebraic results for $K_{2\pi}$ decay in such a simple manner, we are tempted to study the s-wave decays of hyperons using Fig. 2, where the γ_μ coupling of K^* to the baryons is assumed to be of

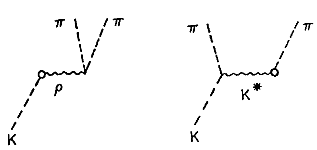

FIG. 1. Mechanism for $K_{2\pi}$ decay.

[1] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); M. Ademollo, Nuovo Cimento **46**, 156 (1966).

[2] Y. Tomozawa, Nuovo Cimento **46**, 707 (1966); J. J. Sakurai, Phys. Rev. Letters **17**, 552 (1966).

[3] L. J. Clavelli, Phys. Rev. **154**, 1509 (1967).

[4] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[5] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).

[6] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).

[7] These relations are expected to hold regardless of whether the weak-interaction Lagrangian is taken to be $d_{6\alpha\beta}J_\mu{}^\alpha J_\mu{}^\beta$ or $\bar{q}\lambda_6 q\pm i\bar{q}\lambda_7\gamma_5 q$.

[8] In the tensor notation the interaction (2) can be written as $\lambda\,\mathrm{Tr}(\lambda_6\{\partial_\mu P,V_\mu\})$. Note that $P=\lambda_\alpha P^\alpha/\sqrt{2}$, $V_\mu=\lambda_\alpha V_\mu{}^\alpha/\sqrt{2}$.

[9] In obtaining Eq. (3) we have assumed $SU(3)$ ($m_\rho=m_{K^*}$, $f_\rho=f_{K^*}$, etc.). At the end of the paper, however, we shall show that (3) must follow from our model even in the broken eightfold way.

[10] In verifying the current-algebraic requirements it is important to keep in mind that energy-momentum conservation holds even when mesons are off the mass shell.

[11] It is easily shown that when the vector-meson pseudoscalar-meson vertex is assumed to transform like the sixth component of an octet, a pseudoscalar meson pole of the type considered by C. Bouchiat and Ph. Meyer [Phys. Letters **22**, 198 (1966)] is absent in the $K_{2\pi}$ amplitude. In this connection it may be mentioned that we have attempted to understand (1a) using $if_{7\alpha\beta}\partial_\mu P^\alpha V_\mu{}^\beta$ in place of (2) without any success.

[12] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964); M. Gell-Mann, *ibid.* **12**, 155 (1964).

the pure F type.[13] We note that this decay mechanism has already been treated in the literature by Lee and Swift.[14–16] Defining the dimensionless s-wave decay amplitudes via

$$\Gamma(B\to B'+\pi^{\pm,0})_{s\text{ wave}} = (|\mathbf{q}_{\text{c.m.}}|/8\pi m_B^2)[(m_B+m_{B}')^2-m_\pi^2]|A(B_{\pm,0})|^2, \quad (5)$$

we get (in addition to the three $|\Delta\mathbf{T}|=\frac{1}{2}$ relations)

$$\frac{A(\Sigma_-^-)}{A(\Lambda_-^0)}=(\tfrac{2}{3})^{1/2}\frac{(m_\Sigma-m_p)}{(m_\Lambda-m_p)} \quad (6a)$$

$$\frac{A(\Xi_-^-)}{A(\Lambda_-^0)}=-\frac{m_\Xi-m_\Lambda}{m\Lambda-m_p} \quad (6b)$$

and

$$A(\Sigma_+^+)=0, \quad (7)$$

from which follows not only a "modified" Lee-Sugawara relation

$$-2\sqrt{2}(m_\Xi-m_\Lambda)^{-1}A(\Xi_-^-)=\sqrt{2}(m_\Lambda-m_p)^{-1}A(\Lambda_-^0)+\sqrt{3}(m_\Sigma-m_p)^{-1}A(\Sigma_-^-) \quad (8)$$

but also the original Lee-Sugawara relation[17]

$$-2\sqrt{2}A(\Xi_-^-)=\sqrt{2}A(\Lambda_-^0)+\sqrt{3}A(\Sigma_-^-) \quad (8')$$

when the Gell-Mann mass formula $2m_N+2m_\Xi-3m_\Lambda-m_\Sigma=0$ is assumed to be valid. We note that all the s-wave hyperon decay amplitudes are uniquely predicted when any one of them [say, $A(\Lambda_-^0)$] is given. The predictions of the K^* dominance model are compared to experiment in Table I, where the s-wave Σ decay and Ξ decay amplitudes predicted from Λ decay are displayed in column 2.[18] We see that (6a) is exactly satisfied while (6b) is satisfied to an accuracy of about 12%. The prediction (7) is, of course, in excellent agreement with observation.

