Investigation of s-Wave $\pi\pi$ Interaction from the Reaction $\pi^- p \to \pi^0\pi^0 n$

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Measurements of the reaction $\pi^- p \to \pi^0\pi^0 n$ at incident momenta of 1.715, 1.889, 2.071, 2.265, and 2.460 GeV/c are presented. They show two important contributions to this reaction: (i) production of the 1238-MeV isobar and a recoil pion, and (ii) peripheral production of the dipion system. We explore the consequences of assuming the latter to be due to the reaction $\pi^-\pi^+ \to \pi^0\pi^0$. This reaction has the advantage that it cannot take place in an $I=1$, $l=1$ state and so should be useful for a study of the s-wave $\pi\pi$ interaction. The measured $\pi\pi$ mass spectrum from peripheral production shows no evidence for any of the relatively narrow s-wave resonances which have been proposed, but shows a broad peak centered at about 600 and about 400 MeV wide. Some evidence that this peak may be a broad s-wave resonance is deduced from a comparison with information about the s-wave interaction deduced from the $\pi^-\pi^+$ asymmetry in the reaction $\pi^- p \to \pi^-\pi^+ n$; conclusions are reached which agree with a number of other experimental results. Against this resonance interpretation we find that a calculation of the peripheral production cross section expected for such a resonance, made by assuming the one-pion-exchange formula with the Ferrari-Selleri form-factor modification, is approximately three times greater than our measured cross section. This may mean that the modification of the one-pion-exchange formula is larger for s-wave $\pi\pi$ interaction than it is for p and d waves.

1. INTRODUCTION

INFORMATION about the pion-pion interaction has been deduced from various sources, of which a major one has been peripheral production of pions in pion-nucleon collisions. This latter source has established an $l=1$, $I=1$ resonance, the ρ meson at a $\pi\pi$ mass close to 750 MeV.[1] Information about the s-wave $\pi\pi$ interaction has been harder to get; a large number of different and not completely compatible conclusions, which we shall summarize in this section, have been drawn from a number of different experiments. In this paper we will present measurements of the reaction

$$\pi^- p \to \pi^0\pi^0 n \tag{1}$$

in which we find strong indication of peripheral production of the two pions, which we take to be due to the reaction

$$\pi^-\pi^+ \to \pi^0\pi^0. \tag{2}$$

This reaction has the considerable advantage that it cannot occur in an $I=1$ state, and can therefore only occur in states of even orbital angular momentum. This reaction should therefore be very suitable for studying the s-wave $\pi\pi$ interaction. Our results argue strongly against there being any relatively narrow resonance in the s-wave $\pi\pi$ interaction with a mass between threshold and 1000 MeV (by relatively narrow we mean having a width between 100 and about 30 MeV: Resonances narrower than about 30 MeV would be lost with our resolution). We see evidence for a broad enhancement in the s-wave $\pi\pi$ cross section stretching from threshold to a mass of about 800 MeV. We will present some evidence that this may be a very broad s-wave resonance. Some preliminary results of this work, based on part of the data, have been published previously.[2] This experiment is part of a general study of neutral final states produced by bombarding protons with π^- mesons of several momenta around 2 GeV/c: Some other results of this work have also been published.[3]

We will now summarize the existing information about the s-wave $\pi\pi$ interaction. Most of this can be divided into three parts: for mass regions near threshold, around 400 MeV, and around 750 MeV. Information near threshold has been deduced from two sources: observation[4] of a final-state interaction in the reaction

$$p+d \to \mathrm{He}^3+\pi\pi,$$

and a dispersion-relation analysis[5] of low-energy pion-nucleon scattering. Both sources indicate a relatively strong s-wave $\pi\pi$ interaction at threshold. If the analyses assume that the $I=0$ s-wave scattering at threshold can be represented by a scattering length, they find for this scattering length values of[4] $(2\pm1)\hbar/\mu c$

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[1] G. Puppi, Ann. Rev. Nucl. Sci. 13, 287 (1963).

[2] I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, W. S. C. Williams, and A. S. Carroll, Nuovo Cimento 39, 979 (1965).
[3] A. S. Carroll, I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, and W. S. C. Williams, Phys. Rev. Letters 16, 288 (1966).
[4] N. E. Booth and A. Abashian, Phys. Rev. 132, 2314 (1963).
[5] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. 128, 1881 (1962).

and[5] $(1.3\pm0.4)h/\mu c$, where μ is the pion mass (the latter of these two results corresponds to a phase shift which rises very rapidly, within a few MeV of threshold, and then levels off at a value of $30\pm6°$). There could be some small difference in the conclusions if some other parametrization of the $I=0$ s-wave $\pi\pi$ scattering were to be used, but the indication of a strong interaction near threshold would presumably remain.

Attempts have also been made to deduce information about $\pi\pi$ scattering at threshold from measurements of pion production in low-energy pion-nucleon collisions (see Booth and Abashian[4] for a summary). However, it has been shown (see, for example, Bransden *et al.*[6]) that at these energies pion production is proceeding almost entirely through one angular-momentum and isotopic-spin state of the $N\pi\pi$ system: the P_{11} state. The interaction is strong in this state and may even be resonant. This argues against a simple mechanism, based on a $\pi\pi$ collision, at these energies, and suggests instead a more general interaction of all three particles with each other. Thus it would seem difficult to obtain information about the threshold $\pi\pi$ interaction from such measurements.

Evidence for a strong $I=0$ s-wave interaction at a $\pi\pi$ mass around 400 MeV has been deduced[7] from the nonuniformity in the density distribution of the events found on the Dalitz plot for the decay of the η meson. The most recent results[8] have been fitted with an s-wave resonance, called the σ meson, of a mass 392 MeV and width 88 MeV. However, this analysis does not seem to demonstrate the need for a resonance of this width, which is scarcely smaller than the range of $\pi\pi$ mass which can be explored in η-meson decay. In support of this remark we note that no particular peaking has been observed in this region of $\pi\pi$ mass[9] in the reaction

$$\pi^- p \to \pi^-\pi^+ n\,. \tag{3}$$

It would seem possible that these results could also be due to a strong s-wave interaction over a wider range of $\pi\pi$ mass. It has, however, been noted that this mechanism for η-meson decay fails to explain the observed branching ratios, so one cannot be sure that one is obtaining any information at all about s-wave $\pi\pi$ interaction.

Around 750 MeV a strong, and possibly resonant, s-wave interaction has been deduced from the asymmetry in the decay of the ρ^0 meson produced in reaction (3): The asymmetry was ascribed to interference between s-wave $\pi\pi$ scattering and the p-wave resonance. As this asymmetry scarcely changes as the $\pi\pi$ mass is varied across the region of the ρ meson, one is led to the conclusion that the s-wave amplitude involved must be largely imaginary. This behavior contrasts with that of the asymmetry in the decay of the charged ρ mesons produced in the reactions:

$$\pi^- p \to \pi^-\pi^0 p\,, \tag{4}$$

$$\pi^+ p \to \pi^+\pi^0 p\,, \tag{5}$$

which asymmetry changes sign as the $\pi\pi$ mass moves through that of the peak of the ρ meson. (A summary of all these results is given by Baton and Regnier.[10]) Such a change in sign of the asymmetry on going through the energy of the p-wave resonance suggests that a largely real s-wave amplitude is involved in reactions (4) and (5). These results are compatible with a small $I=2$ s-wave phase shift, producing a small and largely real s-wave amplitude in reactions (4) and (5), and a large $I=0$ s-wave phase shift, possibly resonant, which combines with $I=2$ amplitude to produce a largely imaginary amplitude in reaction (3). Detailed analyses have been made by two groups. Durand and Chiu[11] use a one-pion-exchange model with absorption and deduce evidence for an s-wave $\pi\pi$ resonance at a mass of 730 MeV and a width of 90 MeV, which they call the ϵ^0 meson. Wolf[12] has used the one-pion-exchange model with the empirical corrections due to Ferrari and Selleri[13] (ascribed by them to form factors) and deduces an s-wave resonance at a mass of 740 MeV with a width of 90 MeV. However, Wolf deduced a width of 170 MeV for the ρ meson in his analysis, and then combined this with the observed variation of the ρ^0-decay asymmetry to deduce the s-wave amplitude. We remark that if the ρ meson has a width smaller than 170 MeV, for which there is considerable evidence, the width deduced for an s-wave resonance would be considerably greater.