Let us now compare these results based on K^* dominance with the predictions of the algebra of currents. About a year ago, Sugawara[19] and Suzuki[20] proved that the Lee-Sugawara relation, together with the vanishing of $A(\Sigma_+^+)$, follows from the algebra of currents provided that the $|\Delta\mathbf{T}|=\frac{1}{2}$ rule is assumed to be valid. More specifically, the current-algebraic approach relates the s-wave hyperon decay amplitudes to the weak-transition masses (which may be visualized as being due to the emission of a weak spurion) as follows:

$$\frac{A(\Sigma_-^-)}{A(\Lambda_-^0)}=-6^{1/2}\left(\frac{-D+F}{D+3F}\right),\quad \frac{A(\Xi_-^-)}{A(\Lambda_-^0)}=\frac{D-3F}{D+3F}, \quad (9)$$

where D and F refer to the couplings of the weak spurion to the baryons. Hara, Nambu, and Schechter[21,22] have speculated that the spurion is "universal" in the sense that the D-to-F ratio that appears in (9) is identical to the D-to-F ratio of the medium-strong (Gell-Mann–Okubo) mass splittings. Using $(m_\Xi-m_N)/(m_\Sigma-m_\Lambda)=-(3F/2D)$ and the Gell-Mann mass formula, it is easy to see that (9) is then equivalent to (6). Thus the algebra of currents supplemented by the hypothesis of the universal spurion coupling leads to exactly the same physical consequences as the K^* dominance model of Lee and Swift[14] as far as the s-wave hyperon decays are concerned.[23]

In the vector-meson dominance model we can relate $K_{2\pi}$ decay to hyperon decay provided that the K^* and $\bar{K}^*$ are coupled universally to the strangeness-changing components of the F spin in the same way as the ρ is

TABLE I. Predictions of the s-wave hyperon decay amplitudes.

	Experiment	Theory from Λ decay	Theory from K_1 decay
$\lvert A(\Lambda_-^0)\rvert$	3.4×10^{-7}	3.4×10^{-7} (input)	2.7×10^{-7}
$\lvert A(\Sigma_-^-)\rvert$	4.0×10^{-7}	4.0×10^{-7}	3.1×10^{-7}
$\lvert A(\Sigma_+^+)\rvert$	$(0\pm0.1)\times10^{-7}$	0	0
$\lvert A(\Sigma_0^+)\rvert$	$\begin{Bmatrix}(3.4\pm0.3)\times10^{-7}\\(2.5\pm0.4)\times10^{-7}\end{Bmatrix}$	2.8×10^{-7}	2.2×10^{-7}
$\lvert A(\Xi_-^-)\rvert$	4.4×10^{-7}	3.9×10^{-7}	3.1×10^{-7}

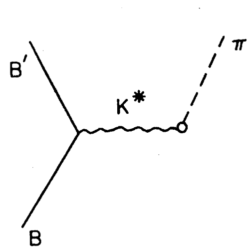

FIG. 2. Mechanism for s-wave hyperon decay.

[13] The $q_\mu\sigma_{\mu\nu}$ coupling of K^* makes no contribution in Fig. 2.

[14] B. W. Lee and A. R. Swift, Phys. Rev. **136**, B229 (1964).

[15] A similar model seems to have been proposed independently by J. Schwinger, Phys. Rev. Letters **12**, 630 (1964).

[16] See also K. Nishijima and L. F. Swank [Phys. Rev. **146**, 1161 (1966)] for a related proposal.

[17] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964).

[18] The experimental data is taken from J. P. Berge, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

[19] H. Sugawara, Phys. Rev. Letters **15**, 870, 997(E) (1965).

[20] M. Suzuki, Phys. Rev. Letters **15**, 986 (1965).

[21] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966).

[22] See also Riazuddin and K. T. Mahanthappa, Phys. Rev. **147**, 972 (1966).