In the simplest one-pion-exchange model, the ρ^0 meson should be produced only in its $m=0$ substate. Then the decay distribution would be just $\cos^2\theta$, so that, if the observed angular distribution has the form $a_0+a_1\cos\theta+a_2\cos^2\theta$, it would seem that information about the s-wave $\pi\pi$ scattering could also be obtained from the constant term a_0. However, it has been found[10] that if a_0 is plotted as a function of $\pi\pi$ mass there is a strong peak with closely the same mass and width as is found in the similar plot of the coefficient of the $\cos^2\theta$ term, a_2. This implies that there is considerable depolarization of the ρ meson from the polarization expected from the simplest one-pion-exchange model, a depolarization which seems to be explained if absorption effects are added to the one-pion-exchange model.[11] One experiment has reported[14] a further peaking of the a_0 coefficient which would indicate an s-wave resonance at a mass of 720 MeV with a width of 50 MeV. Not only

[6] B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, Phys. Rev. **139**, B1566 (1965).
[7] L. M. Brown and P. Singer, Phys. Rev. **133**, B812 (1964).
[8] F. S. Crawford *et al.*, Phys. Rev. Letters **13**, 421 (1964).
[9] L. D. Jacobs and W. Selove, Phys. Rev. Letters **16**, 669 (1966).
[10] J. P. Baton and J. Regnier, Nuovo Cimento **36**, 1149 (1965).
[11] L. Durand and Y. T. Chiu, Phys. Rev. Letters **14**, 329 (1965); **14**, 680 (1965).
[12] G. Wolf, Phys. Letters **19**, 328 (1965).
[13] E. Ferrari and F. Selleri, Phys. Rev. Letters **7**, 387 (1961); F. Selleri, Phys. Letters **3**, 76 (1962).
[14] V. Hagopian *et al.*, Phys. Rev. Letters **14**, 1077 (1965).

does this width seem inconsistent with widths that are deduced from the asymmetry in the angular distribution, but there is also no evidence for such a peaking in another experiment which should also have seen it: Clark *et al.*[15] studied the $\pi^+\pi^-$ mass spectrum from reaction (3) for 6000 events with $|\cos\theta|<0.2$ [although it can be remarked that this experiment was at a lower incident momentum (1.5 GeV/c) than those of Hagopian *et al.*[14] (2.75 and 3.0 GeV/c), and that this may be the cause of the difference].

One further result has been interpreted[16] as due to an s-wave $\pi\pi$ resonance: observation of a peaking in the recoil neutron spectrum from the reaction

$$\pi^- + p \rightarrow n + (X \rightarrow \text{neutral particles})$$

at an incident π-meson momentum of 1.52 GeV/c. The peak corresponds to a meson of a mass of 700 MeV with a width not greater than about 50 MeV. This was named the S^0 meson. Again it can be said that it is surprising, if this peaking is due to an s-wave $\pi\pi$ resonance, that its decay into $\pi^+\pi^-$ was not also seen in the experiment of Clark *et al.*,[15] which was at almost the same energy. The width also seems smaller than is consistent with the observed asymmetry in the decay of the ρ^0 meson.

Information about s-wave $\pi\pi$ scattering has also been deduced from a dispersion-relation analysis[17] of backward pion-nucleon scattering. These authors deduce evidence for a broad s-wave $\pi\pi$ resonance, with a width of the order of 400 MeV.

These different conclusions which have been reached about the s-wave $\pi\pi$ interaction are not all compatible with each other. Some of them do at least indicate that there may be a strong s-wave interaction in all three mass ranges.

We shall return to the ρ^0-decay asymmetry in Sec. 5, making a more phenomenological analysis to compare with our own results.

2. EXPERIMENTAL METHOD

The pion beam was produced by bombarding an internal target in the proton synchrotron Nimrod. This beam traversed the hydrogen target of the University College–Westfield College elastic scattering experiment[18] and was then refocused onto our target, which was a vertical cylinder of 3-cm diameter. The beam was defined by three scintillation counters in coincidence, and then entered the target through a hole in a vase-shaped veto counter surrounding the target. A coincidence between the three beam counters together with no count in the veto counter provided the trigger for the spark chambers. The spark chambers formed five of the six faces of a cube with the target at its center, so that our solid angle for detection of gamma rays was large. The spark chambers had brass plates which were 2.3 radiation lengths thick; the detection efficiency was measured to be $(79\pm1)\%$ for gamma rays normally incident on the chambers. In each chamber the brass plates were preceded by two aluminum foil plates so that charged particles from failures of the veto counter or, as was much more probable in practice, from interactions in the final beam counter could be distinguished from the desired events in which only gamma rays were produced. Each chamber was viewed from two orthogonal directions. The total of ten views of the five chambers was collected together, by a system of field lenses and twenty-eight mirrors, to be recorded by one camera.

Measurements were made for roughly equal amounts of incident beam at each of five momenta: 1.715, 1.889, 2.071, 2.265, and 2.460 GeV/c. Measurements at all five momenta are averaged together in the present work. 1 700 000 photographs were taken, and of these 70 000 showed gamma-ray showers, in numbers ranging from one to ten. In this paper we are concerned with those events in which four gamma rays were produced. A paper containing our results for the remaining events is being prepared.

The photographs were scanned and measured on a digitized protractor. Computer programs reconstructed the event in space and then weighted each event found with the probability that an event of that configuration, but at any azimuthal angle about the incident beam, would have been detected. This detection probability is essentially the product of the probability of each gamma ray converting and the probability of gamma rays not escaping through the open sixth side of the cube as the event is rotated to different azimuthal angles. We then have to correct for the fact that some of the pictures showing four gamma rays are due to events in which more than four gamma rays are produced. We can deduce this background, as far as it is due to events in which gamma rays pass through the chambers without converting or go through the open sixth side of the cube, from those pictures in which more than four gamma rays are seen. In all the results we present here, this background has been determined in this way and subtracted. There is a further background due to low-energy gamma rays which produce showers too small to be recognized.

The failures of gamma rays either to pass through our spark chambers, or, in passing through, to convert, are taken into account by our weighting procedure. It is felt that the effect of nonrecognition of low-energy showers has a negligible effect on the data presented here. This is because with our spark-chamber plate thickness of 3.41 g/cm^2 we estimate our energy cutoff to be in the region 15–20 MeV. Therefore, not more than 3% of all π^0's will decay asymmetrically enough to give a nonrecognizable gamma ray. Furthermore, our kine-

[15] A. R. Clark, J. H. Christenson, J. W. Cronin, and R. Turley, Phys. Rev. **139**, B1556 (1965).
[16] M. Feldman *et al.*, Phys. Rev. Letters **14**, 869 (1965).
[17] C. Lovelace, R. M. Heinz, and A. Donnachie, Phys. Letters **22**, 332 (1966).
[18] E. H. Bellamy *et al.*, Proc. Roy. Soc. (London) **A289**, 509 (1966).

matic analysis routine (see below) rejects such asymmetric decays to an extent that would mean no loss of events unless both the cutoff energy was about 60 MeV *and* the π^0 was virtually at rest in the laboratory. This latter remark implies that since we are dealing here with peripheral events, we have a further argument against any influence from the one uncertainty which our weighting procedure cannot handle.

3. KINEMATICAL ANALYSIS OF FOUR GAMMA-RAY EVENTS

As we know the directions of the four gamma rays but not their energies, and know nothing about the energy or angles of the neutron, we lack seven pieces of kinematic information. On the other hand, we have four constraint equations of energy and momentum and the masses of the two neutral pions, a total of six equations in all, so that it would seem that these events cannot be solved. However, we can proceed because in the decay of a monoenergetic pion there is a very strong peaking in the distribution of opening angles between the two gamma rays at the minimum value allowed kinematically for that pion energy. Thus this opening angle gives a good measure of the energy of the parent pion in a large proportion of decays. At the same time the opening angles are sufficiently small that in a large proportion of events the two pairs of gamma rays from the two pions from reaction (1) are well separated and the identification of which gamma rays should be paired together is obvious. For these events, the bisector of the angle between the two gamma rays in either pair is then a good estimate of the pion direction, so that we have estimates of the directions and energies of both pions, and lack only the angles and energy of the neutron. As we then also have four constraint equations of conservation of energy and momentum, we are now overdetermined by one constraint. Even though these estimates of energies and directions of the two pions are somewhat inaccurate, they are good enough to enable us to reject a large part of the events due to incorrect pairings of gamma rays and to backgrounds

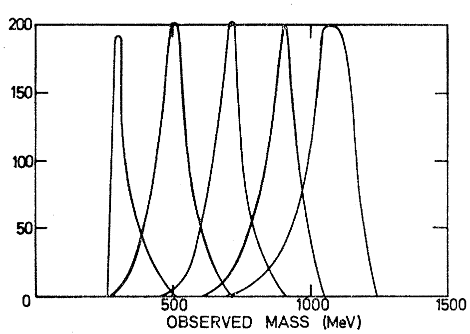

FIG. 1. $\pi\pi$ mass spectra deduced by the kinematical-analysis procedure for events generated by Monte Carlo procedure for reaction $\pi^- p \to n(X^0 \to \pi^0\pi^0)$ for hypothetical X particles of masses of 300, 500, 700, 900, and 1100 MeV, and an incident pion momentum of 2.07 GeV/c.

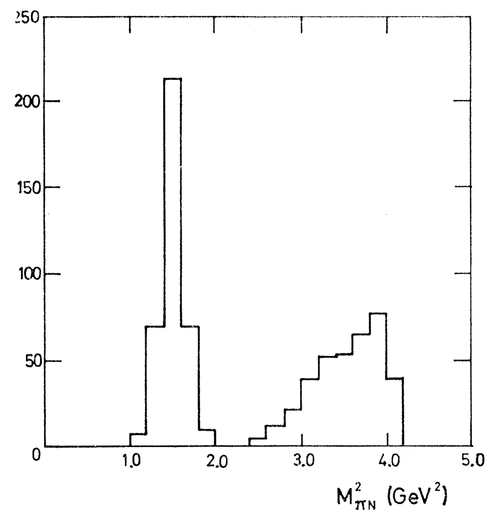

FIG. 2. πn mass spectra deduced by the kinematical-analysis procedure for events generated by Monte Carlo procedure for reaction $\pi^- p \to \pi^0(N^{*0} \to n\pi^0)$ for an N^* of mass 1238 MeV and incident pion momentum of 2.07 GeV/c. For each event, both possible πn masses were calculated: A count of half-weight was then added into the histogram for each mass.