[23] Very recently, G. S. Guralnik, V. S. Mathur, and L. K. Pandit (to be published) have also derived (6a), (6b), and (7) using the current-algebraic techniques and a weak Hamiltonian of the form $i(\lambda_s\partial_\mu j_\mu{}^7+\lambda_p\partial_\mu j_{5\mu}{}^7)$. To the extent that the matrix elements of $j_\mu{}^7$ are dominated by K^*, it is easy to understand why their results for s-wave decays are identical to the results based on K^* dominance. Now, in the standard quark model where the $SU(3)$ breaking part of the strong interaction Lagrangian is as simple as $-\delta m\bar{q}\lambda_8 q$, the divergence of $j_\mu{}^7$ can be shown to be proportional to just $\delta m\bar{q}\lambda_6 q$; thus the model of Guralnik *et al.* is essentially the same as the universal spurion model with a parity-conserving weak Lagrangian of the form $\bar{q}\lambda_6 q$. In this manner we see that the equivalence of the K^* dominance model to the current-algebraic approach based on the universal spurion model is not accidental.

coupled universally to the isospin. The strength of the $K^*\bar{K}\pi$ coupling is now related to that of the $K^*\bar{N}\Lambda$ coupling, etc.; hence we obtain

$$\frac{A(K_1{}^0 \to \pi^+ + \pi^-)}{A(\Lambda_-{}^0)} = \left(\frac{2i}{\sqrt{3}}\right)\frac{(m_K{}^2 - m_\pi{}^2)}{(m_\Lambda - m_p)}. \qquad (10)$$

All the s-wave hyperon decay amplitudes are unambiguously predicted once the K_1 decay rate is given. The theoretically predicted hyperon-decay amplitudes are shown in column 3 of Table I; they seem to be systematically smaller by about 20%.

Let us now go back to Eq. (1). We recall that the current-algebraic derivation of (1) exploits the fact that the amplitude for $K \to \pi$ is related to the $K_{2\pi}$ amplitude in the limit where the four-momentum of one of the pions goes to zero[6,24,25]:

$$(1/2c_\pi)A(K^+ \to \pi^+) = -A(K_1{}^0 \to \pi^+ + \pi^-;\, q(\pi^+) = 0). \qquad (11)$$

Before we examine whether we can derive (11) itself without recourse to the current commutation relations, we find it necessary to speculate on the dynamical origin of the interaction (2). To this end we first write down a current-current interaction that satisfies the $|\Delta\mathbf{T}| = \frac{1}{2}$ rule:

$$\begin{aligned}\mathcal{L}_{\mathrm{N.L.}} = (G_{\mathrm{N.L.}}/\sqrt{2})[&(J_\mu{}^4 - iJ_\mu{}^5)(J_\mu{}^1 + iJ_\mu{}^2) \\ &+ (J_\mu{}^1 - iJ_\mu{}^2)(J_\mu{}^4 + iJ_\mu{}^5) \\ &\qquad - 2J_\mu{}^6\{J_\mu{}^3 + (1/\sqrt{3})J_\mu{}^8\}] \\ = 2(G_{\mathrm{N.L.}}/\sqrt{2}&)d_{6\alpha\beta}J_\mu{}^\alpha J_\mu{}^\beta, \end{aligned} \qquad (12)$$

where[26]

$$J_\mu{}^\alpha = j_\mu{}^\alpha + j_{5\mu}{}^\alpha. \qquad (13)$$

Now, when we take the matrix element of $j_\mu{}^\alpha$ between a vector meson state and the vacuum state, $j_\mu{}^\alpha$ may as well be replaced by the corresponding vector meson field operator[27]

$$j_\mu{}^\alpha \to (m_V{}^2/f_V)V_\mu{}^\alpha. \qquad (14)$$

Similarly, for the axial vector current we take

$$j_{5\mu}{}^\alpha \to c_\pi \partial_\mu P^\alpha, \qquad (15)$$

where c_π is the pion decay constant, numerically equal to 94 MeV [and approximately equal to $-(G_A/G\cos\theta)m_N/G_{\pi NN}$]. We propose to regard the parity-violating vector-meson–pseudoscalar-meson interaction (2) as a phenomenological manifestation of the basic current-current interaction (12) in accordance with the rules (14) and (15). With this assumption our earlier defined constant turns out to be related to $G_{\mathrm{N.L.}}$ via