[for example, events in which the four gamma rays observed were not from reaction (1)]. Thus a systematic analysis procedure was developed (a detailed account has been given in a Rutherford Laboratory report[19]) which should be correct for a large proportion of events. What this procedure does when confronted with all events involving two neutral pions was then found by applying it to events, generated by the Monte Carlo method, due to the reactions

$$\pi^- p \to n(X^0 \to \pi^0\pi^0)\,, \tag{6}$$

$$\pi^- p \to \pi^0(N^{*0} \to n\pi^0)\,, \tag{7}$$

for various masses of X^0 and N^{*0}. As a result of this Monte Carlo study, it was found useful to apply cuts which meant that only approximately 75% of the events were accepted, as we then rejected a large proportion of those events for which the procedure would produce very bad results. (The proportion rejected as a function of mass of X^0 or N^{*0} was determined from the Monte Carlo calculations, and used in correcting the results of the experimental analyses to obtain cross sections.) Some results of these Monte Carlo calculations are shown in Figs. 1, 2, and 3. Figure 1 shows, that if we assume that in reaction (6) a particle X^0, of definite mass, is produced, then the resulting peak in the $\pi\pi$ mass distribution deduced is within ± 15 MeV of the true mass, and the peak has a full width at half-height of above 100 MeV: This is therefore our mass resolution. Figure 2 shows the $n\pi^0$ mass distribution for events produced in reaction (7) with an N^* mass of 1238 MeV, and brings out another problem. As our

[19] A. S. Carroll, N. Middlemas, and W. S. C. Williams, Rutherford Laboratory Report No. RHEL/R 104 (unpublished).

TABLE I. Summary of number of events with four or more gamma rays.

Number of gamma rays	4	5	6	>6
	Raw data			
Total No. of pictures:				
Full target	5757	2012	720	298
Empty target	1502	540	181	76
Events/incident $10^6\pi$ (full target–empty target)	26.9±0.8	9.2±0.5	3.45±0.26	1.4±0.18
	Final weighted events/(10^6 incident π)			
Full target, no backgrounds subtracted	164.44	81.04	37.00	26.46
Full target, background due to higher multiplicities	82.55	36.87	25.33	13.83
Empty target, no backgrounds subtracted	69.76	36.99	16.33	12.57
Empty target, background due to higher multiplicities	41.33	12.40	17.21	4.08
Final result, all backgrounds due to empty target and higher multiplicities subtracted	53.5±6.7	19.6±6.9	12.5±7.0	4.1±8.4

two pions are identical, we can pair either of them with the neutron and calculate the corresponding $n\pi^0$ mass. In our analysis we have calculated both possible $n\pi^0$ masses for each event and given each of these possibilities half the total weight for the event. As a result we see in Fig. 2 that, for events generated with an N^* mass of 1238 MeV, there is a narrow peak at that mass (showing the good mass resolution from our analysis procedure) and a second broader peak due to the projection of the other N^* band on the Dalitz plot (see the Dalitz plot for this reaction shown in Fig. 4). Figure 3 shows that the estimated direction of a dipion X^0 of unique mass produced in reaction (6) is found to be very close to the true direction.

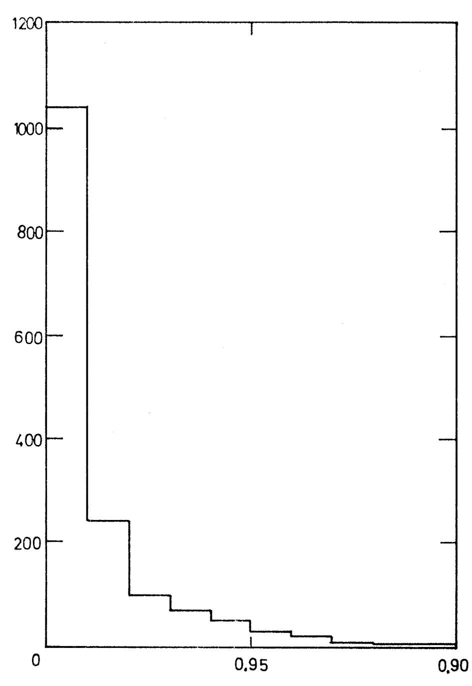

FIG. 3. Results of the kinematical-analysis procedure when applied to events generated by Monte Carlo procedure for reaction $\pi^- p \rightarrow n(X^0 \rightarrow \pi^0\pi^0)$. Histogram of $\cos\epsilon$, where ϵ is the angle between the true direction of motion of X in the over-all c.m. system and the direction estimated by the analysis procedure.

4. EXPERIMENTAL RESULTS

In thinking of statistical accuracies, many people are used to thinking in terms of numbers of events detected. Such a procedure does not give any useful indication in our experiment because of the magnitudes of the backgrounds from incomplete detection of higher multiplicities which have fed into our 4γ data. As a result of this, a meaningful assessment is impossible without taking into account the decreasing detection efficiency of our apparatus for higher multiplicity events, the nature of the kinematical analysis procedure, and the cuts imposed to select our events. All these effects depend strongly on the dynamical variables involved so that these backgrounds have to be determined on an event-by-event basis as described above. Thus the accuracy of the determination of these backgrounds in our 4γ data is determined by the statistical accuracy of our higher multiplicity data. As a result, the statistical accuracy of our final result is much worse than the statistical accuracy of our raw 4γ data, as is illustrated in Table I. The last row in Table I is our deduction of the number of events we would have detected if our apparatus had subtended 4π sr at the target with a detection efficiency of 100% and the target had been pure hydrogen.

Our $\pi^0\pi^0$ fitting procedure does not accept all these 4γ events partly because it only fits about three-quarters of true $\pi^0\pi^0$ events and partly because there is possibly some residue of events which are not $\pi^0\pi^0$ (these will be discussed in a later paper). Numbers of weighted $\pi^0\pi^0$ events per 10^6 incident pions, both total and after cuts, are (another correction factor for the inefficiency of our $\pi^0\pi^0$ fitting procedure has been applied to these numbers, so that they correspond to all $\pi^0\pi^0$ events of these classes):

(i) Total $\pi^0\pi^0$. 41.6±3.0

(ii) Events in peripheral peak, from which those with $1.2 < M_{\pi N}^2 < 1.7$ have been removed. 5.62±0.60

(iii) No. of events in (ii) which fall in region $0.4 < M_{\pi\pi} < 0.8$. 4.32±0.59

The statistical accuracy of the selected events above

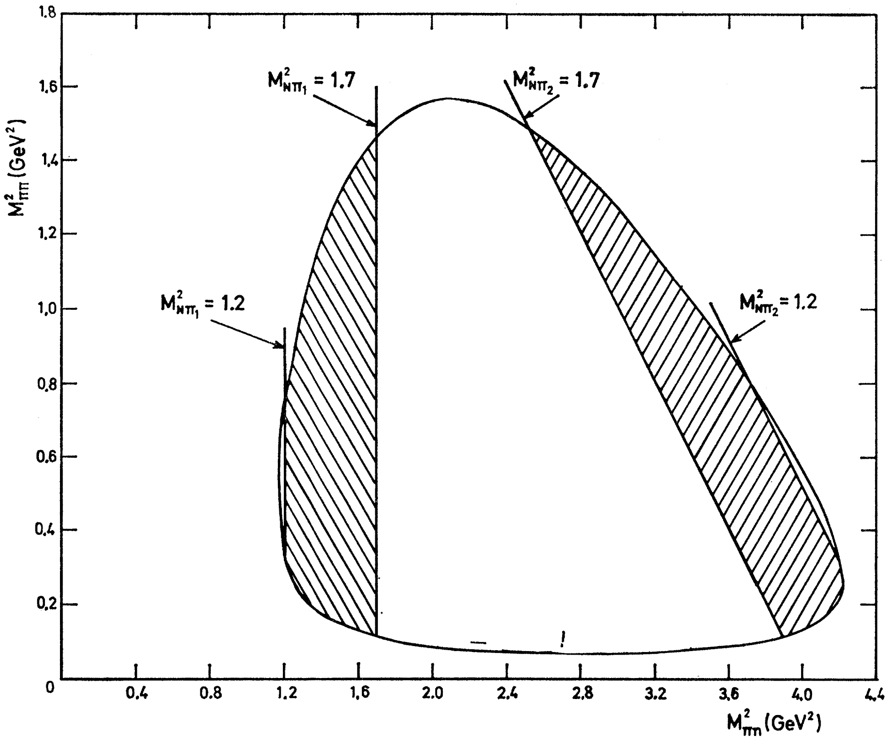

FIG. 4. Kinematical limits of the Dalitz plot for the reaction $\pi^- p \to \pi^0\pi^0 n$ at an incident pion momentum of 2.071 GeV/c. The bands $1.2 < |M_{\pi n}{}^2| < 1.7$ GeV² for either πn combination, whose contents were removed to reject events due to production of the $I=\frac{3}{2}, J=\frac{3}{2}$ isobar, are shown cross-hatched.

relative to the unfitted 4γ events reflects the fact that only a small proportion of the background events can be fitted as $\pi^0\pi^0$ events. The ratio of the background to the foreground contribution falls even further on applying cuts to select events in regions where the foreground is particularly concentrated. For these reasons we have made no further attempt to quote the numbers of events on which our results are based, but have instead kept track of the statistical uncertainties throughout. In all histograms, the backgrounds were subtracted on a bin-by-bin basis, and the uncertainties quoted on the counts in each bin were compiled accordingly.

In Fig. 5 we show the $M_{\pi N}{}^2$ distribution for all our analyzed events. This shows a strong peaking, which we take to be due to production of the $I=\frac{3}{2}, J=\frac{3}{2}$ isobar of mass 1238 MeV in reaction (7). We have presented the angular distribution for production of this N^*, noting a possible relation to the angular distribution for charge-exchange elastic scattering, in a previous paper.[3] Here we have to remove events due to production of this N^* if we wish to study peripheral $\pi\pi$ interactions. We do this by rejecting all events for which

$$1.2 \text{ GeV}^2 < M_{\pi N}{}^2 < 1.7 \text{ GeV}^2$$

for either pion-nucleon combination. We then treat the remaining events as if they were due to reaction (6),

$$\pi^- p \to n(X^0 \to \pi^0\pi^0),$$

and deduce the angular distribution for production of the hypothetical X^0, which we show in Fig. 6. This shows a very strong forward peak tailing off into an approximately isotropic distribution. This forward-angle peak is typical of peripheral reactions: The first bin in the angular distribution contains events with momen-

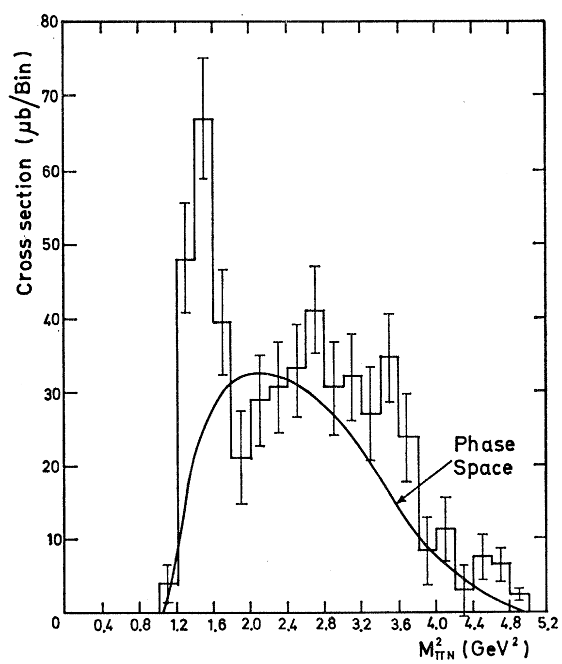

FIG. 5. Observed $\pi^0 n$ mass distribution: mean of results for all five incident momenta. The phase-space curve is the sum of phase-space curves at the five momenta, assuming that cross sections do not vary with momentum. Each $\pi^0 n$ pair has been given a weight of one-half.

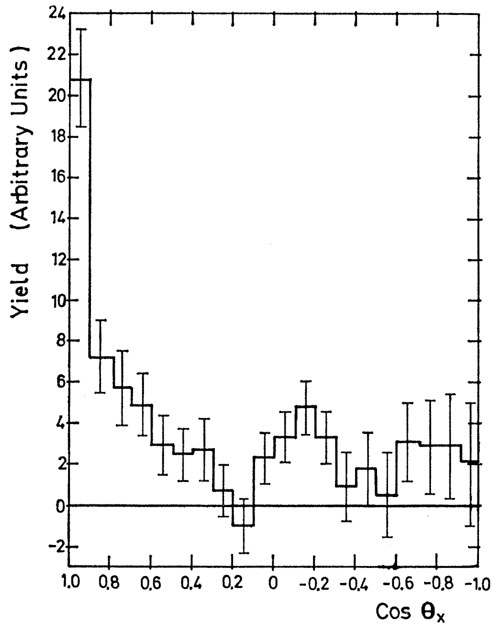

FIG. 6. Observed angular distribution of dipions from the reaction $\pi^- p \to \pi^0\pi^0 n$: Data from which events with $1.2<M_{\pi n}^2<1.7$ GeV, for either πn combinations, have been removed. No correction has been made here for the removal of these events.

tum transfers to the neutron $\Delta^2<7.6\mu^2$ (for almost all possible masses of X^0).

We now examine the $\pi\pi$ mass distribution for events contained in the forward peak. We correct here for the peripheral events which are rejected by the cuts on πn mass by multiplying the number of events remaining in each bin of the $\pi\pi$ mass distribution after these cuts have been applied by the ratio: (total area of strip of Dalitz plot for the chosen $\pi\pi$ mass)/(area of the part of this strip outside N^* bands). In doing this we are assuming that for each possible $\pi\pi$ mass the hypothetical X^0 decays isotropically. To show that this assumption is not unreasonable, Fig. 7 gives the $M_{\pi N}^2$ distribution for those events that fall outside the N^* bands. The phase-space curve shown there is a sum of five phase-space curves, which we calculated by taking the removal of the N^* bands into account, added assuming the cross section for reaction (6) is equal at all our five incident momenta, and then normalized to have the same total cross section as the data shown. A similar phase-space curve calculated on the assumption that the cross section varies as the inverse square of the incident momentum is barely distinguishable from that shown. The experimental results are consistent with this phase-space curve, in agreement with our assumption of isotropic decay of the $\pi\pi$ system.

We have estimated the peripheral $\pi\pi$ mass distribution by taking all events with $\cos\theta_x>0.8$, where θ_x is the angle of production of the $\pi\pi$ system; we then subtracted a background which was taken to be isotropic in θ_x (it was actually calculated as 2/15 times the number of events with $\cos\theta_x<0.5$), and finally applied the correction for removal of the N^* bands. The resulting $\pi\pi$ mass distribution is shown in Fig. 8. In this figure we also show, as a dashed histogram without errors, the results for $\cos\theta_x>0.8$ without subtracting the isotropic background, to show that removal of this isotropic background has made no important change in the $\pi\pi$ mass spectrum; the errors on this dashed histogram are similar to those on the main histogram. One sees clearly that there is no evidence for relatively narrow resonances either at 390 MeV or in the region of 700–740 MeV for which evidence was mentioned in Sec. 1. We also show in Fig. 8 the distribution expected if these events were uniformly distributed over phase space. These phase-space curves have been calculated separately for each of our incident momenta and then added together, assuming either that the cross sections were equal at all five momenta (curve I) or that they varied as the inverse square of the incident momentum (curve II); in each case the resulting curve was normalized so that its area was the same as that of the histogram. It will be noted that there is no important difference between these two curves. We see an indication of departure from uniform population of phase space, with a broad hump in the mass distribution between 400 and 800 MeV, suggesting a strong $\pi\pi$ interaction over all this broad range of energies rather

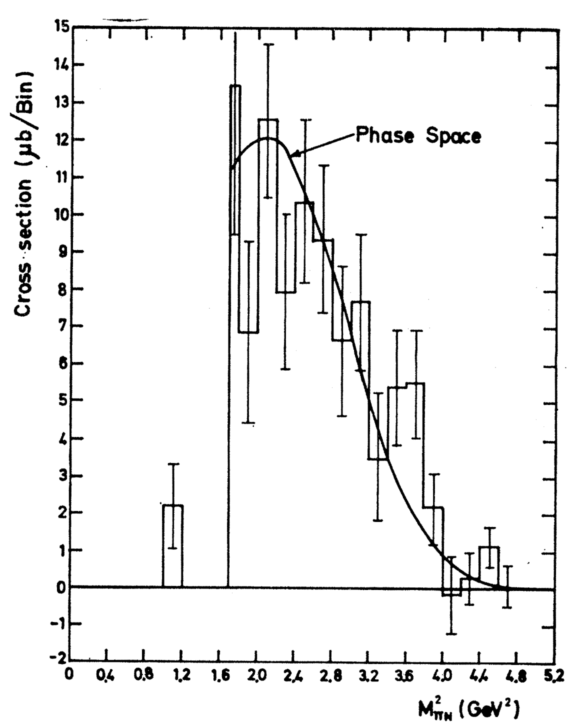

FIG. 7. Observed $\pi^0 n$ mass distribution after removing events which have $1.2<M_{\pi n}^2<1.7$ GeV for either π_n combination. The phase-space curve shown is the sum of five curves, one for each momentum, added assuming the cross section does not vary with momentum. It has been normalized to have the same area as the experimental histogram. Each $\pi^0 n$ pair has been given a weight of one-half.