$$\lambda = 2(G_{\mathrm{N.L.}}/\sqrt{2})c_\pi m_V{}^2/f_V, \qquad (16)$$

and we now see why (3) is insensitive to violation of unitary symmetry.[28] If we subscribe to the rules (14) and (15), it is natural to consider the parity-conserving interaction

$$2(G_{\mathrm{N.L.}}/\sqrt{2})d_{6\alpha\beta}c_\pi{}^2\partial_\mu P^\alpha \partial_\mu P^\beta. \qquad (17)$$

Assuming that the amplitude for $K \to \pi$ is due solely to (17), we get

$$\begin{aligned}A(K^+ \to \pi^+) &= -2(G_{\mathrm{N.L.}}/\sqrt{2})c_\pi{}^2 q(K^+)\cdot q(\pi^+) \\ &= -2(G_{\mathrm{N.L.}}/\sqrt{2})c_\pi{}^2 q(K^+)^2. \end{aligned} \qquad (18)$$

From (3), (16), and (18), we can indeed understand the current-algebraic result (11) without using the current commutation relations.

In closing we would like to comment on the magnitude of $G_{\mathrm{N.L.}}$ that appears in (12). From the K_1 decay rate and the Λ decay rate we obtain $G_{\mathrm{N.L.}}$ as follows:

$$\begin{aligned}G_{\mathrm{N.L.}} &= 1.1\times10^{-5}/m_p{}^2 \quad (\text{from } K_1 \text{ decay}) \\ &= 1.4\times10^{-5}/m_p{}^2 \quad (\text{from } \Lambda \text{ decay}). \end{aligned} \qquad (19)$$

These numbers are remarkably close to the universal Fermi constant $G = 1.0\times10^{-5}/m_p{}^2$. Within the framework of a model in which a $|\Delta\mathbf{T}| = \frac{1}{2}$ interaction is constructed by adding extra neutral currents to the $J_\mu{}^{(\mathrm{ch})\dagger}J_\mu{}^{(\mathrm{ch})}$ interaction, this agreement is disturbing. The reason is that in such a model $G_{\mathrm{N.L.}}$ is expected to be $\sin\theta\cos\theta$ times $10^{-5}/m_p{}^2$, whereas our results based on vector-meson dominance and the substitution rules (14) and (15) indicate that, in sharp contrast with the semileptonic processes, the Cabibbo angle is *not* needed in the nonleptonic decays of hyperons.[29] It appears to us that if the basic hadronic interaction is really of the type $d_{\gamma\alpha\beta}J_\mu{}^\alpha J_\mu{}^\beta$ with $\gamma = 3$, 6, and 8, there is no reason *a priori* why the charged current $a(J_\mu{}^1 + iJ_\mu{}^2) + b(J_\mu{}^4 + iJ_\mu{}^5)$ should be normalized to "unit length" in the sense of Cabibbo.[30]

It is a pleasure to thank Dr. J. Schechter for helpful conversations.

Note added in manuscript. After this paper was completed, a paper by W. W. Wada [Phys. Rev. **138**, 1489 (1965)] came to our attention which also relates the K_1 decay amplitude to the s-wave hyperon decay amplitudes within the framework of the Lee-Swift model.

[24] M. Suzuki, Phys. Rev. **144**, 1154 (1966).

[25] Note that the constant c defined in Ref. 6 is equivalent to our $2c_\pi/m_\pi{}^2$. The first line of Eq. (22) of Ref. 6 should be multiplied by i if the invariant amplitudes are defined in the usual manner.

[26] Our currents are normalized so that $j_\mu{}^\alpha = i\bar{q}\gamma_\mu(\lambda_\alpha/2)q$ and $j_{5\mu}{}^\alpha = i\bar{q}\gamma_\mu\gamma_5(\lambda_\alpha/2)q$ in the quark model.

[27] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[28] The point is that $m_V{}^2$ appearing in the vector-meson–pseudoscalar-meson junction cancels with $1/m_V{}^2$ arising from the vector-meson propagator. A similar cancellation occurs for f_V.

[29] Contrast this with Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966); Y. Hara (to be published).

[30] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).