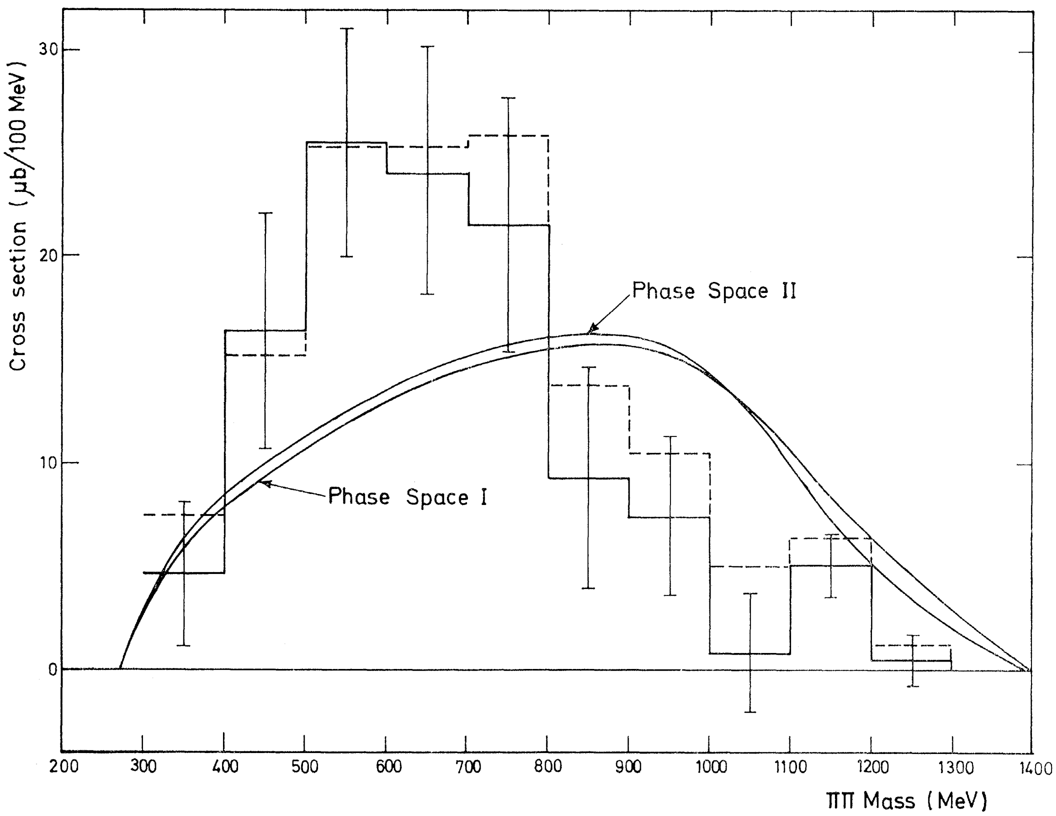

FIG. 8. Peripheral $\pi\pi$ mass spectrum (see text for discussion of selection and correction of data).

than the relatively narrow resonances which have been suggested.

At these relatively low $\pi\pi$ masses one would expect that d-wave scattering would not yet have become important. This surmise is supported by the degree of anisotropy found[10] for the $\pi^-\pi^+$ system in reaction (3): $\cos^3\theta$ and $\cos^4\theta$ terms, which indicate d-wave scattering, contribute only at higher masses. We will therefore assume that we are observing s-wave scattering.

A similar broad peaking in this region of the $\pi\pi$ mass spectrum has been seen[20] in a similar experiment at an incident momentum of 10 GeV/c. However, these authors remark that in this region of their mass spectrum, between 15 and 40% of the events must be background due to feedthrough from events with six gamma rays of which only four have been detected; thus they cannot be certain that this peaking is real. In our results we believe that any residual background of this sort which has not been removed must be very small, so that our conclusion is more certain.

5. EMPIRICAL COMPARISON OF s-WAVE AMPLITUDES

It is traditional to attempt to deduce $\pi\pi$ scattering cross sections from measured peripheral pion production cross sections, by using modifications of the one-pion-exchange model. We shall see, in the following section, that this results in a certain degree of confusion. Therefore, we have attempted a more empirical analysis, comparing the cross section we have measured for the reaction

$$\pi^- p \rightarrow \pi^0\pi^0 n \qquad (1)$$

with the information we can deduce from the asymmetry in the angular distribution of the $\pi^-\pi^+$ system in the reaction

$$\pi^- p \rightarrow \pi^-\pi^+ n\,, \qquad (3)$$

and making a minimum of assumptions.

We assume that the $\pi^-\pi^+$ system is produced only in s-wave and p-wave states. For production of these states in reaction (3) let A = amplitude for feeding either of the $m=\pm1$ substates of p-wave $\pi^+\pi^-$, B = amplitude for feeding the $m=0$ substate of p-wave $\pi^+\pi^-$, C = s-wave amplitude. We will assume that the s wave in reaction (1) is related to C, depending on the isotopic spins involved. Let a, b, and c be the magnitudes of A, B, and C. We assume that the $m=0$ substates of both p wave and s wave are fed coherently, and that the relative phase of B and C is $\delta_p-\delta_s$, where δ_p and δ_s are the effective $l=1$ and $l=0$ scattering $\pi\pi$ phase shifts. These seem to be a minimum set of assumptions that would be contained in any reasonable

[20] M. Wahlig, E. Shibata, D. Gordon, D. Frisch, and I. Mannelli, Phys. Rev. 147, 941 (1966).

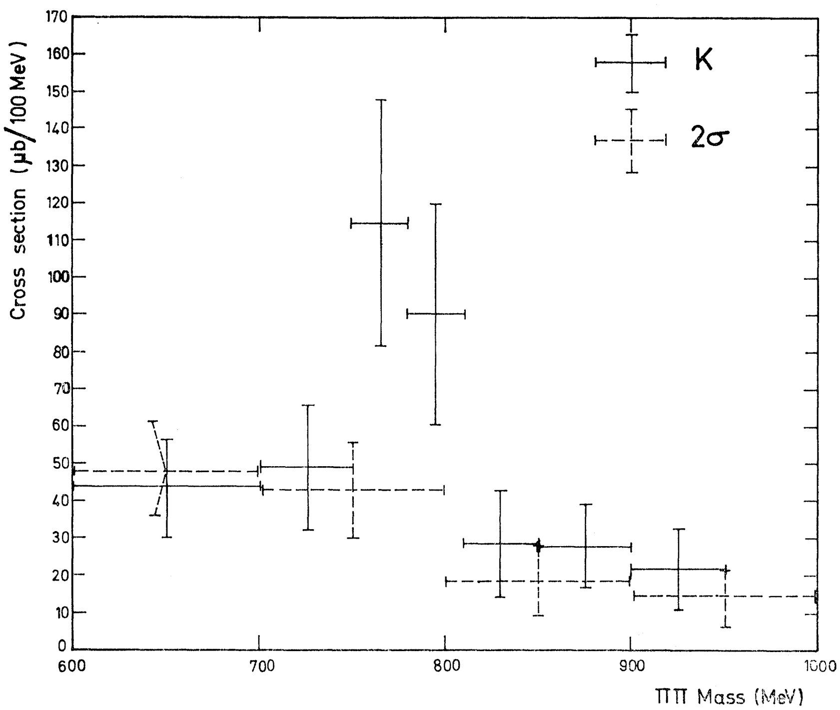

FIG. 9. Comparison between information about s-wave $\pi\pi$ production cross sections from our experiment and that deduced from the decay angular distribution of the ρ^0 meson. K and σ are described in the text.

modification of the one-pion-exchange model. In the simplest versions of the one-pion-exchange model, $A=0$. However, it is found that

$$|A|^2 \simeq 0.15 |B|^2,$$

indicating an appreciable depolarization. The $\pi^-\pi^+$ angular distribution, integrated over the azimuthal angle ϕ, then is

$$(a^2+c^2)+2bc\cos(\delta_p-\delta_s)\cos\theta+(b^2-a^2)\cos^2\theta.$$

We note that the integration over ϕ has enabled us to avoid any uncertainties about the degree of coherence between the feedings of the $m=0$ and the $m=\pm1$ substates; such information cannot easily be obtained by any obvious extension of the one-pion-exchange model. If we neglect c^2 in comparison with a^2 (an assumption which is consistent with the results we obtain), we can then, from the measured angular distributions, deduce values for

$$c^2\cos^2(\delta_p-\delta_s)=\sigma_s\cos^2(\delta_p-\delta_s)=K,$$

where σ_s is the cross section we would obtain for production of s-wave $\pi^-\pi^+$ systems if the p-wave production were zero.

We should then expect σ_s to be related to the peripheral production cross section in reaction (1). However, the s-wave $\pi\pi$ interaction can take place in two states of isotopic spin, $I=0$ and 2. If the phase shifts for scattering in these states (which we assume to be elastic, as the cross sections for $\pi p\rightarrow N\pi\pi\pi$ are so small at these energies) are S_0 and S_2, respectively, then the scattering cross sections for the reaction

$$\pi^-\pi^+\rightarrow\pi^-\pi^+ \quad (8)$$

is $(16/9)\pi\lambda^2|S_0+\frac{1}{2}S_2|^2$, while for the reaction

$$\pi^-\pi^+\rightarrow\pi^0\pi^0$$

it is $(8/9)\pi\lambda^2|S_0-S_2|^2$, where

$$S_0=e^{i\delta_0}\sin\delta_0$$

and

$$S_2=e^{i\delta_2}\sin\delta_2.$$

Thus if σ is the cross section for reaction (2), we should expect

$$K=2\sigma\left|\frac{S_0+\frac{1}{2}S_2}{S_0-S_2}\right|^2\cos^2(\delta_p-\delta_s).$$

Here δ_s is the phase of the s-wave amplitude involved in reaction (8):

$$\tan\delta_s=(\sin^2\delta_0+\tfrac{1}{2}\sin^2\delta_2)/(\cos\delta_0\sin\delta_0+\tfrac{1}{2}\cos\delta_2\sin\delta_2).$$

We have been kindly supplied with detailed measurements[21] of reaction (3) that were made at an incident pion momentum of 2.03 GeV/c, very close to our central momentum, and that therefore are useful for our proposed comparison procedure. We show in Fig. 9 the values of K deduced from these results and, for comparison, those of 2σ from our results. We see a relatively sharp peak in K as a function of $\pi\pi$ mass, which peak is not seen in 2σ. Information about δ_2 is obtained[22]

[21] E. West, J. H. Boyd, A. R. Erwin, and W. D. Walker, Phys. Rev. **149**, 1089 (1966).
[22] N. Armenise *et al.*, Nuovo Cimento **37**, 361 (1965).

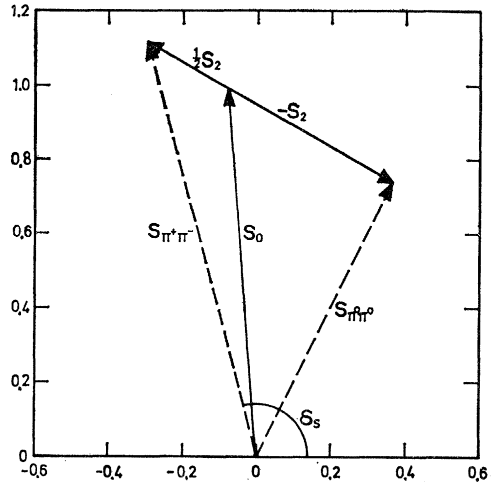

FIG. 10. Example of a conceivable combination of s-wave $\pi\pi$ scattering amplitudes. Here $\delta_0=95°$, $\delta_2=-30°$, giving for the phase of the amplitude involved in the reaction $\pi^-\pi^+\rightarrow\pi^-\pi^+$ the value $\delta_s=105.1°$, and $\sigma(\pi^-\pi^+\rightarrow\pi^-\pi^+)/2\sigma(\pi^-\pi^+\rightarrow\pi^0\pi^0)=2.0$.

from peripheral production in the reaction

$$\pi^+p\rightarrow\pi^+\pi^+n \tag{9}$$

which is taken to be due to the reaction

$$\pi^+\pi^+\rightarrow\pi^+\pi^+ \tag{10}$$

which takes place in an $I=2$ state. This work provides a strong indication that there is no rapid variation of δ_2 with $\pi\pi$ mass. Combining this information with the corresponding lack of such rapid variation of the peripheral $\pi\pi$ mass spectrum found in our experiment, we conclude that the peak in K must be ascribed to a rapid variation in $\cos^2(\delta_p-\delta_s)$ and that this is presumably due to the change in δ_p as one goes through the ρ-meson resonance. One would then conclude that the peak in K would occur where $\delta_p=\delta_s$. At this peak, K is about twice as large as 2σ, implying that the s-wave scattering is not in a pure state of isotopic spin. This value for the ratio $K/2\sigma$ at the peak, and the ratio of peripheral cross sections for reactions (1) and (9) (assuming that these two cross sections can be explained by the same empirical modification of the one-pion-exchange model—see the following sections for further discussion of this point) can be fitted with phase shifts close to $\delta_0=95°$, $\delta_2=-30°$, when $\delta_s=105°$ and $\sigma_s/2\sigma=2.0$, as is shown in Fig. 10. We cannot regard these phase shifts as determined exactly, but we present them here as an example and suggest that this analysis is some indication that the true values of δ_0 and δ_2 may lie somewhere in this region. These values of δ_0 and δ_2 have the merit that they are consistent with conclusions that have been deduced from other experimental results:

(i) The value of δ_s is consistent with having $\delta_s=\delta_p$, for which we have deduced evidence.

(ii) The value of δ_s implies a largely imaginary s-wave amplitude contributing to reaction (3), which is consistent with the $\pi^-\pi^+$ asymmetry observed in that reaction.

(iii) The negative sign for δ_2 agrees with that deduced from the asymmetry in the decay of charged ρ mesons, while the magnitude implies a largely real s-wave amplitude contributing to reactions (4) and (5) in further agreement. Therefore, this analysis provides some support for the suggestion that the broad peak in our peripheral $\pi\pi$ mass spectrum may be a broad s-wave resonance.

6. FERRARI-SELLERI FORM-FACTOR ANALYSIS

Initially, attempts were made to estimate $\pi\pi$ scattering cross sections by assuming that peripheral pion production is due to the one-pion-exchange diagram. However, with the identification of the ρ meson as an $l=1$ resonance, with very small inelasticity, there was then a way to check this procedure, as the $\pi\pi$ scattering cross section at the peak of such an elastic resonance is known. It was found[23] that the $\pi\pi$ scattering cross section estimated from measured peripheral production cross sections by using the one-pion-exchange model was only about one-half of the correct value. Because of this, an empirical modification to the one-pion-exchange cross section was proposed,[13] which had the merit of giving the right value of the $\pi\pi$ scattering cross section at the peak of the ρ resonance for a broad range of incident momenta: 1.59,[24] 2.75,[25] and 4 GeV/c.[26] At 4 GeV/c, a reasonable fit was also obtained[26] to the cross section at the peak of the f^0 meson, assuming it to be an elastic d-wave resonance. It has been proposed[13] that, as the same empirical modification is successful over such a broad range of incident pion momenta, and as it is also successful in fitting pion production in proton-proton collisions at incident kinetic energies from 1 to 3 GeV, its validity might be due to form-factor modifications of the vertices and propagator of the one-pion-exchange diagram. However, an alternative suggestion[27,28] has been that the observed reduction in cross section may be explained by including absorption of the ingoing and outgoing waves in the one- pion-exchange process. This absorption model seem to fit the observed cross section well and provides a reasonable explanation of the observed depolarization of the ρ meson (which would not be explained by a simple form-factor modification).

There has been argument over the relative merits of these models, as to what their relative contributions

[23] J. Alitti *et al.*, Nuovo Cimento **25**, 365 (1962).
[24] J. Alitti *et al.*, Nuovo Cimento **29**, 515 (1963).
[25] J. P. Baton *et al.*, Nuovo Cimento **35**, 713 (1965).
[26] L. Bondar *et al.*, Phys. Letters **5**, 153 (1963).
[27] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).
[28] L. Durand and Y. T. Chiu, Phys. Rev. **139**, B646 (1965).

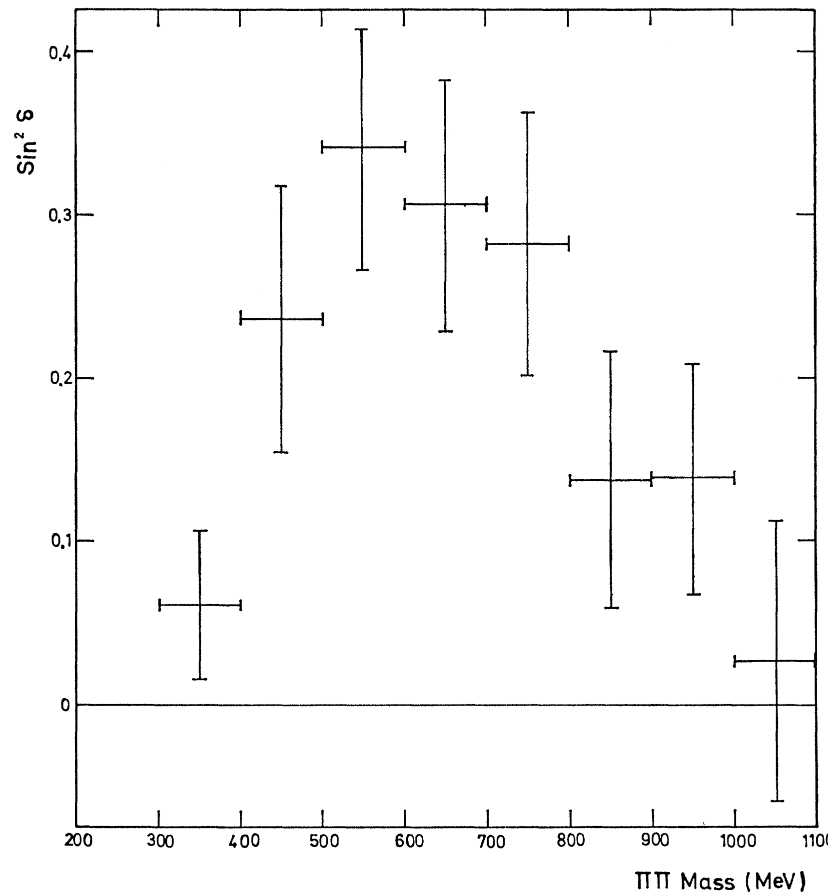

FIG. 11. Values of $\sin^2\delta$ deduced from our peripheral production cross sections, by assuming the one-pion-exchange formula with Ferrari-Selleri form-factor modification and assuming $\sigma(\pi^-\pi^+ \to \pi^0\pi^0) = (8/9)\pi\lambda^2 \sin^2\delta$.

to the reduction in cross section might be. However, it can be argued that we do not need to enter into this discussion, that we can use the Ferrari-Selleri form factor as an empirical modification of the total cross section for peripheral pion production—a modification that has been calibrated by requiring it to fit the cross sections at the peaks of the ρ and f^0 resonances. We therefore explore the use of this idea to attempt to obtain an estimate of the $\pi\pi$ scattering cross section corresponding to our observed peripheral production. We have expressed the s-wave $\pi\pi$ scattering cross section in terms of phase shifts for scattering with $I=0$ and 2 in Sec. 5. Estimates of δ_2 can be obtained from peripheral production in the reaction

$$\pi^+ p \to \pi^+\pi^+ n. \tag{9}$$

There is, however, some further doubt about such an analysis of this reaction. Armenise *et al.*[22] find that the peripheral peak at small angles in the dipion angular distribution for this reaction is wider by a factor of about 2.5 than are the corresponding peripheral peaks in ρ-meson production and in our experiment. Thus there is some suggestion that in this case the peripheral production observed may not be related to $\pi\pi$ scattering in the same way as in reactions (1), (3), and (4). However, one can deduce in general that the two pions in reaction (9) must be in an $I=2$ state and that whatever system is exchanged must have $I=1$. Then whatever is responsible for peripheral production lof an $I=2$ dipion in reaction (9) would also be making reated contributions to reactions (1) and (3). The magnitude of this amplitude S_2 contributing to reactions (1) and (3) would be related in the same way to that for reaction (9) as it would be in a one-pion-exchange model, but the phase of S_2 might not be related to its magnitude as it would be if it were due to one-pion exchange. The asymmetry in charged ρ-meson decay then implies a phase for S_2 which cannot be too far from what we have assumed from the one-pion-exchange model. We can therefore argue that at least we can deduce from reaction (9) an empirical amplitude for the production of an $I=2$ dipion, with the only difficulty being in relating the $I=2$ $\pi\pi$ interaction to this amplitude. For reasons of brevity we will continue to describe what we are doing as if the traditional procedure for extracting $\pi\pi$ scattering were satisfactory. If we then, in the traditional manner, take all events for $\Delta^2 < 15\mu^2$ and compare them with the pion production cross section calculated from a one-pion-exchange formula with Ferrari-Selleri form-factor modification, then we deduce

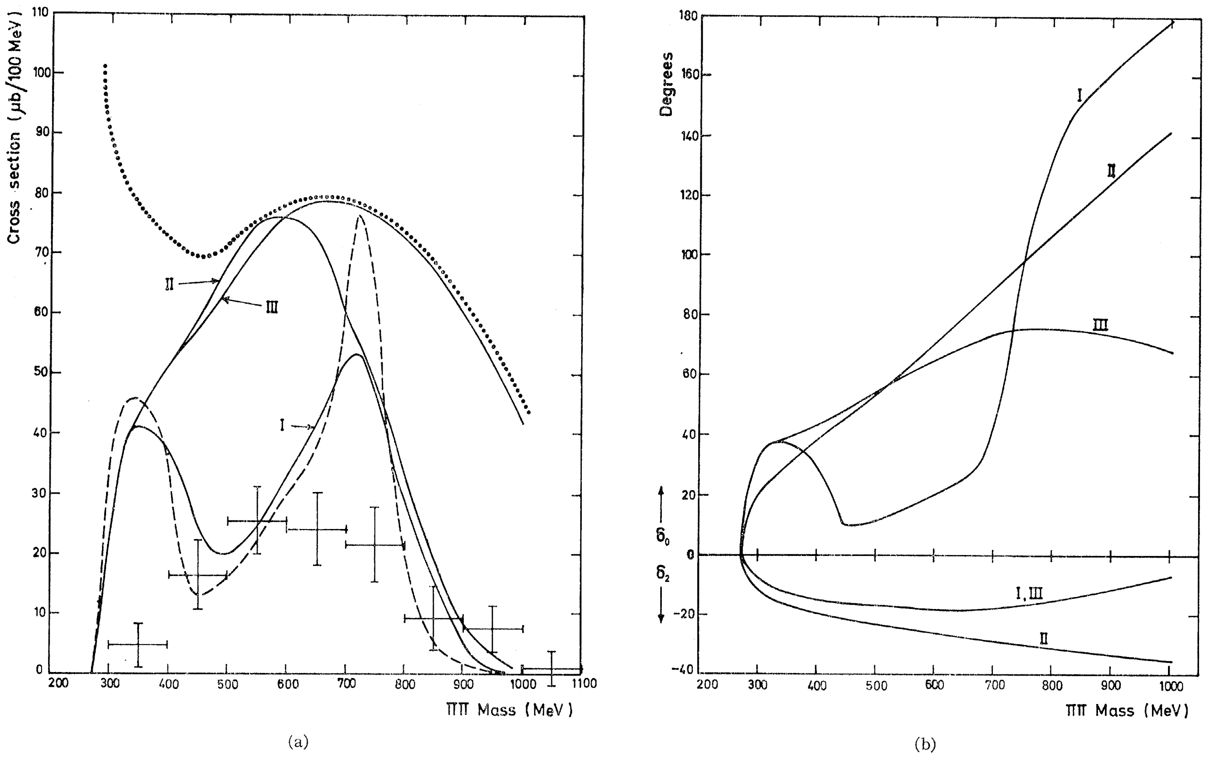

FIG. 12. (a) Comparison of our measured peripheral $\pi\pi$ production cross sections with estimates made using the one-pion-exchange formula with Ferrari-Selleri form-factor modifications, and assuming various $\pi\pi$ scattering phase shifts. Dotted line, assuming either $\delta_0=90°$, $\delta_2=0°$ or $\delta_0=0°$, $\delta_2=90°$. Solid line, three different conceivable sets of phase shifts which are shown in (b). All three sets of results are smeared with our experimental resolution. Dashed line, the calculated cross section for phase-shift set I before smearing with our experimental resolution, shown to illustrate the effect of our resolution on a relatively narrow resonance. The effect of smearing is much smaller for sets II and III. (b) The three phase-shift sets assumed as examples in (a).

a $|\delta_2|$ which rises smoothly through a value of about 13° at a $\pi\pi$ mass of 350 MeV to a value of about 25° at a mass of 850 MeV. (These phase shifts are larger than those quoted by Armenise *et al.*[22]; in their analysis they did not include a Ferrari-Selleri form-factor modification.) The asymmetry in charged ρ-meson decay then indicates that δ_2 is negative. We have taken this magnitude and form of δ_2 as a guide.

We now analyze our results using the one-pion-exchange formula with Ferrari-Selleri form-factor modification. If first we assume that the cross section for the reaction

$$\pi^-\pi^+ \longrightarrow \pi^0\pi^0 \qquad (2)$$

is $(8/9)\pi\lambda^2 \sin^2\delta$, we deduce from our results the values of $\sin^2\delta$ shown in Fig. 11. This cross section would be expected if one had scattering in only one of two possible isotopic-spin states, and provides a useful indication of the magnitude of s-wave $\pi\pi$ scattering our peripheral production implies. We see that our peripheral production cross section is smaller than that calculated for an s-wave resonance, but not by orders of magnitude.

As a further indication, we compare in Fig. 12(a) our peripheral production cross section with cross sections calculated from the one-pion-exchange formula with Ferrari-Selleri modifications, assuming various s-wave phase shifts as examples. Firstly we show the production cross section calculated assuming the cross section for reaction (2) to be $(8/9)\pi\lambda^2$: If we had an s-wave resonance in only one isotopic-spin state the production cross section should rise to this curve. We also show production cross sections calculated from three sets of conceivable phase shifts [shown in Fig. 12(b)]. Set I is the phase shifts proposed by Wolf[12] and includes an $I=0$ resonance at 740 MeV with a width of 90 MeV. For this case we show two curves, one smeared with our experimental resolution and the other before this smearing. (This shows the effect of our experimental resolution on a relatively narrow resonance. For sets II and III, which are representative of broader resonances, the effect of this smearing is almost negligible.) We see that our cross sections are not consistent with such a rapid variation of the phase shift, justifying our earlier remark ruling out relatively narrow resonances. Sets II and III illustrate effects of interference between scattering in the two isotopic-spin states: With the negative δ_2 one has constructive interference for $\delta_0<90°$ and destructive interference for $\delta_0>90°$, these effects being larger for a larger δ_2. Set II includes a broad resonance in the $I=0$ state with phase shifts around 750 MeV similar to those deduced in Sec. 5. Both sets II and III produce cross sections much larger than that observed, which argues against a resonance interpretation of our results. However, we note that set II does produce a peak similar to that observed: If the calculated cross sections are divided by a factor of 3 we obtain a curve in good agreement with the experimental results.

A possible way to obtain a smaller calculated cross section is for δ_0 and δ_2 to have the same sign, with neither going up to 90° where the interference would be destructive. A reasonable fit can be obtained with $|\delta_2|\simeq 25°$ and $|\delta_0|\simeq 60°$ over a broad region of $\pi\pi$ mass. With δ_2 negative, this implies a large negative δ_0; however, such a negative δ_0 is not consistent with the asymmetry in the angular distribution of low-mass dipions[9] from reaction (3), and the actual values of the phase shifts correspond to amplitudes which fit badly with our analysis in Sec. 5.

7. CONCLUSION

We have found evidence for peripheral dipion production in the reaction

$$\pi^- p \rightarrow \pi^0 \pi^0 n.$$

Such peripheral production in this reaction should provide a useful means of studying the s-wave $\pi\pi$ interaction. The $\pi\pi$ mass spectrum in the peripheral production shows no indication of any of the relatively narrow s-wave resonances that have been proposed; instead, we find a broad peak between masses of 400 and 800 MeV. It is not clear whether this indicates a broad resonance in the s-wave $\pi\pi$ interaction. A phenomenological comparison of our peripheral production cross sections with information about s-wave $\pi\pi$ interactions deduced from the asymmetry in ρ^0-meson decay suggests that the $I=0$ s-wave phase shift is not far from 90° for masses in the neighborhood of 750 MeV, and leads to conclusions in agreement with those deduced from other phenomena. If, however, we estimate the production cross sections by assuming for s-wave $\pi\pi$ interactions the same empirical modification of the one-pion-exchange model as is found to be successful in the case of p-wave and d-wave $\pi\pi$ scattering, then we calculate, for the case of a broad $I=0$ s-wave resonance interfering with an $I=2$ s-wave amplitude in agreement with other experimental indications, a production cross section which is approximately 3 times our experimental result. This can either be taken as an argument against an $I=0$ s-wave resonance, or as an argument against using the same empirical modification of the one-pion-exchange formula for the case of s-wave scattering as one does for p-wave and d-wave scattering. There seems to be no other evidence for or against the latter of these two possibilities; however, it is a plausible hypothesis, so that with the evidence for an s-wave resonance deduced in Sec. 5, and from the way the conclusions of that analysis agree with several other phenomena, it seems the more attractive possibility.

We note that if the empirical modification of the one-pion-exchange formula is so much greater for the case of s-wave interaction, we have also to increase the magnitude of δ_2 deduced from reaction (9). If again we deduce δ_2 by taking all events with $\Delta^2 < 15\mu^2$ as being due to $I=2$ s-wave $\pi\pi$ scattering, and include this further factor of 3 in the empirical modification, we find that the magnitude of δ_2 deduced is increased, typically to $\delta_2 \simeq -40°$ at a $\pi\pi$ mass of 750 MeV. There then would still seem to be no evidence for even a broad resonance in the $I=2$ s-wave $\pi\pi$ state.

It has been suggested[29,30] that δ_0 may fall from 0° at threshold and pass through $-90°$ at a $\pi\pi$ mass somewhere near 750 MeV. Our results and analysis cannot distinguish between this possibility and a resonance in which δ_0 rises through $+90°$. In principle it should be possible to distinguish between these possibilities by comparing the peripheral production cross sections in reactions (1), (3), and (4) at low $\pi\pi$ masses where only s-wave scattering could be contributing. Such an analysis has been attempted by comparing our results for reaction (1) with those of West *et al.*[20] for reactions (3) and (4). Unfortunately, it was not possible to reach a definite conclusion, as the statistical accuracy was not good enough. Such a negative δ_0 disagrees with the observed $\pi^-\pi^+$ asymmetry[9] in reaction (3) at low $\pi\pi$ masses.

As an alternative possibility, it has been suggested[31] that δ_0 may decrease from zero at threshold to $-270°$ at a mass around 750 MeV. Such behavior would produce a double-peaked mass spectrum in our experiment, similar to that deduced from set I in Fig. 13(a). Our results would therefore seem to argue against this possibility.

Alexanian and Wellner[29] have also suggested that δ_2 may also fall through a value of $-90°$. Our arguments against a broad $I=2$ s-wave resonance could also seem to argue against this possibility, as does the behavior of the $\rho^\pm$ asymmetries in reactions (4) and (5).

We conclude that there is some evidence for a broad s-wave $\pi\pi$ resonance, of width about 400 MeV. We note that such a resonance is very similar to that proposed by Lovelace *et al.*,[17] and may be responsible for several of the reports of strong s-wave interaction at different $\pi\pi$ masses which were listed in Sec. 1.

ACKNOWLEDGMENTS

This work was done at the proton synchrotron Nimrod of the Rutherford High Energy Laboratory. We are indebted to many people at that laboratory,

[29] M. Alexanian and M. Wellner, Phys. Rev. **140**, B1079 (1965).
[30] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).
[31] L. F. Cook, Phys. Rev. Letters **17**, 212 (1966).

and in particular to Dr. G. H. Stafford and D. C. Salter, and to Dr. L. C. W. Hobbis and all his Nimrod Division. The pion beam was shared with the University College–Westfield College group (University of London): We enjoyed a very amicable collaboration with them and wish to thank them for all their help to us. Our data were analyzed at the Atlas Computer Laboratory: We are particularly indebted to J. Sparrow and H. Hurst for help in this. We are also indebted to Mrs. J. Huxtable and her scanners in Oxford and to Dr. W. D. Walker for communicating his results to us in advance of publication.

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New Determination of the Branching Ratios of K^+-Meson Decay in Emulsion*

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A new experiment has been carried out to measure the branching ratios for K^+ decay in emulsion under improved conditions. The mesons were brought to rest in a 14-cc volume within a large stack of nuclear-research emulsion. The stack was designed so that the secondaries of the longest range could be followed to rest if they were emitted within certain cones. Some 700 such K^+ decays were chosen as the sample. The principal method for identifying the secondaries was by following the tracks to rest, thus avoiding many sources of systematic error. Ionization measurements were used to resolve ambiguities. The over-all scanning efficiency was found to be higher than 95%. The observed branching ratios for the $K_{\mu 2}$, $K_{\mu 3}$, $K_{\pi 2}$, τ, τ', and K_{e3} modes are (61.8±2.9), (5.4±0.9), (19.3±1.6), (6.0±0.4), (2.3±0.6), and (5.3±0.9)%, respectively. These results tend to reconcile the discrepancies between emulsion measurements and heavy-liquid-chamber data.

I. INTRODUCTION

THE branching ratios for the decay modes of the K^+ meson have been measured several times in the past dozen years by different research groups.[1–6] Discrepancies exist not only between data obtained by use of different kinds of detectors, but also between data obtained from similar detectors under different experimental conditions. The difference between the $K_{\mu 2}/K_{\pi 2}$ ratio obtained from the xenon bubble chamber[1,2] and from some of the early emulsion measurements[3,4] has appeared so great that it even has been put forward as evidence for a "shadow universe."[7]

To carry out our measurement of the branching ratios we have gone back, with improved techniques, to nuclear research emulsion. Track-following was employed on a scale never before undertaken. Stack size was the greatest ever used for this purpose. Better blob density was achieved and more uniform development was accomplished than in previous experiments. In addition, several new methods were introduced for the reduction of bias and for the calculation of scanning efficiencies.

It is appropriate, before describing our experiment, to review some of the previous experiments.

The two experiments performed by the Birge group[3] and Alexander *et al.*[4] have been cited[1,2,7] as providing the most precise emulsion data for the K^+ branching ratios. In Birge's pioneering experiment the sample size was moderate (149 $K_{\mu 2}$ and 77 $K_{\pi 2}$ being found for the major modes), and different batches of samples were used to determine different decay modes. Track-following, the most direct method, was employed in identifying 97 events, and blob counting at the K^+ decay point was used to determine 185 events. The stack and the sample used by Alexander *et al.* were larger, but the identification of secondaries was based almost entirely upon blob counting and scattering meas-

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[1] Byron P. Roe, Daniel Sinclair, John L. Brown, Donald A. Glaser, John A. Kadyk, and George H. Trilling, Phys. Rev. Letters 7, 346 (1961).

[2] Francis S. Shaklee, Gary L. Jensen, Byron P. Roe, and Daniel Sinclair, Phys. Rev. 136, B1423 (1964).

[3] R. W. Birge, D. H. Perkins, J. R. Peterson, D. H. Stork, and M. N. Whitehead, Nuovo Cimento 4, 834 (1956).

[4] G. Alexander, R. H. W. Johnson, and C. O'Ceallaigh, Nuovo Cimento 6, 478 (1957).

[5] T. E. Hoang, M. F. Kaplon, and G. Yekutieli, Phys. Rev. 102, 1185 (1956).

[6] P. K. Aditya, J. K. Bøggild, K. H. Hansen, J. E. Hooper, and M. Scharff, in Proceedings of the Cosmic Ray Symposium Ahmedabad, India, 1960, NP-11232, p. 373 (unpublished).

[7] Allen E. Everett, Phys. Rev. Letters 14, 615 (1965).